Simple symmetry breaking in a chiral SU(4) × SU(4) model of pseudoscalar mesons*

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We calculate the effects of simple symmetry breaking on the mass spectrum, neutral isoscalar mixing angles, and decays of pseudoscalar mesons using a chiral SU(4) nonlinear model to describe the mesons. We find that in addition to quark-mass-type terms, a simple U(1)-breaking term will induce a reasonable isoscalar mass spectrum, give a low value for the $\eta' \rightarrow \eta\pi\pi$ decay, and is the dominant contribution to the η'' (paracharmonium) decay into three isoscalars. We also calculate the effects of symmetry breaking on hadronic currents and thus on the weak decays of new mesons.

I. INTRODUCTION

In this paper we shall construct and discuss a chiral U(4) × U(4) phenomenological Lagrangian[1-3] for the 16-plet of pseudoscalar mesons. Since symmetry is broken in lowest order by the masses of particles, we expect to see most clearly the structure of symmetry breaking in the mass spectrum of the lowest-lying baryons—the pseudoscalars. The nonlinearity of the model we use lets us calculate these symmetry-breaking mass terms *and* the effects of these terms on higher-order processes that can be compared with experiment.

In Sec. II we discuss the construction of the model and its symmetry-breaking terms. We calculate the entire mass spectrum and the three isoscalar-meson mixing angles. We also construct the Noether vector and axial-vector currents from the basic Lagrangian density. In addition we also calculate the strong decays of the isoscalar mesons and discuss the effects of the symmetry-breaking terms on these decays.

In Sec. III we discuss the weak decays. The currents computed in Sec. II are used to compute the CP-conserving decays of the mesons, and the results are compared to experiment.

II. CONSTRUCTION OF THE STRONG LAGRANGIAN

The phenomenological Lagrangian density we use is a straightforward extension to U(4) of U(2) models of Nishijima[4] and Gursey,[5] and the U(3) model of Cronin.[6] The pseudoscalar mesons transform nonlinearly under chiral U(4) in such a way that the auxiliary meson matrix function $M(\Phi)$ satisfies

$$M(\Phi)M^\dagger(\Phi) = M^\dagger(\Phi)M(\Phi) = 1 \tag{2.1}$$

and

$$M^\dagger(\Phi) = M(-\Phi). \tag{2.2}$$

The matrix $M(\Phi)$ transforms according to the representation $(4_L, 4_R^*)$ of $U(4)_L \times U(4)_R$. The Hermitian conjugate matrix $M^\dagger(\Phi)$ transforms as $(4_L^*, 4_R)$. In these expressions Φ is the 4 × 4 Hermitian pseudoscalar-meson matrix ϕ_a^b $(a, b = 1, 2, 3, 4)$. We use the standard identifications $\pi^+ = \phi_1^2$, $K^+ = \phi_1^3$, $D^+ = \phi_4^2$, $F^+ = \phi_4^3$, etc. The three neutral isoscalar members of the pseudoscalar multiplet are denoted as η, η', and η''.

The meson matrix M is now expanded in a power series in ϕ_a^b as

$$M_a^b = \sum_n a_n (i\phi_a^b/f)^n. \tag{2.3}$$

The expansion coefficients a_n are considered to be real and independent of ϕ_a^b while f is a parameter of dimension (mass) which will later be identified with the (unrenormalized) pion decay constant.

Using (2.1) and without loss of generality we can write (2.3) as

$$M_a^b = \delta_a^b + \frac{2i}{f}\phi_a^b - \frac{2}{f^2}\phi_c^b - \frac{ia_3}{f^3}\phi_a^c\phi_d^b + \frac{2(a_3-1)}{f^4}\phi_a^c\phi_c^d\phi_d^e\phi_e^b + \cdots, \tag{2.4}$$

where repeated indices denote summation. It would then seem that up to order four in ϕ there is only arbitrary constant, a_3, in the expansion. In fact, by invoking the theorem of Chisholm,[7] all the S-matrix elements are independent of the value of a_3 in (2.4). Thus, to order four in ϕ *all* nonlinear chiral models are equivalent.[8] For calculational purposes it is useful to let $a_3 = \frac{4}{3}$, which is equivalent to letting

$$M_a^b = \exp\left(\frac{2i}{f}\phi_a^b\right). \tag{2.5}$$

We now construct the Lagrangian density with the four-quark model as our guide. We shall add to the Lagrangian a quark-mass–type symmetry-breaking term which transforms as $(4, 4^*) + (4^*, 4)$ and an additional term which breaks the larger U(4) × U(4) symmetry of the model to SU(4) × SU(4).

Thus we write the Lagrangian density as

$$\mathcal{L} = -\frac{f^2}{8}\left[\partial_\mu M_a^b \partial_\mu M_b^{\dagger a} - A_a^b (M_b^a + M_b^{\dagger a}) - U(\det M + \det M^\dagger)\right], \tag{2.6}$$

where

$$A_a^b \equiv A_a \delta_a^b \quad \text{(no sum)}. \tag{2.7}$$

Expanding the matrix M_a^b we see that

$$\mathcal{L}_{\text{free}} = -\tfrac{1}{2}(\partial_\mu \phi_a^b \partial_\mu \phi_b^a + A_a \phi_a^b \phi_b^a + U \phi_a^a \phi_b^b). \tag{2.8}$$

We now note that

$$m^2(\phi_a^b) = \tfrac{1}{2}(A_a + A_b) \quad (a \neq b) \tag{2.9}$$

so that assuming isotopic-spin invariance we have

$$\pi^2 = A_1 = A_2, \quad D^2 = \tfrac{1}{2}(A_1 + A_4),$$
$$K^2 = \tfrac{1}{2}(A_1 + A_3), \quad F^2 = \tfrac{1}{2}(A_3 + A_4),$$

where for simplicity of notation we have substituted the particle symbol for its mass. From (2.9) we also see the mass-squared sum rule

$$F^2 = D^2 + K^2 - \pi^2. \tag{2.10}$$

Using as input

$$\pi^2 \equiv 1, \quad K^2 = 13.60, \quad D^2 = 191.1, \tag{2.11}$$

we see that

$$A_1 = A_2 = \pi^2 = 1, \quad A_3 = 2K^2 - \pi^2 = 26.20, \quad A_4 = 2D^2 - \pi^2 = 381.2, \tag{2.12}$$

and $F^2 = 203.7$. For the four neutral pseudoscalars

$$(\eta_1, \eta_2, \eta_3, \eta_4) \equiv (\pi^0, \eta, \eta', \eta'') \tag{2.13}$$

(2.8) gives us the following mass-squared matrix (in a nondiagonal basis):

$$\mathfrak{M}_{ab} = \begin{bmatrix} A_1 + U & U & U & U \\ U & A_1 + U & U & U \\ U & U & A_3 + U & U \\ U & U & U & A_4 + U \end{bmatrix}. \tag{2.14}$$

If we had taken the entire Lagrangian density to be U(4) × U(4) invariant (so that $U = 0$), then (2.14) takes the form

$$\begin{bmatrix} \pi^2 & & 0 & \\ & \pi^2 & & \\ 0 & & 2K^2 - \pi^2 & \\ & & & 2D^2 - \pi^2 \end{bmatrix},$$

so that one of the isoscalars is degenerate with the pion. Hence we must have a nondiagonal term to construct a more realistic theory.

It is now necessary to solve for the eigenvalues $(\pi^{02}, \eta^2, \eta'^2, \eta''^2)$ of (2.14) which satisfy

$$\det(\mathfrak{M} - \lambda \cdot 1) = \prod_a (\lambda - \eta_a{}^2) = 0, \tag{2.15}$$

where the η_a's are defined in (2.13). Since we have assumed isotropic-spin invariance, we can divide out $(\lambda - \pi^2)$ from both sides of (2.15), which leaves us with

$$U = \frac{(\lambda - A_1)(\lambda - A_3)(\lambda - A_4)}{2(\lambda - A_3)(\lambda - A_4) + (\lambda - A_1)(\lambda - A_4) + (\lambda - A_1)(\lambda - A_3)}. \tag{2.16}$$

This equation is linear in U and cubic in λ so that one value of U will specify all three roots of the equation and thus the three isoscalar masses. A plot of the three solutions as a function of U is given in Fig. 1.

To complete this picture it is now necessary to specify the η-η', η-η'', and η'-η'' mixing angles x, y, and z. Thus in matrix notation we have

$$(\eta_a) = \sum_b R(x, y, z)_{ab}(\phi_b^b), \tag{2.17}$$

where the mixing-angle matrix $R(x, y, z)$ is

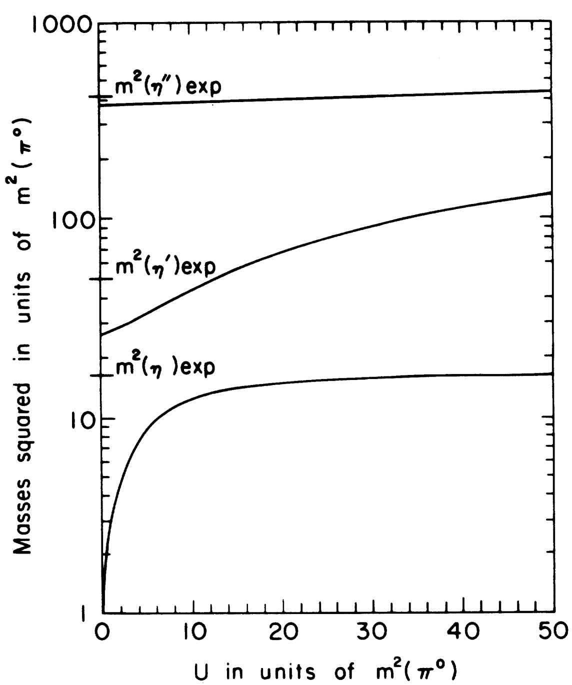

FIG. 1. A plot of the η, η', and η'' masses squared as a function of U, all in units of $m^2(\pi^0)$.

$$R(x,y,z)=\begin{bmatrix}1&0&0&0\\0&\cos y&0&-\sin y\\0&0&1&0\\0&\sin y&0&\cos y\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\cos z&-\sin z\\0&0&\sin z&\cos z\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&\cos x&-\sin x&0\\0&\sin x&\cos x&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1/\sqrt{2}&-1/\sqrt{2}&0&0\\1/\sqrt{6}&1/\sqrt{6}&-2/\sqrt{6}&0\\1/\sqrt{3}&1/\sqrt{3}&1/\sqrt{3}&0\\0&0&0&1\end{bmatrix}. \tag{2.18}$$

Here we have taken the paracharmonium picture of the η'' so that the unmixed η'' is ϕ_4^4. The unmixed η and η' are then members of the SU(3) octet and singlet, respectively. The matrix elements of (2.14) can now be expressed in terms of the masses and mixing angles as follows:

$$\begin{aligned}
\mathfrak{M}_{11}&=\frac{\pi^2}{2}+\frac{\eta^2}{6}(x_-^2\cos^2y+x_+^2\sin^2y\sin^2z)+\frac{\eta'^2}{6}x_+^2\cos^2z+\frac{\eta''^2}{6}(x_-^2\sin^2y+x_+^2\cos^2y\sin^2z)+\frac{x_+x_-}{6}(\eta''^2-\eta^2)\sin 2y\sin z,\\
\mathfrak{M}_{33}&=\frac{\eta^2}{3}(x_+^2\cos^2y+x_-^2\sin^2y\sin^2z)+\frac{\eta'^2}{3}x^2\cos^2z+\frac{\eta''^2}{3}(x_+^2\sin^2y+x_-^2\cos^2y\sin^2z)-\frac{x_+x_-}{3}(\eta''^2-\eta^2)\sin 2y\sin z,\\
\mathfrak{M}_{44}&=\eta^2\sin^2y\cos^2z+\eta'^2\sin^2z+\eta''^2\cos^2y\cos^2z,\\
\mathfrak{M}_{13}&=\frac{-\eta^2x_+x_-}{3\sqrt{2}}(\cos^2y-\sin^2y\sin^2z)+\frac{\eta'^2x_+x_-}{3\sqrt{2}}\cos^2z-\frac{\eta''^2x_+x_-}{3\sqrt{2}}(\sin^2y-\cos^2y\sin^2z)\\
&\quad+\frac{1}{6\sqrt{2}}(\eta''^2-\eta^2)(x_-^2-x_+^2)\sin 2y\sin z,\\
\mathfrak{M}_{14}&=\frac{\eta^2x_+}{2\sqrt{6}}\sin^2y\sin 2z-\frac{\eta'^2x_+}{2\sqrt{6}}\sin 2z+\frac{\eta''^2x_+}{2\sqrt{6}}\cos^2y\sin 2z+\frac{x_-}{2\sqrt{6}}(\eta''^2-\eta^2)\sin 2y\cos z,\\
\mathfrak{M}_{34}&=\frac{\eta^2x_-}{2\sqrt{3}}\sin^2y\sin 2z-\frac{\eta'^2x_-}{2\sqrt{3}}\sin 2z+\frac{\eta''^2x_-}{2\sqrt{3}}\cos^2y\sin 2z-\frac{x_+}{2\sqrt{3}}(\eta''^2-\eta^2)\sin 2y\cos z,
\end{aligned}\tag{2.19}$$

where

$$x_+=\sqrt{2}\cos x+\sin x,\quad x_-=\cos x-\sqrt{2}\sin x,\quad x_+^2+x_-^2=3. \tag{2.20}$$

We can now solve for the mixing angles in terms of known masses for specific values of U. From the trace of (2.14) and (2.12) we see that

$$U=\tfrac{1}{4}[\eta''^2+\eta'^2+\eta^2+\pi^2-2(D^2+K^2)]. \tag{2.21}$$

We then solve for the mixing angles x, y, and z which are given by

$$\cos y=\left(\frac{\Delta}{\Delta''}\,\frac{3U^2/\tilde{\Delta}+\Delta+\tilde{\Delta}}{3U^2/\tilde{\Delta}+2\Delta+\tilde{\Delta}-\Delta''}\right)^{1/2}, \tag{2.22}$$

$$\cos z=\left(\frac{\tilde{\Delta}}{\Delta''\cos^2y-\Delta}\right)^{1/2}, \tag{2.23}$$

and

$$\sin x=-\frac{1}{2\sqrt{3}}\,\frac{\Delta''}{U}\sin 2y\cos z, \tag{2.24}$$

where we have defined

$$\Delta=\eta'^2-\eta^2,\quad \Delta''=\eta''^2-\eta^2,\quad \tilde{\Delta}=2D^2-\pi^2+U-\eta'^2. \tag{2.25}$$

Let us now look at the isoscalar mass spectrum and mixing angles for a specific choice of U, the only remaining free parameter in the theory. As can be seen from Fig. 1 it is impossible to exactly fit the η-η'-η'' masses, but one can get a reasonably good fit for $10<U<30$. Using a least-squares fit we find that for $U=18.50$ we have

$$\eta^2(16.54)=14.82,\quad \eta'^2(50.35)=64.83,\quad \eta''^2(440)=402.7, \tag{2.26}$$

where the experimental masses squared are in parentheses. To complete the picture, we find the mixing angles to be

$$x=-14.2^\circ,\quad y=1.2^\circ,\quad z=5.3^\circ. \tag{2.27}$$

These values of the masses and mixing angles seem quite reasonable considering the simplicity of the model used. An exact fit could probably be obtained in a more realistic model by slightly varying the parameters in (2.6).[9] Thus we expect the general Lagrangian in (2.6) to give us the general features of a 16-plet of pseudoscalar mesons obeying simply-broken chiral symmetry.

In addition to the mass matrix we can also construct the vector and axial-vector Noether currents from (2.6). They are seen to be

$$(V_a^b)_\alpha = \frac{if^2}{4}[M, \partial_\alpha M^\dagger]_a^b = i(\phi_a^c \overleftrightarrow{\partial}_\alpha \phi_c^b) + \cdots \tag{2.28}$$

and

$$(P_a^b)_\alpha = \frac{if^2}{4}\{M, \partial_\alpha M^\dagger\}_a^b = f\partial_\alpha \phi_a^b + \frac{1}{2f}[(4-a_3)\phi_a^c \partial_\alpha \phi_c^d \phi_d^b - a_3(\partial_\alpha \phi_a^c \phi_c^d \phi_d^b + \phi_a^c \phi_c^d \partial_\alpha \phi_d^b)] + \cdots . \tag{2.29}$$

From (2.29) we see that f, the pion decay constant $\approx m_\pi$, is also equal to the K, D, and F decay constants.[10] Reference 6 gives a detailed connection between this type of model and the current algebra including partial conservation of axial-vector current (PCAC).

We shall now apply (2.6) to some strong interactions of mesons, most notably the strong decays of the isoscalars. Expanding (2.6) to terms of order 4, we see that

$$\mathcal{L}_s(\phi^4) = \frac{1}{2f^2}\{2(a_3-1)\partial_\alpha\phi_a^b\partial_\alpha\phi_b^c\phi_c^d\phi_d^a + (a_3-2)\partial_\alpha\phi_a^b\phi_b^c\partial\phi_c^d\phi_d^a + (a_3-1)A_a\phi_a^b\phi_b^c\phi_c^d\phi_d^a + \tfrac{1}{3}U[\phi_a^a\phi_b^b\phi_c^c\phi_d^d + (3a_3-4)\,\phi_a^a\phi_b^c\phi_c^d\phi_d^b]\}. \tag{2.30}$$

Calculating to lowest order in $1/f$ (tree diagrams), we find that

$$T(\eta' \to \eta\pi^+\pi^-) = -\frac{4}{f^2}(R_{21})(R_{31})\pi^2, \tag{2.31}$$

where R_{ab} is defined in (2.17) and (2.18). Calculating the η' width, using as input (2.26) and (2.27), we find that

$$\Gamma(\eta' \to \eta\pi^+\pi^-) = 2.07 \text{ keV}, \tag{2.32}$$

which is well below the upper limit of 450 keV.

With the definition $S_i \equiv -(P-k_i)^2$, we calculate the strong-decay T matrices for η as follows[3]:

$$T(\eta'' \to \eta_a\pi^+\pi^-) = -\frac{4}{f^2}(R_{a1})(R_{41})\pi^2, \tag{2.33}$$

$$T(\eta''(P) \to \eta_a(k_1) + K^+(k_2) + K^-(k_3)) = \frac{1}{3f^2}[(R_{41}-R_{43})(R_{a1}-R_{a3})(3S_1 - \eta''^2 - \eta_a{}^2 - 2K^2)] - \frac{2}{3f^2}[R_{41}R_{a1}(K^2+\pi^2) + R_{43}R_{a3}(3K^2-\pi^2)\,(R_{a1}R_{43} + R_{a3}R_{41})K^2], \tag{2.34}$$

$$T(\eta'' \to \eta_a\eta_b\eta_c) = \frac{4}{f^2}\Big\{\sum_i (R_{ai}R_{bi}R_{ci}R_{4i})\pi^2 + 2(R_{a3})(R_{b3})(R_{c3})(R_{43})(K^2-\pi^2) + 2(R_{a4})(R_{b4})(R_{c4})(R_{44})(D^2-\pi^2) + U\Big[\sum_i (R_{ai})\Big]\Big[\sum_i (R_{bi})\Big]\Big[\sum_i (R_{ci})\Big]\Big[\sum_i (R_{4i})\Big]\Big\}. \tag{2.35}$$

The widths for the appropriate decays are listed in Table I.

As can be seen from (2.8), (2.14), and (2.31) to (2.35) the U(1)-breaking parameter U initially is determined by the isoscalar masses, and also contributes only to the decay η'' to three isoscalars. In (2.30) we see that the U(1) term gives rise to an explicitly Okubo-Zweig-Iizuka (OZI)-rule-breaking term not dependent on the isoscalar mixing angles. Since no analog exists in SU(3) to the $\eta'' \to \eta_a\eta_b\eta_c$ decay, we now have a test for this type of symmetry-breaking term. As has been shown in Ref. 6, there are other types of symmetry-breaking terms that will lead to off-diagonal elements in (2.14) such as $(M_a^a - M_a^{\dagger a})^2$, but these terms lead to a high value for the $\eta' \to \eta\pi\pi$ mode. The U(1)-breaking term, however, gives a small value for the $\eta' \to \eta\pi\pi$ decay while also giving a reasonable η-η'-η'' mass spectrum. Another feature of the U(1) term is that in the current-algebra approximation it contributes to the electromagnetic decay of $\eta \to 3\pi$, while the symmetry-breaking terms of Ref. 6 do not.[11]

Equation (2.30) also describes meson-meson scattering amplitudes and thus scattering lengths. These are completely calculated and discussed in Ref. 6, and the extension to U(4) is straightforward. It should be noted that in this model there is no strong F-π scattering amplitude since there are no quark flavors common to both mesons.

TABLE I. Strong-decay widths of the η''.

Decay mode	Width (MeV)
$\eta'' \to \eta\pi^+\pi^-$	1.62×10^{-2}
$\eta'' \to \eta'\pi^+\pi^-$	6.68×10^{-3}
$\eta'' \to \pi^- K^+\bar{K}^0$	1.06×10^{-1}
$\eta'' \to \eta K^+K^-$	3.08×10^{-2}
$\eta'' \to \eta' K^+K^-$	1.30×10^{-1}
$\eta'' \to \eta\eta\eta$	1.52×10^{-1}
$\eta'' \to \eta\eta\eta'$	1.40
$\eta \to \eta\eta'\eta'$	2.25×10^{1}

III. WEAK DECAYS OF MESONS

Using the Glashow-Iliopoulos-Maiani (GIM) model[12] for weak interactions, and using the axial-vector hadronic current in (2.29), we construct the following effective Lagrangian density for the semileptonic decays of the pseudoscalar mesons:

$$\mathcal{L}_w(\pi) = i\frac{G}{\sqrt{2}}\cos\theta_C f\,\partial_\alpha \pi^- \bar{\mu}\gamma_\alpha(1+\gamma_5)\nu_\mu\,, \tag{3.1}$$

$$\mathcal{L}_w(K) = i\frac{G}{\sqrt{2}}\sin\theta_C f\,\partial_\alpha K^- \bar{\mu}\gamma_\alpha(1+\gamma_5)\nu_\mu\,, \tag{3.2}$$

$$\mathcal{L}_w(D) = -i\frac{G}{\sqrt{2}}\sin\theta_C f\,\partial_\alpha D^- \bar{\mu}\gamma_\alpha(1+\gamma_5)\nu_\mu\,, \tag{3.3}$$

$$\mathcal{L}_w(F) = i\frac{G}{\sqrt{2}}\cos\theta_C f\,\partial_\alpha F^- \bar{\mu}\gamma_\alpha(1+\gamma_5)\nu_\mu\,. \tag{3.4}$$

In the above G is the Fermi constant, θ_C is the Cabibbo angle, and f is the pseudoscalar decay constant. If a massive charged lepton (L) of mass $\approx$1.8 GeV (Ref. 13) exists, then (3.3) and (3.4) with $\bar{\mu}\gamma_\alpha(1+\gamma_5)\nu_\mu$ replaced by $\bar{L}\gamma_\alpha(1\pm\gamma_5)\nu$ would describe the $F, D \to L\nu$ decays.[14] The lepton decay rates of the F and D are listed in Table II.

Now using the vector hadronic current in (2.28) we see that for the dominant semileptonic decays K_{l3}, D_{l3}, and F_{l3}, the hadronic part of the matrix elements[15] are

$$(4P_0q_0)^{1/2}\langle\pi^-(q)\,|\,(V_1^3)_\alpha\,|\,K_S(P)\rangle = \frac{1}{\sqrt{2}}Q_\alpha\,, \tag{3.5}$$

$$(4P_0q_0)^{1/2}\langle\bar{K}^0(q)\,|\,(V_4^3)_\alpha\,|\,D^+(P)\rangle = Q_\alpha\,, \tag{3.6}$$

$$(4P_0q_0)^{1/2}\langle K^-(q)\,|\,(V_4^3)_\alpha\,|\,D^0(P)\rangle = Q_\alpha\,, \tag{3.7}$$

$$(4P_0q_0)^{1/2}\langle\eta_a(q)\,|\,(V_4^3)_\alpha\,|\,F^+(P)\rangle = R_{a3}Q_\alpha\,, \tag{3.8}$$

where

$$Q_\alpha \equiv (P+q)_\alpha\,. \tag{3.9}$$

Using (3.5) we find that $\Gamma(K_S \to \pi^+e^-\nu) = 7.34\times 10^6$ sec^{-1} as compared with the experimental value of 7.52×10^6 sec^{-1}. The other semileptonic rates are listed in Table II.

For the nonleptonic decays[16-23] we shall assume that the effective weak Lagrangian density is proportional to the 20-dimensional GIM interaction as follows.

TABLE II. Weak-decay rates for two- and three-body final states of charmed mesons. Here L is a heavy meson of mass ~1.8 GeV.

D^+ Rates in sec^{-1}	D^0 Rates in sec^{-1}	F^+ Rates in sec^{-1}
$\Gamma(D^+\to\mu^+\nu) = 1.57\times 10^8$		$\Gamma(F^+\to\mu^+\nu) = 3.08\times 10^9$
$\Gamma(D^+\to L^+\nu) = 4.72\times 10^8$		$\Gamma(F^+\to L^+\nu) = 3.35\times 10^{10}$
$\Gamma(D^+\to\bar{K}^0e^+\nu) = 1.09\times 10^{11}$	$\Gamma(D^0\to K^-e^+\nu) = 1.09\times 10^{11}$	$\Gamma(F^+\to\eta e^+\nu) = 5.05\times 10^{10}$
$\Gamma(D^+\to\bar{K}^0\mu^+\nu) = 1.06\times 10^{11}$	$\Gamma(D^0\to K^-\mu^+\nu) = 1.06\times 10^{11}$	$\Gamma(F^+\to\eta\mu^+\nu) = 4.91\times 10^{10}$
		$\Gamma(F^+\to\eta' e^+\nu) = 2.03\times 10^{10}$
		$\Gamma(F^+\to\eta'\mu^+\nu) = 1.91\times 10^{10}$
$\Gamma(D^+\to\bar{K}^0\pi^+) = 2.25\times 10^{10}$	$\Gamma(D^0\to K^-\pi^+) = 5.12\times 10^{12}$	$\Gamma(F^+\to K^+K^0) = 4.83\times 10^{12}$
	$\Gamma(D^0\to\bar{K}^0\pi^0) = 2.89\times 10^{12}$	$\Gamma(F^+\to\pi^+\eta) = 1.89\times 10^{12}$
	$\Gamma(D^0\to\bar{K}^0\eta) = 1.23\times 10^{12}$	$\Gamma(F^+\to\pi^+\eta') = 1.51\times 10^{12}$
	$\Gamma(D_0\to\bar{K}^0\eta') = 3.06\times 10^{11}$	$\Gamma(F^+\to\pi^+\pi^0) = 0$
$\Gamma(D^+\to K^-\pi^+\pi^+) = 6.16\times 10^{11}$	$\Gamma(D^0\to K^-\pi^+\eta) = 3.02\times 10^{11}$	$\Gamma(F^+\to K^+K^0\pi^0) = 3.29\times 10^{11}$
$\Gamma(D^+\to\bar{K}^0\pi^+\pi^0) = 2.00\times 10^{11}$	$\Gamma(D^0\to K^-\pi^+\pi^0) = 2.47\times 10^{11}$	$\Gamma(F^+\to K^+K^-\pi^+) = 2.66\times 10^{11}$
$\Gamma(D^+\to\bar{K}^0\pi^+\eta) = 2.28\times 10^{10}$	$\Gamma(D^0\to\bar{K}^0\pi^+\pi^-) = 2.03\times 10^{11}$	$\Gamma(F^+\to\pi^+\pi^0\eta) = 1.77\times 10^{11}$
$\Gamma(D^+\to\bar{K}^0\pi^+\eta') = 6.84\times 10^9$	$\Gamma(D^0\to\bar{K}^0\pi^0\eta) = 1.89\times 10^{11}$	$\Gamma(F^+\to K^0\bar{K}^0\pi^+) = 1.09\times 10^{11}$
	$\Gamma(D^0\to\bar{K}^0\eta\eta) = 9.91\times 10^9$	$\Gamma(F^+\to\pi^+\pi^0\eta') = 7.00\times 10^{10}$
	$\Gamma(D^0\to\bar{K}^0\pi^0\eta') = 2.50\times 10^9$	$\Gamma(F^+\to K^+\bar{K}^0\eta) = 5.20\times 10^{10}$
		$\Gamma(F^+\to\pi^+\eta\eta') = 1.62\times 10^{10}$
		$\Gamma(F^+\to\pi^+\eta\eta) = 1.56\times 10^{10}$
		$\Gamma(F^+\to\pi^-\pi^+\pi^+) = 7.85\times 10^7$
		$\Gamma(F\to\pi^+\pi^0\pi^0) = 2.49\times 10^7$

$$\mathfrak{L}_w = \frac{G}{2\sqrt{2}}\bar{x}[\sin\theta_C \cos\theta_C (j^a_{2\alpha} j^3_{a\alpha} - 2j^4_{2\alpha} j^3_{4\alpha} + 2j^3_{2\alpha} j^4_{4\alpha}) + \cos^2\theta_C (j^3_{4\alpha} j^1_{2\alpha} - j^3_{2\alpha} j^1_{4\alpha}) - \sin^2\theta_C (j^2_{4\alpha} j^1_{3\alpha} - j^1_{4\alpha} j^2_{3\alpha}) + \sin\theta_C \cos\theta_C (j^3_{4\alpha} j^1_{3\alpha} - j^1_{4\alpha} j^3_{3\alpha} - j^2_{4\alpha} j^3_{8\alpha} + j^1_{4\alpha} j^2_{2\alpha})] + \text{H.c.} \tag{3.10}$$

Here $\bar{x}$ is a dimensionless parameter to be taken from the $K_S \to 2\pi$ rate. The currents j^b_a are the left-handed currents

$$j^b_{a\alpha} = (V^b_a)_\alpha + (P^b_a)_\alpha - \tfrac{1}{4}\delta^b_a((V^c_c)_\alpha + (P^c_c)_\alpha), \tag{3.11}$$

where $(V^b_a)_\alpha$ and $(P^b_a)_\alpha$ are defined in (2.28) and (2.29). We have chosen the 20-dimensional part of the GIM interaction since it contains the $\Delta I = \frac{1}{2}$ octet part of the SU(3) $|\Delta S| = 1$, $\Delta C = 0$ nonleptonic decays.

Some typical two-body decay matrices are given below:

$$T(K^0_1 \to \pi^+\pi^-) = \frac{G}{2}\bar{x}\sin\theta_C\cos\theta_C f(K^2 - \pi^2), \tag{3.12}$$

$$T(D^0 \to K^-\pi^+) = -i\frac{G}{\sqrt{2}}\bar{x}\cos^2\theta_C f(D^2 - \pi^2), \tag{3.13}$$

$$T(D^0 \to K^0\eta_a) = -i\frac{G}{\sqrt{2}}\bar{x}\cos^2\theta_C \times f[(R_{a2} - R_{a3})(\eta_a{}^2 - K^2) + (R_{a1} - R_{a4})(D^2 - \eta_a{}^2)], \tag{3.14}$$

$$T(D^+ \to \bar{K}^0\pi^+) = i\frac{G}{2\sqrt{2}}\bar{x}\cos^2\theta_C f(K^2 - \pi^2), \tag{3.15}$$

$$T(F^+ \to \pi^+\pi^0) = 0, \tag{3.16}$$

$$T(F^+ \to K^+\bar{K}^0) = i\frac{G}{2\sqrt{2}}\bar{x}\cos^2\theta_C f(F^2 - K^2), \tag{3.17}$$

$$T(F^+ \to \pi^+\eta_a) = i\frac{G}{2\sqrt{2}}\bar{x}\cos^2\theta_C f(R_{a4} - R_{a3})(F^2 - \eta_a{}^2). \tag{3.18}$$

From the experimental rate for (3.12) we find that

$$\bar{x} \approx 2.14/\sin\theta_C\cos\theta_C. \tag{3.19}$$

The decay rates for the nonleptonic two-body final states are listed in Table II.

It is interesting to note the suppression of the $F^+ \to \pi^+\pi^0$ and $D^+ \to \bar{K}^0\pi^+$ decays. Both of these suppressions can be understood in terms of Bose symmetry for the final states.[21] For the $F^+ \to \pi^+\pi^0$ mode, Bose symmetry requires the 2π state to be an $I = 2$ state, while the F^+ is a singlet and the effective Lagrangian density for a $\Delta S = \Delta C$ decay has only a $\Delta I = 1$ part. Since we are assuming isotopic-spin invariance, $T(F^+ \to \pi^+\pi^0) = 0$. We have a similar situation for the $D^+ \to \bar{K}^0\pi^+$ mode, but in this case we are dealing with V spin. The D^0 has V spin $= 0$ while the $\bar{K}^0$ and π^+ both have V spin $= \frac{1}{2}$. Bose symmetry requires the $\bar{K}^0\pi^+$ to be in a V spin $= 1$ state, while the Lagrangian density has only a $V = 0$ part. Since SU(3) symmetry is broken in the mass spectrum, the rate for $D^+ \to \bar{K}^0\pi^+$ is proportional to the SU(3) mass breaking.

Finally contributions to the three-body nonleptonic decay amplitudes in tree approximation arise from both four-point weak vertices and from two-point weak vertices and four-point strong vertices. The relevant four-point strong interaction is given by (2.30). As an example of this, consider the decay $D^0 \to \bar{K}^0\pi^+\pi^-$ shown in Fig. 2. First there is the direct four-meson term from the current-current VV and PP terms in (3.10). This is zeroth order in $1/f$. In addition we have strong K-K-π-π, D-D-π-π, and D-K-F-π vertices of order $1/f^2$ in combination with weak D-K and F-π terms of order f^2. These are the only contributions to zeroth order in $1/f$ and all must, in general, be considered in calculating an amplitude. The relevant three-body T matrices are listed below:

$$T(K^+(P) \to \pi^-(k_1) + \pi^+(k_2) + \pi^+(k_3)) = -\frac{G}{2\sqrt{2}}\bar{x}\sin\theta_C\cos\theta_C(-S_1 + K^2 + \pi^2); \tag{3.20}$$

for the D^+ we have

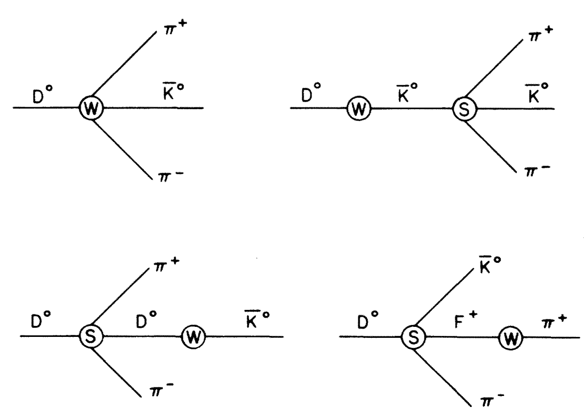

FIG. 2. Feynman diagrams in tree order for the decay $D^0 \to \bar{K}^0\pi^+\pi^-$. The S represents a strong vertex, while W represents a weak one.

$$T(D^+(p)\to K^-(k_1)+\pi^+(k_2)+\pi^+(k_3)) = -\frac{G}{2\sqrt{2}}\bar{x}\cos^2\theta_C\left(1-\frac{2\pi^2}{F^2-\pi^2}\right)(-S_1+D^2+K^2), \tag{3.21}$$

$$T(D^+(P)\to \bar{K}^0(k_1)+\pi^+(k_2)+\eta_a(k_3))$$

$$= -\frac{G}{2\sqrt{2}}\bar{x}\cos^2\theta_C\Big\{(R_{a1}-R_{a2})(S_3-S_2)+(R_{a3}-R_{a2})(S_1-S_2)$$
$$+\tfrac{1}{3}[(S_1-\pi^2-\eta_a{}^2)(2R_{a2}-R_{a3}-R_{a4})+(S_1-K^2-D^2)(R_{a1}+R_{a2}-2R_{a4})$$
$$+(S_2-D^2-\pi^2)(-R_{a2}-R_{a3}+2R_{a4})+(S_2-K^2-\eta_a{}^2)(R_{a1}-2R_{a2}+R_{a4})$$
$$+2(S_3-\pi^2-K^2)(R_{a3}-R_{a1})]$$
$$+\frac{1}{3}\frac{K^2}{D^2-K^2}[R_{a4}(-3S_1+3D^2+K^2+\pi^2+\eta_a{}^2)+R_{a2}(-3S_2+2D^2+K^2+2\pi^2+\eta_a{}^2)$$
$$+R_{a1}(-3S_3+2D^2+K^2+2\pi^2+\eta_a{}^2)]$$
$$-\frac{1}{3}\frac{\pi^2}{F^2-\pi^2}[R_{a2}(-3S_1+2D^2+2K^2+\pi^2\eta_a{}^2)+R_{a4}(-3S_2+3D^2+2K^2+\eta_a{}^2)$$
$$+R_{a3}(-3S_3+2D^2+3K^2+\eta_a{}^2)]\Big\}; \tag{3.22}$$

for the D^0 we have

$$T(D^0(p)\to\pi^-(k_1)+\pi^+(k_2)+\bar{K}^0(q_3)) = \frac{G}{2\sqrt{2}}\bar{x}\cos^2\theta_C\left(S_1\frac{K^2}{D^2-K^2}-S_2\frac{D^2}{D^2-K^2}+S_3\frac{F^2}{F^2-\pi^2}-\pi^2\frac{F^2+\pi^2}{F^2-\pi^2}\right), \tag{3.23}$$

$$T(D^0(p)K^-(k_1)+\pi^+(k_2)+\eta_a(k_2))$$

$$= \frac{G}{2\sqrt{2}}\bar{x}\cos^2\theta_C\Big\{(R_{a1}-R_{a2})(S_3-S_2)+(R_{a1}-R_{a4})(S_2-S_1)$$
$$+\tfrac{1}{3}[(S_1-\eta_a{}^2-\pi^2)(2R_{a1}-R_{a3}-R_{a4})+(S_1-D^2-K^2)(R_{a1}+R_{a2}-2R_{a3})$$
$$+2(S_2-D^2-\pi^2)(R_{a4}-R_{a2})+(S_3-K^2-\pi^2)(2R_{a3}-R_{a1}-R_{a4})$$
$$+(S_3-\eta_a{}^2-D^2)(R_{a2}+R_{a3}-2R_{a1})]$$
$$+\frac{1}{3}\frac{D^2}{D^2-K^2}[R_{a3}(-3S_1+D^2+3K^2+\pi^2+\eta_a{}^2)+R_{a2}(-3S_2+D^2+2K^2+2\pi^2+\eta_a{}^2)$$
$$+R_{a1}(-3S_3+D^2+2K^2+2\pi^2+\eta_a{}^2)]$$
$$-\frac{1}{3}\frac{\pi^2}{F^2-\pi^2}[R_{a1}(-3S_1+2D^2+2K^2+\pi^2+\eta_a{}^2)+R_{a4}(-3S_2+3D^2+2K^2+\eta_a{}^2)$$
$$+R_{a3}(-3S_3+2D^2+3K^2+\eta_a{}^2)]\Big\}, \tag{3.24}$$

$$T(D^0(p)\to\bar{K}^0(k_1)+\eta_a(k_2)+\eta_b(k_3))$$

$$= \frac{G}{2\sqrt{2}}\bar{x}\cos^2\theta_C\Big\{(R_{a1}-R_{a4})(R_{b3}-R_{b4})(S_1-S_3)+(R_{b1}-R_{b4})(R_{a3}-R_{a4})(S_1-S_2)$$
$$+(R_{a3}-R_{a2})(R_{b3}-R_{b2})(S_1-\tfrac{1}{3}D^2-K^2)+(R_{a1}-R_{a4})(R_{b1}-R_{b4})(S_1-D^2-\tfrac{1}{3}K^2)$$
$$+\frac{1}{3}\frac{K^2}{D^2-K^2}[(R_{a4}-R_{a1})(R_{b4}-R_{b1})(3S_1+D^2-K^2-\eta_a{}^2-\eta_b{}^2)+2R_{a4}R_{b4}(2D^2-\pi^2)+2R_{a1}R_{b1}\pi^2]$$
$$-\frac{1}{3}\frac{D^2}{D^2-K^2}[(R_{a3}-R_{a2})(R_{b3}-R_{b2})(3S_1-D^2+K^2-\eta_a{}^2-\eta_b{}^2)+2R_{a3}R_{b3}(2K^2-\pi^2)+2R_{a2}R_{b2}\pi^2]\Big\}; \tag{3.25}$$

for the F^+ we have

$$T(F^+(p)\to\pi^-(k_1)+K^+(k_2)+\pi^+(k_3)) = \frac{G}{2\sqrt{2}}\bar{x}\cos^2\theta_C\frac{2\pi^2}{F^2-\pi^2}(-S_1+F^2+\pi^2), \tag{3.26}$$

$$T(F^+(p)\to\pi^+(k_1)+K^+(k_2)+K^-(k_2))=-\frac{G}{2\sqrt{2}}\bar{x}\cos^2\theta_C(-S_1+K^2), \tag{3.27}$$

$$T(F^+(p)\to K^0(k_1)+\bar{K}^0(k_2)+\pi^+(k_2))=\frac{G}{2\sqrt{2}}\bar{x}\cos^2\theta_C\left[-\frac{\pi^2}{F^2-\pi^2}S_1+\frac{F^2}{F^2-\pi^2}S_2-\frac{D^2}{D^2-K^2}S_3+\frac{K^2}{D^2-K^2}(F^2-\tfrac{1}{3}\pi^2)\right], \tag{3.28}$$

$$\begin{aligned}
&T(F^+(p)\to K^+(k_1)+\bar{K}^0(k_2)+\eta_a(k_3))\\
&=\frac{G}{2\sqrt{2}}\bar{x}\cos^2\theta_C\Big\{(R_{a3}-R_{a4})(S_2-S_1)+(R_{a3}-R_{a2})(S_2-S_3)\\
&\quad+\tfrac{1}{3}[(S_1-F^2-K^2)(2R_{a1}-R_{a2}-R_{a3})+(S_1-\eta_a{}^2-K^2)(R_{a1}-2R_{a3}+R_{a4})+2(S_2-F^2-K^2)(R_{a2}-R_{a4})\\
&\quad+(S_3-F^2-\eta_a{}^2)(-R_{a1}-R_{a2}+2R_{a3})+(S_3-2K^2)(-2R_{a1}+R_{a3}+R_{a4})]\\
&\quad+\frac{1}{3}\frac{F^2}{F^2-\pi^2}[(R_{a1}+R_{a2})(-3S_1+F^2+3K^2+\pi^2+\eta_a{}^2)+(R_{a3})(-3S_3+D^2+4K^2+\eta_a{}^2)]\\
&\quad+\frac{1}{3}\frac{K^2}{D^2-K^2}[R_{a3}(-3S_1+2F^2+3K^2+\eta_a{}^2)+R_{a4}(-3S_2+3F^2+K^2+\pi^2+\eta_a{}^2)\\
&\qquad+R_{a1}(-3S_3+2F^2+2K^2+\pi^2+\eta_a{}^2)]\Big\},
\end{aligned} \tag{3.29}$$

$$\begin{aligned}
&T(F^+(p)\to\pi^+(k_1)+\eta_a(k_2)+\eta_b(k_3))\\
&=\frac{G}{2\sqrt{2}}\bar{x}\cos^2\theta_C\Big\{(R_{a1}-R_{a2})(R_{b3}-R_{b4})(S_2-S_1)+(R_{a3}-R_{a4})(R_{b1}-R_{b2})(S_3-S_1)\\
&\quad+(R_{a1}-R_{a2})(R_{b1}-R_{b2})(-S_1+\tfrac{1}{3}F^2+\pi^2)+(R_{a3}-R_{a4})(R_{b3}-R_{b4})(-S_1+F^2+\tfrac{1}{3}\pi^2)\\
&\quad+\frac{1}{3}\frac{F^2}{F^2-\pi^2}[(R_{a1}-R_{a2})(R_{b1}-R_{b2})(3S_1-F^2-3\pi^2-\eta_a{}^2-\eta_b{}^2)+6\pi^2(R_{a1}R_{b1}+R_{a2}R_{b2})]\\
&\quad+\frac{1}{3}\frac{\pi^2}{F^2-\pi^2}[(R_{a4}-R_{a3})(R_{b4}-R_{b3})(3S_1-3F^2-\pi^2-\eta_a{}^2-\eta_b{}^2)+(4D^2+4F^2-2\pi^2)R_{a4}R_{b4}\\
&\qquad+(4K^2+4F^2-2\pi^2)R_{a3}R_{b3}]\Big\}.
\end{aligned} \tag{3.30}$$

Calculation for the $K\to 3\pi$ mode in (3.20) gives the standard result that $\Gamma(K^+\to\pi^-\pi^+\pi^+)=3.82\times10^6\ \text{sec}^{-1}$, which is about 15% below the experimental value of $4.52\times10^6\ \text{sec}^{-1}$. The rest of the three-body decay rates are listed in Table II.

One of the methods used to look for charmed particles is a search for peaks in invariant-mass spectra. In the case of the D^0 and D^+ mesons such peaks have been found in the $K^-\pi^+$ and the $K^-\pi^+\pi^+$ systems.[24,25] Reference to Table II shows that $\Gamma(D^0\to K^-\pi^+)$ and $\Gamma(D^+\to K^-\pi^+\pi^+)$ are each, respectively, the largest rates in the D^0 and D^+ systems. The rate $D^+\to K^-\pi^+\pi^+$ is not enhanced in this model, but rather it has a relatively large branching ratio due to the suppression of the two-body state $D^+\to\bar{K}^0\pi^+$. A search for the F^+ meson would then seem easiest in the $K^+\bar{K}^0$ mode since it has the largest rate, or in the more detectable $\pi^+K^+K^-$ mode. The $\pi^+\pi^+\pi^-$ mode is greatly suppressed and it should not be experimentally detectable at this time. In general the search for the F^+ meson should be much harder than the searches for the D^0 and D^+ since (1) one must look in a three-body charged *final* state in lieu of a two-body state as in D^0 decay and (2) there is no dominant three-body charged final state as was the case with the D^+.

Another possible test for charmed mesons is ν_μ-induced μ^-e^+ events. Again such events have been seen.[26-28] Here the e^+'s are assumed to have come from semileptonic decays of either F^+'s, D^+'s, or D^0's. Reference to Table II shows that only the D^+ has an appreciable semileptonic branching ratio. Again this is due to the suppression of the $D^+\to\bar{K}^0\pi^+$ mode. Therefore each time a μ^-e^+ final state is observed, it has most probably come from

$$\nu_\mu+N\to\mu^-+D^++\text{hadrons}$$
$$D^+\to e^++\bar{K}^0+\nu_e.$$

Thus each μ^-e^+ event should be associated with *at least* one neutral kaon. Experimentally there are now 1.8 ± 0.7 neutral K's per each μ^-e^+ event.[30]

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