PHYSICAL REVIEW D VOLUME 16, NUMBER 7 1 OCTOBER 1977

# Parity violation in the decay $\Sigma^0 \to \Lambda + e^+ + e^-$

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(Received 23 June 1977)

Neutral weak currents should contribute to the rate for $\Sigma^0 \to \Lambda + e^+ + e^-$; in particular, they should make themselves known as a parity violation in the decay spectrum. The branching ratio for parity violation becomes infinite as the $\Lambda$ momentum, $p_\Lambda$, approaches its kinematic limit, $\Delta = M_\Sigma - M_\Lambda$, roughly as $10^{-6}(\Delta/p_\Lambda)$. Neutral weak currents have never been detected in neutrinoless processes. Although a signal as small as $10^{-6}(\Delta/p_\Lambda)$ is beyond the reach of present-day experiments, recent experiments on atomic spectra indicate that standard models in the electron sector, at least, may be wrong. Finding no effect in $\Sigma^0$ decay of the order we predict could confirm this conclusion.

## I. INTRODUCTION

Weak neutral currents have been studied for several years now, and the strangeness-conserving ones have been shown to exist in many experiments. Very little is known about them, however, in the purely electron sector. In fact, almost nothing is known about neutral weak currents not involving neutrinos. In this note we calculate some parity-violating effects which must be found in the decay $\Sigma^0 \to \Lambda + e^+ + e^-$ if anything like the standard $V-A$ models are correct. We calculate the effect in a fairly model-independent way, isolating those factors which are likely to be of order unity. We give examples of these numbers in the Weinberg-Salam model.[1] We present the notation and basic steps in some detail, in part to clear up some confusion in the literature, and in part so that the reader can apply the method to related effects without having to start all over again.

## II. NOTATION AND KINEMATICS

We shall use established notation when possible, following most closely that of Alff *et al.*[2] The $\Sigma^0$, $\Lambda$, and electron masses will be denoted by $M_\Sigma$, $M_\Lambda$, and $m$, respectively, and the four-momenta denoted by $p'$, $p$, $p_-$, and $p_+$. Define

$$M = \tfrac{1}{2}(M_\Sigma + M_\Lambda), \quad \Delta = M_\Sigma - M_\Lambda. \tag{2.1}$$

The four-momentum transferred to the electron-positron pair is

$$q = p_+ + p_- = p' - p. \tag{2.2}$$

We are looking for a parity-violating correlation of the form $\vec{\sigma}\cdot\vec{K}$, where $\vec{K}$ is one of the final momenta and $\vec{\sigma}$ is the $\Lambda$ spin matrix. To do the calculation covariantly, it is convenient to introduce the spin four-vector $s = (s^0, \vec{s})$ such that $\hat{s} = \vec{s}/|\vec{s}|$ is in the direction of the $\Lambda$ spin, and $s$ is constrained by $s\cdot s = -1$, $s\cdot p = 0$. Then a measure of parity violation is obtained by comparing the rate for $\hat{s}$ in two opposite directions.

We shall do the calculation in the $\Sigma^0$ rest frame, averaging over the $\Sigma^0$ spins. In an actual experiment, the $\Sigma^0$ will be polarized and one will have to do a more complicated computation; but we shall get the order of magnitude and basic features of the spectrum right.

In our notation, states are normalized to Dirac $\delta$ functions in momentum space. The $\Sigma^0$ and $\Lambda$ spinors are normalized to $\bar{u}u = 1$. To project out a given $\hat{s}$, one replaces $u(p)$ by $\frac{1}{2}(1+\gamma_5\gamma\cdot s)u(p)$.[3] The rate, for given $\hat{s}$, is the rate for the $\Lambda$ spin to be in the direction $\hat{s}$ in the $\Lambda$ rest frame. In the $\Sigma^0$ rest frame, the $\Lambda$ is fairly nonrelativistic, so we shall not bother about the small difference.

Provided all spins are summed or averaged, the rate, as in any four-point function, depends on two invariants. If $\hat{s}$ is chosen in the direction of some momentum, e.g. the positron momentum, the spectrum will still depend on two parameters only. Let $E_\pm = p^0_\pm$. Then two conventional variables are

$$x = (q^2)^{1/2} \tag{2.3}$$

and

$$y = (E_+ - E_-)/|\vec{p}|. \tag{2.4}$$

Here $x$ is the effective mass of the lepton pair and ranges from $2m$ to $\Delta$. Some further useful kinematic relationships can be found in the Appendix.

## III. THE ELECTROMAGNETIC DECAY OF THE $\Sigma^0$

The principal decay mode of the $\Sigma^0$ is $\Sigma^0 \to \Lambda + \gamma$. Coleman and Glashow pointed out in 1961 that if the electromagnetic current is an SU(3) octet, the lifetime could be related to the well-known neutron magnetic moment.[4] The branching ratio for $\Gamma \to \Lambda + e^+ + e^-$ compared to $\Gamma \to \Lambda + \gamma$ was calculated in 1958 by Feinberg[5] in an attempt to use it to determine the $\Sigma^0$-$\Lambda$ relative parity. The detailed spectrum was more fully investigated by Feldman and

Fulton,[6] Byers and Burkhardt,[7] Evans,[8] and others.[9] An experiment by Alff *et al.* in 1964 showed that the data definitely required the relative parity to be even. (In 1961 Dreitlein and Primakoff[10] even suggested studying the correlations of the spins of the $\Sigma^0$ and the $\Lambda$ and the photon polarization in the principal decay mode to look for parity-violating effects.)

Thus this section contains no new results. We include it for completeness, to develop the notation,[11] and to discuss some approximations and features of the spectrum which may make parity-violating effects observable. The matrix element of the electromagnetic current between the vacuum and the final lepton pair is

$$\langle e^+e^-|j^\mu_{\mathrm{em}}(0)|\rangle = \frac{m}{(2\pi)^3(E_+E_-)^{1/2}}L^\mu , \tag{3.1}$$

where

$$L^\mu = \bar{v}(e^+)\gamma^\mu u(e^-). \tag{3.2}$$

The matrix element of the current between the $\Sigma^0$ and the $\Lambda$ has the form

$$(2\pi)^{-3}(M_\Lambda/E)T^\mu ,$$

where $E$ is the $\Lambda$ energy. Both $T^\mu$ and $L^\mu$ are manifestly covariant and are vectors. The most general form of $T^\mu$ is

$$T^\mu = \bar{u}(p)[f_1\gamma^\mu + if_2q_\nu\sigma^{\mu\nu}/(2M) + f_3q^\mu]u(p'). \tag{3.3}$$

We make the standard approximation that only $f_2$ is important. There are two related, but not identical, arguments. The first is that if SU(3) were an exact symmetry $f_3$ would vanish because, in an eigenstate of SU(3), it transforms differently under charge conjugation, and $f_1$ would vanish because the electric charge coupling is pure $F$ type, while the $\Sigma^0$ and $\Lambda$ are both neutral. There is no such restriction on $f_2$, which in the SU(3) limit is $D$-type coupling. Or, ignoring SU(3), one may argue that $T^\mu$ is at any rate conserved, so that $f_3 = -f_1\Delta/q^2$. None of the form factors should have a pole at $q^2 = 0$. Therefore, $\gamma^\mu$ and $q^\mu$ must occur in $T^\mu$ in the combination

$$\frac{V(q^2)}{M^2}(q^2\gamma^\mu - \Delta q^\mu), \tag{3.4}$$

where $V$ is dimensionless and finite at $q^2 = 0$. If $V(q^2)$ varies on the scale $q^2/M^2$, it may be safely ignored compared to $f_2(q^2)$. We mention all this simply to point out that the standard approximations, while plausible, are not rigorous. In the same spirit we take $f_2$ to be a constant. It is dimensionless and plausibly of order unity.

The total rate for the principal decay mode $\Sigma^0 \to \Lambda + \gamma$ can now be computed in terms of the single parameter $f_2$. The result is

$$\Gamma_{\Lambda\gamma} = \alpha f_2^{\,2}\Delta^3 M/M_\Sigma^{\,3}. \tag{3.5}$$

The coefficient can be related to the neutron magnetic moment $\mu_N$ by[4]

$$\frac{f_2}{2M} = -\frac{\sqrt{3}}{2}\mu_N . \tag{3.6}$$

From the observation that $\mu_N = -1.91/2M_N$, we obtain $\Gamma_{\Lambda\gamma} = 6.21\times10^{-3}$ MeV, corresponding to a lifetime of about $10^{-19}$ sec. Recent experiments are not in disagreement with this prediction.[12]

The Dalitz-pair mode, $\Sigma^0 \to \Lambda + e^+ + e^-$, which is the subject of the present paper, is down by a factor of $\alpha$. We outline the calculation of the branching ratio $\Gamma_{\Lambda e^+e^-}/\Gamma_{\Lambda\gamma}$, including its dependence on $x$ and $y$.

The decay amplitude is proportional to an invariant $\mathfrak{M}'$ which is

$$\mathfrak{M}' = -\frac{e^2}{q^2}L_\mu T^\mu . \tag{3.7}$$

For a given $\Lambda$ spin $\hat{s}$, let us define

$$|\mathfrak{M}|^2 = \tfrac{1}{2}\sum|\mathfrak{M}'|^2 , \tag{3.8}$$

where the sum is over all the spins, but the projection matrix $(1+\gamma_5\gamma\cdot s)/2$ is included in the definition of $T^\mu$ (we anticipate, of course, the calculation in Sec. IV). The differential decay rate is

$$d\Gamma = \frac{8M_\Lambda}{(2\pi)^5}m^2|\mathfrak{M}|^2 d\bar{\Gamma}, \tag{3.9}$$

where

$$d\bar{\Gamma} = \frac{d^3p_+}{2E_+}\frac{d^3p_-}{2E_-}\frac{d^3p_\Lambda}{2E}\delta_4(p+p_++p_--p') \tag{3.10}$$

and

$$\frac{\partial^2\bar{\Gamma}}{\partial x\partial y} = \frac{\pi^2xp_\Lambda}{2M_\Sigma} . \tag{3.11}$$

Here, $p_\Lambda$ is a function of $x$ alone (see Appendix). The physics lies in the calculation of $|\mathfrak{M}|^2$. Write

$$m^2|\mathfrak{M}|^2 = (e/x)^4 T_{\mu\nu}L^{\mu\nu}, \tag{3.12}$$

where

$$L^{\mu\nu} = \sum_{\text{spins}} m^2 L^\mu L^{\nu*} = p_+^\mu p_-^\nu + p_-^\mu p_+^\nu - g^{\mu\nu}(p_+\cdot p_- + m^2) \tag{3.13}$$

and

$$T_{\mu\nu} = \tfrac{1}{2}\sum_{\text{spins}} T_\mu T_\nu^*$$
$$= \frac{f_2^{\,2}}{4M_\Lambda M_\Sigma}\left\{-\frac{x^2}{8M^2}Q_\mu Q_\nu + \tfrac{1}{2}(x^2-\Delta^2)g_{\mu\nu} + \frac{\Delta}{4M}(Q_\mu q_\nu + q_\mu Q_\nu) - \tfrac{1}{2}q_\mu q_\nu \right.$$
$$\left. + is^\alpha\left[\epsilon_{\alpha\mu\nu\beta}(\tfrac{1}{2}Q^\beta\Delta - q^\beta M) - \frac{1}{8M}(q^\beta Q^\gamma - Q^\beta q^\gamma)(\epsilon_{\alpha\mu\beta\gamma}Q_\gamma - \epsilon_{\alpha\nu\beta\gamma}Q_\mu)\right]\right\}, \tag{3.14}$$

where $Q = p + p'$. Many of the terms in (3.14) vanish when contracted with $L^{\mu\nu}$, since $q_\mu L^{\mu\nu} = 0$. We exhibit the term linear in $s^\alpha$ explicitly here for use in the next section.

From Eqs. (3.9) through (3.14), we obtain the differential decay rate:

$$\frac{\partial^2\Gamma}{\partial x\partial y} = \frac{\alpha^2 f_2^{\,2} p_\Lambda}{\pi x^3 M_\Sigma^{\,2}}\,\frac{(\Delta^2 - x^2)}{2}\left[\frac{x^2}{8M^2}(4M^2 - x^2)(y^2 - 1) + (x^2 + 2m^2)\right]. \tag{3.15}$$

Notice that since $p_\Lambda$ is proportional to $(\Delta^2 - x^2)^{1/2}$ as $x \to \Delta$ (see Appendix), the expression (3.15) vanishes like $(\Delta^2 - x^2)^{3/2}$ in that limit. The coefficient $f_2$ cancels out in the ratio

$$\rho(x, y) = \frac{1}{\Gamma_{\Lambda\gamma}}\frac{\partial^2\Gamma}{\partial x\partial y}. \tag{3.16}$$

Our expression (3.15) is in agreement with previous calculations.[5,6,8,9]

The total branching ratio is

$$\rho = \int_{2m}^{\Delta} dx \int_{-(1-4m^2/x^2)^{1/2}}^{(1-4m^2/x^2)^{1/2}} dy\,\rho(x, y)$$
$$\approx \tfrac{1}{184}. \tag{3.17}$$

Although the ratio (3.17) was the number first calculated, it is not as interesting as the detailed spectrum, since, because of the factor $x^{-3}$ in (3.15) a substantial contribution to $\rho$ comes from the part of the spectrum near $x = 2m$. As far as we know, the spectrum (3.15) is in agreement with experiment.

## IV. CONTRIBUTION OF THE WEAK NEUTRAL CURRENT

In general, we expect that there are weak neutral currents, described by a few parameters, which can be evaluated in particular models. But one should not put too much emphasis on their precise values. The question here is: What is the order of magnitude of the parity violating effects and where in the spectrum are they largest? The important exercise is therefore to isolate those parameters which are of order unity.

We follow the notation and calculational scheme outlined in the preceding sections. We assume that the weak currents, both hadronic and leptonic, are vector and axial-vector only. Then, analogous to $L^\mu$ in Eq. (3.2), there is a weak leptonic current

$$L_5^\mu = \bar{v}(e^+)\gamma^\mu(g_V + g_A\gamma_5)u(e^-). \tag{4.1}$$

Equation (4.1) simply defines $g_A$ and $g_V$. There is also a weak hadron current, whose matrix element $W^\mu$ is defined analogously to $T^\mu$. It may have both a vector and an axial-vector part. The vector part has the same general form as $T^\mu$ in Eq. (3.3). If either SU(3) or conserved vector current (CVC) for weak neutral currents is a good approximation, this part can be written

$$\bar{u}(p)\frac{if_2'}{2M}q_\nu\sigma^{\mu\nu}u(p'), \tag{4.2}$$

where $f_2'$ is not necessarily equal to $f_2$, but is also probably of order unity. The axial-vector part of $W^\mu$ may have a term

$$C\bar{u}(p)\gamma_\mu\gamma_5 u(p'). \tag{4.3}$$

The only other possibility is a term of the form $\bar{u}(p)iq_\nu\sigma^{\mu\nu}\gamma_5 u(p')$. Such a term is a "second-class current," and we assume, without much experimental evidence, that it is negligible. This is equivalent to assuming that the weak neutral axial-vector current is a member of an isotopic triplet, all of whose terms transform the same way under $G$ parity, and is true in most models. Finally, a term proportional to $q_\mu\gamma_5$ would be negligible when contracted with the lepton current. Thus the only new dimensionless parameters are $f_2'$, $C$, $g_A$, and $g_V$.

The decay rate is related to $|\mathfrak{M}|^2$, and so to $\mathfrak{M}'$, just as in Sec. III. The invariant matrix element is

$$\mathfrak{M}' = \frac{-e^2}{x^2}T_\mu L^\mu + \frac{G}{\sqrt{2}}W_\mu L_5^\mu. \tag{4.4}$$

The interesting effect comes from the interference term. Define

$$L_5^{\mu\nu} = \sum_{\text{spins}} m^2 L^\mu L^{\nu *}$$
$$= g_V[p_+^\mu p_-^\nu + p_-^\mu p_-^\nu - g^{\mu\nu}(p_+\cdot p_- + m^2)]$$
$$+ ig_A\epsilon^{\alpha\mu\beta\nu}p_{+\alpha}p_{-\beta} \tag{4.5}$$

and

$$W_{\mu\nu} = \tfrac{1}{2}\sum_{\text{spins}} T_\mu W_\nu^*.$$

Then, with the inclusion of the weak effects, Eq. (3.12) is replaced by

$$m^2|\mathfrak{M}|^2=\left(\frac{e}{x}\right)^4 T_{\mu\nu}L^{\mu\nu}-\frac{e^2}{x^2}\frac{G}{\sqrt{2}}(W_{\mu\nu}L_5^{\mu\nu}+\text{c.c.}) \tag{4.6}$$

plus terms of order $G^2$.

The tensor $W_{\mu\nu}$ has two parts: the vector part $W_{\mu\nu}^{(1)}$ and the axial-vector part $W_{\mu\nu}^{(5)}$. The vector part $W_{\mu\nu}^{(1)}$ is identical to the expression (3.14), with $f_2{}^2$ replaced by $f_2 f_2'$.

For completeness, we write $W_{\mu\nu}^{(5)}$:

$$W_{\mu\nu}^{(5)}=\frac{f_2C}{4M_\Lambda M_\Sigma}\Big[-i\epsilon_{\mu\alpha\nu\beta}p'^\alpha p^\beta+\frac{Q_\mu}{2M}(s\cdot p'p_\nu-s_\nu p\cdot p')-M_\Sigma(s_\mu p_\nu-p_\mu s_\nu)$$
$$+M_\Lambda(s_\mu p'_\nu+p'_\mu s_\nu-g_{\mu\nu}s\cdot p')-\frac{M_\Lambda M_\Sigma}{2M}Q_\mu s_\nu\Big]. \tag{4.7}$$

Next, to calculate the rate we must compute $W_{\mu\nu}L_5^{\mu\nu}$. We need keep terms linear in $s$ only (because others will not contribute to a reflection asymmetry) and real (because the complex conjugate is to be added). The result will have terms proportional to $s\cdot p_+$, $s\cdot p_-$, and $s\cdot p'$. Since $s\cdot p=0$, and $p'=p+p_+ +p_-$, we may eliminate terms proportional to $s\cdot p_-$ in favor of $s\cdot p_+$ and $s\cdot p'$. The result for $W_{\mu\nu}L_5^{\mu\nu}$ is

$$\frac{-f_2f_2'g_A}{4M_\Sigma M_\Lambda}\left\{s\cdot p_+\frac{x^2}{2M}(\Delta^2-x^2)+s\cdot p'\left[\frac{x^2}{4M}(x^2-\Delta^2)\frac{M_\Sigma p_\Lambda y x^2}{2M}\right]\right\}$$
$$\frac{+f_2Cg_V}{4M_\Lambda M_\Delta}\left\{s\cdot p_+p_\Lambda y\frac{M_\Sigma}{M}(M\Delta-x^2/2)\;\frac{+s\cdot p'}{2M}\left[M_\Sigma p_\Lambda y(\tfrac{1}{2}x^2-M\Delta)-M_\Sigma{}^2p_\Lambda{}^2y^2+x^2(M^2-\Delta^2/4)+2Mm^2(2M-\Delta)\right]\right\} \tag{4.8}$$

plus terms independent of $s$, plus imaginary terms. For a given $\hat{s}$, the parity-violating part of the differential decay rate is

$$\frac{\partial^2\Gamma}{\partial x\partial y}=\frac{8M_\Lambda}{(2\pi)^5}\frac{\pi^2xp_\Lambda}{2M_\Sigma}\frac{e^2}{x^2}\sqrt{2}\,GW_{\mu\nu}L_5^{\mu\nu}, \tag{4.9}$$

where $W_{\mu\nu}L_5^{\mu\nu}$ is given in the expression (4.8).

## V. EVALUATION OF THE RESULT

All the expressions above are given without approximation, except that the form factors are taken to be constants. Now we begin to make some approximations. The vector $s$ has components

$$s^0=\frac{\hat{s}\cdot\vec{p}}{E}[1-(\hat{s}\cdot\vec{p})^2/E^2]^{-1/2}, \tag{5.1}$$

$$\vec{s}=\hat{s}[1-(\hat{s}\cdot\vec{p})^2/E^2]^{-1/2}. \tag{5.2}$$

Since $\hat{s}$ is a unit vector, $\vec{s}$ is of order unity, while $s^0$ is of order $\Delta/M$. In the $\Sigma^0$ rest frame, $s\cdot p' =M_\Sigma s^0$, which is therefore of order $\Delta$. Since $|\vec{p}_+| \approx E_+$, $s\cdot p_+$ is of order $\Delta$ also. Since both $x$ and $p_\Lambda$ are never greater than $\Delta$, the first term in (4.8) is smaller than the second, roughly of the order $\Delta/M$, or about 7%, assuming that $g_A$ and $g_V$ are of the same order of magnitude.

In the spirit of our approximations, we replace, in the second term in (4.8), $M\Delta-x^2/2$ by $M\Delta$, and $(M-\Delta)$ by $M$. The term in $m^2$ can be ignored except for very small values of $x$. But we are not interested in the spectrum near $x=2m$, because near $x=\Delta$ the electromagnetic rate vanishes as $(\Delta^2-x^2)^{3/2}$, while the parity violation vanishes only as $(\Delta^2-x^2)$. Therefore, we look at the rate near $x=\Delta$, and predict that while both rates vanish there, the branching ratio for parity violation becomes infinite.

We have already estimated from SU(3) that $f_2$ is about 2. The coefficient $C$ is related to the charged axial current which mediates the semileptonic decay $\Sigma^-\to\Lambda+e^-+\bar{\nu}$. Its magnitude is about 0.2. Finally, $g_V$ is given in the Weinberg-Salam model as $\frac{1}{2}-\sin^2\theta_W$. If the Weinberg angle $\theta_W$ is 35°, $g_V$ is −0.158. There is a peculiar sensitivity here. If $\theta_W$ were exactly 30°, $g_V$ would be zero and the effect would vanish.

As an example, take $\hat{s}$ along the positron direction. Then, ignoring terms of order $\Delta/M$ compared to terms of order unity, $\vec{s}\approx\hat{s}\approx\hat{p}_+$, $s\cdot p_+ =-\hat{p}_+\cdot\vec{p}_+=-|\vec{p}_+|$, and $s\cdot p'=M_\Sigma\hat{p}_+\cdot\vec{p}/E$. For $|\vec{p}_+|$ of the order $\Delta$,

$$|\vec{p}_+|\approx E_+=\frac{x^2+2M\Delta}{4M_\Sigma}\frac{py}{2}\approx\frac{\Delta+py}{2}. \tag{5.3}$$

Furthermore,

$$\vec{p}_+\cdot\vec{p}=(E_-{}^2-E_+{}^2-p_\Lambda{}^2)/2$$
$$=-p_\Lambda y(M_\Sigma-E)-p_\Lambda{}^2/2 \tag{5.4}$$

so that, approximately,

$$s\cdot p'=-\frac{p_\Lambda y\Delta-p_\Lambda{}^2}{\Delta+p_\Lambda y}. \tag{5.5}$$

Thus, for $\vec{s}$ along the positron direction,

$$\frac{\partial^2 \Gamma_+}{\partial x \partial y} = \frac{\sqrt{2}\, f_2 C g_V G \alpha p_\Lambda}{8\pi^2 x M^2} F(x, y), \tag{5.6}$$

where $\Gamma_+$ is the $s$-dependent part only of the interference term, $p_\Lambda$ is the $\Lambda$ three-momentum ($\approx[\Delta^2 - x^2]^{1/2}$), and $F(x, y)$ is given approximately by

$$-\tfrac{1}{2}(\Delta + p_\Lambda y) p_\Lambda y M \Delta - \frac{p_\Lambda M}{2}\left(\frac{p_\Lambda + y\Delta}{\Delta + p_\Lambda y}\right) \times (x^2 - \Delta p_\Lambda y - p_\Lambda{}^2 y^2). \tag{5.7}$$

Similarly, for $\vec{s}$ along $-\vec{p}_+$

$$\frac{\partial^2 \Gamma_-}{\partial x \partial y} = -\frac{\partial^2 \Gamma_+}{\partial x \partial y}. \tag{5.8}$$

The expression (5.7) is approximate, terms of relative order $\Delta/M$ having been omitted. Moreover, it is not valid when either the electron or positron momentum is nonrelativistic. We define as a measure of the parity violation

$$P(x, y) = \frac{\dfrac{\partial^2 \Gamma_+}{\partial x \partial y} - \dfrac{\partial^2 \Gamma_-}{\partial x \partial y}}{\dfrac{\partial^2 \Gamma_+}{\partial x \partial y} + \dfrac{\partial^2 \Gamma_-}{\partial x \partial y}}. \tag{5.9}$$

For the denominator one can use the electromagnetic decay rate Eq. (3.15). This rate, away from $x^2 = m^2$, we approximate as

$$\frac{\alpha^2 f_2{}^2 p_\Lambda}{\pi x^3 M_\Sigma{}^2} \frac{(\Delta^2 - x^2)}{2} x^2 \left(\frac{y^2 - 1}{2} + 1\right) = \frac{\alpha^2 f_2{}^2 p_\Lambda}{\pi x M_\Sigma{}^3} \frac{(\Delta^2 - x^2)}{4} (y^2 + 1) \tag{5.10}$$

and

$$P(x, y) = \frac{\sqrt{2}\, C g_V G M}{4\alpha f_2 p_\Lambda (1 + y^2)} \times \left[ y\Delta(\Delta + p_\Lambda y) + \frac{(p_\Lambda + \Delta y)}{(\Delta + p_\Lambda y)}(x^2 - \Delta p_\Lambda{}^2 y^2)\right]. \tag{5.11}$$

This branching ratio $P(x, y)$ is largest for small $p_\Lambda$, or $x$ near $\Delta$. In fact, it becomes infinite as $p_\Lambda \to 0$. In that limit, the bracket in (5.11) becomes just $2\Delta^2 y$, and

$$P(x, y) = \frac{C g_V}{f_2}\left(\frac{G M \Delta}{\sqrt{2}\,\alpha}\right)\left(\frac{y}{1 + y^2}\right)\left(\frac{\Delta}{p}\right). \tag{5.12}$$

The constant $GM\Delta/\sqrt{2}\,\alpha$ is about $10^{-4}$. From (3.6), $f_2$ is about 2.03. If we take $C = 0.2$ from semileptonic charged-$\Sigma$ decay, and $g_V$ from the Weinberg-Salam model to be $-0.16$,

$$P(x, y) \approx -1.6 \times 10^{-6} \frac{y^2}{1 + y^2}\left(\frac{\Delta}{p_\Lambda}\right). \tag{5.13}$$

## VI. CONCLUSION

The approximate expression (5.13) gives the magnitude of the correlation expected between the $\Lambda$ spin and the positron momentum in the decay $\Sigma^0 \to \Lambda + e^+ + e^-$. More exact expressions can be calculated from the detailed expressions in Secs. IV and V. The effect is too small to be seen in present experiments, but might be possible sometime in the future.[13] In any event, more detailed calculations would be needed to take into account the polarization of the $\Sigma^0$ in the production process. In this calculation we have for convenience taken the $\Sigma^0$ to be unpolarized in its own rest frame.

The important point is the order of magnitude of the result, which has assumed that the axial-vector currents can be taken by isospin from charged $\Sigma$ beta decay for the hyperons, and from a Weinberg-Salam–type model for the electrons. Neither of these assumptions to our knowledge has as yet any experimental basis. Indeed, recent experiments in atoms[14] fail to show parity violation at the level expected by the Weinberg-Salam–type models. Therefore, failure to see parity violation in $\Sigma^0$ decay at the level predicted by (5.13) would be further evidence that the weak neutral currents do not have the $V, A$ structure which the charged ones have.

## ACKNOWLEDGMENT

One of us (E. A.) would like to thank B. W. Lee for the hospitality of the Theory Group at Fermi National Accelerator Laboratory, and E. Paschos who collaborated with him on a preliminary version of this work.

## APPENDIX: KINEMATICS

The $\Sigma^0$, $\Lambda$, electron and positron four-momenta are denoted $p'$, $p$, $p_-$, and $p_+$, respectively. These are related to the energies and three-momenta by $p' = (M_\Sigma, 0)$, $p = (E, \vec{p})$, $p_\pm = (E_\pm, \vec{p}_\pm)$. We do all our calculations in the $\Sigma^0$ rest frame using two conventional variables[2]

$$x = (q^2)^{1/2} \tag{A1}$$

and

$$y = (E_+ - E_-)/|\vec{p}|, \tag{A2}$$

where $q = p_+ + p_- = p' - p$ is the four-momentum transferred to the lepton pair. We often use the constants

$$M = \tfrac{1}{2}(M_\Sigma + M_\Lambda) \approx 1153 \text{ MeV} \tag{A3}$$

and

$$\Delta = (M_\Sigma - M_\Lambda) \approx 77 \text{ MeV}. \tag{A4}$$

A useful auxiliary vector is $Q = p' + p$,

$$Q^2 = 4M^2 + \Delta^2 - x^2 . \tag{A5}$$

Useful relations are

$$M_\Sigma{}^2 + M_\Lambda{}^2 = \tfrac{1}{2}(Q^2 + q^2) = \tfrac{1}{2}(4M^2 + \Delta^2), \tag{A6}$$

$$E = (2M^2 + \Delta^2/2 - x^2)/(2M_\Sigma), \tag{A7}$$

$$p_\Lambda = |\vec{p}| = (\Delta^2 - x^2)^{1/2}(4M^2 - x^2)^{1/2}/(2M_\Sigma) \approx (\Delta^2 - x^2)^{1/2} . \tag{A8}$$

The last approximation, which we used to get our final result, is not valid for $x \approx 2m$ ($m$ is the electron mass).

The spin four-vector is $s = (s^0, \vec{s})$, where

$$s^0 = a\frac{\hat{s}\cdot\vec{p}}{E}, \quad \vec{s} = a\hat{s} \tag{A9}$$

where $\hat{s}$ is a unit vector in the direction of the $\Lambda$ spin, and

$$a = [1 - (\hat{s}\cdot\vec{p})^2/E^2]^{1/2} \approx 1 . \tag{A10}$$

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*Work supported in part by the National Science Foundation.

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