PHYSICAL REVIEW D VOLUME 16, NUMBER 1 1 JULY 1977

Baryon electromagnetic mass differences in the charmed-quark model*

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(Received 18 February 1977)

We consider a model in which baryon electromagnetic mass differences arise from intrinsic quark-mass differences plus Coulomb and magnetic-moment interactions between quarks. We obtain several inequalities relating the mass splittings of charmed and uncharmed baryons. By making additional assumptions about the properties of the baryon wave functions, we are able to obtain predictions of the electromagnetic mass differences of all charmed baryons belonging to the 120 multiplet of SU(8).

With the recent discovery of charmed baryons,[1,2] it may not be too long before the electromagnetic mass splittings of some of these particles are measured. Therefore, it appears to be an appropriate time to estimate their electromagnetic mass difference. We shall also give some results for uncharmed baryons so as to be able to compare with existing experiments.

We shall work within the framework of the quark model, using methods analogous to those we have previously applied to uncharmed hadrons[3] and to charmed mesons.[4] A number of previous calculations have been made of charmed-baryon electromagnetic mass splittings using the quark model.[5-9] We shall compare our results with those of previous works, and shall point out where our assumptions differ from those made in earlier calculations.

We restrict ourselves to a model with only four quark flavors, although we can easily generalize our results to include additional quarks. As we have previously noted,[3] we do not need to consider explicitly the color degree of freedom in quarks. We denote the four quarks as usual by u, d, s, and c. Different members of a baryon isospin multiplet differ only in their u and d quarks; the s and c quarks carry strangeness and charm, respectively, and have isospin zero.

In the model, isospin symmetry is broken in two ways: first by an intrinsic effective mass difference ϵ between the u and d quarks, which we do not attempt to explain, and second by the Coulomb and magnetic interactions of the quarks. This model of the electromagnetic interaction has been used many times in the literature, the papers by Kuo and Yao[10] and Miyamoto[11] being among the earliest. Later papers using these ideas include those by Gal and Scheck,[12] Itoh *et al.*,[6] Ono,[7] and our own previous papers.[3,4]

Following all these authors, we write the electromagnetic interaction V_{ij} between the ith and jth quarks as

$$V_{ij}=Q_iQ_j/r_{ij}-(8\pi/3)\vec{\mu}_i\cdot\vec{\mu}_j\delta(\vec{r}_{ij}), \qquad (1)$$

where Q_i and $\vec{\mu}_i$ are the charge and magnetic moment, respectively, of the ith quark, and r_{ij} is the distance between the ith and jth quark.

This interaction contains too many parameters to be useful. Therefore, following Greenberg,[13] De Rújula *et al.*,[14] and others, we take the quark magnetic moments to be proportional to their charges and inversely proportional to their effective masses when bound in hadrons. The assumption that the quark moments are proportional to their charges leads to the well-known prediction that the ratio of the neutron moment μ_n to the proton moment μ_p is $\mu_n/\mu_p=-2/3$, in rather good agreement with experiment. However, unless we take the quark moments inversely proportional to their masses, the Λ moment is predicted to be significantly larger than the experimental value. In fact, we need to take the ratio $m_u/m_s=0.7\pm0.1.$ in order for the calculated Λ moment to equal its experimental value. If the quarks had Dirac magnetic moments, the masses of the u and s quarks would be

$$m_u=336\text{ MeV},\quad m_s=480\pm70\text{ MeV}. \qquad (2)$$

Because no magnetic moment of a charmed baryon has yet been measured, we cannot get an estimate for the mass of the charmed quark in a similar way. However, we note that the values of m_u and m_s given in Eq. (2) are not very far from the values deduced from strong-interaction spectroscopy.[14,15] Such spectroscopy leads to estimates that the charmed quark has a mass between 1.3 and 1.7 GeV. For definiteness, we take

$$m_c=1660\pm30\text{ MeV}, \qquad (3)$$

the value estimated by De Rújula *et al.*[14] Then we have $m_u/m_c=0.2$.

With these results, it is straightforward to obtain expressions for the electromagnetic mass differences of the charmed baryons in terms of ϵ and the expectation values of V_{ij}. The values of $\langle V_{ij}\rangle$ depend on assumptions about the properties of the strong-interaction wave functions unperturbed by the electromagnetic interaction. In our paper on uncharmed hadrons,[3] we used rather general wave functions, our principal assumption being that these wave functions are symmetric under the interchange of the coordinates (excluding color coordinates) of any two identical quarks (that is, quarks having the same flavor). We then obtained several inequalities relating the electromagnetic mass splittings of different isospin multiplets.

We apply the same methods[3] to charmed baryons belonging to the 120-plet of SU(8). Then we obtain expressions for the charmed-baryon mass differences in terms of ϵ and the expectation values $\langle r_{ij}\rangle$ and $\langle\delta(\vec{r}_{ij})\rangle$. These expectation values are in general different for each baryon except for members of the same isospin multiplet. However, because $\langle r_{ij}\rangle$ and $\langle\delta(\vec{r}_{ij})\rangle$ are both positive-definite quantities, we can obtain one inequality relating a charmed-baryon electromagnetic mass difference to an uncharmed mass difference. This inequality is

$$\Xi_{cc}^{+} - \Xi_{cc}^{++} < \Xi^{-} - \Xi^{0} = 6.4 \pm 0.6 \text{ MeV}. \qquad (4)$$

Our notation is that the symbol for a baryon denotes its mass. We use a subscript c on the symbol for each charmed quark which replaces a strange quark in the baryon, but otherwise keep the conventional symbol. Thus, for example, Ξ^0 contains uss quarks, while Ξ_{cc}^{++} contains ucc. Similar notation has been used previously in the literature.[16]

We can obtain several additional inequalities if we assume that the magnetic interaction energy between two quarks in a baryon is smaller than the Coulomb interaction energy. The model requires this for the nucleon, or the calculated neutron-proton mass difference would be larger than the experimental value.[3] It is plausible that this qualitative feature should hold for quarks in all baryons belonging to the 120-plet of SU(8), even though SU(8) is a broken symmetry. If so, we obtain

$$\Sigma_c^{+} - \Sigma_c^{++} < \Xi^{-} - \Xi^{0}, \qquad (5a)$$

$$\Sigma_c^{*+} - \Sigma_c^{*++} < \Xi^{-} - \Xi^{0}, \qquad (5b)$$

$$\Xi_{cc}^{*+} - \Xi_{cc}^{*++} < \Xi^{-} - \Xi^{0}, \qquad (5c)$$

$$\Sigma_c^{+} - \Sigma_c^{++} < \Sigma_c^{0} - \Sigma_c^{+}, \qquad (5d)$$

$$\Sigma_c^{*+} - \Sigma_c^{*++} < \Sigma_c^{*0} - \Sigma_c^{*+}. \qquad (5e)$$

An asterisk on the symbol for a particle denotes a member of the symmetric 20-plet of SU(4) which contains the usual SU(3) decuplet. The absence of an asterisk denotes a member of the 20-plet of mixed symmetry which contains the usual SU(3) octet. Of these inequalities, (5a) is likely to be the one most amenable to experimental test in the near future.

In order to obtain further relations we need to assume more about the baryon wave functions. To be specific, let us assume that the expectation values $\langle 1/r_{ij}\rangle$ and $\langle\delta(\vec{r}_{ij})\rangle$ with respect to a baryon wave function depend only on the flavors and spin configurations of the ith and jth quarks, and not on the third quark in the baryon. This assumption bears a certain resemblance to the assumption made earlier by Rubinstein,[17] Gal and Scheck,[12] Franklin,[5] and others that baryon masses arise from the sum of two-body quark interaction energies. With our assumption we obtain the following mass relations involving charmed baryons:

$$n - p = \Sigma_c^{0} - \Sigma_c^{++} + \Xi_{cc}^{++} - \Xi_{cc}^{+} \qquad (6a)$$

$$= \Sigma_c^{*0} - \Sigma_c^{*++} + \Xi_{cc}^{*++} - \Xi_{cc}^{*+}, \qquad (6b)$$

$$\Sigma^{+} + \Sigma^{-} - 2\Sigma^{0} = \Sigma_c^{++} + \Sigma_c^{0} - 2\Sigma_c^{+} \qquad (6c)$$

$$= \Sigma_c^{*++} + \Sigma_c^{*0} - 2\Sigma_c^{*+}, \qquad (6d)$$

$$\Sigma^{*+} - \Sigma^{*-} + 2(\Xi^{*-} - \Xi^{*0}) = \Sigma_c^{*++} - \Sigma_c^{*0} + 2(\Xi_c^{*0} - \Xi_c^{*+}). \qquad (6e)$$

These mass relations (plus one other) have been previously obtained by Franklin,[5] who assumed two-body quark interaction energies. Thus, the fact that we have used the specific form of Eq. (1) for the quark-quark electromagnetic interaction has not enabled us to obtain any new mass relations beyond those obtained by Franklin. For a more detailed discussion of the differences between our assumptions and those of Franklin, see Ref. 3.

To obtain further mass relations, let us consider the consequences of assuming that the expectation values $\langle 1/r_{ij}\rangle$ and $\langle\delta(\mathrm{r}_{ij})\rangle$ are independent of quark flavor and spin. We shall discuss this assumption further after we exhibit our results. With this approximation, the expression for all the baryon electromagnetic mass differences can be written in terms of three unknown parameters ϵ, C, and M defined by

$$\begin{aligned} \epsilon &= m_d - m_u, \\ C &= \alpha\langle 1/r\rangle/3, \\ M &= 2\pi\alpha\langle\delta(\vec{r})\rangle/(9m_u^2), \end{aligned} \qquad (7)$$

where α is the fine-structure constant. We are neglecting ϵ in the expression for M, as including ϵ would give rise only to a second-order effect in the mass splitting. These definitions of C and M differ by a factor of 3 from ones used in our earlier papers.[3,4] We also introduce two parameters x and y which are considered to be known. They are

$$x = m_u/m_s \cong 0.7, \quad y = m_u/m_c \cong 0.2. \qquad (8)$$

In terms of our parameters, the expression for the uncharmed- and charmed-baryon electromagnetic mass differences in the SU(4) 20-plet of mixed symmetry are

$$n - p = \epsilon - C + M\,, \tag{9a}$$

$$\Sigma^0 - \Sigma^+ = \epsilon - C + 2(1+x)M\,, \tag{9b}$$

$$\Sigma^- - \Sigma^0 = \epsilon + 2C + (2x-1)M\,, \tag{9c}$$

$$\Xi^- - \Xi^0 = \epsilon + 2C + 4xM\,, \tag{9d}$$

$$\Sigma_c^+ - \Sigma_c^{++} = \epsilon - 4C + 2(1-2y)M\,, \tag{9e}$$

$$\Sigma_c^0 - \Sigma_c^+ = \epsilon - C - (1+4y)M\,, \tag{9f}$$

$$\Xi_c^0 - \Xi_c^+ = \epsilon - C - (x+4y)M\,, \tag{9g}$$

$$\Xi_c'^0 - \Xi_c'^+ = \epsilon - C + xM\,, \tag{9h}$$

$$\Xi_{cc}^+ - \Xi_{cc}^+ = \epsilon - 4C - 8yM\,. \tag{9i}$$

The charmed baryon Ξ_c belongs to an SU(3) sextet while Ξ_c' belongs to an SU(3) antitriplet.

The electromagnetic mass differences in the symmetric 20-plet are

$$\Delta^+ - \Delta^{++} = \epsilon - 4C + 4M\,, \tag{10a}$$

$$\Delta^0 - \Delta^+ = \epsilon - C + M\,, \tag{10b}$$

$$\Delta^- - \Delta^0 = \epsilon + 2C - 2M\,, \tag{10c}$$

$$\Sigma^{*0} - \Sigma^{*+} = \epsilon - C + (2-x)M\,, \tag{10d}$$

$$\Sigma^{*-} - \Sigma^0 = \epsilon + 2C - (1+x)M\,, \tag{10e}$$

$$\Xi^{*-} - \Xi^{*0} = \epsilon + 2C - 2xM\,, \tag{10f}$$

$$\Sigma_c^{*+} - \Sigma_c^{*++} = \epsilon - 4C + 2(1+y)M\,, \tag{10g}$$

$$\Sigma_c^{*0} - \Sigma_c^{*+} = \epsilon - C + (2y-1)M\,, \tag{10h}$$

$$\Xi_c^{*0} - \Xi_c^{*+} = \epsilon - C + (2y-x)M\,, \tag{10i}$$

$$\Xi_{cc}^{*+} - \Xi_{cc}^{*++} = \epsilon - 4C + 4yM\,. \tag{10j}$$

From Eqs. (9a), (9b), (9c) we can solve for the unknown parameters ϵ, C, and M in terms of the known parameters x and y and the known mass differences $n-p$, $\Sigma^0-\Sigma^+$, and $\Sigma^- - \Sigma^0$. We obtain

$$\epsilon = 1.9 \text{ MeV}, \quad C = 1.35 \text{ MeV}, \quad M = 0.75 \text{ MeV}\,. \tag{11}$$

Note that both C and M are positive numbers. Because of the definitions of C and M [given in Eq. (7)], the model requires this. Thus, we are encouraged that there is no obvious incompatibility between the model and the data.

We can now use the values of ϵ, C, and M from Eq. (11) to obtain values of all the other mass differences of Eq. (9) and (10). Our values are given in Table I. Also given in Table I are the experimental mass differences from the Particle Data Group.[18] The calculated values of the mass differences of the uncharmed baryons are included in Table I to give the reader an idea of how well the model works in cases that can be compared to experiment. In fact, it appears to work rather well.

Our calculated values of the mass differences agree in sign with the values of Itoh *et al.*[6] and of Ono,[7] but the magnitudes are different. The calculated values of Lane and Weinberg[9] and of Deshpande *et al.*[8] often differ from ours in sign as well

TABLE I. Baryon electromagnetic mass differences in MeV. The symbol for a baryon stands for its mass. Our notation for the charmed baryons is explained in the text.

	This calculation	Experiment from Ref. 18
$n-p$	input	1.3
$\Sigma^- - \Sigma^+$	input	8.0
$\Sigma^- - \Sigma^0$	input	4.9
$\Xi^- - \Xi^0$	6.7	6.4 ± 0.6
$\Sigma_c^+ - \Sigma_c^{++}$	−2.6	
$\Sigma_c^0 - \Sigma_c^+$	−0.8	
$\Xi_c^0 - \Xi_c^+$	−0.6	
$\Xi_c'^0 - \Xi_c'^+$	1.1	
$\Xi_{cc}^+ - \Xi_{cc}^{++}$	−4.7	
$\Delta^0 - \Delta^{++}$	0.8	1.4 ± 0.4
$\Delta^0 - \Delta^+$	1.3	
$\Delta^- - \Delta^0$	3.1	
$\Sigma^{*-} - \Sigma^{*+}$	4.9	4.1 ± 1.3
$\Sigma^{*-} - \Sigma^{*0}$	3.3	
$\Xi^{*-} - \Xi^{*0}$	3.5	3.3 ± 0.7
$\Sigma_c^{*+} - \Sigma_c^{*++}$	−1.7	
$\Sigma_c^{*0} - \Sigma_c^{*+}$	0.1	
$\Xi_c^{*0} - \Xi_c^{*+}$	0.3	
$\Xi_{cc}^{*+} - \Sigma_{cc}^{*++}$	−2.9	

TABLE II. A comparison of the calculated values in MeV of the mass difference of the charmed baryons Σ_c^{++} and Σ_c^0. This particular mass difference is chosen as it is probably most amenable to measurement in the not-too-distant future.

Reference	$\Sigma_c^{++} - \Sigma_c^0$
This calculation	3.4
Itoh *et al.*	6.5
Ono	6.1
Lane and Weinberg	−6
Deshpande *et al.*	−3 to −18
Chan	0.4
Kalman	−2.7

as in magnitude. The best hope for distinguishing among these predictions in the near future will probably come from measurement of the mass difference $\Sigma_c^{++} - \Sigma_c^0$, as both these particles may be stable against strong and electromagnetic decay. Our calculated value of this mass difference, together with the values calculated by several other authors, is given in Table II. From Table II it appears that there are as many different predictions as there are calculations. A good measurement of the mass difference between the Σ_c^{++} and Σ_c^0 would therefore serve to distinguish between various versions of the charmed-quark model.

We now discuss the assumptions we used in obtaining our calculated mass differences of Table I, and compare with the assumptions used by Itoh *et al.*,[6] Ono,[7] and Lane and Weinberg.[9] Our assumption that the expectation values $\langle 1/r_{ij}\rangle$ and $\langle \delta(\vec{r}_{ij})\rangle$ are independent of flavor and spin is essentially equivalent to the assumption that the baryon unperturbed wave functions are SU(8)-invariant. Because of the large mass difference between the c quark and the other quarks, this assumption is certainly questionable. However, not only did Itoh *et al.* make this same assumption, but they also assumed that the quark magnetic moments are independent of their masses. If we set $x=y=1$ in Eqs. (9) and (10), our results reduce to those of Itoh *et al.* However, we believe it is more plausible to assume that the quark moments are inversely proportional to their masses.[13,14,19]

On the other hand, Ono did not assume full SU(8) invariance of the baryon spatial wave functions. Instead, he assumed a specific functional form for these wave functions, namely, that they are harmonic-oscillator wave functions. He also assumed that the quark moments are inversely proportional to their masses. Lane and Weinberg omitted the magnetic interaction altogether, and also used a larger value of the effective mass difference between the d and u quarks. Deshpande *et al.*[8] used the MIT bag model, and the assumptions are quite different from ours. In the absence of some detailed assumption about the behavior of the expectation values of V_{ij}, the model does not enable one to calculate the electromagnetic mass differences of all the charmed baryons belonging to the 120-plet.

In this paper we started with minimal assumptions and obtained only the one inequality (4). By progressively strengthening our assumptions, we were able to obtain the additional inequalities (5), the mass relations (6), and finally the predictions of Table I. Although the predictions of Table I could be altered by changing some of the detailed assumptions of our model, it is hard to see how this model could survive if the inequalities (4) and (5) should turn out to disagree with experiment.

After this work was completed, we came upon a paper by Chan,[20] with calculations of baryon mass differences. Also, subsequent to our work, Kalman[21] has calculated baryon mass differences in a model making use of noncompact groups. Chan's and Kalman's results are both different from ours, and their predictions for $\Sigma_c^{++} - \Sigma_c^0$ are included in Table II.

We should like to thank Calvin Kalman for interesting discussions.

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*Work supported in part by the Energy Research and Development Administration.

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