Nonleptonic decays in a renormalizable gauge model of chiral symmetry

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We present a model based on chiral $SU(3)\times SU(3)$ for the nonleptonic decays of mesons and baryons. The model is fully gauge-invariant and renormalizable. The basic fields in the model are the spin-1 and spin-0 mesons. All symmetry-breaking effects, including the nonleptonic weak mixings, are achieved through the spontaneous-symmetry-breaking mechanism. Our choice of the Higgs-Kibble scalars automatically ensures octet dominance for the nonleptonic weak vertex. Since we find it hard to include baryons in the model, a phenomenological treatment of the baryon decays, assuming SU(3) invariance of the baryonic couplings, is presented. In the model we calculate the two-pion and three-pion decays of the kaon, the s-wave amplitudes for the hyperon decays, and the K_L-K_S mass difference. The results for $K\to 2\pi$ and $K\to 3\pi$ decay widths are in excellent agreement with experiment. The slope parameter for the $K\to 3\pi$ decay, however, comes out with the wrong sign. The s-wave amplitudes for the hyperon decays agree reasonably well with experiment. The estimate for the K_L-K_S mass difference is of the correct order of magnitude. $K^+\to\pi^+\pi^0$ decay is calculated by using the known current-algebra estimate of $\eta\pi^0$ mixing. The decay width obtained by us is rather low. Our conclusion is that $\eta\pi^0$ mixing alone is not sufficient to explain the K^+ decay. On extending the model to chiral $SU(4)\times SU(4)$, we predict the existence of a component transforming as the $\underline{15}$ representation of SU(4) in the nonleptonic Hamiltonian. Therefore, the nonleptonic decays of charmed mesons will provide a definitive test for the model.

I. INTRODUCTION

In weak-interaction phenomenology the nonleptonic decays have long occupied an important position. This is due to the difficulty in theoretically implementing the $\Delta I=\frac{1}{2}$ selection rule exhibited by these decays. All the available data on the nonleptonic decays [excepting the data on the $(K\to 3\pi)$-decay slope parameters] satisfy the $\Delta I=\frac{1}{2}$ rule within a few percent.[1] The ordinary current $\times$ current theory (that explains semileptonic decays so well) requires a considerable admixture of $\Delta I=\frac{3}{2}$ in the nonleptonic decay Hamiltonian. Traditionally the $\Delta I=\frac{1}{2}$ rule, or its SU(3) generalization, the octet-dominance phenomenon, has been explained either by attributing it to the dynamics of strong interactions, or by invoking the existence of neutral hadronic currents that do not couple to the leptons and result in a pure $\Delta I=\frac{1}{2}$ character of the nonleptonic Hamiltonian.[2]

During the last few years, the major problem of the current $\times$ current theory, its nonrenormalizability, has been solved by embedding it, as the lowest-order phenomenology, in the unified gauge theories of weak and electromagnetic interactions. However, the explanation of the octet-dominance phenomenon still proceeds on the traditional lines. Many authors have exhibited the dynamical suppression of the $\Delta I=\frac{3}{2}$ and enhancement of the $\Delta I=\frac{1}{2}$ parts of the weak-interaction Hamiltonian, especially as a consequence of the asymptotic freedom of the non-Abelian gauge theories of strong interactions.[3] There has also been attempts to obtain the pure SU(3)-octet structure of the nonleptonic Hamiltonian by introducing neutral vector bosons.[4] But in all these attempts to obtain the nonleptonic decays from the current $\times$ current theory, the predicted decay rates turn out to be an order of magnitude smaller than the actual values.[5]

Besides these conventional solutions, there is another way of approaching the $\Delta I=\frac{1}{2}$ problem. One need not insist on the current $\times$ current form of the Hamiltonian for the nonleptonic decays. This implies treating the semileptonic and the nonleptonic processes on different footings. The $\Delta I=\frac{1}{2}$ rule is thus looked upon as a basic symmetry of the nonleptonic Hamiltonian, and not as merely the design of strong-interaction dynamics or of fortuitous cancellations. One must then search for other mechanisms in the weak-interaction model that may give rise to an octet-dominant nonleptonic Hamiltonian.[6] In the context of gauge-symmetric models, Lee and Treiman[7] were the first to explore this possibility. Taking an eight-quark version of the Georgi-Glashow model,[8] they showed that the nonleptonic decays can proceed through the Higgs-Kibble scalar exchange, and that this process will compete successfully with the vector-boson exchange (i.e., current $\times$ current contribution) provided the masses of the Higgs scalars are smaller than or at least comparable to the other masses in the model. In a realistic model with physical hadrons (rather than quarks) as basic fields, the Lee-Treiman approach requires the Higgs-Kibble scalar-meson masses to be of the same order of magnitude as the hadron masses. This is an unpalatable feature of the Higgs-Kibble scalar-exchange mechanism.

Here we have the objective of showing (i) that a purely octet-type, renormalizable, nonleptonic Hamiltonian can be constructed, and (ii) that if this Hamiltonian were the only term contributing to the nonleptonic processes, a good fit to most of the known experimental data can be obtained.

We have ignored all discussion of leptons and have avoided adding the nonleptonic contributions of the current × current term that would have arisen if we had tried to include leptons in our scheme in any of the known ways. For example, if we follow the unified lepton-hadron models of Bars, Halpern, and Yoshimura,[9] we shall end up with $W^{\pm}$ mesons reproducing the conventional current × current theory results. The effects of adding the current-current contribution are about 20% violation of the $\Delta I=\frac{1}{2}$ rule in the hyperon decays and an order-of-magnitude larger value for the width $\Gamma(K^+\to\pi^+\pi^0)$. These consequences are clearly unacceptable.

With this fact in view we have adopted the philosophy that the nonleptonic interaction Hamiltonian is primarily octet-type and contains no significant admixture of current-current terms. We are not the first to suggest this; Marshak *et al.*[10] have already recorded this possibility. Some works in the literature show that the suppression of the $\Delta I=\frac{3}{2}$ piece of the current-current contribution comes from strong-interaction dynamics (e.g., Mathur and Yen, Ref. 3). But this suppression alone is not enough to explain the observed widths. As Salam[5] has noted, the $\Delta I=\frac{1}{2}$ component of current-current theory gives amplitudes which are much lower than the observed ones. One needs a mechanism for enhancing the $\Delta I=\frac{1}{2}$ contribution. The mechanism proposed in the present work can be considered as the required enhancement term.

The model we are going to discuss is based on chiral SU(3) × SU(3) symmetry. It can be trivially extended to chiral SU(4) × SU(4) symmetry. This extension will not affect the results presented here. However, it will have interesting consequences for the nonleptonic weak decays of charmed mesons. The SU(4) version of this model predicts the nonleptonic Hamiltonian to transform as the $\underline{15}$ representation of SU(4), while the effective nonleptonic Hamiltonian generated by the current-current interaction is known to have no component transforming as the $\underline{15}$ representation. Hence presence of a piece transforming as the $\underline{15}$ representation in the nonleptonic decays of charmed mesons will be positive evidence in favor of our model.

In this paper we generate the $\Delta I=\frac{1}{2}$ symmetric nonleptonic vertex in a realistic model of strong interactions of spin-$0^{\pm}$ and spin-$1^{\pm}$ mesons proposed earlier by one of us (A.K.K.).[11] (This model is described in Sec. II.) This is achieved by adding to the strong-interaction Lagrangian terms that violate strangeness when Higgs-Kibble scalars are allowed to develop nonzero vacuum expectation values (VEV's). As only bilinear combinations of Higgs-Kibble scalars couple to the hadrons, the requirement of renormalizability guarantees octet dominance for interactions of hadrons in the case when Higgs-Kibble scalars are triplets of hadronic SU(3).[12] By making a suitable choice for the VEV's of the scalar fields it is possible to arrange that the couplings involving only hadrons remain SU(2) invariant and conserve strangeness, the nonleptonic interactions arising solely from the mixing of various particles. The weak-interaction model is discussed in Sec. III.

The weak-interaction Lagrangian thus generated is used to study $K\to 2\pi$ and $K\to 3\pi$ decays. Since there is no $\Delta I=\frac{3}{2}$ term in the nonleptonic vertex, the decay width for $K^+\to\pi^+\pi^0$ is zero if we consider weak interaction alone. An attempt is made to study this decay by attributing it to the electromagnetic effects. Parity-violating decays of baryons are considered phenomenologically. We find it difficult to write the model with baryons as basic fields,[13] so we assume the baryonic couplings to be SU(3) invariant and we attribute the parity-violating parts in the amplitudes for hyperon decays to the K^*- and S_K-pole terms. The K_L-K_S mass difference is also evaluated as a second-order weak effect.

Our results for $K_S^0\to 2\pi$ and $K\to 3\pi$ decay widths agree with experiment. The predicted slope parameters for $K\to 3\pi$ decays are in rough agreement with experiment as to their magnitude, but have the wrong signs. However, this is a piece of data that seems to violate the octet-dominance hypothesis. For the K_L-K_S mass difference our numbers are of the correct order of magnitude. Parity-violating baryonic decays are fitted within 20% of the experimental values. This result is in conformity with the earlier current-algebra predictions using K^* pole alone.[10] One place where our analysis fails completely is the K^+ decay. Using a current-algebra estimate of the electromagnetic mixing given by Cicogna *et al.*,[14] we obtain the K^+ decay width to be orders of magnitude smaller than the experimental value. The electromagnetic mixing alone is apparently not sufficient to explain the K^+ decay.

The paper is organized as follows. In Sec. II we sketch the strong-interaction model and fit various masses and parameters in the model. Section III extends the strong model of Sec. II to the weak case. In Sec. IV, parity-violating nonleptonic effects, viz., $K\to 2\pi$ decay and s-wave baryonic decays, are discussed. Section V deals with the $K\to 3\pi$ decays. The K_L-K_S mass difference is calculated in the same section. Finally, in Sec. VI, we present a discussion of the model postulated in this paper.

II. THE STRONG-INTERACTION MODEL

We start with the strong-interaction part of the model proposed earlier by one of the authors.[11] We redevelop it here for the sake of completeness.

The model is written for scalar and pseudoscalar ($0^\pm$) meson nonets with vector and axial-vector ($1^\pm$) mesons as gauge fields. The full symmetry group of the Lagrangian is

$$G \sim SU(3)_L \otimes SU(3)_R \otimes U(1)_{V_0} \otimes U(1)_{K_Q} \otimes U(1)_{K_Y}.$$

The $SU(3)_L \otimes SU(3)_R$ is the usual chiral-symmetry group of hadrons. The scalar and pseudoscalar nonets u_k and v_k ($k=0,1,\ldots 8$), respectively, are assumed to transform as $(3,3^*)$ and $(3^*,3)$ representations of the chiral group. We define

$$M=\sum_{k=0}^{8}\frac{\lambda_k}{\sqrt{2}}(u_k+iv_k),$$

which transforms as

$$M'=\exp\left(+i\sum_{k=1}^{8}\epsilon_k^L\frac{\lambda_k}{2}\right)M\exp\left(-i\sum_{k=1}^{8}\epsilon_k^R\frac{\lambda_k}{2}\right).$$

The other pieces in G are $U(1)_{V_0}$, the symmetry under the ninth vector transformation, and $U(1)_{K_Q}$, $U(1)_{K_Y}$, the symmetry groups belonging to the new quantum numbers K_Q, K_Y that must be introduced in order to obtain a suitable charge assignment for Higgs-Kibble scalars. K_Q, K_Y shall be defined later in the text. The invariance of the Lagrangian under the ninth axial-vector transformation is not insisted upon, because a study of the meson masses shows that the term $(\det M+\det M^\dagger)$ which breaks the symmetry under this transformation must be added to the Lagrangian if the η mass is to be explained satisfactorily.

Gauge fields. The whole group G with the exception of $U(1)_{K_Q}$ and $U(1)_{K_Y}$ is gauged. We do not need to gauge the subgroups $U(1)_{K_Q}$ and $U(1)_{K_Y}$ because we are not interested in the photon couplings. The gauging requires introduction of 17 spin-1 fields. We choose $X_\mu^{L,k}, X_\mu^{R,k}$ ($k=1$ to 8) corresponding to $SU(3)_L\otimes SU(3)_R$ and V_μ^0 corresponding to $U(1)_{V_0}$. We define the matrices

$$\hat{X}_\mu^L=\sum_{k=1}^{8}X_\mu^{L,k}\frac{\lambda_k}{2},$$
$$\hat{X}_\mu^R=\sum_{k=1}^{8}X_\mu^{R,k}\frac{\lambda_k}{2}.$$

Further, the vector and the axial-vector fields may be defined as

$$\hat{V}_\mu=\frac{1}{\sqrt{2}}(\hat{X}_\mu^L+\hat{X}_\mu^R)\equiv\frac{1}{\sqrt{2}}\sum_{k=1}^{8}V_\mu^k\lambda_k,$$

$$\hat{A}_\mu=\frac{1}{\sqrt{2}}(\hat{X}_\mu^L-\hat{X}_\mu^R)\equiv\frac{1}{\sqrt{2}}\sum_{k=1}^{8}A_\mu^k\lambda_k,$$

respectively.

Higgs-Kibble fields. Higgs-Kibble fields are chosen to be 3×3 matrices $\Phi_{L,R}$ whose columns $\Phi_{L,R}^{(a)}$ ($a=1,2,3$) are triplets transforming as $(3,1)$ and $(1,3)$, respectively, under $SU(3)\otimes SU(3)$. Since $\Phi^{(a)}$'s are triplets with fractional charges in general, and these are to be assigned nonzero VEV's, we must redefine the charge and hypercharge operators if these are to be conserved. We define

$$Q=F_3+\frac{1}{\sqrt{3}}F_8+K_Q,$$

$$Y=\frac{2}{\sqrt{3}}F_8+K_Y.$$

The new quantum numbers K_Q, K_Y are zero for known hadrons. For Higgs-Kibble scalar fields, however, we choose

$$\Phi_{L,R}^{(1)}:\ K_Y=-\tfrac{1}{3},\quad K_Q=-\tfrac{2}{3},$$
$$\Phi_{L,R}^{(2)}:\ K_Y=-\tfrac{1}{3},\quad K_Q=+\tfrac{1}{3},$$
$$\Phi_{L,R}^{(3)}:\ K_Y=+\tfrac{2}{3},\quad K_Q=+\tfrac{1}{3}.$$

This assignment makes the charges and hypercharges of the diagonal elements of Φ's zero.

The Lagrangian of the model is

$$L=L_1+L_2+L_3,$$

where L_1 is the Lagrangian of $0^\pm$ fields, L_2 is the Lagrangian of $1^\pm$ fields and the Higgs-Kibble fields, and L_3 is the interaction Lagrangian of $0^\pm$ fields and the auxiliary Higgs-Kibble fields. These pieces are written in detail below:

$$L_1=-\tfrac{1}{2}\operatorname{Tr}\{D_\mu M^\dagger D_\mu M\}-\tfrac{1}{2}\mu_0^2\operatorname{Tr}\{M^\dagger M\}$$
$$-\tfrac{1}{2}a(\det M+\det M^\dagger)-b\operatorname{Tr}\{M^\dagger M M^\dagger M\}$$
$$-c(\operatorname{Tr}\{M^\dagger M\})^2,$$

where

$$D_\mu^M=\partial_\mu M-ig(\hat{X}_\mu^L M-M\hat{X}_\mu^R);$$

$$L_2=-\tfrac{1}{2}\operatorname{Tr}\{D_\mu\Phi_L^\dagger D_\mu\Phi_L\}-\tfrac{1}{2}\operatorname{Tr}\{D_\mu\Phi_R^\dagger D_\mu\Phi_R\}$$
$$-V(\Phi_L,\Phi_R)-\tfrac{1}{4}\operatorname{Tr}\{\hat{X}_{\mu\nu}^L\hat{X}_{\mu\nu}^L+\hat{X}_{\mu\nu}^R\hat{X}_{\mu\nu}^R\}$$
$$-\tfrac{1}{4}(\partial_\mu V_\nu^0-\partial_\nu V_\mu^0)^2,$$

where

$$\hat{X}^L_{\mu\nu} = \partial_\mu \hat{X}^L_\nu - \partial_\nu \hat{X}^L_\mu - ig[\hat{X}^L_\mu, \hat{X}^L_\nu],$$

$$D_\mu \Phi_L = \partial_\mu \Phi_L - i\left(g\hat{X}^L_\mu + g_0 \frac{\lambda_0}{2} V^0_\mu\right)\Phi_L$$

(similar definitions hold for $X^R_\mu, D_\mu \Phi_R$);

$$L_3 = -\mathrm{Tr}\{A(\Phi_L^\dagger M \Phi_R)\} + \mathrm{H.c.},$$

where A is the 3×3 matrix

$$A = \begin{pmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}.$$

Of the auxiliary Higgs-Kibble scalars, the "would-be" Goldstone bosons are eliminated by the Higgs-Kibble mechanism.[15] The masses of the remaining Φ fields are assumed to be very large, so that the Φ and M mixing coming from L_3 can be ignored. In this limit of taking the masses of the partners of the would be Goldstone bosons to infinity, L_3 reproduces the Gell-Mann–Oakes–Renner[16] and Glashow-Weinberg[17] type $(3, 3^*)\oplus(3^*, 3)$ symmetry breaking.[18]

VEV's and consistency conditions. To generate the masses of gauge fields and of the pseudoscalar nonet, the following VEV's are given to the Higgs-Kibble fields:

$$\langle\Phi_L\rangle_0 = \langle\Phi_R\rangle_0 = \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix}.$$

As usual, new fields with zero VEV's are defined such that

$$\Phi'_{L,R} = \Phi_{L,R} - \langle\Phi_{L,R}\rangle_0 .$$

On neglecting the mixings between M and Φ', i.e., on assuming the remaining Φ''s to be highly massive, we are left with only a linear term in M, viz., $\frac{1}{2}\mathrm{Tr}\{A'(M+M^\dagger)\}$, where A' is a new constant matrix involving A and f's. The presence of linear terms in M indicates that M also has nonzero VEV's. We set

$$\langle M\rangle_0 = \begin{pmatrix} \rho_1 & 0 & 0 \\ 0 & \rho_2 & 0 \\ 0 & 0 & \rho_3 \end{pmatrix} \equiv \rho .$$

Define $M' = M - \langle M\rangle_0$ such that $\langle M'\rangle_0 = 0$. Equating the linear terms in M' to zero, we get the consistency conditions

$$\mu_0{}^2\rho + a(\det\rho)\rho^{-1} + 4b\rho^3 + 4c\,\mathrm{Tr}\{\rho^2\}\rho = A' . \qquad (2.1)$$

In this model we do not consider the isospin-violating effects, so we set $\rho_1=\rho_2$, $\epsilon_1=\epsilon_2$, and $f_1=f_2$.

Spin-0 and spin-1 mixings. When the Lagrangian is written in terms of M' there is a mixing between spin-1 and spin-0 fields arising from the $\mathrm{Tr}\{D_\mu M^\dagger D_\mu M\}$ term in the Lagrangian. This mixing is of the type $V_\mu \partial_\mu u$ and $A_\mu \partial_\mu v$, and is removed by defining new fields

$$\tilde{V}^i_\mu = V^i_\mu - C^V_{ij}\partial_\mu u_j ,$$
$$\tilde{A}^i_\mu = A^i_\mu - C^A_{ij}\partial_\mu v_j . \qquad (2.2)$$

The coefficients C are determined by ensuring that there is no mixing between the spin-0 and the new spin-1 fields. This redefinition modifies the kinetic-energy terms for the spin-0 fields. The correct kinetic-energy expressions are restored by renormalizing spin-0 fields as

$$\left.\begin{aligned} S_i &= Z_{S_i}{}^{-1/2} u_i \\ P_i &= Z_{P_i}{}^{-1/2} v_i \end{aligned}\right\} \quad \text{for } i \neq 0, 8,$$

and

$$\left.\begin{aligned} S_i &= \zeta^S_{ij} u_j \\ P_i &= \zeta^P_{ij} v_j \end{aligned}\right\} \quad \text{for } i = 0 \text{ and } 8 .$$

ζ's are defined so that there is no mixing between singlet and octet fields and the kinetic-energy expressions have the correct form.

Masses and parameters. The vector and axial-vector meson masses are

$$m_\rho{}^2 = g^2 f_1{}^2 ,$$
$$m_{K^*}{}^2 = \tfrac{1}{2}g^2(f_1{}^2+f_3{}^2) + \tfrac{1}{2}g^2(\rho_1-\rho_3)^2 ,$$
$$m_{A_1}{}^2 = g^2 f_1{}^2 + 2g^2\rho_1{}^2 ,$$
$$m_{K_A}{}^2 = \tfrac{1}{2}g^2(f_1{}^2+f_3{}^2) + \tfrac{1}{2}g^2(\rho_1+\rho_3)^2 ,$$
$$m_{A_8}{}^2 = \tfrac{1}{3}g^2(f_1{}^2+2f_3{}^2) + \tfrac{2}{3}g^2(\rho_1{}^2+2\rho_3{}^2) .$$

The mass matrix for the eight and zeroth mesons is

$$(m_V{}^2)_{00} = \tfrac{1}{3}(2g^2 f_1{}^2 + g^2 f_3{}^2)(g_0/g)^2 ,$$
$$(m_V{}^2)_{08} = \tfrac{1}{3}\sqrt{2}\,(g^2 f_1{}^2 - g^2 f_3{}^2)(g_0/g) ,$$
$$(m_V{}^2)_{88} = \tfrac{1}{3}(g^2 f_1{}^2 + 2g^2 f_3{}^2) .$$

The eigenvalues of this matrix give $m_\omega{}^2$ and $m_\phi{}^2$. The scalar- and pseudoscalar-meson masses are

$$m_{S_\pi}{}^2 = Z_{S_\pi}(\mu^2 + 12b\rho_1{}^2 - a\rho_3) ,$$
$$m_{S_K}{}^2 = Z_{S_K}[\mu^2 + 4b(\rho_1{}^2 + \rho_1\rho_3 + \rho_3{}^2) - a\rho_1] ,$$
$$m_\pi{}^2 = Z_\pi(\mu^2 + 4b\rho_1{}^2 + a\rho_3) ,$$
$$m_K{}^2 = Z_K[\mu^2 + 4b(\rho_1{}^2 - \rho_1\rho_3 + \rho_3{}^2) + a\rho_1] ,$$

where

$$\mu^2 = \mu_0{}^2 + 4c(2\rho_1{}^2 + \rho_3{}^2) ;$$

and the renormalization constants are

$$Z_{S_\pi}=1\,,$$

$$Z_{S_K}=1+\frac{(\rho_1-\rho_3)^2}{f_1^{\,2}+f_3^{\,2}}\,,$$

$$Z_\pi=1+\frac{2\rho_1^{\,2}}{f_1^{\,2}}\,,$$

$$Z_K=1+\frac{(\rho_1+\rho_3)^2}{f_1^{\,2}+f_3^{\,2}}\,.$$

The neutral-scalar-meson masses, $m_{S_\eta}^{\ 2}$ and $m_{S_{\eta'}}^{\ 2}$, are obtained by diagonalizing the matrix:

$$(m_S^{\,2})_{88}=[\mu^2-\tfrac{1}{3}a\rho_1(4-\delta)+4b\rho_1^{\,2}(1+2\delta^2)+\tfrac{16}{3}c\rho_1^{\,2}(1-\delta)^2]\,,$$

$$(m_S^{\,2})_{00}=[\mu^2+\tfrac{2}{3}a\rho_1(2+\delta)+4b\rho_1^{\,2}(2+\delta^2)+\tfrac{8}{3}c\rho_1^{\,2}(2+\delta)^2]\,,$$

$$(m_S^{\,2})_{08}=\frac{\rho_1-\rho_3}{3\sqrt{2}}[24b(\rho_1+\rho_3)+16c(2\rho_1+\rho_3)-2a]\,.$$

Here δ is (ρ_3/ρ_1). The neutral-pseudoscalar-meson masses are determined by simultaneously diagonalizing the kinetic-energy matrix K and the mass matrix:

$$(K)_{88}=1-2g^2\rho_1^{\,2}(1+2\delta)^2/9m_{A_8}^{\ 2}\,,$$

$$(K)_{00}=1-4g^2\rho_1^{\,2}(1-\delta)^2/9m_{A_8}^{\ 2}\,,$$

$$(K)_{08}=-2\sqrt{2}\,g^2\rho_1^{\,2}(1-\delta)(1+2\delta)/9m_{A_8}^{\ 2}\,,$$

$$(m_P^{\,2})_{88}=[\mu^2+\tfrac{1}{3}a\rho_1(4-\delta)+\tfrac{4}{3}b\rho_1^{\,2}(1+2\delta^2)]\,,$$

$$(m_P^{\,2})_{00}=[\mu^2-\tfrac{2}{3}a\rho_1(2+\delta)+\tfrac{4}{3}b\rho_1^{\,2}(2+\delta^2)]\,,$$

$$(m_P^{\,2})_{08}=\left[\frac{2}{3\sqrt{2}}a\rho_1(1-\delta)+\frac{8}{3\sqrt{2}}b\rho_1^{\,2}(1-\delta^2)\right]\,.$$

Inputs. The parameters $gf_1, gf_3, g\rho_1$ are fixed by taking m_ρ, m_{K^*}, and m_{A_1} (1100 MeV) as inputs. g_0/g is fixed by using $(m_\omega^{\ 2}+m_\phi^{\ 2})$ as input. μ^2, b, a are fixed by taking m_π, m_K, and $(m_\eta^{\ 2}+m_{X^0}^{\ 2})$ as inputs. $\delta=\rho_3/\rho_1$ is varied over a suitable range. The best values are obtained for $\delta=1/0.80$. g is fixed from the $K^*K\pi$ width.

All masses come out to be in very good agreement with experimental numbers. The predicted masses and their experimental values are tabulated in Table I.

III. THE WEAK-INTERACTION MODEL

In extending the strong-interaction model of Sec. II to weak interactions, we have two objectives in mind. First, we want to obtain mixings between various particles, so as to generate an "octet-dominant" nonleptonic vertex. Second, we do not want this mixing to break the SU(2) invariance of the strong couplings involving only hadrons; i.e., we want to keep the SU(2)-invariant strong vertices of Sec. II intact.[19] The first of our objectives can be met by any one of the following prescriptions.

(i) Add additional terms in L_3 that violate total Y as defined by us.[20] These are the terms like $\Phi_L^{(2)\dagger}M\Phi_R^{(3)}$, etc. All of these terms can be included by redefining the constant matrix A as

$$A=\begin{pmatrix}\epsilon_1 & 0 & 0\\ 0 & \epsilon_2 & \epsilon_4\\ 0 & \epsilon_5 & \epsilon_3\end{pmatrix}.$$

(ii) Make the VEV's nonzero for even those Φ's for which total Y as defined by us is not zero, viz.,

$$\langle\Phi_L\rangle_0=\langle\Phi_R^\dagger\rangle_0=\begin{pmatrix}f_1 & 0 & 0\\ 0 & f_2 & f_4\\ 0 & f_5 & f_3\end{pmatrix}.$$

It is easy to see that either of the above prescriptions will generate a strangeness-violating weak vertex. Since only bilinear combinations of Higgs-Kibble scalars couple to the hadrons, and our Higgs-Kibble scalars are only triplets under SU(3), octet dominance of the weak vertex is guaranteed in the second case. However, on carrying out any one of the above prescriptions we

TABLE I. Meson masses in MeV. Experimental values for m_π, m_K, $(m_\eta^{\ 2}+m_{X^0}^{\ 2})$, m_ρ, m_{K^*}, $(m_\omega^{\ 2}+m_\phi^{\ 2})$, and m_{A_1} are used as input to fix various parameters. Experimental values are taken from Ref. 1.

	Predicted value	Experimental value		Predicted value	Experimental value
π	140	140	ρ	770	770 ± 10
K	494	494	K^*	892	892.2
η	548	548.8	ω	795	782.7
η'	958	957.6	ϕ	1010	1019.7
S_π	979	976 ± 10	A_1	1100	~1100
S_K	1019	~1100	K_A	1252	1242 ± 10
S_η	1050	993 ± 5	A_8	1304	1286 ± 10
$S_{\eta'}$	718	~700			

shall end up with terms linear in u_6 and v_7 and we shall be forced to assign nondiagonal VEV's to M. This will defeat our second objective of keeping the hadron vertices intact. We must demand that coefficients of u_6 and v_7 remain zero when we generate weak mixings. This can be done in accordance with the consistency condition (2.1) only if we execute both the prescriptions detailed above simultaneously. Thus we change both A and Φ as indicated, and assume $f_4, f_5 \ll f_1, f_2, f_3$ and $\epsilon_4, \epsilon_5 \ll \epsilon_1, \epsilon_2, \epsilon_3$. This assumption is only a statement of the fact that the weak mixings are much weaker than the strong couplings.

The nonleptonic Lagrangian that we obtain is depicted below. The parity-conserving part is

$$L^{\rm NL}_{\rm p.c.} = -\tfrac{1}{2}g^2(f'+f'') \times \bar{S}[V_{\rho\mu}V_{K\mu} + (V_{11\mu}+V_{33\mu})V_{K\mu} + A_{\rho\mu}A_{K\mu} + (A_{11\mu}+A_{33\mu})A_{K\mu}] + \text{H.c.}$$

and the parity-violating part is

$$L^{\rm NL}_{\rm p.v.} = -\tfrac{1}{2}g^2(f'-f'') \times \bar{S}[V_{\rho\mu}A_{K\mu} + (V_{11\mu}+V_{33\mu})A_{K\mu} + A_{\rho\mu}V_{K\mu} + (A_{11\mu}+A_{33\mu})V_{K\mu}] + \text{H.c.},$$

where

$$\bar{S} = (0,1),$$

$$V_\rho = \vec{V}_\rho\cdot\vec{\tau}, \quad V_K = \begin{pmatrix} V_{K^+} \\ V_{K^0} \end{pmatrix}.$$

$$V_{11} = \frac{g_0}{g}\frac{V_0}{\sqrt{3}} + \frac{V_8}{\sqrt{6}}, \quad V_{33} = \frac{g_0}{g}\frac{V_0}{\sqrt{3}} - \frac{2V_8}{\sqrt{6}},$$

$$A_\rho = \vec{A}_\rho\cdot\vec{\tau}, \quad A_K = \begin{pmatrix} A_{K^+} \\ A_{K^0} \end{pmatrix},$$

$$A_{11} = \frac{A_8}{\sqrt{6}}, \quad A_{33} = -\frac{2A_8}{\sqrt{6}},$$

and

$$f' = \tfrac{1}{2}(f_2 f_5 + f_4 f_3), \quad \text{and } f'' = \tfrac{1}{2}(f_2 f_4 + f_5 f_3).$$

The V_μ and A_μ are to be expressed in terms of physical fields through Eqs. (2.2).

We may remark here that having fixed all the strong parameters from masses in Sec. II, we here encounter only two unknown parameters, i.e.,

$$g^{\rm NL}_{\rm p.c.} = (f'+f'') \text{ and } g^{\rm NL}_{\rm p.v.} = (f'-f''),$$

one each for the parity-conserving and parity-violating parts. We fix these by introducing one $K^0_S \to 2\pi$ width and one $K\to 3\pi$ width as inputs.

IV. PARITY-VIOLATING EFFECTS

In this section, first we discuss $K^0_S \to 2\pi$ decays. One of the neutral kaon decay widths is used to fit the parity-violating nonleptonic parameter in the model, viz., $g^{\rm NL}_{\rm p.v.}$. K^+ decay is calculated using this parameter and current-algebra estimate of the strength of electromagnetic coupling. Parity-violating baryonic decays are also discussed.

A. $K^0_S \to 2\pi$

The decay width for the two-meson decay process is given by

$$\Gamma(K^n \to \pi^a\pi^b) = \frac{k_{\rm c.m.}}{8\pi m_{K_n}{}^2}|m|^2. \tag{4.1}$$

Here, m_{K_n} is the mass of the decaying kaon. $k_{\rm c.m.}$ is the center-of-mass momentum of the final particles given by

$$k_{\rm c.m.}{}^2 = \frac{[m_{K_n}{}^2 - (m_{\pi_a}+m_{\pi_b})^2][m_{K_n}{}^2 - (m_{\pi_a}-m_{\pi_b})^2]}{4m_{K_n}{}^2} \tag{4.2}$$

and m is the S-matrix element defined by

$$\langle\pi^a\pi^b|S|K^n\rangle = i(2\pi)^4\delta(k_n - p_a - p_b)\frac{1}{(2\pi)^{9/2}}\frac{m}{(8\omega_k\omega_a\omega_b)^{1/2}}. \tag{4.3}$$

The graphs contibuting to K_S decay are drawn in Fig. 1. Of these the contribution of ρ-pole graph is zero. Of the other graphs a typical matrix element is transcribed below:

$$\langle\pi^a\pi^b|m|K^n\rangle = \tfrac{1}{4}g^2[\tau^a,\tau^b]_{2n}g^{\rm NL}_{\rm p.v.}C_\pi Z_\pi{}^{1/2} \times\frac{1}{m_{K^*}{}^2}[g^{(1)}_{K^*K\pi}m_K{}^2 + g^{(2)}_{K^*K\pi}(2m_\pi{}^2 - m_K{}^2)]. \tag{4.4}$$

Equation (4.4) is the matrix element for the K^*-pole contribution. In this equation C_π, Z_π are the renormalization constants introduced in Sec. II. $g_{K^*K\pi}$'s are strong coupling constants. τ's are Pauli spin matrices. The widths $\Gamma(K^0_S\to\pi^+\pi^-)$ and $\Gamma(K^0_S\to 2\pi^0)$ are related by a factor of 2. We have the relation

$$\Gamma(K_S\to\pi^+\pi^-) = \tfrac{1}{2}\Gamma(K_S\to 2\pi^0),$$

which follows from the $\Delta I = \frac{1}{2}$ rule.

B. $K^+\to\pi^+\pi^0$

The $\Delta I=\frac{1}{2}$ rule, built into the model, prohibits all of the diagrams in Fig. 1 from contributing to

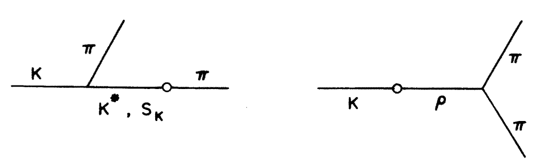

FIG. 1. Feynman graphs contributing to the two-pion decay of neutral kaon. 0 denotes weak parity-violating vertex.

K^+ decay. As can be readily seen from (4.4) the isospin factor is zero for $K^n \equiv K^+$. Since we do not introduce any current × current term in the model, only violations of $\Delta I = \frac{1}{2}$ can arise from the electromagnetic mixing.

We assume electromagnetic mixing of π^0 to be of the type $g^{\text{em}}_{\eta\pi^0}\eta\pi^0$. The graphs that contribute to K^+ decay are shown in Fig. 2(a). The graphs of Fig. 2(b) are proportional to the π^+-π^0 mass difference and are taken into account by using the experimental value of $m_{\pi^+} - m_{\pi^0}$ and using estimates of other (strong) vertices from the model. On evaluation of the matrix element with $g^{\text{em}}_{\eta\pi} = 0.79 \times 10^4/\sqrt{3}$ (Ref. 14) we obtain

$$\frac{\Gamma(K^+ \to \pi^+\pi^0)}{\Gamma(K^0_S \to 2\pi^0)} = 1/17\,000\,,$$

which is to be compared with the experimental number 1/670. In the context of our model electromagnetic effects do not seem to be sufficient to account for the $K^+ \to \pi^+\pi^0$ decay.

C. Baryon decays

Since it is not feasible to start with baryons as the basic fields in the model (see Ref. 13), we treat their decays phenomenologically. We assume SU(3) invariance of the baronic couplings. The coupling of the vector mesons to the baryons is of the form[21]

$$\frac{ig}{\sqrt{2}}\bar{B}\gamma_\mu[V_\mu, B]\,. \tag{4.5}$$

B stands for baryon field. g is the strong coupling constant already described in the case of mesons. Use of the same strong coupling constant as used in the case of mesons is justified on the basis of the conservation of vector charge. For the K^* pole, Eq. (4.5) becomes

$$ig_{B'BK^*}(B'\gamma_\lambda B)K^*_\lambda\,. \tag{4.6}$$

The coupling of scalar mesons to baryons is assumed to be

$$\text{Tr}\{D(\bar{B}[B,S]_+) + F(\bar{B}[S,B]_-)\}\,, \tag{4.7}$$

where D and F are fixed from experimental scalar-meson coupling strengths.

The baryon decays then proceed through K^* and S_K poles. The graphs contributing to the decay processes are shown in Fig. 3. In these the S_K-π, K^*-π couplings are the nonleptonic couplings already evaluated in Sec. IV A.

On evaluating the decay amplitudes, we observe (Table II) that the S_K-pole contribution is negligible compared to the K^*-pole contribution. There is some ambiguity as to the experimental values of scalar-meson couplings. But none of the available values raises the S_K-pole contribution to any significant level. The models ascribing the S_K pole an important role are thus not favored by our analysis.[22]

The total predicted amplitudes of baryon decays are systematically lower than the experimental value by about 20%.

At this point one can try to estimate the effect of the current-current contribution. We have done this in context of the Bars, Halpern, and Yoshimura model.[9] There is no new contribution to $K_S \to 2\pi^0$ decay coming from the current-current interaction. So the parameter $g^{\text{NL}}_{\text{p.v.}}$, which was fixed from $\Gamma(K_S \to 2\pi^0)$ remains unchanged. The ratio $\Gamma(K_S \to \pi^+\pi^-)/\Gamma(K_S \to 2\pi)$ changes to 0.73. This in itself is not very far off from the experimental value 0.69. However, the width $\Gamma(K^+ \to \pi^+\pi^0)$ comes out approximately ten times larger than the experimental value. For the baryon decays there is about 20% *violation* of the $\Delta I = \frac{1}{2}$ rule. These results are clearly unacceptable. One can, at best, only add the $\Delta I = \frac{1}{2}$ part of the cur-

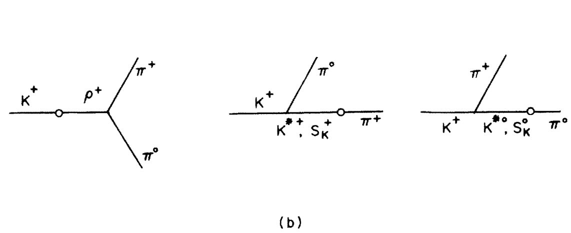

FIG. 2. Feynman graphs showing contributions to $K^+ \to \pi^+\pi^0$ decay. Graphs in Fig. 2(a) are proportional to the η-π^0 mixing and those in Fig. 2(b) are proportional to π^+-π^0 mass difference.

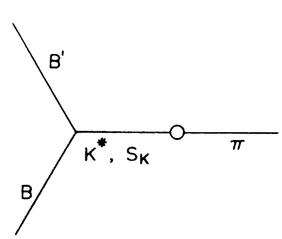

FIG. 3. Feynman graph giving rise to s-wave baryonic decays.

rent-current interaction, assuming the $\Delta I = \frac{3}{2}$ part to be suppressed by the strong-interaction dynamics. However, this is a small correction.[5] We are ignoring it in our further analysis.

V. PARITY-CONSERVING EFFECTS

In this section decay widths and slope parameters for the 3π decays of kaon and the K_L^0-K_S^0 mass difference are calculated.

A. $K\to 3\pi$ decay width

We consider the three-pion decay

$$K_m(k)\to \pi_a(p_a)+\pi_b(p_b)+\pi_c(p_c)\,. \qquad (5.1)$$

The graphs contributing to this process are drawn in Fig. 4. The amplitude A of the process is written as a function of Mandlestam's variables s,t,u defined as

$$\begin{aligned} s &= -(k-p_c)^2 = m_K^2 + m_{\pi_c}^2 - 2\omega_c\cdot m_K\,,\\ t &= -(k-p_a)^2 = m_K^2 + m_{\pi_a}^2 - 2\omega_a\cdot m_K\,,\\ u &= -(k-p_b)^2 = m_K^2 + m_{\pi_b}^2 - 2\omega_b\cdot m_K\,. \end{aligned} \qquad (5.2)$$

In terms of the S-matrix element of the process (5.1) the amplitude $A(s,t,u)$ is defined as

$$\langle \pi_a\pi_b\pi_c|S|K_m\rangle = i\,(2\pi)^4\delta^4(k-p_a-p_b-p_c)(1/2\pi)^6 \times \frac{1}{(16\omega_k\omega_a\omega_b\omega_c)^{1/2}}A(s,t,u)\,. \qquad (5.3)$$

The decay width for the $K\to 3\pi$ process is then written as[23]

$$\Gamma(K_m\to\pi_a\pi_b\pi_c) = \frac{1}{64\pi^3 m_K}\int_{m_{\pi_b}}^{\omega_{\max}} d\omega_b \int_{\omega_-}^{\omega_+} d\omega_a |A|^2\,,$$

where

$$\omega_{\max} = \frac{m_K^2 + m_{\pi_b}^2 - (m_{\pi_a}+m_{\pi_c})^2}{2m_K}$$

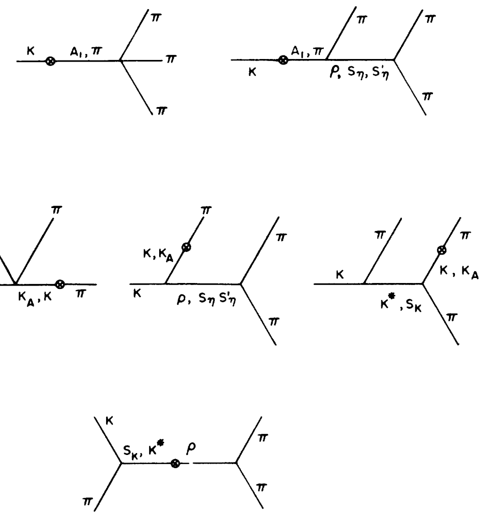

FIG. 4. Feynman graphs contributing to $K\to 3\pi$ decay. $\otimes$ denotes parity-conserving weak vertex.

and $\omega_\pm$ are the solutions of the quadratic equation

$$X\omega^2 + Y\omega + Z = 0\,,$$

where

$$\begin{aligned} X &= 4[(\omega_b - m_K)^2 + \omega_b^2 + m_{\pi_b}^2]\,,\\ Y &= 4(\omega_b - m_K)Q\,,\\ Z &= Q^2 + 4m_{\pi_a}^2(\omega_b^2 - m_{\pi_b}^2)\,, \end{aligned}$$

and

$$Q = m_K^2 - m_{\pi_c}^2 + m_{\pi_a}^2 + m_{\pi_b}^2 - 2m_K\omega_b\,.$$

Isospin structure of $A(s,t,u)$. Since we have the $\Delta I = \frac{1}{2}$ rule, the final pions can be only in the $I=0$ or $I=1$ state. The $I=0$ state is not relevant to K_L and K^+ decays.[24] So the final pions must be in the $I=1$ state and the amplitude for the process $K_m \to \pi_a\pi_b\pi_c$ can be written in terms of only one amplitude $f(s,t,u)$ as follows:

TABLE II. The s-wave baryon decay parameters. $g_{\text{p.v.}}^{\text{NL}}$ is fitted from the $K^0\to 2\pi$ width. All amplitudes are dimensionless numbers. The definition of the s-wave amplitude A and the experimental values are taken from Ref. 1. Our sign convention is the same as the one followed in Ref. 13. Input BBS couplings are $NNS^* = 7.9$ MeV and $NNS = 1.0$ MeV [G. Ebel *et al.*, Nucl. Phys. B33, 317 (1971)].

Process	K^*-pole contribution	S_K-pole contribution	Total amplitude (predicted)	Total amplitude (experimental)
$\Lambda^0\to p\pi^-$	1.13	0.01	1.14	1.48
$\Xi^-\to\Lambda\pi^-$	−1.30	−0.02	−1.32	−2.04
$\Sigma^+\to n\pi^+$	0.00	0.00	0.00	0.06
$\Sigma^-\to n\pi^-$	1.31	0.02	1.33	1.93
$\Sigma^+\to p\pi^0$	−0.93	−0.01	−0.94	−1.48

$$A(K_m \to \pi_a\pi_b\pi_c) = (\tfrac{1}{2})^{1/2}(\vec{A}\circ\vec{\tau})_{2m}K_m\,,$$

where

$$\vec{A}(s,t,u) = f(s,t,u)(\vec{\pi}_b\circ\vec{\pi}_c)\vec{\pi}_a + f(s,u,t)(\vec{\pi}_a\cdot\vec{\pi}_c)\vec{\pi}_b + f(t,s,u)(\vec{\pi}_b\circ\vec{\pi}_a)\vec{\pi}_c\,. \tag{5.4}$$

Defining functions F_S and F_N, symmetric and antisymmetric, respectively, under interchange of π_a, π_b, π_c, viz.,

$$F_S = \tfrac{1}{3}[f(s,t,u) + f(s,u,t) + f(t,s,u)]\,,$$
$$F_n = \tfrac{1}{3}[2f(s,t,u) - f(s,u,t) - f(t,s,u)]\,; \tag{5.5}$$

the amplitudes for the four K-decay processes can be expressed as

$$A(K^+ \to \pi^+\pi^-\pi^+) = \frac{1}{\sqrt{2}}(2F_S - F_N)\,,$$
$$A(K^+ \to \pi^+\pi^0\pi^0) = \frac{1}{\sqrt{2}}(F_S + F_N)\,,$$
$$A(K_L \to \pi^0\pi^+\pi^-) = -(F_S + F_N)\,,$$
$$A(K_L \to 3\pi^0) = -\frac{1}{\sqrt{6}}(3F_S)\,. \tag{5.6}$$

On integrating over the whole phase space the interference of F_S and F_N vanishes for $K^+ \to \pi^+\pi^-\pi^+$ and $K_L \to 3\pi^0$. It is small, approximately 2 to 4%, for $K^+ \to \pi^+\pi^0\pi^0$ and $K_L \to \pi^+\pi^-\pi^0$. The contribution of F_N to the decay rates is of the order of 1%. The widths are calculated by performing phase-space integrals separately for each process, taking into account the available phase space. The parameter $g^{\rm NL}_{\rm p.c.}$ is fixed from one of the decay widths. As displayed in Table III, all decay widths are in good agreement with experiment. *We emphasize that the correct prediction of the branching ratios is a success of the model beyond the consequences of the $\Delta I = \frac{1}{2}$ rule alone.* As is obvious from (5.6), the $\Delta I = \frac{1}{2}$ rule does not relate all the amplitudes with one another.[25]

B. Slope parameters

The slope parameter, σ of Dalitz plot, is defined through a linear expansion of the amplitude A in the variable t as

$$A = A(0)\left[1 + \frac{\sigma}{2m_\pi^2}(t - t_0)\right]\,,$$

TABLE III. $K \to 3\pi$ decay widths in MeV. Experimental values are taken from Ref. 1.

Process	Predicted decay width	Experimental decay width
$K^+ \to \pi^+\pi^0\pi^0$	0.9205×10^{-15}	0.9205×10^{-15} (input)
$K^+ \to \pi^-\pi^+\pi^+$	0.282×10^{-14}	0.2974×10^{-14}
$K_L \to \pi^0\pi^+\pi^-$	0.152×10^{-14}	0.1512×10^{-14}
$K_L \to \pi^0\pi^0\pi^0$	0.287×10^{-14}	0.2707×10^{-14}

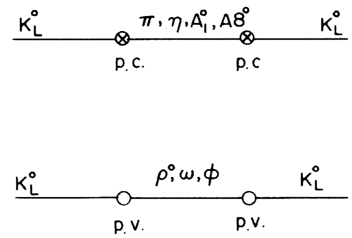

FIG. 5. K^0_L self-energy graphs.

where $t_0 = \frac{1}{3}m_K^2 + m_\pi^2 = (s_0 = u_0)$ is the symmetric point $s = u = t = \frac{1}{3}(m_K^2 + 3m_\pi^2)$, and $A(0)$ is the amplitude at this point. We compute the slope parameter by differentiating A with respect to t at $t = t_0$. Though the model gives excellent results for total widths, the situation in the case of slope parameter is not encouraging. We obtain values correct in order of magnitude, but wrong in sign. (Table IV). It must be emphasized here that the slope-parameter prediction is independent of the strength of weak vertex assumed. In the present model, the slope parameter depends only on the structure of the weak nonleptonic vertex and the strong vertices. The situation is not entirely hopeless since, in principle, one can add terms of form

$$(\Phi_L^{(2)\dagger}MM^\dagger\Phi_L^{(3)} + L \to R)\,,$$

which satisfy all requirements we have imposed on the model (see Sec. III). This means one extra parameter in the model which can be used to fit the slope parameter. However, we will not discuss this possibility any more.

C. K^0_L-K^0_S mass difference

We compute the K^0_L-K^0_S mass difference by considering it as a second-order weak effect, and using the weak coupling constants evaluated in Secs. IV and V A.

The K^0_L-K^0_S mass difference can be written as

$$\Delta m = (m_{K_L} - m_{K_S}) = (\Delta E)_L - (\Delta E)_S\,,$$

where $(\Delta E)_L$ and $(\Delta E)_S$ are self-energies of K_L and K_S arising due to second-order weak processes. As is well known, only pole-term contribu-

TABLE IV. Slope parameters for the $K \to 3\pi$ decays. Experimental values are taken from Ref. 1.

Process	Predicted slope parameter	Experimental slope parameter
$K^+ \to \pi^+\pi^0\pi^0$	-0.27	0.52
$K^+ \to \pi^-\pi^+\pi^+$	0.135	-0.213
$K_L \to \pi^0\pi^+\pi^-$	-0.27	0.61

tions to the self-energy are important[26]. In case of K_S, the only pole terms consistent with CP invariance are S_{π^0} and S_η poles. However, in our model there is no K-S_{π^0} or K-S_η coupling. Hence, all the mass difference arises due to K_L self-energy terms. The graphs contributing to this self-energy are depicted in Fig. 5. On computing all the graphs we obtain $\Delta m \simeq 3\times 10^{10}$ sec^{-1}, which is of the same order of magnitude as the experimental value 0.5398×10^{10} sec^{-1} (see Ref. 1), and has the correct sign.

VI. CONCLUSION

We have developed a realistic, fully gauge-invariant model for the nonleptonic weak decays. Treating the $\Delta I = \frac{1}{2}$ rule as a basic symmetry of the nonleptonic decays we have arranged the nonleptonic vertex in the model to be of explicitly octet type. We have tried to reproduce most of the available information about the nonleptonic decays from the model. In this we have been modestly successful. While the two-pion decays of the neutral kaon, s-wave baryonic decays, parity-conserving kaon decays, and K_L^0-K_S^0 mass difference are explained reasonably well by the model, it fails to explain the data on $K^+ \to \pi^+\pi^0$ decay.

One aspect of the nonleptonic decays that we have completely neglected is the p-wave baryonic decays. A phenomenological treatment of these decays, on the lines of the s-wave decays, would have required computing complicated loop diagrams with baryon poles (Fig. 6). This is, perhaps, stretching the phenomenology too far. An explanation of these decays must await the development of a gauge-invariant model with baryons as basic fields.

We may remark here that the fit of the theory with the experimental information can be improved by adding the $\Delta I = \frac{1}{2}$ part of the current × current interaction to the interaction proposed in this model. In the case of baryon decays, a fit along these lines has already been tried by some authors.[27] They include both the K^*-pole and the octet part of the usual current-algebra contribution. However, this type of exercise in fitting shall increase the number of parameters and destroy the simplicity of our model.

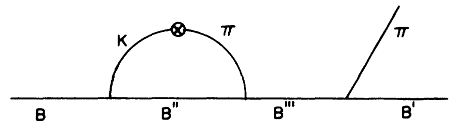

FIG. 6. A typical Feynman diagram for the $B \to B'\pi$ parity-conserving (p-wave) amplitude.

The model proposed in the text can be easily extended to chiral SU(4) × SU(4): Start with basic fields M transforming as $(4, 4^*) + (4^*, 4)$ of the SU(4 × SU(4); for the Higgs fields choose four quartets $\Phi_{L,R}^{(a)}$, $a = 1$ to 4. The Higgs fields now transform as (4, 1) and (1, 4) representations. Going through all the other manipulations, we shall end up with a nonleptonic Hamiltonian that transforms as the $\underline{15}$ representation of SU(4). This is in marked contrast to the current × current interaction. The effective nonleptonic Hamiltonian in the latter case does not have any component transforming as the $\underline{15}$ representation.[28] If, experimentally, the nonleptonic decays of charmed mesons exhibit any $\underline{15}$ dimensional component, it shall be definite evidence in favor of the model proposed here.

ACKNOWLEDGMENT

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[1]See, for the latest $\Delta I = \frac{1}{2}$ fits, Particle Data Group, Phys. Lett. 50B, 1 (1974).

[2]For an early review of this problem, see Roger F. Dashen, Steven F. Frautschi, Murray Gell-Mann, and Yasuo Hara, *The Eightfold Way*, edited by M. Gell-Mann and Y. Ne'eman (Benjamin, New York, 1964), p. 254.

[3]See, for example, V. S. Mathur and K. C. Yen, Phys. Rev. D 8, 3569 (1973); M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33, 108 (1973); G. Altarelli and L. Maiani, Phys. Lett. 52B, 351 (1974).

[4]See, for example, A. Pais, Phys. Rev. Lett. 29, 1712 (1972).

[5]See, A. Salam, Phys. Lett. 8, 216 (1964). This paper refers to B. D'Espagnat, in *Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960*, edited by E. C. G. Sudarshan, J. H. Tinlet, and A. C. Melissines (Interscience, New York, 1960), p. 589; H. Ticho, Brookhaven Conference on Weak Interactions, 1963 (unpublished).

[6]A. Salam (Ref. 5) proposes such an octet-invariant piece in addition to the current × current Hamiltonian.

[7]B. W. Lee and S. B. Treiman, Phys. Rev. D 7, 1211 (1973).

[8]B. W. Lee, J. Primack, and S. B. Treiman, Phys. Rev. D 7, 510 (1973).

[9]I. Bars, M. B. Halpern, and Y. Yoshimura, Phys. Rev. D 7, 1233 (1973).

[10]See R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969), p. 576.

[11]A. K. Kapoor, I. I. T., Kanpur report (unpublished); and Ph.D. thesis, 1975 (unpublished). This model is the same as the strong and electromagnetic parts of the model proposed in Ref. 9.

[12]This possibility was suggested in Ref. 11.

[13]The representations in which the $\frac{1}{2}^+$ octet of baryons can be accommodated are $(1,8)\oplus(8,1)$ and $(3,3)\oplus(3^*,3)$. In both of these cases it is not possible to reproduce masses and coupling constants in the tree approximation to the same accuracy as for mesons. This problem was discussed in Ref. 11.

[14]G. Cicogna, F. Strocchi, and H. Vargara-Caffarelli, Phys. Rev. Lett. 29, 1702 (1972).

[15]P. W. Higgs, Phys. Rev. Lett. 12, 132 (1964); Phys. Rev. 145, 1156 (1966); T. W. B. Kibble, *ibid.* 155, 1554 (1967).

[16]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[17]S. L. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968).

[18]Another renormalizable mixing between $0^\pm$ scalars and Higgs-Kibble fields is possible in this model. We can have

$$L_3' = -\mathrm{Tr}\{B_1\Phi_L^\dagger MM^\dagger\Phi_L + B_2\Phi_R^\dagger M^\dagger M\Phi_R\},$$

where B_1, B_2 are constant matrices. The L_3' term on the completion of Higgs-Kibble mechanism adds a $(1,8)\oplus(8,1)$ breaking term to the Lagrangian. The model with $L_3'+L_3$ is then the spontaneous breaking version of the model developed by R. Arnowitt, M. H. Friedman, P. Nath, and R. Suitor [Phys. Rev. Lett. 26, 104 (1971)] and K. Schilcher [Phys. Rev. D 4, 237 (1971)].

[19]This is possible only in a fully gauge-invariant model. In the models where an explicit symmetry-breaking term is added the SU(2) shall have to be necessarily broken at hadron vertices [see, for example, A. K. Kapoor, Phys. Rev. D 11, 1841 (1974)].

[20]To conserve charge and hypercharge not only is it necessary to redefine the quantum numbers of Φ by adding K_Q and K_Y, but also only those couplings of Φ must be written that conserve K_Q and K_Y. Note that in Sec. II we have scrupulously avoided adding terms like $\Phi_L^{(1)\dagger} M\Phi_R^{(2)}$ or $\Phi_L^{(2)\dagger} M\Phi_R^{(3)}$ that violate K_Q and K_Y, and hence total charge and hypercharge, as defined by us. These terms have been kept out merely by chosing A to be a diagonal matrix. In the case of the weak interactions we relax this condition.

[21]This coupling is the same as one would get if baryons were included in the model and minimal couplings with the gauge fields were written.

[22]Bruce H. J. McKellar and Peter Pick, Phys. Rev. D 8, 265 (1973).

[23]See, G. Källén, *Elementary Particle Physics* (Addison-Wesley, Reading, Mass. (1965), p. 196.

[24]See, for this remark, and also for the rest of the discussion in this section, Ref. 10, p. 549.

[25]Marshak *et al.* (Ref. 10, p. 553) also get the correct branching ratios. However, their analysis includes in addition to the $\Delta I = \frac{1}{2}$ rule, many other assumptions which boil down to making F_N contribution to widths zero.

[26]Marshak *et al.* (Ref. 10, pp. 618–624).

[27]See, for example, Joseph Schecter, Phys. Rev. 174, 1829 (1968).

[28]M. B. Einhorn and C. Quigg, Phys. Rev. D 12, 2015 (1975).