D^{*+}-D^{*0} mass difference*

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Born-term and first-intermediate-state contributions to the D^{*+}-D^{*0} mass splitting are calculated using pole-dominated form factors. A self-coupled Lagrangian for the interactions of the vector mesons is introduced and yields a magnetic-type γVV coupling, with unit anomalous magnetic moment κ for the charged vector mesons. Using a tadpole term determined from the K^{*+}-K^{*0} mass difference, we find $m_{D^{*+}} - m_{D^{*0}} = 4.8$ MeV, in close agreement with the value 5.8 MeV previously found for the D^+-D^0 splitting. Results for the ρ and K^* are also presented.

I. INTRODUCTION

Recently, several authors have discussed the electromagnetic mass splittings of the charmed mesons discovered in experiments at SPEAR.[1] The interest in this problem has been motivated by the peculiar fact[2] that, if the peak at 2.00 GeV in the mass spectrum of the particles recoiling against the $D(1865)$ is taken to be the charmed vector meson D^*, then the D^*-D mass splitting is almost exactly equal to the pion mass m_π. The D and D^* electromagnetic mass differences then become crucial for determining the relative amounts of charged and neutral D's in the final state resulting from the decay $D^* \to D\pi$. Taking a more general point of view, electromagnetic mass differences should provide important clues towards an understanding of hadron dynamics.

Analyses of the D^+-D^0 mass splitting based on the charmonium model yield predictions[2-7] anywhere between 1 and 15 MeV. The difficulty here lies in the ambiguity of the nonrelativistic potential to be employed. An alternative approach is to use the Cottingham formula.[8] The Born-term and first-intermediate-state contributions to the electromagnetic self-mass can be calculated using pole-dominated form factors and SU(4) symmetry. The remaining continuum contributions are summarized in terms of a tadpole term[9] which transforms like the third component of isospin and which is determined from $\Delta m_K \equiv m_{K^+} - m_{K^0}$. This method has been applied[10] to $D(1865)$, with the result $\Delta m_D = 5.8$ MeV. As a by-product, the form factors can be used to determine cross sections for production of pairs of charmed mesons in e^+e^- annihilation.

In the present work, we apply the approach of Ref. 10 to the problem of the D^{*+}-D^{*0} mass splitting. Since the most general $\gamma D^* D^*$ coupling involves three independent form factors, calculation of the Born term requires stronger assumptions than are necessary in the case of the D. For definiteness and simplicity, we assume in what follows that the D^* possesses no quadrupole-type coupling to the photon. This is further motivated in Sec. II, where a self-coupled Lagrangian for the strong interactions of the vector mesons is shown to provide a magnetic-type coupling with unit anomalous magnetic moment. The off-shell dependence of the form factors is given by vector-meson dominance, and is the same for both electric and magnetic form factors. In Sec. III we obtain the Born contribution to the self-mass of a vector meson with arbitrary anomalous magnetic moment κ, and discuss the conditions necessary for convergence of the resulting integrals. We find that when $\kappa \neq 0$, the form factor must fall off faster than $(q^2)^{-1}$ at large q^2. Taking $\kappa = 0$, and using pole-dominated form factors, we calculate the Born contribution to the D^* and K^* electromagnetic mass splittings. The D contribution to the D^* mass splitting is calculated using an SU(4)-symmetry-breaking mechanism proposed in Ref. 10, and the corresponding results are obtained for the K^*. The tadpole term is then determined from the K^* mass splitting, with the result $m_{D^{*+}} - m_{D^{*0}} = 5.99$ MeV. In Sec. IV, we deal with the divergence problems arising in the case $\kappa \neq 0$. A "radially excited" multiplet of vector mesons V' is introduced and assumed to couple to the D^* and K^* in an SU(4)-invariant way. The relative coupling of the V and V' multiplets to the D^* and K^* is then fixed by requiring that the electromagnetic mass splittings be finite. This approach has the advantage of cutting off the integral at large q^2 while retaining the pole-dominance picture. We find that, for $\kappa = 0$, the introduction of the V' multiplet lowers Δm_{D^*} from 5.99 MeV to 5.09 MeV. The κ-dependent terms appear with small coefficients, and for the "canonical" value $\kappa = 1$ suggested by the self-coupled Lagrangian we find that $\Delta m_{D^*} = 4.81$ MeV. Our conclusions are given in Sec. V.

II. γVV COUPLING

To carry out a calculation of Δm_{D^*} in the spirit of Ref. 10, one must specify the $\gamma D^* D^*$ vertex. In the following, we shall assume that the electromagnetic current matrix element is given by[11]

$$\langle V_i(q_2)|J_\mu^\gamma(0)|V_i(q_1)\rangle = \frac{e}{(2\pi)^3} F_i(q^2)[(\epsilon_1\cdot\epsilon_2)(q_1+q_2)_\mu - (-\kappa q_1 + q_2 + \kappa q_2)\cdot\epsilon_1\epsilon_{2\mu} - (-\kappa q_2 + q_1 + \kappa q_1)\cdot\epsilon_2\epsilon_{1\mu}]. \quad (2.1)$$

Here V_i ($i=0,\ldots,15$) are assumed to form a U(4) 16-plet of vector mesons, and $\kappa F_i(0)$ is the anomalous magnetic moment of V_i. A form such as (2.1) in which the electric and magnetic form factors have the same q^2 dependence is natural in a vector-dominance picture.

Since there are no data on magnetic moments of vector mesons, we can determine κ only by choosing a particular model for the γVV vertex. For this purpose, and to further motivate the form (2.1), we assume the following self-coupled Lagrangian for the strong interactions of the 16-plet of vector mesons V_i ($i=0,\ldots,15$):

$$\mathcal{L}_{st} = -\tfrac{1}{4} G_{\mu\nu}^i G_i^{\mu\nu} + \tfrac{1}{2} m^2 \phi_\mu^i \phi_i^\mu. \quad (2.2)$$

Here m is the vector-meson mass, and

$$G_{\mu\nu}^i = \partial_\mu \phi_\nu^i - \partial_\nu \phi_\mu^i + g f_{ijk} \phi_\mu^j \phi_\nu^k, \quad (2.3)$$

where g is a coupling constant. The Lagrangian (2.2) is invariant under the space-time-independent SU(4) transformation

$$\phi_\mu^i \to \phi_\mu^i + f_{ijk}\epsilon_j \phi_\mu^k. \quad (2.4)$$

Varying (2.2) with respect to ϕ_μ^i produces the Euler-Lagrange equations

$$(\Box + m^2)\phi_\mu^i = g J_\mu^i \quad (2.5)$$

and the subsidiary condition

$$\partial^\mu \phi_\mu^i = 0. \quad (2.6)$$

The conserved current J_μ^i is given by

$$J_\mu^i = f_{ijk}[\partial^\nu(\phi_\mu^j \phi_\nu^k) + (\partial^\nu \phi_\mu^j - \partial_\mu \phi_j^\nu)\phi_\nu^k] + g f_{ilj} f_{lkm} \phi_\mu^k \phi_\nu^j \phi_m^\nu. \quad (2.7)$$

We now impose the vector-dominance condition[12]

$$\langle V_i(q_2)|J_\mu^{\gamma j}(0)|V_k(q_1)\rangle = \frac{m^2}{g}\langle V_i(q_2)|\phi_\mu^j(0)|V_k(q_1)\rangle, \quad (2.8)$$

where the electromagnetic current is given by

$$J_\mu^\gamma = J_\mu^{\gamma 3} + \frac{1}{\sqrt{3}} J_\mu^{\gamma 8} - \left(\frac{2}{3}\right)^{1/2} J_\mu^{\gamma 15} + \frac{\sqrt{2}}{3} J_\mu^{\gamma 0}. \quad (2.9)$$

Using (2.5), we get

$$\langle V_i(q_2)|J_\mu^{\gamma j}(0)|V_k(q_1)\rangle = \frac{m^2}{m^2-q^2}\langle V_i(q_2)|J_\mu^j(0)|V_k(q_1)\rangle. \quad (2.10)$$

By substituting (2.7) in (2.10), we find that the electromagnetic current matrix element is of the form (2.1) with $\kappa=1$. The value $\kappa=1$ is a consequence of the trilinear coupling in (2.2), and the SU(4) invariance of the VVV vertex is guaranteed. It is intriguing to note that if the μe events[13] in e^+e^- annihilation at SPEAR are assumed to be due to the production and decay of a pair of heavy vector mesons, then $\kappa=1$ provides[14] reasonable agreement with data on the energy spectrum and collinearity angle of the μe pair.

III. RESULTS FOR $\kappa = 0$

In this section we write down the expressions for the Born-term and first-intermediate-state contributions to the vector-meson mass splittings, and use pole-dominated form factors to evaluate them. Using the general magnetic-moment-type coupling (2.1), the Born term for the electromagnetic self-mass of the vector meson V is given by

$$\delta m = \frac{\alpha}{48\pi m}\int_0^\infty dq^2 [F(-q^2)]^2 \{3(1+6\kappa)y + \tfrac{1}{2}(2+6\kappa-\kappa^2)y^2 - \tfrac{1}{4}\kappa^2 y^3 + [12 - (1+12\kappa+4\kappa^2)y - (1+3\kappa)y^2 + \tfrac{1}{4}\kappa^2 y^3](1+4/y)^{1/2}\}, \quad (3.1)$$

where m is the vector-meson mass and $y \equiv q^2/m^2$. If $\kappa=0$, the integral converges provided that

$$F(q^2) \underset{q^2\to\infty}{=} O((q^2)^{-\alpha}), \quad \alpha > \tfrac{1}{2}; \quad (3.2)$$

however, when $\kappa \neq 0$ we require

$$F(q^2) \underset{q^2\to\infty}{=} O((q^2)^{-\alpha}), \quad \alpha > 1 \quad (3.3)$$

for convergence. Taking a spectral representation[15] for the form factor,

$$F(q^2) = \int_0^\infty dc\, \rho(c) \frac{cm^2}{cm^2 - q^2}, \quad (3.4)$$

we see that (3.3) is satisfied if

$$\int_0^\infty c\rho(c)\, dc = 0. \quad (3.5)$$

Substituting (3.4) in (3.1) we obtain for the Born term

$$\delta m = \frac{\alpha m}{48\pi}\int_0^\infty dc_1 \int_0^\infty dc_2\, \rho(c_1)\rho(c_2)\, \frac{c_1 c_2}{c_2 - c_1} \times\{f(c_2)-f(c_1)+\kappa[f_1(c_2)-f_1(c_1)] + \kappa^2[f_2(c_2)-f_2(c_1)]\}, \quad (3.6)$$

where

$$f(c) = c(c-3)\ln c - 2c - (c+3)(4-c)^2 w(c),$$
$$f_1(c) = -\tfrac{3}{4}c[(6-c)\ln c + 26 - 3c + (4-c)^2 w(c)],$$
$$f_2(c) = -\tfrac{1}{8}c[\tfrac{1}{2}c(2-c)\ln c + 9\ln 2 + \tfrac{977}{60} + c + \tfrac{1}{2}(4+c)(4-c)^2 w(c)], \quad (3.7)$$

with

$$w(c) = \int_0^1 dx\,(x^2 - cx + c)^{-1}. \quad (3.8)$$

In a vector-dominance picture for $F(q^2)$, the natural asymptotic behavior is $F(q^2)\propto(q^2)^{-1}$ at large q^2. According to (3.3), this means that the self-mass diverges unless $\kappa=0$. Therefore, the easiest way of obtaining a finite self-mass is to set $\kappa=0$ with a form factor dominated by a few low-lying vector mesons. We first discuss this case, and return to the general case $\kappa\neq 0$ in Sec. IV.

Assuming SU(4) symmetry for the 16-plet couplings and ideal ω-ϕ mixing we get[10]

$$\rho_{D^*}(c) = \tfrac{1}{2}[\pm\delta(c-c_\rho) - \tfrac{1}{3}\delta(c-c_\omega) + \tfrac{4}{3}\delta(c-c_\psi)], \quad (3.9)$$

where the upper (lower) sign refers to the D^{*+} (D^{*0}) and $c_i \equiv (m_i/m_{D^*})^2$. The Born-term contribution to the D^* mass splitting is then given by

$$(\Delta m_{D^*})_{\rm Born} = \frac{m_{D^*}\alpha c_\rho}{48\pi}\left(-\frac{1}{3}\frac{c_\omega}{c_\omega - c_\rho}[f(c_\omega)-f(c_\rho)] + \frac{4}{3}\frac{c_\psi}{c_\psi - c_\rho}[f(c_\psi)-f(c_\rho)]\right)$$
$$= 2.67\ \text{MeV}. \quad (3.10)$$

This may be compared with the value 2.91 MeV obtained[10] for the Born term for Δm_D.

To estimate the D intermediate-state contribution to Δm_{D^*}, we write the current matrix element as

$$\langle P_j(q_2)|J_\mu^\gamma(0)|V_i(q_1)\rangle = \frac{g_{PV}e^2}{(2\pi)^3}F_{ji}(q^2)\epsilon_{\lambda\mu\nu\rho}\epsilon_1^\lambda q_1^\nu q_2^\rho, \quad (3.11)$$

where P_j ($j=0,\ldots,15$) are a U(14) 16-plet of pseudoscalar mesons. The coupling constant g_{PV} has dimensions of $(\text{mass})^{-1}$, and we have previously suggested[10] that it can be written as

$$g_{PV} = f_{PV}/m_V, \quad (3.12)$$

where f_{PV} is an SU(4)-symmetric, dimensionless constant, and m_V is the *produced* vector-meson mass. Note that (3.12) corresponds to SU(4) breaking only in the coupling constant and not in the form factor, as is suggested by the strong anomaly framework.[10,16] The constant f_{PV} can be determined from the decay $\omega^-\to\pi^-\gamma$, with the result $f_{PV}=2.03$. The form factor $F_{ji}(q^2)$ in (3.11) is normalized at $q^2=0$ in terms of SU(4) Clebsch-Gordan coefficients. Taking a spectral representation (3.4) for F_{ji}, the P intermediate-state contribution to the self-mass of V is given by[15,17]

$$\delta m = -\frac{f_{PV}^2 m\alpha}{48\pi}\int_0^\infty dc_1\int_0^\infty dc_2\,\rho(c_1)\rho(c_2)\,\frac{c_1c_2}{c_2-c_1}\times[U(b,c_2)-U(b,c_1)], \quad (3.13)$$

where

$$2cU(b,c) = 2c^2 + (b-c)[6c-(b-c)^2]\ln[(1+b)/c] + 2b^3\ln(1+b^{-1}) + [4c-(b-c)^2]^2 w(b,c), \quad (3.14)$$

$$w(b,c) = \int_0^1 dx\,(x^2+bx-cx+c)^{-1}, \quad (3.15)$$

and

$$b = m_P^2/m^2 - 1. \quad (3.16)$$

Here m is the vector-meson mass and m_P is the pseudoscalar-meson mass. Again assuming ideal ω-ϕ mixing, we find that the spectral function for the D^* is given by

$$\rho_{DD^*}(c) = \tfrac{1}{2}[\pm\delta(c-c_\rho) - \tfrac{1}{3}\delta(c-c_\omega) - \tfrac{4}{3}\delta(c-c_\psi)]. \quad (3.17)$$

Substituting (3.17) in (3.13), we find that the D intermediate-state contribution to the D^* mass difference is given by

$$(\Delta m_{D^*})_D = \frac{m_\rho^2\alpha f_{PV}^2}{144\pi m_{D^*}}\left(\frac{c_\omega}{c_\omega-c_\rho}[U(b,c_\omega)-U(b,c_\rho)] + \frac{4c_\psi}{c_\psi-c_\rho}[U(b,c_\psi)-U(b,c_\rho)]\right) + (\text{imaginary part})$$
$$= 0.57\ \text{MeV}. \quad (3.18)$$

The imaginary part in (3.18) simply reflects the fact that D is lighter than D^*, so that D^* can undergo the decay $D^*\to D\gamma$. In fact, using (3.13) we find the expected result that for D^{*+}

$$\delta m = \text{Re}\,\delta m - \tfrac{1}{2}i\Gamma(D^{*+}\to D^+\gamma), \quad (3.19)$$

where

$$\Gamma(D^{*+}\to D^{+}\gamma)=\left(\frac{g_{PV}}{3}\right)^3\left(\frac{\alpha}{24}\right)\left(\frac{m_{D^*}{}^2-m_D{}^2}{m_{D^*}}\right)^3 \qquad (3.20)$$
$$=0.61 \text{ keV}.$$

Similar calculations may be carried out for the $K^{*+}-K^{*0}$ mass difference. The spectral functions corresponding to (3.9) and (3.17) are

$$\rho_{K^*}(c)=\tfrac{1}{2}[\pm\delta(c-c_\rho)+\tfrac{1}{3}\delta(c-c_\omega)+\tfrac{2}{3}\delta(c-c_\phi)] \quad (3.21)$$

and

$$\rho_{KK^*}(c)=\tfrac{1}{2}[\pm\delta(c-c_\rho)+\tfrac{1}{3}\delta(c-c_\omega)-\tfrac{2}{3}\delta(c-c_\phi)], \quad (3.22)$$

respectively. Using these, we find

$$(\Delta m_{K^*})_{\rm Born}=1.71 \text{ MeV},$$
$$(\Delta m_{K^*})_K=0.34 \text{ MeV}. \qquad (3.23)$$

These may be compared with the values 2.19 MeV and 0.26 MeV, respectively, obtained[10] for Δm_K. That the K intermediate-state contribution in (3.23) is relatively large is due to the small K mass. A result analogous to (3.19) holds for the K^*, with $\Gamma(K^{*+}\to K^+\gamma)=41.6$ keV.

Using the observed[18] $\Delta m_{K^*}=-4.1\pm0.6$ MeV and assuming that the tadpole transforms like the third component of isospin for *quadratic* masses, we find 2.75 MeV for the tadpole contribution to Δm_{D^*}. Adding the Born, D intermediate state, and tadpole terms gives $\Delta m_{D^*}=5.99$ MeV, compared with 5.8 MeV for Δm_D found previously.[10] These results are summarized in the second column of Table I.

IV. RESULTS FOR $\kappa\neq 0$

In this section, we return to the general case $\kappa\neq0$. We impose the condition that electromagnetic mass differences must be finite, although unobservable *self-masses* may or may not diverge.[19] In terms of the spectral function this means that, for example, for the D^*

$$\int_0^\infty c[\rho^+(c)-\rho^0(c)]dc=0. \qquad (4.1)$$

Any modification of (3.9) satisfying (4.1) must be somewhat arbitrary. However, we notice that (3.9) almost satisfies (4.1) in the sense that at large q^2 the isoscalar part is much larger than the isovector part owing to the large ψ mass. This suggests that the isovector contribution can be cut off at large q^2 without seriously damaging the low-q^2 behavior of $F_{D^*}(q^2)$. To do so within the pole-dominance framework, additional vector-meson poles must be included in the form factor. A natural way of doing this is to introduce another 16-plet of vector mesons V', with $I_3=Y=C=0$ members ρ', ω', ϕ', and ψ'. There is evidence[18] for resonance structure in e^+e^- annihilation at $E_{\rm c.m.}=1600$ MeV, which we identify with the ρ', and for ψ' we take $\psi'(3684)$. The ω' and ϕ' have not yet been observed, possibly because they have large widths. For definiteness, we take $m_{\omega'}=1.7$ GeV and $m_{\phi'}=2.0$ GeV, although our results will not be sensitive to these masses. Assuming SU(4) invariance of the couplings, the V' contribution to $\rho_{D^*}(c)$ will have the same form as (3.9), within a normalization factor. Requiring the form factor to be properly normalized and to satisfy (4.1) fixes the relative normalization of the V and V' contributions and yields the result

$$\rho_{D^*}(c)=\frac{1}{2(c_{\rho'}-c_\rho)}\{c_{\rho'}[\pm\delta(c-c_\rho)-\tfrac{1}{3}\delta(c-c_\omega)+\tfrac{4}{3}\delta(c-c_\psi)]-c_\rho[\pm\delta(c-c_{\rho'})-\tfrac{1}{3}\delta(c-c_{\omega'})+\tfrac{4}{3}\delta(c-c_{\psi'})]\}. \qquad (4.2)$$

The V' multiplet is coupled to $D^*\bar{D}^*$ with about $\frac{1}{4}$ the strength of the V multiplet.

Substituting (4.2) in (3.6) gives

$$(\Delta m_{D^*})_{\rm Born}=(2.00-0.19\kappa-0.04\kappa^2) \text{ MeV}. \quad (4.3)$$

For $\kappa=0$, the value 2.00 MeV is not substantially different from the 2.67 MeV obtained by using (3.9), which suggests that our method of securing a finite mass difference preserves the main features of (3.9). Notice that the κ-dependent terms in (4.3) are negative; this is a general feature of a magnetic-moment coupling.

Using a spectral function analogous to (4.2) for the K^*, we find that

$$(\Delta m_{K^*})_{\rm Born}=(1.20-0.08\kappa-0.04\kappa^2) \text{ MeV}. \quad (4.4)$$

Taking $\kappa=0$, the tadpole contribution to Δm_{D^*} is found to be 2.52 MeV, giving 5.09 MeV for Δm_{D^*}. These results appear in the third column of Table I.

TABLE I. Contributions to the K^*, D^*, and ρ electromagnetic mass splittings in MeV.

	$\kappa=0$ (V)	$\kappa=0$ ($V+V'$)	$\kappa=1$ ($V+V'$)
$(\Delta m_{K^*})_{\rm Born}$	1.71	1.20	1.08
$(\Delta m_{K^*})_K$	0.34	0.34	0.34
$(\Delta m_{D^*})_{\rm Born}$	2.67	2.00	1.77
$(\Delta m_{D^*})_D$	0.57	0.57	0.57
$(\Delta m_{D^*})_{\rm tad}$	2.75	2.52	2.47
Δm_{D^*}	5.99	5.09	4.81
$(\Delta m_\rho)_{\rm Born}$	1.52	3.43	3.36
$(\Delta m_\rho)_\eta$	0.25	0.25	0.25
Δm_ρ	1.77	3.68	3.61

Taking $\kappa = 1$ in (4.3) and (4.4), as suggested by the self-coupled Lagrangian of Sec. II, yields the results shown in Table I. We see that the introduction of a nonzero anomalous magnetic moment reduces the mass differences, but not drastically.

We can also apply these ideas to the ρ^+-ρ^0 mass difference. The ρ^0 Born term vanishes, and the form factor for the ρ^+ Born term is dominated by the ρ and ρ' only:

$$\rho_{\rho^+}(c) = \frac{1}{c_{\rho'} - c_\rho}\left[c_{\rho'}\,\delta(c - c_\rho) - c_\rho \delta(c - c_{\rho'})\right]. \quad (4.5)$$

For arbitrary κ we find

$$(\Delta m_\rho)_{\rm Born} = (3.43 - 0.06\kappa - 0.01\kappa^2)\ {\rm MeV}. \quad (4.6)$$

The π intermediate-state contribution is identical for ρ^+ and ρ^0, and so does not contribute to the mass difference. The only other light intermediate state is η, which does not contribute to ρ^+ and lowers the ρ^0 self-mass. The tadpole term vanishes. Our results for the ρ are summarized in Table I. Notice that the effect of adding the ρ' to the ρ is quite drastic, in contrast to the situation for the K^* and D^*. The reason is that the latter mass differences involve the product of the isoscalar and the isovector form factors, so the ρ and ρ' contributions tend to cancel. The ρ mass difference involves the square of the isovector form factor, so the ρ and ρ' contributions add. Based on experimental data, Δm_ρ is quoted[18,20] as (-2.4 ± 2.1) MeV, although the quality of the data suggests that this value could change substantially.

V. CONCLUSIONS

In this paper, we have performed a detailed analysis of the ρ, K^*, and D^* electromagnetic mass differences by calculating the Born-term, first intermediate-state, and tadpole contributions. Our approach is similar to that applied in Ref. 10 to the problem of the D^+-D^0 mass difference. However, calculation of the Born term is complicated in the spin-1 case by the need to take into account possible magnetic and quadrupole form factors, and by the attendant convergence problems of the resulting integrals. To deal with the latter problem, we have introduced a "radially excited" multiplet of vector mesons V', which couples to the K^* and D^* with about $\frac{1}{4}$ the strength of the V multiplet. Presumably, the V' multiplet should also contribute to the D mass difference, although $V'PP$ couplings are expected[10,21] to be small.

The main conclusion to be drawn from our work is that, for the K^* and the D^*, the Born and first intermediate-state contributions do not differ greatly from the corresponding quantities for K and D. In all cases, terms proportional to κ and κ^2 have small coefficients. Finally, we find $\Delta m_{D^*} \approx 5$ MeV, which is consistent[22] with the value expected[10] for Δm_D.

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