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Empirical formula for inclusive proton spectra between 10 and 300 GeV*

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An empirical formula which well represents the proton spectra in the inclusive reaction $p + \mathrm{Be} \to p +$ anything between 10 and 300 GeV is constructed.

I. INTRODUCTION

For many purposes, such as planning high-energy experiments, designing high-energy secondary beams, calculating radiation shields, designing total absorption nuclear-cascade detectors, calculating the energy deposition of secondary particles in superconducting magnets, etc., it is useful to construct simple analytical expressions for secondary productions in high-energy collisions. Previous papers deal with pion, kaon, and antiproton productions.[1,2] Constructed in this paper is an empirical formula which gives an excellent representation of the inclusive proton spectra in high-energy collisions between 10 and 300 GeV.

II. SCALING BEHAVIOR OF THE LONGITUDINAL-MOMENTUM DISTRIBUTION

The longitudinal-momentum distribution of the secondary protons appears to be consistent with the linear scaling of secondary pions described in a previous paper.[1] Namely, the longitudinal-momentum spectra as the function of X is linearly proportional to the incident momentum p_i, where $X = p_l/p_m$; p_l is the longitudinal momentum and p_m is the maximum kinematically allowed value of p_l or essentially p_i. Henceforth, we replace p_m by p_i. Such a behavior is illustrated in Fig. 1, in which the 0° production data from Dekkers *et al.*[3] are plotted as the function of X for three incident momenta, 11.8, 18.8, and 23.1 GeV/c. The curves are from formula (1). It is seen that the spectrum moves up linearly with the incident momentum.

The longitudinal-momentum distribution can therefore take the form

$$\frac{d^2\sigma}{dp\,d\Omega}(0^\circ) = A p_i \exp(BX^C), \tag{1}$$

where A is the normalization constant, and B and C are constants determining the scaling functions $\exp(BX^C)$. A least-squares fitting program determined the constants: $A = 1.30$, $B = 5.354$, and $C = 0.6324$. Later, the double differential cross section in units of mb/(GeV/c) sr per nucleus will be converted into number of protons /(GeV/c) sr by dividing by 227 mb, the p-Be absorption cross section.[4]

III. NORMALIZATION OF DATA FROM VARIOUS EXPERIMENTS

Data from five more experiments were utilized in the subsequent analysis of the angular dependence of the momentum spectra. Unfortunately, of the six experiments, no two have overlapping points to allow a direct comparison of the normalization.

The data from Lundy *et al.*[5] at 13.4 GeV/c and those from Allaby *et al.*[6] at 19.2 GeV/c allow one to make reasonably accurate extrapolations to 0° production that would then allow one to check the normalization by means of the scaling behavior discussed in the preceding section. It was immediately noticed that the data of Lundy *et al.* are lower than those of Dekkers *et al.* by a factor of

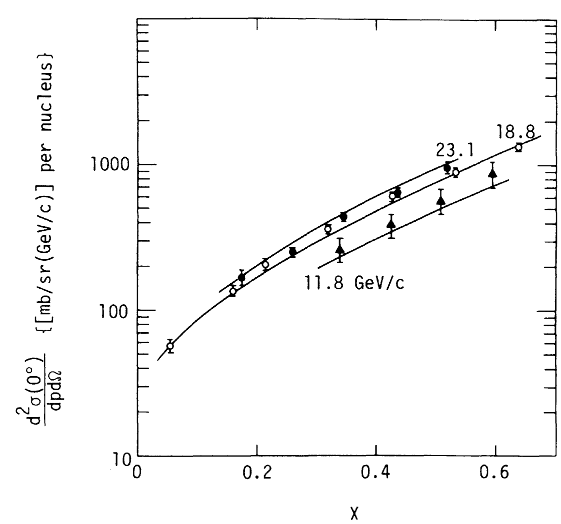

FIG. 1. The 0° production data from Dekkers *et al.* (Ref. 3), plotted as a function of X, indicate a linear scaling with incident momenta. The curves are from formula (1).

~1.5, and the data of Allaby *et al.* are higher than those of Dekkers *et al.* by a factor of ~1.5. Therefore, as an average, the data of Dekkers *et al.* were chosen as the standard of normalization, although there is no guarantee that this is the correct choice. The difficulties with the normalization of this type of experiments are well known. Sometimes the data can be off by a factor of 2.5 to 5, as noted previously.[7]

The final values of the normalization factor for each experiment were determined during the least-squares analysis after the forms of the angular dependence were chosen. The result is that the data of Lundy *et al.*, Allaby *et al.*, Baker *et al.*,[8] and Aubert *et al.*[9] have to be renormalized by factors of 1.5, 0.67, 0.71, and 1.43, respectively with respect to the data of Dekkers *et al.* The external target efficiency of Refs. 8 and 9 is taken as 37%. The internal target efficiency of Ref. 10 is taken as 50% but the data are renormalized by a factor of 1.43.

IV. ANGULAR DEPENDENCE OF THE MOMENTUM SPECTRA

Attempts were made, but to no avail, to represent the angular dependence of the momentum spectra by a simple exponential of the transverse-momentum distribution as has been done successfully in the case of pion production.[1] Instead, a somewhat complicated expression has to be constructed in order to fully reproduce the details of the data.

$$Y=\frac{d^2N}{dp\,d\Omega}=0.005\,726p_i\exp[5.354(p/p_i)^{0.6324}-D\theta^E(p^E-F\cos^G\theta)](1-p/p_i)^{H\theta p/p_i} \qquad (2)$$

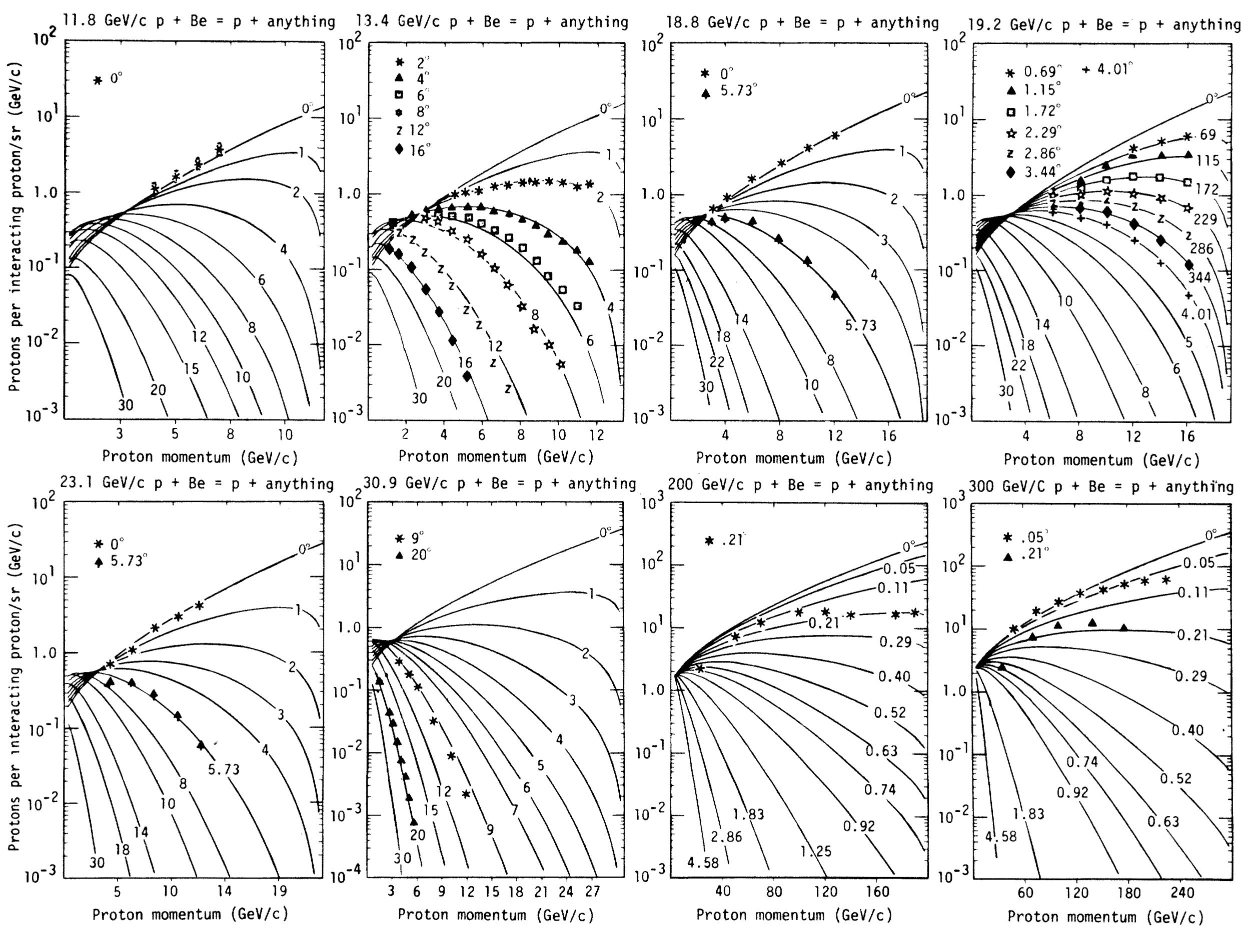

FIG. 2. The inclusive proton spectra from $p+\mathrm{Be}\rightarrow p+$anything, between 10 and 300 GeV where the data exist. The curves were generated with formula (2) after fitting to the data.

gives the number of protons /(GeV/c) sr, where p_i and p are incident and secondary momenta in GeV/c, θ is the production angle in rad. D and E determine the general feature of the transverse-momentum distribution, while F and G take care of the crossing over of the spectra at low momenta. The last characteristic, although not explicit from the limited p-Be data, is clearly demonstrated by the more detailed data of p-p collisions[11] and is indispensable in the present fitting. Such crossing-over characteristic is also clearly seen in the inclusive pion data in p-Be collisions. The last term with the parameter H is required to obtain good fitting to the nonzero-angle large-momentum data. This term has no effect on the longitudinal-momentum distribution because it becomes one when θ becomes zero. It also satisfies the kinematical constraint that the spectra vanish as p becomes p_i.

Least-squares analyses were again carried out to determine the remaining parameters D through H, by minimizing the quantity

$$Q = \frac{1}{F}\sum_i \left(\frac{\log_{10}Y_i^e - \log_{10}Y_i^c}{\Delta Y_i^e/Y_i^e}\right)^2 , \qquad (3)$$

where the superscripts e and c refer to the experimental and calculated values, respectively, ΔY_i^e is the error pertaining to ith exprimental datum, and F is the number of degrees of freedom. All data are assigned 20% errors except where the errors are explicitly provided by the experimenters. The result of the analysis is $Q=0.23$, indicating an excellent fit with $D=3.508$, $E=1.15$, $F=4.129$, $G=37.34$, and $H=20.27$.

V. DISCUSSION

As can be seen from Figs. 1 and 2 , formula (2) fits all data below 30 GeV/c quite well. It is, however, reliable to no better than a factor of 2 for energies above 100 GeV. The fit can be improved when more detailed data above 100 GeV become available. Although it involves no deeper physics other than the scaling, formula (2) provides a simpler yet more accurate way to calculate the inclusive proton spectra than the computer program using the thermodynamical model.[12]

Since there are only minor differences in the inclusive spectra from light elements, formula (2) should be a good approximation for light elements other than Be. Also, for small angles near the kinematic limit of the momentum, there are pronounced elastic or quasielastic peaks. They are not accounted for in the present formula.

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