Discrepancy analysis for SU(3)

J. Hamilton
Nordita, Copenhagen, Denmark

G. C. Oades
Institute of Physics, Århus, Denmark
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We show how to extract much information relevant to SU(3) symmetry from a partial-wave analysis of $\bar{K}N \to \bar{K}N$, $\bar{K}N \to \pi\Sigma$, and $\bar{K}N \to \pi\Lambda$ up to 1.9 GeV (center-of-mass system). The method uses the discrepancy function of dispersion theory, and the results provide checks on a considerable number of couplings between hadrons.

I. INTRODUCTION AND SUMMARY

The low- and medium-energy baryon bound states and resonances are well described the the L-excitation quark model SU(6)×O(3), using three quarks.[1] However, one can inquire whether SU(3) properties can be observed in the scattering amplitudes away from the resonances, and also one can ask whether there is any trace of SU(3) in scattering states where there is a repulsion between the particles. The answer to the first question is that one can indeed see SU(3) properties at nonresonant energies by using the discrepancy functions,[2] but only in the amplitudes where there is a strong attraction; to the second question the answer is "probably not."

Data

We use the analysis of the experimental $Y=0$, P- and D-wave processes $\bar{K}N \to \bar{K}N$, $\bar{K}N \to \pi\Sigma$, and $\bar{K}N \to \pi\Lambda$ up to 1.9 GeV [center-of-mass system (c.m.s.) energy] by Lea, Martin, Moorhouse, and Oades[3] (to be called LMMO). They use a K-matrix method to fit the data, and we have several remarks on the form used. However, the main problem is determining the unseen amplitudes $\pi\Sigma \to \pi\Sigma$, $\pi\Sigma \to \pi\Lambda$, and $\pi\Lambda \to \pi\Lambda$, and we look into the principles involved. It is convenient to use the eigenstates of $K(s)$; an eigenstate having eigenvalue $\lambda(s) \simeq 0$ is called a null state. Numerous null states occur in the data fit, and this means that the background parts of the amplitudes are of very simple form.

If there are several null states in one partial wave, there will be ambiguity in the unseen amplitudes; this happens in the fit of LMMO in two cases for isospin $I=1$. Elsewhere this ambiguity does not occur.

The factorized K-matrix fit of LMMO is equivalent to using a relativistic effective-range multichannel formula having only a few parameters. Such a formula makes use of causality (analyticity) and unitarity, and the requirement that K be factorized at the resonances is an important feature. The data fitting works because there is a fairly clear distinction between the several parts of an amplitude (resonant, other varying parts, small background terms). It (usually) determines the unseen amplitudes because we see that there cannot be an eigenstate of $K(s)$ which does not contain $\bar{K}N$.

Discrepancies

It is difficult to see SU(3) properties in the partial-wave amplitudes $F_{ij}(s)$ themselves, apart from the resonances, or bound states (i,j are channel indices). However, the discrepancy functions $\Delta_{ij}(s)$, which were valuable in exploring the dynamics of the πN system,[2] have no physical cut, and they are much smoother functions of s than are $F_{ij}(s)$. They should be better for our purpose.

The functions $\Delta_{ij}(s)$ are given solely by the experimental data, and they are in fact the experimental estimates of the exchange forces in the $\bar{K}N \to \bar{K}N$, $\pi\Sigma \to \pi\Sigma$, $\bar{K}N \to \pi\Lambda$, etc., partial waves. Moreover, the $\Delta_{ij}(s)$ cannot be affected by any argument about Castillejo-Dalitz-Dyson (CDD) poles; one can only argue whether $\Delta_{ij}(s)$ are evaluated accurately.

The discrepancies $\Delta_{ij}(s)$ relate to the PB (i.e., $0^-\frac{1}{2}^+$) channels, and only those ($\bar{K}N, \pi\Sigma, \pi\Lambda$) which are open over most of our energy range. Since there are four or five PB channels in pure SU(3), the $\Delta_{ij}(s)$ cannot show the full SU(3) symmetry, but only a truncated version of SU(3). We distinguish between this *truncated* SU(3) *symmetry and intrinsic* SU(3) *breaking*. The intrinsic breaking can arise because the ranges and the couplings of the exchange forces may depart from pure SU(3) values; also there will be additional intrinsic SU(3) breaking in the amplitudes $F_{ij}(s)$ due to the spread in PB thresholds, and because inelastic (i.e., other) processes also have spread thresholds.

Conclusions

There are five main results of our work:

(i) Those eigenstates $\vec{v}^{(k)}(s)$ of $\Delta_{ij}(s)$ having large positive eigenvalues $\lambda_k(s)$ (corresponding to strong attraction) show SU(3) clearly. This is not a resonance property since $\Delta_{ij}(s)$ exist at all energies and they show no bump or discontinuity at the resonances.

(ii) The set L of states having large positive eigenvalues $\lambda_k(s)$ is identical with the set Th where theory predicts strong attractive exchange forces in $PB \to PB$ scattering. Also, each of set L contains a bound state or resonance (below 1.95 GeV).

The set R of all resonances (below 1.95 GeV) contains a few more states than L, for example, $(10, D_{3/2})$. Earlier theoretical studies[4,5] have suggested that in set $(R-L)$ strong attractive interactions are found in other meson-baryon channels, such as PD (i.e., $0^-\frac{3}{2}^+$).

(iii) This result deals with symmetry breaking. The bordered matrix theorem of Ref. 6 enables us to understand why SU(3) breaking is fairly small in the set L of states, even though all the PB channels are not present ($\eta\Sigma, \eta\Lambda, K\Xi$ are missing). In the remaining states SU(3) breaking is large, and most of the $\lambda_k(s)$ are small. This is partly due to not having all the PB channels, but there is a further effect. Theoretical calculations show that repulsive eigenvalues of $\Delta_{ij}(s)$ should occur in some $Y=0$, $I=1$ states, but none are seen.

(iv) Except for the resonant states at resonance energies, SU(3) breaking is worse in the partial-wave amplitudes $F_{ij}(s)$ than in the $\Delta_{ij}(s)$.

(v) Our results should make it possible to confirm, or redetermine, some of the basic couplings used in calculating the exchange forces. For example, there is the choice between whether the pseudoscalar, or the equivalent pseudovector, PBB couplings obey SU(3) best, and there are problems concerning the VBB couplings,[7] and so on.

Our results should also make it possible to predict some partial waves, and they may suggest improved ways of analyzing experimental data.

In Sec. II we discuss the data analysis and display the discrepancy eigenstates; in Sec. III the theory of PB exchange forces is summarized; in Sec. IV we discuss SU(3) repulsive states and we also propose new ways of analyzing the experimental data.

II. DATA AND DISCREPANCIES

Our work is based on the K-matrix fit of LMMO[3] to experimental data on $\bar{K}N \to \bar{K}N$, $\bar{K}N \to \pi\Sigma$ and, $\bar{K}N \to \pi\Lambda$ from 1.52 GeV to 1.9 GeV (c.m.s. energy). Other analyses including newer data are by now available (see for example Ref. 8) but these do not give the K-matrix elements we need to reconstruct the unseen amplitudes for $\pi\Sigma \to \pi\Sigma$, $\pi\Sigma \to \pi\Lambda$, and $\pi\Lambda \to \pi\Lambda$. We believe that our use of the older LMMO analysis is justified since the newer results do not show large changes from the LMMO results in the region of interest to us.

Here we use the normalization in which

$$T^{-1} = K^{-1} - iQ\,, \tag{1}$$

where Q is the diagonal momentum matrix. For a single channel this normalization corresponds to $T = (e^{2i\delta} - 1)/2iq$. An account of K and its use in data fitting are found in pages 394–399 of Ref. 9 and particularly in papers quoted there; the reader might also consult Refs. 10, 11, and 12. In their fit LMMO use the parametric form

$$Q^{1/2}KQ^{1/2} = C_0\vec{\mu}^0 \times \vec{\mu}^0(W_0 - W) + \sum_{r=1}^{n} C_r \frac{\vec{\mu}^{(r)} \times \vec{\mu}^{(r)}}{W_r - W} \tag{2}$$

with $W = s^{1/2}$ the c.m.s. energy. C_0, C_r ($C_r > 0$), W_0, W_r, and the constant unit vectors (in channel space) $\vec{\mu}^0$, $\vec{\mu}^{(r)}$ are real parameters; $\times$ means outer product. The factorization property of Eq. (2) is *necessary* in the case of a resonance (or bound-state) pole so as to ensure that the resonance (or bound state) only occurs in a single eigenstate. Factorization is not necessary for the background terms but may give a more flexible fitting function for large $|W|$.

There are two comments:

(a) If W_r is below the thresholds the constraint $C_r > 0$ means that the term $(W_r - W)^{-1}$ will represent a bound-state pole, or a repulsive-driving-force pole, but not an attractive-driving-force pole. This is a bias in fitting, but it should increase the reliability of our main results. We explicitly demonstrate attractive driving forces, and we fail to find repulsive forces.

(b) In place of Eq. (1) we take the form

$$T = K(I - iQK)^{-1} \tag{3}$$

and we *extrapolate* the results of LMMO down to the $\pi\Lambda$ threshold (1255 MeV) using simple formulas consistent with Eq. (3). [In this extrapolation the measured width and branching ratios of $D_{03}(1520)$ and the pole approximation for $P_{13}(1385)$ are used.]

In the fitting by LMMO (1.52 to 1.9 GeV) a channel is added to allow for inelastic processes (i.e., $KN \to x$, where $x \neq \bar{K}N, \pi\Sigma, \pi\Lambda$), so for $I = 0, 1$ the fits are, respectively, 3, 4 channels. Good fits were obtained[3] with at most two pole terms $(W_r - W)^{-1}$ in Eq. (2), plus the $(W_0 - W)$ term; almost always $W_0 > 1.9$ GeV. When Eq. (2) has $(m+1)$ terms in an n-channel fit, there will be $(n-m-1)$, or more, null states [i.e., eigenstates of $K(s)$ having eigenvalue $\lambda(s) \simeq 0$]. For P_{13} and D_{15} there are two such null states while elsewhere there is never more than one null state.

The unseen amplitudes

An important problem is that of the unseen amplitudes, i.e., $\pi\Sigma \to \pi\Sigma$, $\pi\Sigma \to \pi\Lambda$, etc. These are constrained by unitarity,[13] such restrictions automatically being satisfied by the K-matrix fit which also supplies further restrictions via the analytic form of Eq. (2) and the small number of parameters used in the fit. As an example consider a resonance at W_1 in a 3-channel problem. We can find C_1 and $\vec{\mu}$ using only T_{11}, T_{12}. A fit over the resonance region gives the total width (C_1), and the branching ratio at W_1 gives μ_1^2, $\mu_1\mu_2$; so μ_3 is determined (up to a sign).

Possible ambiguities still remain due to null states. (To simplify the discussion we ignore differences between the channel momenta so that the eigenstates of T and K are directly related.) If there is no null state in the analysis the fit to $K(s)$ is completely determined; if there is a null state with an eigenvector $\vec{v}$ which has no $\bar{K}N$ component then by changing the eigenvalue from zero we can alter the unseen amplitudes without affecting the fit to the data. In the case where there is only one null state $\vec{v}$ will in general have a $\bar{K}N$ component and this ambiguity will not exist. In the case of two null states it will in general be possible to choose a combination of $\vec{v}_1$ and $\vec{v}_2$ which has no $\bar{K}N$ component, and the unseen amplitudes are then subject to ambiguity. Null states and states where ambiguity can occur are shown in Table I. Note that the ambiguity concerns the unseen amplitudes, and it cannot contain any resonance which couples to $\bar{K}N$.

TABLE I. Analysis of the fits of LMMO (Ref. 3).

State	P_{01}	P_{03}	D_{03}	D_{05}	P_{11}	P_{13}	D_{13}	D_{15}
Terms in Eq. (2)	3	3	3	2	3	2	3	2
No. of null states	0	1	0	1	1	2	1	2
Ambiguity	No	No	No	No	No	Yes	No	Yes

Discrepancies

We must use the reduced partial-wave amplitudes,

$$F_{ij}(s) = T_{ij}(s)/(q_i q_j)^l ,$$

with l the orbital angular momentum and q_i $(i=1, \ldots, N)$ the channel momentum. Also i,j refer only to the PB channels $\bar{K}N, \pi\Sigma, \pi\Lambda$. The discrepancy matrix $\Delta_{ij}(s)$ is defined by

$$\Delta_{ij}(s) = \mathrm{Re}F_{ij}(s) - \frac{P}{\pi}\int_{s_0}^{\infty} ds' \, \frac{\mathrm{Im}F_{ij}(s')}{s'-s} . \qquad (5)$$

All $F_{ij}(s)$ have physical cuts reaching down to s_0, the $\pi\Lambda$ threshold, $\mathrm{Im}F_{ij}(s)$ being determined by unitarity in the region between the physical threshold and s_0. In states P_{01}, P_{11} we also subtract from the right-hand-side of Eq. (5) the bound-state pole terms $(s-M_\Lambda^2)^{-1}$, $(s-M_\Sigma^2)^{-1}$, respectively. The discrepancies $\Delta_{ij}(s)$ are given solely by the data.

By dispersion theory, $\Delta_{ij}(s)$ are the left-hand cut (or driving-force) terms in the dispersion relation for $F_{ij}(s)$. Thus $\Delta_{ij}(s)$ are regular and smoothly varying for $s > s_0$, and $\Delta_{ij}(s) = O(s^{-1})$ as $s \to \infty$. They are ideal functions to examine for symmetry structure.

Eigenstates of $\Delta(s)$

In pure SU(3) for $Y=0$, $I=0$ there are four PB states and five for $I=1$. Imagine forming $\Delta_{ij}(s)$ with respect to PB states in pure SU(3). The eigenstates of $\Delta_{ij}(s)$ would be the PB representations of SU(3), i.e., $27, 8_1, 8_2, 1$ for $I=0$, and $27, 10, \overline{10}, 8_1, 8_2$ for $I=1$ (where $8_1, 8_2$ are some linear combinations of 8_d and 8_f). The eigenvalues of $\Delta_{ij}(s)$ would be the pure SU(3) exchange forces in these representations.

In practice we use the results of LMMO[3] to evaluate $\Delta_{ij}(s)$ over $\bar{K}N, \pi\Sigma$ for $I=0$, and over $\bar{K}N, \pi\Sigma, \pi\Lambda$ for $I=1$. Even though these are not the full PB SU(3) channels, we can still demonstrate SU(3).

It is simplest to look at the eigenstates of $\Delta_{ij}(s)$ for physical s. Typical values are shown in Table II. Some of the values at 1600 MeV show disturbances which are attributed to an imperfect join of the LMMO[3] values to the extrapolating formulas at 1.52 GeV.

A general feature of the results is the slowness of the rotation of the eigenvectors of $\Delta(s)$ as s varies. This tells us that (in any partial wave) the dominant exchange forces acting in the various channels are of much the same range.

We can give some estimate of the effect of the possible ambiguities due to null vectors. Calculating the eigenstates of T_{ij} coming from LMMO,[3] we find that $\vec{v}$ varies very little from 1650 MeV to 1850 MeV. The projection of $\vec{v}$ in $(\bar{K}N, \pi\Sigma, \pi\Lambda)$ space is $\vec{v}' = (0, 0.72, 0.69)$ for P_{13}, and $\vec{v}' = (0, 0.62, 0.78)$ for D_{15}. Since $\vec{v}'$ is almost constant, the ambiguity will add to $\Delta_{ij}(s)$ a term approximately of the form

TABLE II. Typical values of the eigenstates of $\Delta_{ij}(s)$. The units are $\hbar = c = m_\pi = 1$.

State	Eigenvalues and eigenvectors $W = 1600$ MeV	$W = 1650$ MeV	$W = 1700$ MeV
		$10^2\lambda$ ($\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$)	
P_{13}	8.0 (0.43, −0.61, 0.66)	7.0 (0.42, −0.62, 0.66)	6.2 (0.41, −0.62, 0.66)
	0.58 (0.81, 0.59, 0.02)	0.47 (0.81, 0.59, 0.04)	0.37 (0.81, 0.59, 0.05)
	−0.01 (0.40, −0.53, −0.75)	−0.01 (0.41, −0.52, −0.75)	0 (0.42, −0.51, −0.75)
		$10^4\lambda$ ($\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$)	
D_{15}	11.2 (0.92, 0.31, −0.25)	6.9 (0.90, 0.25, −0.37)	4.8 (0.87, 0.19−0.45)
	4.0 (−0.01, 0.64, 0.77)	3.3 (0.12, 0.66, 0.74)	2.4 (0.22, 0.68, 0.70)
	−0.02 (0.40, −0.70, 0.59)	−0.01 (0.42, −0.71, 0.57)	0 (0.44, −0.71, 0.55)
		$10^2\lambda$ ($\bar{K}N$, $\pi\Sigma$)	
P_{03}	1.50 (0.81, 0.58)	1.17 (0.79, 0.60)	0.93 (0.78, 0.62)
	0.08 (0.58, −0.81)	0.08 (0.60, −0.79)	0.07 (0.62, −0.78)
		$10^4\lambda$ ($\bar{K}N$, $\pi\Sigma$)	
D_{03}	21.1 (0.39, 0.92)	16.8 (0.56, 0.83)	13.7 (0.64, 0.76)
	6.3 (0.92, −0.39)	6.3 (0.83, −0.56)	5.2 (0.76, −0.64)

$$D(s)\,\vec{v}'\times\vec{v}', \tag{6}$$

where $D(s)$ is some function of s.

Consider $\vec{u}^{(1)}$ the leading eigenvector of $\Delta_{ij}(s)$ for P_{13} (or D_{15}) in Table II [$\vec{u}^{(1)}(s)$ corresponds to the highest eigenvalue $\lambda_1(s)$.] It happens that

$$\vec{u}^{(1)}\cdot\vec{v}' \simeq 0, \tag{6$'$}$$

almost exactly for P_{13} and roughly for D_{15}. Since (as we shall see) the eigenvalues $\lambda_1(s)$ are large, Eq. (6′) implies that, on adding the term in Eq. (6) to $\Delta_{ij}(s)$, there will only be a small change in the highest eigenstates $[\lambda_1(s), \vec{u}^{(1)}(s)]$ for P_{13} and D_{15}. The other eigenstates of P_{13} and D_{15} may, however, be considerably altered by the ambiguity.

The eigenvalues $\lambda_k(s)$ of $\Delta_{ij}(s)$ vary slowly with s, and we quote in Table III the values at 1650 MeV (which should be free of end effects).

TABLE III. Eigenvalues $\lambda_k(s)$ of $\Delta_{ij}(s)$ at 1650 MeV (λ_2 and λ_3 for P_{13} and D_{15} may be subject to ambiguity). The units are $\hbar = c = m_\pi = 1$.

		λ_1	λ_2	
$I=0$	$P_{1/2}$	11.1×10^{-2}	2.5×10^{-2}	
	$P_{3/2}$	1.17×10^{-2}	0.08×10^{-2}	
	$D_{3/2}$	16.8×10^{-4}	6.3×10^{-4}	
	$D_{5/2}$	4.1×10^{-4}	-0.7×10^{-4}	
		λ_1	λ_2	λ_3
$I=1$	$P_{1/2}$	4.7×10^{-2}	0.61×10^{-2}	0.36×10^{-2}
	$P_{3/2}$	7.0×10^{-2}	0.5×10^{-2}	0
	$D_{3/2}$	13.6×10^{-4}	2.6×10^{-4}	1.1×10^{-4}
	$D_{5/2}$	6.9×10^{-4}	3.3×10^{-4}	0

Large eigenvalues of $\Delta_{ij}(s)$ and SU(3)

We need some measure of a strong attractive interaction. By comparing with the well known exchange forces in the πN system,[14] or by using the unitary limit,[15] we can say that, at 1650 MeV, $\lambda \gtrsim 4\times10^{-2}$ for a P wave and $\lambda \gtrsim 0.7\times10^{-3}$ for a D wave indicate strong attraction. (Units are $\hbar = c = m_\pi = 1$.)

The strongly attractive cases in Table II are

$$I=0;\quad P_{1/2}, D_{3/2}:\ \lambda_1 = 1.7\times10^{-3},\ \lambda_2 = 0.63\times10^{-3},$$

$$I=1;\quad P_{1/2}, P_{3/2}, D_{3/2}, D_{5/2}.$$

The second D_{03} eigenstate is included although it is a little under our criterion for strong attraction.

The values of the corresponding $\Delta_{ij}(s)$ eigenstates and the SU(3) partial waves they suggest are given in Table IV. For octets the state is $\gamma 8_d + \delta 8_f$, and

$$\delta/\gamma = \frac{3}{\sqrt{5}}\;\frac{\alpha}{1-\alpha}$$

defines the f/d ratio α.

The experimental analysis therefore shows clearly that strongly attractive Δ_{ij} eigenstates occur in the partial waves:

$$(1; D_{3/2}),\quad (10; P_{3/2}),$$
$$(8; P_{1/2}, \alpha\simeq 0.3),\quad (8; D_{3/2}, \alpha \simeq 0.7). \tag{7}$$

Looking also at D_{05} we see there is a moderately strong attractive discrepancy for $(8; D_{5/2}, \alpha \simeq -0.5)$.[16] No other eigenvalues are large. [Notice that the second D_{03} eigenvector $\vec{\mu}^{(2)}$ must be orthogonal to $\vec{\mu}^{(1)}$, which is approximately a singlet; if

$\vec{\mu}^{(2)}$ could not give an 8 (with a reasonable α) we would expect $\lambda_2(s)$ to be small.]

The SU(3) nature of the results is supported by the πN P-wave discrepancy analysis[17] which showed strong attraction in P_{11} and P_{33} [respectively, in $(8;P_{1/2})$ and $(10;P_{3/2})$]. Another comparison with experimental data[14] shows that there is a strong attractive discrepancy in πN D_{13} [i.e., $(8;D_{3/2})$].[18]

Agreement with theory

The states in Eq. (7) plus the $(8;D_{5/2})$ are precisely those in which the PB exchange calculations (Sec. III) give strong attractive interactions. Moreover the values of α for the octets agree with theory.

We have considered whether this good agreement with theory could somehow be produced spuriously by the nature of the data analysis of LMMO.[3] That seems very improbable. Except at resonances the eigenstates of $T_{ij}(s)$ have no obvious systematic properties, and the dominant eigenstates tend to rotate as s varies. Moreover the Λ and Σ poles, and $Y_1^*(1385)$, and even $\Lambda(1520)$ all lie below the range of LMMO's analysis, but they do concern the discrepancies. It is significant that both the $I=0$ and $I=1$ terms in the octets show up in the discrepancies.

We should remark that a large positive discrepancy extending over a considerable range of energy will produce correlations between $\text{Re}T_{ij}(s)$ and $\text{Im}T_{ij}(s)$ which are big and are very different from the case of zero discrepancy; it is not surprising that such large correlations can be clearly detected.

TABLE IV. SU(3) properties of the strong attractive discrepancy eigenstates $\vec{u}(s)$ (1650 MeV).

State $I=0$	Eigenvector $(\bar{K}N, \pi\Sigma)$	Indicated SU(3) state	Comments
$P_{1/2}$	(0.49, −0.87)	8 ($\alpha \simeq 0.15$)	Not 1, 27 poor fit
$D_{3/2}$	(0.56, 0.83)	1	Could be 8 ($\alpha \to \infty$), Not 27
$D_{3/2}$	(0.83, −0.56)	8 ($\alpha \simeq 0.5$)	Could be 27, Not 1
$I=1$	$(\bar{K}N, \pi\Sigma, \pi\Lambda)$		
$P_{1/2}$	(0.29, −0.85, −0.45)	8 ($\alpha \simeq 0.35$)	Not 10, 27; $\overline{10}$ poor fit
$P_{3/2}$	(0.42, −0.62, 0.66)	10	Not $\overline{10}$, 27, 8
$D_{3/2}$	(0.34, 0.94, 0.04)	8 ($\alpha \simeq 0.8$)	Not 10, $\overline{10}$, 27
$D_{5/2}$	(0.90, 0.25, −0.37)	8 ($\alpha \simeq -0.5$)	Not 10, $\overline{10}$, 27

III. EXCHANGE FORCES IN $PB \to PB$

The theory of exchange forces in PB systems has been discussed by various authors,[19] and Golowich[20] has come nearest to completeness. However, we need a more concise description, and above all we require the magnitudes of the different interactions, evaluated in a realistic fashion using modern information on the coupling constants, etc. Such calculations can only be done by using the reduced partial-wave amplitudes[21] [like $F_{ij}(s)$ above].

We use the PB exchange forces because the discrepancies $\Delta_{ij}(s)$ are evaluated over PB channels. This does not imply that PB exchanges give the whole dynamics at our energies. For example, it is known[5] that the PD $(0^-\frac{3}{2}^+)$ channel is also important for the $(8;D_{5/2})$ resonance.

We are dealing with the peripheral region where (for $l \geq 1$) only long- and medium-range interactions need be considered; these come from $V(1^-)$, $B(\frac{1}{2}^+)$, $D(\frac{3}{2}^+)$, and $\epsilon(0^+)$ exchanges. We assume that ϵ ($m_\epsilon \sim 500$–700 MeV) is mainly a singlet (or can effectively be treated as a singlet since its mass is low); we have evidence for this assumption. The numerous couplings needed for the calculations have been given (and sometimes determined) in Ref. 22.

The structure of the interactions (and so most of the low-energy PB behavior) will depend on two sets of coefficients:

(a) the representation ratios (analogous to the charge ratios in πN,[15] and
(b) the angular momenta ratios.

These tell us which exchanges are strong or weak, attractive or repulsive, in the physical states.

The representation ratios are given first for pure SU(3), while the angular momentum ratios come from integrations over the actual cuts.[22]

V exchange

The strength of V exchange in the various representations is given in Table V. Here $V(s)$, $T(s)$ are standard left-hand-cut terms; they vary slowly in the physical region of s.

The dependence of V and T on (J,l) is found in two independent ways using the results of Ref. 22:

(i) From $I=0$ and $I=1$ $\bar{K}N \to \bar{K}N$. This involves ω and ρ exchange, and the ρ coupling is derived from a $K\pi \to K\pi$ analysis, etc.

(ii) From $I=0$ (or 1) $\pi\Sigma \to \pi\Sigma$ and $I=1$ $\pi\Lambda \to \pi\Sigma$. Only ρ exchange occurs, and there are certain assumptions about the strengths of the $\rho\Sigma\Sigma$, $\rho\Sigma\Lambda$ couplings.[22]

The qualitative features from methods (i), (ii) agree. We write $l_\pm$ for $J=l\pm\frac{1}{2}$. For $l=1,2,3$ (P, D,F waves) the results are the following:

$T(l_-)$ = strong attractive

(being somewhat above the unitary limit used in Sec. II) and

$V(l_-)$ = medium attractive

(below the unitary limit).

Roughly, we have

$$V(l_-)/T(l_-)\simeq 0.4\,, \tag{8a}$$

$$T(l_+)\simeq -V(l_-)\,, \tag{8b}$$

$$V(l_+)/V(l_-)\simeq 0.4\,, \tag{8c}$$

$$V(l_+)+\tfrac{1}{4}T(l_+)=\text{small}\,. \tag{8d}$$

Octet example

The interaction in the octet state $(\gamma 8_d+\delta 8_f)$ is thus, for l_-,

$$3V(l_-)+(0.75+3.35\gamma\delta)T(l_-)\,. \tag{9a}$$

This is strongly attractive for $\gamma\simeq\delta\simeq 1/\sqrt{2}$, but is small for $\gamma\delta\simeq -\frac{1}{2}$. The interaction is approximately

$$3.35\gamma\delta T(l_+) \tag{9b}$$

for l_+. For $\gamma\delta\simeq -\frac{1}{2}$ this is fairly strongly attractive.

The large value of $G^T_{\rho NN}/G^V_{\rho NN}$ is closely related to the results in Eqs. (8a)–(9b).

B, D, and ϵ exchanges

The representation ratios of these exchanges [for pure SU(3)] are given in Table VI. For the PBB coupling we use $\alpha_{PBB}=\frac{1}{3}$. Here $B(s)$, $D(s)$, and $\epsilon(s)$ are standard left-hand-cut terms which vary slowly for physical s. The angular momentum properties are again based on practical calculations in Ref. 22.

TABLE V. V exchange in any partial wave according to pure SU(3).

Representation	Vector coupling $\alpha_V=1$	Tensor coupling $\alpha_T=0.25$
27	$-2V$	$-\frac{1}{2}T$
10	0	$-\frac{3}{2}T$
$\overline{10}$	0	$+\frac{3}{2}T$
$8_d\to 8_d$	$3V$	$\frac{3}{4}T$
$8_f\to 8_f$	$3V$	$\frac{3}{4}T$
$8_d\to 8_f$	0	$\frac{3}{4}\sqrt{5}\,T$
1	$6V$	$\frac{3}{2}T$

TABLE VI. B, D, and ϵ exchange in any state, according to pure SU(3).

Representation	B exchange	D exchange	ϵ exchange
27	0.35 B	0.33 D	ϵ
10	B	D	ϵ
$\overline{10}$	−0.2 B	D	ϵ
$8_d\to 8_d$	−0.53 B	$2D$	ϵ
$8_f\to 8_f$	−0.28 B	0	ϵ
$8_d\to 8_f$	0	2.4 D	0
1	0.55 B	−5 D	ϵ

For $l\geq 1$ these give the following rough rules

B exchange:

$$\text{odd } l\begin{cases} B(l_+)=\text{strong attractive}\\ B(l_-)\simeq -\dfrac{1}{2l}B(l_+)\,,\end{cases} \tag{10a}$$

$$\text{even } l,\ \text{reverse the sign of } B(l_\pm); \tag{10b}$$

D exchange:

$$\text{odd } l\begin{cases} 2D(l_-)=\text{medium attractive}\\ D(l_+)\simeq\frac{1}{4}D(l_-)\,;\end{cases} \tag{11a}$$

$$\text{even } l,\ \text{reverse the sign of } D(l_\pm) \tag{11b}$$

ϵ exchange:

$\epsilon(s)$ = medium attractive

$\epsilon(l_+)\simeq\epsilon(l_-)$.

The sign reversals in Eqs. (10b), (11b) are due to the space nature of the u-channel interactions.

Applications of the theory

Using the pure SU(3) coefficients in Tables V, VI we see unambiguously that there must be strong attractions in the states:

l_+, l odd; 10 (B exchange),

l_-; $8(\gamma\delta\simeq\frac{1}{2})$ (V exchange),

l_-, l even; 1 (V,B,D exchange).

The dominant exchanges are indicated. All other states have medium attraction, or less. The octet eigenstates are chosen to make the V exchange, Eq. (9a), maximum; $\gamma\delta\simeq\frac{1}{2}$ gives $\alpha\simeq\overline{\alpha}=0.43$. So the most attractive PB eigenstates are $(l=1,2)$

$$(10;P_{3/2}),\quad (8;P_{1/2}),\quad (8;D_{3/2}),\quad (1;D_{3/2}) \tag{12}$$

with $\alpha\simeq\overline{\alpha}$ for the octets. This agrees well with the strongly attractive eigenstates of $\Delta_{ij}(s)$ found in Sec. II.

However, $\Delta_{ij}(s)$ are evaluated over the "open" channels only, i.e., $(\bar{K}N, \pi\Sigma)$ for $I=0$ and $(\bar{K}N, \pi\Sigma, \pi\Lambda)$ for $I=1$, so $\Delta_{ij}(s)$ will not obey pure SU(3). Let $H_{ij}(s)$ be the exchange-force interaction matrix for pure SU(3), as we have just discussed, and let $(H_{ij}(s))_{\text{open}}$ be the interaction matrix over the "open" channels. In Ref. 6 and in detailed calculations in Ref. 22, $(H_{ij}(s))_{\text{open}}$ is evaluated, including reasonable symmetry breakings of the various couplings. It turns out that the largest positive eigenvalues $(\lambda_1(s))_{\text{open}}$ of $(H_{ij}(s))_{\text{open}}$ are somewhat less than the corresponding largest eigenvalues $\lambda_1(s)$ of $H_{ij}(s)$, but $(\lambda_1(s))_{\text{open}}$ are still strongly attractive. Let $(\vec{\omega}^{(1)})_{\text{open}}$ and $\vec{\omega}^{(1)}$ be the corresponding eigenvectors of $(H_{ij})_{\text{open}}$ and H_{ij}. By using a theorem on bordered matrices, it is shown in Ref. 6 that the projection of $\vec{\omega}^{(1)}$ in the space of the open channels is almost in the same direction as $(\vec{\omega}^{(1)})_{\text{open}}$. (The several conditions required for this result to hold are indeed obeyed by the physical system.)

This means that there is good agreement between the strongly attractive eigenstates of $\Delta_{ij}(s)$ [Eq. (7)] and those [Eq. (12)] of the calculated interaction matrix $(H_{ij}(s))_{\text{open}}$. We see below that there is also agreement for $(8;D_{5/2})$.

Mixing of representations

In going from $H_{ij}(s)$ to $(H_{ij}(s))_{\text{open}}$ five eigenstates are reduced to three for $I=1$, and four are reduced to two for $I=0$. We have just seen that when there is strong attraction $(\lambda_1(s))_{\text{open}}$ and $(\vec{\omega}^{(1)})_{\text{open}}$ are particularly related to $\lambda_1(s)$ and $\vec{\omega}^{(1)}$; Fig. 1 shows possible relations between the other eigenstates in that case. In general the second (and third) eigenstates of $(H_{ij}(s))_{\text{open}}$ will not show SU(3) features because of their mixed origin [Figs. 1(a), 1(b)]. However, in D_{03} both $\lambda_1(s)$ and $\lambda_2(s)$ are strongly attractive (and well separated) and a possible scheme is suggested in Fig. 1(c); both $(\vec{\omega}^{(1)})_{\text{open}}$ and $(\vec{\omega}^{(2)})_{\text{open}}$ can obey SU(3) because in $(\bar{K}N, \pi\Sigma)$ space the 1 and 8 ($\alpha \simeq 0.5$) vectors are nearly orthogonal.

$(8;D_{5/2})$ and 10

Using Table V, VI and Eqs. (9), (10) we see that $(8;D_{5/2})$ with $\gamma\delta \simeq -\frac{1}{2}$ would have medium/strong attraction. The detailed calculations in Ref. 22 show that this is also true in the open-channel matrix $(H_{ij})_{\text{open}}$. Also $\gamma=-\delta=1/\sqrt{2}$ gives $\alpha=-2.9$ (but we look closer at α below). For $I=1$ the open-channel branching ratios vary little for α between -0.5 and -3, so the PB exchange calculations are in good agreement with the highest eigenstate of $\Delta_{ij}(s)$ for $I=1, D_{5/2}$ (c.f. the last lines in Tables III, IV). However, the attraction in $(8;D_{5/2})$ is not

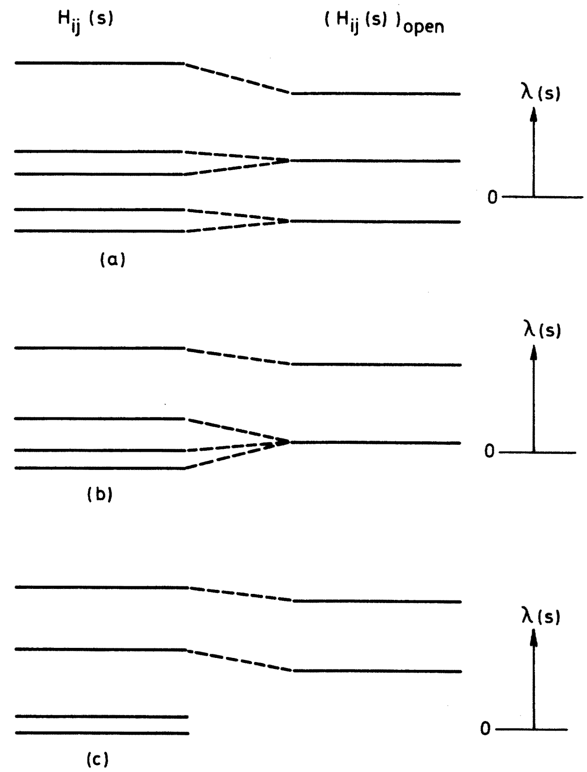

FIG. 1. Relation between eigenstates of the full PB system and the truncated systems. (a) Case of $I=1$, (b) one type of $I=0$, (c) type for $I=0$, D_{03}.

especially strong, so the methods of Ref. 6 will not work so well, and indeed SU(3) breaking is noticeable in $I=0, D_{5/2}$; the highest eigenvalue of $\Delta_{ij}(s)$ is too small (Table III) and the eigenvector is $(8; \alpha \simeq +0.2)$.

The other medium strong attraction expected from Tables V, VI and Eqs. (8)–(11) is in $(\overline{10};P_{1/2})$. But this cannot show up in our data because the dominant attraction in $Y=0$ P_{11} is the $(8;P_{1/2})$, and the projections in $(\bar{K}N, \pi\Sigma, \pi\Lambda)$ space of $\overline{10}$ and $(8; \alpha \simeq 0.3)$ are far from orthogonal. Therefore the symmetry breaking (or symmetry truncation) on going to the open channels [Fig. 1(a)] must mix $\overline{10}$ with other representations, and will necessarily suppress the second eigenvalue $\lambda_2(s)$ of $\Delta_{ij}(s)$ for P_{11}. Table III confirms this.

In the discrepancy analysis for $Y=+1$,[17] only $\pi N \to \pi N$ was used, so $\overline{10}$ cannot be found there. However, the analysis of $I=0$ KN scattering shows a pronounced feature in P_{01} (Refs. 23, 24) around 1800 MeV (Fig. 2); this does not appear in the $T=1$ amplitude P_{11}, so the phenomenon must be in $\overline{10}$. This behavior is consistent with a medium-strength attraction in $\overline{10}$, as predicted by theory.

f/d ratios for the octets

Remembering the dominance of the V and B exchanges, Tables V, VI show that the *sign* of δ/γ is determined by the V-exchange $8_d \to 8_f$ term $(3\sqrt{5}/4)T$; this gives $\delta/\gamma > 0$ for l_- octets, and $\delta/\gamma < 0$ for l_+. The B (and D) exchanges will modify the crude values of α given above, which came from using Eqs. (9a), (9b). [We must arrange that $\lambda_1(s)$ is maximum.]

B exchange is greatest for $\delta = 0$ and it is attractive in $(8; P_{1/2})$ and repulsive in $(8; D_{3/2})$, (but it is not large). So we get $\alpha < \bar{\alpha} = 0.43$ for $(8; P_{1/2})$ and $\alpha > \bar{\alpha}$ for $(8; D_{3/2})$. D exchange tends to give the same effects. In $(8; D_{5/2})$ B exchange is strongly attractive, so α moves from -3.0 towards 0; D exchange is small here.

The f/d ratios of the leading Δ eigenstates should be close to the f/d ratios given by the measured branching ratios of Λ, Σ and of the $(8; D_{3/2})$ resonances. There should presumably be similar close agreement between the measured f/d values and the value from $\Delta(s)$ for the $(8; D_{5/2})$ resonances, even though we believe that there is considerable coupling to PD and other channels.[5] The experimental values from the resonances,[25] the best values for Δ eigenstates (Tables IV), and the SU(6) resonance predictions are given in Table VII.

IV. OTHER PROPERTIES

SU(3) repulsions

The pure SU(3) scheme in Tables V, VI predicts moderate repulsions in the PB states (a) $(10; P_{1/2})$, (b) $(27; P_{1/2})$, (c) $(8_2; P_{3/2})$, (d) $(27; D_{3/2})$, (e) $(10; D_{5/2})$. (Here 8_2 indicates the octet with lowest eigenvalue.) Each, except (b), is the most repulsive eigenstate of $H_{ij}(s)$ for its partial wave.

We see [(a) + (b)] in the low-energy behavior of the πN amplitude P_{31},[14] while (b) and (d) are seen in the low-energy behavior of the $\bar{K}N$ amplitudes P_{11}, D_{13}.[24] However, in the case $Y=0$ the list of eigenvalues of $\Delta_{ij}(s)$ in Table III shows none negative (within errors). The practical calculations of Ref. 22 over the open PB channels show no repulsion in the $I=0$ states but in the $I=1$ states $(H_{ij}(s))_{\text{open}}$ has repulsive eigenvalues, P_{11} and D_{15} being clear cases.

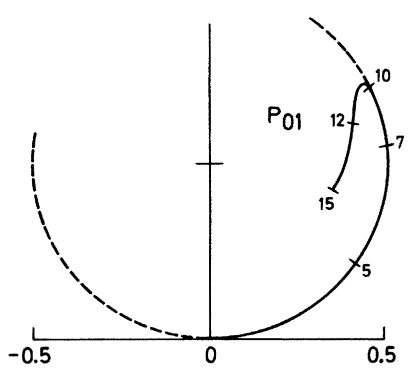

FIG. 2. A typical Argand diagram for the $\bar{K}N$ partial wave P_{01} (Ref. 23).

TABLE VII. f/d values of resonance and discrepancy octets.

	$(8; P_{1/2})$	$(8; D_{3/2})$	$(8; D_{5/2})$
Resonances (expt. values)	0.35	0.7	−0.15
Eigenstates of $\Delta_{ij}(s)$ (Table IV)	0.3	0.65	−0.5
SU(6) prediction	0.4	0.62	−0.5

In Sec. II a possible ambiguity in the $D_{5/2}$ $(I=1)$ discrepancy was discussed. However, the vector $\vec{v}'$ of Eq. (6) is very different from the 10 representation, so this cannot be the cause of losing the repulsion.

There may be various explanations for this lack of SU(3) structure, but it is clear that some intrinsic symmetry breaking is present here, over and above the truncation effects caused by going from $H_{ij}(s)$ to $(H_{ij}(s))_{\text{open}}$. This is in sharp contrast to the strongly attractive eigenstates of $\Delta_{ij}(s)$ which obey SU(3) well.

Eigenstates of $F_{ij}(s)$

We have examined the eigenstates of the reduced partial-wave amplitudes $F_{ij}(s)$ in the energy region (1.5 to 1.9 GeV) of LMMO's analysis. Away from the resonance energies these eigenstates generally do not show any simple SU(3) properties.

Analysis of data

Our work suggests improvements in the future analysis of experimental data. The importance of null states should be realized, especially in connection with the possible ambiguity in the unseen amplitudes.

The success of the LMMO analysis[3] depends on having only a few parameters in $K(s)$, Eq. (2). A more sophisticated data analysis would require greater flexibility in parametrizing $K(s)$, and the increase in parameters might well lead to difficulties. We suggest that some theoretical constraints should be used.

The minimum theory is implied if one constrains the solutions of the data analysis by requiring that the discrepancies $\Delta_{ij}(s)$ [Eq. (5)] should have slowly rotating eigenvectors $\vec{u}^{(k)}(s)$ in the physical region. Remembering that there is no trouble about unseen amplitudes at, or near, a resonance (unless it is completely decoupled from $\bar{K}N$) the requirement of slow rotation of the $\vec{u}^{(k)}(s)$ will spread the stabilizing effect of the resonance to other energies.

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