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Angular distribution of dileptons in high-energy hadron collisions*

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We give the general form of the angular distribution of lepton pairs produced in hadron collisions, where each dilepton is assumed to come from the decay of a virtual photon. The Drell-Yan model extended to include parton transverse momentum (with on-shell quarks) is then used to calculate the three coefficients in the distribution.

I. INTRODUCTION

We consider the angular distribution of high-mass muon pairs produced in high-energy collisions of unpolarized hadrons. We will first discuss the form of the general angular distribution, assuming only that the dimuon arises from the decay of a virtual photon. As we will see, the angular distribution is described by three coefficients, $A_n(s, Q^2, y, \vec{Q}_T^{\,2})$ with $n=0,1,2$. We will then discuss these coefficients in the Drell-Yan[1] parton model extended to take transverse momentum into account.

In the Drell-Yan model, the virtual photon that decays into the dimuon is assumed to arise from the annihilation of a quark parton from one of the two hadrons with an antiquark from the other hadron. As Drell and Yan pointed out, one test of the model is to measure the angular distribution of the muons. At asymptotic energies the quarks move parallel to the incident hadrons, so the angular distribution of the dimuons is $1+\cos^2\theta$, where θ is the angle between the muon momentum and the beam axis in the dimuon center-of-mass frame.

A complication arises from the unexpectedly large dimuon transverse momenta $\vec{Q}_T$ observed in recent experiments.[2-5] For instance, at a dimuon mass $(Q^2)^{1/2}$ in the range 4 to 10 GeV the average transverse momentum is[2] $Q_T \sim 1.5$ GeV. Independent of any model, one notes that if $\vec{Q}_T^{\,2}$ were negligible in comparison to Q^2, then the beam momentum and target momentum would be collinear in the dimuon center-of-mass frame, and the angular distribution could be a function only of the angle θ between the muon momentum and a z axis placed parallel to the beam momentum. However, when Q_T is not negligible, the beam and target momenta are not collinear in the dimuon center-of-mass system, and the angular distribution can depend on two polar angles θ and ϕ. Thus the possibilities for experimental information are richer.

One must, of course, define the axes with respect to which the angles θ and ϕ are to be measured. A convenient choice is to place the z axis half way between the beam and target axes, and to measure the azimuthal angle ϕ with respect to the plane formed by the beam and target axes.

As we will see in Sec. II, the general angular distribution with respect to these angles is

$$\frac{dN}{d\Omega} \propto 1+\cos^2\theta + (\tfrac{1}{2}-\tfrac{3}{2}\cos^2\theta)A_0 + 2\cos\theta\sin\theta\cos\phi\, A_1 + \tfrac{1}{2}\sin^2\theta\cos 2\phi\, A_2\,,$$

where the three coefficients A_n are functions of s, Q^2, y, and $\vec{Q}_T^{\,2}$. When $\vec{Q}_T=0$, the scattering has azimuthal symmetry and one finds $A_1=A_2=0$.

The Drell-Yan model applies in the limit $Q^2\to\infty$ with Q^2/s, $\vec{Q}_T$, and the dimuon rapidity y held fixed. It predicts $A_n \to 0$ in this limit. In any reasonable model one would expect power-law corrections to this limit:

$$\begin{aligned} A_0 &= Q^{-2} f_0(Q^2/s, y, \vec{Q}_T^{\,2}) + O(Q^{-4})\,,\\ A_1 &= Q^{-1} f_1(Q^2/s, y, \vec{Q}_T^{\,2}) + O(Q^{-3})\,,\\ A_2 &= Q^{-2} f_2(Q^2/s, y, \vec{Q}_T^{\,2}) + O(Q^{-4})\,. \end{aligned}$$

It would clearly be useful to have some theoretical control over how large the correction coefficients f_n are. We present a parton-model prediction for these coefficients in Sec. III.

The parton-model prediction arises as follows. Imagine the time evolution of the scattering process as described by time-ordered perturbation theory in the dimuon center-of-mass frame. Two quarks with opposite momenta annihilate to form two muons. Since we are using time-ordered perturbation theory, these quarks are on their mass shells, and we take them to have zero mass. The quark axis will, in general, not lie either along the beam momentum or the target momentum or along the z axis defined above. Instead the quark axis will be randomly distributed near these axes according to a distribution function that is related to the transverse-momentum distributions of the partons within each hadron. Thus the $1+\cos^2\theta$ distribution of the angle between the muon axis and the quark axis is smeared according to the parton transverse-momentum distributions.

The angular-distribution coefficients A_n are thus related in this model to the parton transverse-momentum distributions. These distributions can be estimated from the dimuon $\vec{Q}_T$ distribution measured in the same experiment. For instance, a typical parton transverse momentum seems to be about 1 GeV.

It seems unlikely that this parton model tells the whole story about $1/Q$ corrections to the $1+\cos^2\theta$ angular distribution. For instance, there is the competing bremsstrahlung process in which a quark from one hadron emits a photon that decays to the muon pair; the quark then annihilates with an antiquark from the other hadron to form a gluon. There are also corrections to the model from initial- and final-state interactions between the partons in one hadron and those in the other. It is difficult to give a reliable theoretical analysis of the other contributions to the angular coefficients A_n. They appear to be of the same order in $1/Q$ as the contributions in the model of this paper.

In our opinion, this problem would benefit from further theoretical study. For instance, the competing bremsstrahlung process involves a large energy denominator and large momentum transfer in the subprocess quark + antiquark $\rightarrow$ gluon. Does this imply that bremsstrahlung process is suppressed by a factor of the quark-gluon running coupling constant at a momentum of roughly Q? It would also be of interest to see what angular distributions are predicted by other models.

In the meantime, the quark-annihilation model discussed in this paper has the virtues that it is a simple extension of the Drell-Yan model and has some predictive power. In particular, Eq. (4.4) gives a relation in the model between directly measurable quantities.

This paper is laid out as follows. In Sec. II we set up the kinematics and discuss the angular distribution assuming only a one-photon model. In Sec. III we relate the angular distribution to the parton distribution using the parton model discussed above. In Sec. IV we integrate over $\vec{Q}_T$ and ϕ to obtain a particularly simple result, (4.4). Finally, conclusions and some discussion are contained in Sec. V.

II. KINEMATICS

We will use two coordinate frames in this section: (a) a center-of-mass frame of the incident hadrons, which we will normally simply call the c.m. frame; (b) a particular rest frame $\mathcal{O}'$ of the muon pair.

We choose the z axis of the c.m. frame along the beam direction and, for any vector with components V^ν in the c.m. frame, we define $\vec{V}_T=(V^1,V^2)$ and $V^\pm = 2^{-1/2}(V^0 \pm V^3)$. It will be convenient not to specify the x- and y axes and to use a notation that is manifestly covariant under rotations about the z axis. Denote momentum components in the c.m. frame by

P_A^ν for beam momentum,

P_B^ν for target momentum,

l^ν for μ^- momentum,

$\bar{l}^\nu$ for μ^+ momentum,

$Q^\nu \equiv l^\nu + \bar{l}^\nu$ for dimuon momentum.

We make the approximation $P_A{}^2 = P_B{}^2 = l^2 = \bar{l}^2 = 0$ throughout this paper. This amounts to neglecting $M_A{}^2$ and $M_B{}^2$ in comparison to s and neglecting $M_\mu{}^2$ in comparison to Q^2.

By measuring the two muon momenta one can determine five Lorentz-invariant kinematic variables: the dimuon mass Q, rapidity $y=\frac{1}{2}\ln(Q^+/Q^-)$, transverse momentum squared $\vec{Q}_T{}^2$, and the polar angles θ and ϕ with respect to appropriately chosen axes in a rest frame of the dimuon.

We find it useful to choose these axes, which define the $\mathcal{O}'$ reference frame, as follows: In general the beam momentum $\vec{P}'_A$ and target momentum $\vec{P}'_B$ will not be collinear. [As we will see later, the angle between $\vec{P}'_A$ and $-\vec{P}'_B$ is θ_{AB}, where $\tan(\frac{1}{2}\theta_{AB}) = |\vec{Q}_T|/Q$.] Let us choose the z axis of the $\mathcal{O}'$ frame so that it bisects the angle between $\vec{P}'_A$ and $-\vec{P}'_B$, as shown in Fig. 1. Let θ be the angle between the muon momenta and this z axis. Let the azimuthal angle ϕ of $\vec{l}'$ be measured relative to a transverse unit vector $\hat{q}_T$ that lies in the $(\vec{P}'_A, \vec{P}'_B)$ plane in the direction away from $(\vec{P}'_A+\vec{P}'_B)_T$. Thus $\vec{l}'\cdot\hat{q}_T = |\vec{l}'|\sin\theta\cos\phi$. (We again do not specify the direction of the x and y axes of the $\mathcal{O}'$ coordinate system, preferring to maintain a notation that is manifestly covariant under rotations about the z axis.)

These muon polar angles θ and ϕ are given in

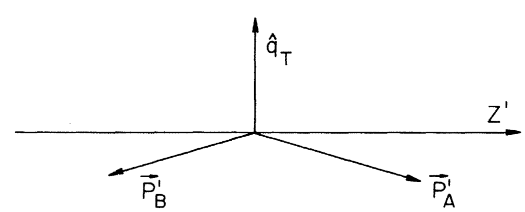

FIG. 1. Illustration of the definition of the reference frame $\mathcal{O}'$.

terms of c.m. frame variables by

$$\cos\theta = 2Q^{-1}(Q^2+\vec{Q}_T{}^2)^{-1/2}(l^+\bar{l}^- - l^-\bar{l}^+)\,,$$
$$\sin^2\theta = Q^{-2}\vec{\Delta}_T{}^2 - Q^{-2}(Q^2+\vec{Q}_T{}^2)^{-1}(\vec{\Delta}_T\cdot\vec{Q}_T)^2\,, \qquad (2.1)$$
$$\tan\phi = \frac{(Q^2+\vec{Q}_T{}^2)^{1/2}}{Q}\,\frac{\vec{\Delta}_T\cdot\hat{R}_T}{\vec{\Delta}_T\cdot\hat{Q}_T}\,,$$

where $\Delta^j = l^j - \bar{l}^j$, $\hat{Q}_T$ is a transverse unit vector in the direction of $\vec{Q}_T$, and $\hat{R}_T$ is a transverse unit vector in the direction of $\vec{P}_A\times\vec{Q}$. (Notice that the right-hand sides of these equations are manifestly invariant under z boosts, so that the equations apply equally well using laboratory-frame variables.)

Assuming that the muon pair arises from the decay of a virtual photon, the muon angular distribution must have the form

$$\frac{dN}{d\Omega} = \frac{d\sigma}{dQ^2dy\,d\vec{Q}_T d\Omega}\Big/\frac{d\sigma}{dQ^2dy\,d\vec{Q}_T}$$
$$= \frac{3}{16\pi}(1+\hat{l}'^i\hat{l}'^jT^{ij})\,, \qquad (2.2)$$

where $\hat{l}' = \vec{l}'/|\vec{l}'|$ and the lower-case latin indices i,j are summed from 1 to 3. The array T^{ij} is a tensor under rotations in the dimuon rest frame and depends on Q^2 and the vectors $\vec{P}'_A$ and $\vec{P}'_B$ but not, of course, on $\hat{l}$. It is normalized to $T^{ii}=1$, so that $\int d\Omega(dN/d\Omega)=1$.

Since T^{ij} depends on the two vectors $\vec{P}'_A$ and $\vec{P}'_B$, covariance under rotations and parity imply that it can be decomposed in terms of three scalar coefficients. For example, one might write

$$T^{ij} = \alpha P'^i_A P'^j_A + \beta P'^i_B P'^j_B + \gamma(P'^i_A P'^j_B + P'^i_B P'^j_A)$$
$$+ (1-\alpha\vec{P}'^2_A - \beta\vec{P}'^2_B - 2\gamma\vec{P}'_A\cdot\vec{P}'_B)\delta^{ij}\,.$$

A more useful set of three scalar coefficients, A_0, A_1, and A_2, can be defined by writing, in the $\mathcal{O}'$ coordinate system,

$$T^{3I} = \hat{q}^I A_1\,,$$
$$T^{IJ} = \tfrac{1}{2}\delta^{IJ}A_0 + (\hat{q}^I\hat{q}^J - \tfrac{1}{2}\delta^{IJ})A_2\,, \qquad (2.3)$$
$$T^{33} = 1 - A_0\,,$$

where the capital latin indices take the values 1, 2. The corresponding angular distribution is

$$\frac{16\pi}{3}\frac{dN}{d\Omega} = 1+\cos^2\theta + (\tfrac{1}{2}-\tfrac{3}{2}\cos^2\theta)A_0$$
$$+ 2\cos\theta\sin\theta\cos\phi A_1 + \tfrac{1}{2}\sin^2\theta\cos 2\phi A_2\,. \qquad (2.4)$$

The coefficients A_n are functions of the variables s, Q^2, y, and $\vec{Q}_T{}^2$.

Notice that if $\vec{Q}_T=0$, so that $\vec{P}'_A$ and $\vec{P}'_B$ both lie along the z axis in the $\mathcal{O}'$ frame, there is no preferred direction for the transverse unit vector $\hat{q}_T$. Thus $dN/d\Omega$ must be independent of ϕ, and, accordingly, A_1 and A_2 must vanish at $\vec{Q}_T=0$. (In fact, one expects[6] that $A_n \propto |\vec{Q}_T|^n$ as $|\vec{Q}_T|\to 0$.) Therefore, if transverse momenta are neglected, one expects in general an angular distribution $1+\frac{1}{2}A_0 + (1-\frac{3}{2}A_0)\cos^2\theta$. One must have a model for the process in order to predict A_0. Drell and Yan predicted[1] that $A_0=0$ in their parton model (neglecting the effects of parton transverse momentum). As we will see in the next section, the Drell-Yan model with parton transverse momenta included leads to two changes from this prediction. First, $\vec{Q}_T$ need not vanish, so that two new coefficients, A_1 and A_2, appear. Second, A_0 need not vanish.

III. THE ANGULAR DISTRIBUTION IN A PARTON MODEL

In this section we seek to evaluate the coefficients A_0, A_1, and A_2 in the extended Drell-Yan model described in the Introduction. These coefficients vanish if partons have no transverse momentum. As we will see, if k_T is a typical parton transverse momentum, A_0 and A_2 vanish like $k_T{}^2/Q^2$, and A_1 like k_T/Q, as $k_T/Q\to 0$. The Drell-Yan model describes the leading behavior of the cross section as $k_T/Q\to 0$; thus we will be able to express the *leading* behavior of A_n for small k_T/Q in terms of parton distribution functions.

In the Drell-Yan model, one views the muon pair as arising from the annihilation of a quark of type a, with momentum k_a^μ, from hadron A with a corresponding antiquark b, with momentum k_b^μ, from hadron B. The quarks are approximately on mass shell and we take them to have negligible mass. Initial-state interactions between hadrons A and B and final-state interactions among the leftover quarks are ignored in the model.[7] Thus the cross section is

$$\frac{d\sigma}{d^4Q d\Omega} = \sum_a \int d^3k_a d^3k_b L_a(\vec{k}_a,\vec{k}_b)\frac{d\sigma(a+b\to\mu^+\mu^-)}{d^4Qd\Omega}\,. \qquad (3.1)$$

Here $L_a(\vec{k}_a,\vec{k}_b)$ is the luminosity of the quark-antiquark colliding beam (per unit $d\vec{k}_a d\vec{k}_b$) divided by the hadron beam luminosity, $d\sigma(a+b\to\mu^+\mu^-)/d^4Qd\Omega$ is the quark-antiquark cross section, and there is a sum over all flavors and colors of quarks a, with $b=\bar{a}$.

We discuss the quark-antiquark cross section first. It is

$$\frac{d\sigma(a+b\to\mu^+\mu^-)}{d^4Qd\Omega} = \frac{4\pi\alpha^2}{3}\frac{e_a{}^2}{Q^2}\delta^4(k_a^\nu + k_b^\nu - Q^\nu)$$
$$\times\frac{3}{16\pi}(1+\hat{l}'^i\hat{l}'^jt^{ij})\,, \qquad (3.2)$$

where

$$t^{ij}=(k_a'-k_b')^i(k_a'-k_b')^j/(\vec{k}_a'-\vec{k}_b')^2\,. \tag{3.3}$$

The primes here indicate vector components in the muon-pair rest frame $\mathcal{O}'$ described in the previous section. This is the cross section with unpolarized quarks. (Symmetry under parity times time reversal forbids a net quark polarization.)

In order to use this result, we must reexpress t^{ij} in terms of the components of k_a^μ and k_b^μ in the c.m. frame. This task is simplified by the fact that in order to extract the coefficients A_n to lowest order in transverse momenta we need only evaluate t^{3I} and t^{IJ} (for $I,J=1,2$) to lowest order in transverse momenta.

To begin, one verifies that the $\mathcal{O}'$ reference frame is reached from the c.m. frame by the following two steps. First, boost along the z axis to an intermediate frame $\mathcal{O}^*$ in which $(Q^3)^*=0$. The vectors $\vec{P}_A^*$ and $\vec{P}_B^*$ thus remain parallel to the z^* axis, while $\vec{Q}_T^*=\vec{Q}_T$ remains nonzero. Second, boost in the $-\vec{Q}_T$ direction through a boost angle ω with $\sinh\omega=|\vec{Q}_T|/Q$ to the $\mathcal{O}'$ frame. In the $\mathcal{O}'$ frame, $\vec{Q}'=0$ and $(Q^0)'=Q$. The vectors $\vec{P}_A'$ and $\vec{P}_B'$ now make equal angles $\phi'=\arctan(|\vec{Q}_T|/Q)$ with the z' axis. This method of obtaining $\mathcal{O}'$ defines its transverse axes in terms of those of the c.m. frame. Thus the vector $\hat{q}_T$, defined in the previous section to be a transverse unit vector lying in the plane of $\vec{P}_A'$ and $\vec{P}_B'$ in the direction away from $(\vec{P}_A'+\vec{P}_B')_T$, is in fact equal to $\vec{Q}_T/|\vec{Q}_T|$, as suggested by the notation chosen for it.

Now consider the vector $(k_a'-k_b')^\mu$ in the $\mathcal{O}'$ frame. Its $\mu=0$ component is zero and its three-vector part has length $|\vec{k}_a'-\vec{k}_b'|=Q$. The boost through the small angle ω relates $\mathcal{O}'$ components to $\mathcal{O}^*$ components[8]:

$$\vec{k}_a'-\vec{k}_b'=\vec{k}_a^*-\vec{k}_b^*+\hat{q}_T[\hat{q}_T\cdot(\vec{k}_a^*-\vec{k}_b^*)_T]\left(\frac{1}{\cosh\omega}-1\right).$$

Since $\cosh\omega-1=O(\vec{Q}_T^{\,2}/Q^2)=O((\vec{k}_{aT}+\vec{k}_{bT})^2/Q^2)$, we can neglect the second term above. We also note that transverse components of $\vec{k}_a-\vec{k}_b$ are unchanged by the z boost from the $\mathcal{O}^*$ frame to the c.m. frame. Thus

$$(k_a'-k_b')^I\approx(k_a-k_b)^I,\quad I=1,2\,, \tag{3.4}$$

where the unprimed vectors denote components in the c.m. frame. Finally, for $(k_a'-k_b')^3$ we have

$$\begin{aligned}(k_a'-k_b')^3&=[Q^2-(\vec{k}_{aT}'-\vec{k}_{bT}')^2]^{1/2}\\&\approx[Q^2-(\vec{k}_{aT}-\vec{k}_{bT})^2]^{1/2}\,,\end{aligned}$$

or, to lowest order in transverse momentum,

$$(k_a'-k_b')^3\approx Q\,. \tag{3.5}$$

Using (3.4) and (3.5) we can write t^{ij} in terms of c.m. variables as $(I,J=1,2)$:

$$\begin{aligned}t^{IJ}&\approx Q^{-2}(k_a-k_b)^I(k_a-k_b)^J\,,\\t^{I3}&\approx Q^{-1}(k_a-k_b)^I\,.\end{aligned} \tag{3.6}$$

For the sake of completeness we note that T^{33} is given, correct to order $k_T^{\,2}/Q^2$, by

$$t^{33}=1-t^{II}\approx1-(\vec{k}_{aT}-\vec{k}_{bT})^2/Q^2\,.$$

We can now return to the cross section $d\sigma/d^4Q\,d\Omega$. We define for any function $f(\vec{k}_a,\vec{k}_b)$ of the quark momenta the expectation value $\langle f\rangle$ weighted according to the probability that a muon pair of a given Q^μ was produced by quarks of momentum $\vec{k}_a$ and $\vec{k}_b$.

$$\begin{aligned}\langle f\rangle=N\sum_a e_a^{\,2}\int d^3k_a d^3k_b\, f(\vec{k}_a,\vec{k}_b)L_a(\vec{k}_a,\vec{k}_b)\\\times\delta^4(k_a^\nu+k_b^\nu-Q^\nu)\,.\end{aligned} \tag{3.7}$$

The normalization N is such that $\langle 1\rangle=1$. With this notation, the cross section [Eqs. (3.1) and (3.2)] is

$$\frac{d\sigma}{d^4Q\,d\Omega}=\frac{d\sigma}{d^4Q}\frac{3}{16\pi}(1+\hat{l}'^i\hat{l}'^jT^{ij})\,, \tag{3.8}$$

where

$$T^{ij}=\langle t^{ij}\rangle \tag{3.9}$$

and

$$\begin{aligned}\frac{d\sigma}{d^4Q}=\frac{4\pi\alpha^2}{3}\frac{1}{Q^2}\sum_a e_a^{\,2}\int d^3k_a d^3k_b L_a(\vec{k}_a,\vec{k}_b)\\\times\delta^4(k_a^\nu+k_b^\nu-Q^\nu)\,.\end{aligned}$$

The coefficients A_n that determine the angular distribution can be extracted by using their definition, Eq. (2.3):

$$\begin{aligned}A_0&=\langle t^{II}\rangle\\A_1&=\langle\hat{q}_T^I t^{3I}\rangle\\A_2&=2\langle\hat{q}_T^I\hat{q}_T^J t^{IJ}\rangle-A_0\,.\end{aligned}$$

Using $\hat{q}_T=\vec{Q}_T/|\vec{Q}_T|=(\vec{k}_{aT}+\vec{k}_{bT})/|\vec{Q}_T|$ and the expressions (3.6) for t^{ij}, we obtain

$$\begin{aligned}A_0&\approx Q^{-2}\langle(\vec{k}_{aT}-\vec{k}_{bT})^2\rangle\,,\\A_1&\approx Q^{-1}|\vec{Q}_T|^{-1}\langle\vec{k}_{aT}^{\,2}-\vec{k}_{bT}^{\,2}\rangle\,,\\A_2&\approx2Q^{-2}|\vec{Q}_T|^{-2}\langle[\vec{k}_{aT}^{\,2}-\vec{k}_{bT}^{\,2}]^2\rangle-A_0\,.\end{aligned} \tag{3.10}$$

For k_T/Q small, A_0 and A_2 are of order $(k_T/Q)^2$ and A_1 is of order (k_T/Q); the corrections to Eq. (3.10) are smaller by a factor $(k_T/Q)^2$.

The cross section $d\sigma/d^4Q$ and the expectation values $\langle f\rangle$ are expressed in terms of the quark-antiquark luminosity function $L(\vec{k}_a,\vec{k}_b)$, about which nothing has been said so far. Consider hadron A, which has a large + component of momentum $P_A^+=[\tfrac{1}{2}s]^{1/2}$ and has $\vec{P}_{AT}=P_A^-=0$. Let $\mathcal{P}_{a/A}(x_a,\vec{k}_{aT}^{\,2})$ be the probability of finding a parton of type a in A

carrying transverse momentum $\vec{k}_{aT}$ and + component of momentum $k_a^+ = x_a P_A^+$. Similarly, hadron B has $P_B^- = [\frac{1}{2}s]^{1/2}$ and we define $\mathcal{P}_{b/B}(x_b, \vec{k}_{bT}^2)$ with $k_b^- = x_b P_B^-$.

In terms of $\mathcal{P}(x, \vec{k}_T^2)$ we have, in the Drell-Yan model,

$$d^3k_a d^3k_b L(\vec{k}_a, \vec{k}_b)\delta^4(k_a^\nu + k_b^\nu - Q^\nu)$$

$$\approx \frac{2}{s} dx_a \delta(x_a - Q^+/P_A^+) dx_b \delta(x_b - Q^-/P_b^-) d^2k_{aT} d^2k_{bT}$$

$$\times \mathcal{P}_{a/A}(x_a, \vec{k}_{aT}^2)\mathcal{P}_{b/B}(x_b, \vec{k}_{bT}^2)\delta(\vec{k}_{aT} + \vec{k}_{bT} - \vec{Q}_T).$$

There are corrections to this relation that are smaller by factors of k_T/Q. However, in order to use Eq. (3.10) to calculate the A_n to leading order, we need only this leading approximation to L.

(There is, for instance, a relativistic correction to L due to the fact that the quark and antiquark do not collide head-on in the c.m. frame. Such k_T^2/Q^2 corrections cannot be reliably calculated within the model since it is kinematically inconsistent to assume that a massless hadron consists of free massless constituents with $\vec{k}_{aT} \neq 0$. Thus it is fortunate that the corrections are not needed.)

We thus obtain[9]

$$\frac{d\sigma}{dQ^2 dy d^2Q_T} \approx \frac{4\pi\alpha^2}{3sQ^2} \sum_a e_a^2 \int d^2k_{aT} \int d^2k_{bT} \delta(\vec{k}_{aT} + \vec{k}_{bT} - \vec{Q}_T)\mathcal{P}_{a/A}(x_a, \vec{k}_{aT}^2)\mathcal{P}_{\bar{a}/B}(x_b, \vec{k}_{bT}^2), \quad (3.11)$$

with $x_a x_b = Q^2/s$ and $x_a/x_b = Q^+/Q^- = e^{2y}$. The expectation values defined in Eq. (3.7) can be replaced by

$$\langle f \rangle = \mathfrak{N} \int d^2k_{aT} \int d^2k_{bT} f(\vec{k}_{aT}, \vec{k}_{bT})\delta(\vec{k}_{aT} + \vec{k}_{bT} - \vec{Q}_T) \times \sum_a e_a^2 \mathcal{P}_{a/A}(x_a, \vec{k}_{aT}^2) \times \mathcal{P}_{\bar{a}/B}(x_b, \vec{k}_{bT}^2). \quad (3.12)$$

IV. DISTRIBUTIONS INTEGRATED OVER $\vec{Q}_T$

The calculation of the coefficients A_n in (3.10) would require a detailed knowledge of the parton distribution functions. It is convenient to integrate the results over $\vec{Q}_T$ so as to eliminate the $\delta(\vec{k}_{aT} + \vec{k}_{bT} - \vec{Q}_T)$ in (3.12) and the $\vec{Q}_T$ dependence of the coefficients. In the case of A_0 we find an interesting result involving measurable quantities, while for A_1 we find a result in terms of the expectation value of $\vec{k}_{aT}^2 - \vec{k}_{bT}^2$.

First we define

$$\mathcal{A}_0 = \left(\frac{d\sigma}{dQ^2 dy}\right)^{-1} \int d^2Q_T \frac{d\sigma}{dQ^2 dy d^2Q_T} A_0.$$

Then

$$\mathcal{A}_0 \approx \langle\langle(\vec{k}_{aT} - \vec{k}_{bT})^2\rangle\rangle/Q^2, \quad (4.1)$$

where

$$\langle\langle f \rangle\rangle \equiv \mathfrak{N} \int d^2k_{aT} \int d^2k_{bT} f(\vec{k}_{aT}, \vec{k}_{bT}) \times \sum_a e_a^2 \mathcal{P}_{a/A}(x_a, \vec{k}_{aT}^2) \times \mathcal{P}_{\bar{a}/B}(x_b, \vec{k}_{bT}^2). \quad (4.2)$$

We can simplify (4.1) because there is no angular correlation between $\vec{k}_{aT}$ and $\vec{k}_{bT}$. Since $\langle\langle \vec{k}_{aT} \cdot \vec{k}_{bT} \rangle\rangle = 0$, we have

$$\mathcal{A}_0 \approx \langle\langle(\vec{k}_{aT} + \vec{k}_{bT})^2\rangle\rangle/Q^2 = \langle\langle \vec{Q}_T^2 \rangle\rangle/Q^2. \quad (4.3)$$

Using Eq. (2.4) integrated over ϕ to eliminate A_1 and A_2, we can write this result as

$$\left(\frac{d\sigma}{dQ^2 dy}\right)^{-1} \frac{d\sigma}{dQ^2 dy\, d\cos\theta} = \frac{3}{8}\left[\left(1 + \frac{1}{2}\frac{\langle\langle \vec{Q}_T^2 \rangle\rangle}{Q^2}\right) + \left(1 - \frac{3}{2}\frac{\langle\langle \vec{Q}_T^2 \rangle\rangle}{Q^2}\right)\cos^2\theta\right]. \quad (4.4)$$

Notice that $\langle\langle \vec{Q}_T^2 \rangle\rangle$ is the *measured* average of $\vec{Q}_T^2$:

$$\langle\langle \vec{Q}_T^2 \rangle\rangle = \left(\frac{d\sigma}{dQ^2 dy}\right)^{-1} \int d^2Q_T \vec{Q}_T^2 \frac{d\sigma}{dQ^2 dy d^2Q_T}. \quad (4.5)$$

Thus (4.4) provides a test of the parton model used in this paper that is independent of any specific model for the parton distributions $\mathcal{P}(x, \vec{k}_T^2)$.

To obtain a simple result for A_1, we weight the cross section by $|Q_T|$. Define

$$\mathcal{A}_1 = \left(\frac{d\sigma}{dQ^2 dy}\right)^{-1} \int d^2Q_T \frac{d\sigma}{dQ^2 dy d^2Q_T} \frac{|\vec{Q}_T|}{Q} A_1, \quad (4.6)$$

which is a weighted average of the azimuthal asymmetry. Then, using Eq. (3.10) we find

$$\mathcal{A}_1 \approx \langle\langle \vec{k}_{aT}^2 - \vec{k}_{bT}^2 \rangle\rangle/Q^2. \quad (4.7)$$

Thus $\mathcal{A}_1$ measures the difference between the mean square transverse momenta of the annihilating partons.

A third relation, involving A_2 integrated with a factor $\vec{Q}_T^2$, can be derived, but it does not seem to have such a straightforward interpretation.

V. CONCLUSIONS

We have given the general form (2.4) of the angular distribution, assuming only that the dimuon is formed from a single virtual photon. We have then related the coefficients A_0, A_1, and A_2 that appear in this general expression to parton distribution functions, working to lowest order in transverse momenta. [See Eq. (3.10).]

These coefficients A_n are calculated in an extended Drell-Yan model. The original model was proposed to describe muon pair production in the limit $s\to\infty$, $Q^2\to\infty$ with Q^2/s fixed and $\vec{Q}_T^2$ fixed or integrated over. In this limit, the coefficients A_n vanish and the angular distribution becomes $1+\cos^2\theta$. The coefficients A_n describe corrections to the Drell-Yan angular distribution to first and second order in $|\vec{Q}_T|/Q$. We have argued that these corrections can be calculated within the extended model in a straightforward fashion and with no apparent ambiguities. (This is essentially because we are calculating each A_n to the lowest order in $\vec{Q}_T^2/Q^2$ at which it does not vanish.) Nevertheless, one can well worry, as discussed in the Introduction, that our analysis stretches the Drell-Yan model beyond the limits of its validity. Despite these misgivings, we proceed on the grounds that any reasonable model is better than no model at all as a guide for the interpretation of experimental results.

Let us use the experimental and theoretical information at hand to estimate the sign and approximate magnitudes of the coefficients A_n in this model. The coefficient A_0 is

$$A_0 \approx Q^{-2}\langle(\vec{k}_{aT}-\vec{k}_{bT})^2\rangle$$

in the model. Clearly, $A_0>0$. Typical dimuon transverse momenta observed at beam energy 400 GeV and $Q\sim 4-10$ GeV are[2] $Q_T\sim 1.5$ GeV. Thus typical parton transverse momenta must be of order $k_T\sim 1$ GeV, and we expect

$$A_0\sim 2\ \mathrm{GeV}^2/Q^2\,.$$

A precise result in the model for the average value $\mathcal{A}_0$ of A_0 (integrated over $\vec{Q}_T$) is given in Eq. (4.3):

$$\mathcal{A}_0=\langle\!\langle\vec{Q}_T^2\rangle\!\rangle/Q^2\,,$$

where $\langle\!\langle\vec{Q}_T^2\rangle\!\rangle$ is the measured average $\vec{Q}_T^2$. The coefficient A_1 is

$$A_1\approx Q^{-1}|Q_T|^{-1}\langle\vec{k}_{aT}^2-\vec{k}_{bT}^2\rangle$$

in the model. For the reasons given above, we expect the magnitude of A_1 to be roughly

$$|A_1|\sim 1\ \mathrm{GeV}/Q\,.$$

The sign of A_1 is quite interesting. If the beam partons that contribute to the cross section at a given Q^2 and y have more transverse momentum than the target partons, then $A_1>0$. Suppose, for example, that the dimuon rapidity y is positive. Since the momentum fractions of the observed partons obey $x_a/x_b=e^{2y}$, the condition $y>0$ implies that the beam parton has greater x than the target parton. The kinetic "energy" k^- of a (right-moving) parton is $k^-=\vec{k}_t^{\,2}/2\times P^+$. In order to keep the "energy" of wee (i.e., small-x) partons from being too large one must demand that the transverse momentum of wee partons is small. Thus one expects[10] that the typical transverse momentum of partons with momentum fraction x is an increasing function of x. (In other words the large-x valence quarks form a tight bundle in transverse position space, surrounded by a large cloud of wee sea quarks.) Experimental support for this expectation comes from the observation that the average dimuon transverse momentum $\langle\!\langle|Q_T|\rangle\!\rangle$ increases as $Q^2/s=x_ax_b$ increases.[2,3] Therefore, we expect that A_1 is positive for positive y and negative for negative y. It will be very interesting to see if this qualitative prediction is borne out. The final coefficient A_2 is

$$A_2\approx 2Q^{-2}|\vec{Q}_T|^{-2}\langle(\vec{k}_{aT}^2-\vec{k}_{bT}^2)^2\rangle-A_0$$

in the model. Thus we expect A_2+A_0 to be positive and of order $2\ \mathrm{GeV}^2/Q^2$.

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