PHYSICAL REVIEW D VOLUME 16, NUMBER 7 1 OCTOBER 1977

Minimal rule and fragmentation model

Margaret M. Malone and Shui-Yin Lo
School of Physics, University of Melbourne, Parkville, Vic. 3052, Australia
(Received 14 June 1976; revised manuscript received 14 December 1976)

The minimal rule for hadron scattering, proposed by Chou and Yang as a basic part of their fragmentation model, is tested for high-energy pp and π^-p data. We also calculate charge transfer for π^-p. In addition, we investigate the behavior of the fragmentation contribution to inclusive distributions in the central region, the rate of increase allowed by the minimal rule for extra energy-dependent contributions there, and factorization within the fragmentation concept.

I. INTRODUCTION

Several years ago Chou and Yang[1] proposed a minimal rule for hadron collisions, which states that only infinitesimal longitudinal momentum transfer is allowed at asymptotic energies. The rule is supposed to hold independent of a detailed dynamical mechanism and hence is model independent. It is a necessary condition for the success of the fragmentation model.

This minimal rule has been expressed in the form of a sum rule and tested[2] for CERN ISR data at $\sqrt{s}=53$ GeV. Now that new experiments have provided a more complete data set for pp and π^-p collisions, we can compare the fragmentation model to experiment more fully. In particular, we have again calculated the minimal sum rule for pp and also π^-p and a range of energies (Sec. II), and found that the model's asymptotic prediction is well satisfied (at least within 10%) at the highest available energies. We also investigated a charge sum rule for π^-p collisions. The fragmentation model requires[3] that charge transfer drop to zero at asymptotic energies. We calculated how much charge is "carried through" in π^-p collisions by the pion and found that at these energies (up to 40 GeV) there is still significant deviation from the fragmentation prediction. π^-p is a convenient case to study as the initial charges are not symmetric, although the experimentally available energies are still relatively low.

The experimental verification of the minimal rule as calculated in Sec. II provides a solid basis for its incorporation into the specific form of the fragmentation model. This is outlined in Sec. 3, for single-particle distributions.

It is known that the fragmentation mechanism is probably not sufficient to account for behavior of inclusive spectra in the central region. However, even if it is not dominant, the basic features of the model dictate that there should be a fragmentation contribution there, and in Sec. IV we investigate its importance by observing the energy dependence of the difference between particle and antiparticle cross sections at $x=0$. Now the verification of the minimal rule sum (in Sec. II) is not affected if other processes apart from fragmentation contribute to the central region, and in fact it can impose an upper bound on the amount of nonscaling mechanism that can contribute to inclusive cross sections in a finite region of rapidity around $y^*=0$.

We also examine the problem of factorization in fragmentation (Sec. V) and derive the criteria for when the cross section approximately factorizes.

II. SUM-RULE CALCULATIONS

(i) In the fragmentation model hadron-hadron collisions are viewed[1] as two extended objects passing through each other and breaking into fragments in the process. At infinite incident energies the longitudinal momentum transfer is predicted to be only infinitesimal.[4] Therefore one can write the minimal rule

$$X_R=\sum_{x_i>0} x_i = 1 \quad \text{as } s\to\infty \tag{2.1}$$

for processes

$$a+b\to a^+ + b^+ , \tag{2.2}$$

where $a^+=\sum_i a_i$, $b^+=\sum_j b_j$ with sums over all stable products in the right (R) hemisphere. The Feynman variable $x_i=p_{i\parallel}/p_0^*$ is the ratio of the longitudinal momentum of a_i to a in the c.m. system. Note that kinematics alone only requires $|X_R|\le 1$; for example[2] if all secondary particles are produced at rest in the c.m. system then $X_R=0$, or if the two clusters are massive $M_{a^+}+M_{b^+}=\sqrt{2\alpha}P_0^*$ then $X_R=1-\alpha$.

Equation (1) can in principle be tested from present high-energy inclusive data by writing

$$X_R=\frac{1}{\sigma_{\rm tot}}\sum_y\left(\int_0^1 x\frac{d\sigma}{dx}\,dx\right)_y , \tag{2.3}$$

where the sum is over all stable particles y (with $x>0$). A lower bound[2] for X_R from pp collisions

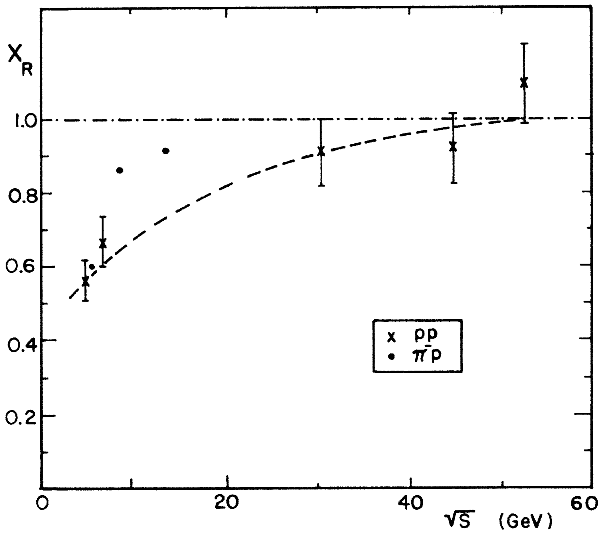

FIG. 1. Minimal rule sum X_R versus total energy $\sqrt{s}$ for pp (crosses) and π^-p (dots). The dashed line is only to guide the eye.

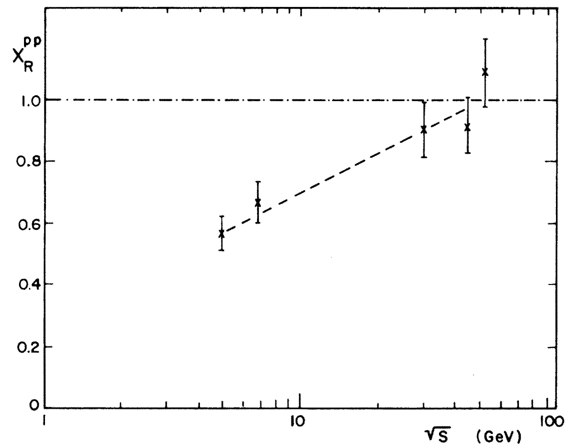

FIG. 2. $X_R^{pp}(\sqrt{s})$ on a logarithmic scale.

was found to be

$$X_R^{pp} > 0.958 \tag{2.4}$$

for data at the then highest available ISR energies (≤ 53 GeV, depending on the product type). We have recalculated this value with more complete data, and also evaluated X_R^{pp} at other energies.[5] The pp points in Figs. 1 and 2 show the behavior of X_R^{pp} as a function of available energy $\sqrt{s}$. Within data and calculation errors the minimal-rule prediction $X_R \to 1$ is well satisfied at the highest present ISR energies. At intermediate energies the increase in X_R^{pp} with $\sqrt{s}$ appears to be logarithmic, as shown in Fig. 2.

Table I shows the values calculated for X_R, together with the contributions to the sum rule[2] by different species of product particles. Errors are estimated to be not greater than 10% throughout, and the centered dots mean that the higher-energy data were not available. The last row shows the "rough estimates" of "fragmentation fractions" for $p \xrightarrow{p} x \cdots$ as calculated by Chou and Yang.[6] Their sum rule calculates $\sum_i M_t^{-1}(E^i - p_{i\parallel})$ in the laboratory frame. The sum is over the fractions of E_0 which become energies of fragments. Asymptotically,

$$\sum_i M_t^{-1}(E^i - p_{i\parallel}) \simeq \sum_i x_i, \quad s \to \infty \tag{2.5}$$

where the right-hand side is calculated in the c.m. frame (R hemisphere) as before. Our calculated values of $\sum_i x_i$ agree quite well with their estimates of $\sum_i M_t^{-1}(E^i - p_{i\parallel})$.

We repeated the minimal-rule calculation for π^-p as a case of unlike projectile and target. The results are shown in Table II and Fig. 1. The points for $X_R^{\pi^-p}$ are only lower bounds as the data set[7] is very incomplete and so do not have error bars; however, it can be seen that the minimal rule is at least very close to being satisfied at these energies. The values for $X_R^{\pi^-p}$ are higher than those for X_R^{pp} at comparable energies, possibly indicating relative transparency of the pion.

(ii) At finite incoming energy the fragmentation model also requires that charge exchange go to zero for hadron-hadron collisions.[3] Explicitly this means that for processes $a + b \to a^+ + b^+$, the

TABLE I. X_R^i versus $\sqrt{s}$, where X_R^i are the fractions of X_R^{pp} for $pp \to c_i +$ (anything).

$\sqrt{s}$ (GeV)	p	n	All π's	K's	Σ's	$\bar{p}$	Λ's	X_R^{pp}
4.93	0.41		0.15	0.004	0.003		0.01	0.57
6.84	0.42	0.03	0.20	0.01	0.01	0.001	0.01	0.67
30.6	0.40	0.06	0.38	0.03	0.01	0.01	•••	0.91
45	0.40	•••	0.37	0.04	•••	0.01	•••	0.92
53	0.48	0.11	0.41	•••	•••	•••	0.02	1.09
Chou and Yang	0.40	0.12	0.40	0.05				

TABLE II. As in Table I, for $X_R^{\pi^- p}$.

$\sqrt{s}$ (GeV)	π^-	π^+	π^0	p	Λ's	K's	$X_R^{\pi^- p}$
5.56	>0.253	0.100	0.078	0.005	0.002	0.036	>0.606
8.72	0.386	0.144	0.144	≥0.006	0.006	0.124	>0.858
13.73	0.488	···	···	···	···	···	>0.910

total charge of a^+ (R-hemisphere products) should be the same as a. One can in principle examine this hypothesis event by event, but a more convenient way is to take the set of single-charged-particle inclusive cross sections and evaluate the charge of a^+ by means of a sum rule. In particular, we used the $\pi^- p$ data[7] to calculate the charge sum rule, defined in analogy to Eq. (3):

$$Q_R = \frac{1}{\sigma_{\text{tot}}} \sum_c \int_0^1 Q_c \frac{d\sigma^c}{dx} dx , \qquad (2.6)$$

where the sum is over all stable (charged) products, charge Q^c (R, L stand for right or left gap particles, respectively, as always) from $\pi^- p \to c + \cdots$. We chose $\pi^- p$ for definite reasons. The pp interactions will automatically yield $Q_R = Q_L = 1$, owing to symmetry properties independent of whether there is charge transfer or not. The $\pi^+ p$ data is also possible, but since the two particles are positive $Q_L + Q_R = 2$, and the charge exchange would not show up as prominently as in $\pi^- p$ processes. Now for $\pi^- p$, $Q_R = -1$ for zero charge transfer, so one expects in the fragmentation model that $Q_R(\sqrt{s})$ should approach -1 as $\sqrt{s} \to \infty$. Only two sets of data were suitable; the results are shown in Table III (errors are estimated ±10%). It is obvious that these values do not yet show any trend towards -1, and so charge exchange is still an important effect at these energies. Therefore, in the terminology of the fragmentation model, at low energy the "spill-over effect[3]" still contributes. More realistically, the low $|Q_R|$ probably indicates the presence of other nonfragmentary central processes, e.g., clusters. The 100-GeV data set for $\pi^- p$ will be important to establish any asymptotic trend, when available. The small drop in $|Q_R^{\pi^- p}|$ going from incident laboratory momentum 16 GeV/c to 40 GeV/c, as shown in Table III, is due to the unusually large $d\sigma^{\pi^+}/dx$ (40 GeV) distribution at $x \simeq 1$ from Serpukhov, and is probably not a real effect.

Figure 3 shows the behavior of the sum charge of particles with momentum $>x_1$:

$$|Q_R^{\pi^- p}(x_1)| = -\frac{1}{\sigma_{\text{tot}}} \sum_i \int_{x_1}^1 Q^i \frac{d\sigma^i}{dx} dx , \quad 0 \leq x_1 \leq 1 \qquad (2.7)$$

at 16 GeV/c incident laboratory momentum, as x_1 increases from zero. This means that one is looking at the decrease in $|Q_R|$ as first the central region and then larger x are gradually excluded from the sum. $|Q_R|$ drops steeply away from its full value [as in (2.4)] in the small-x region and then flattens out, indicating that by far the largest charge contribution comes from the central region. This will maximize the effect of small-x spill-over and central processes in reducing $|Q_R(0)|$.

III. FRAGMENTATION MODEL FOR SINGLE-PARTICLE SPECTRA

Following Refs. 8 and 9, we denote the limiting single-particle distributions by

$$\rho_1(x)dx = \lim_{s\to\infty} \left(\frac{d\sigma}{dx}\right) dx , \qquad (3.1)$$

and then, using the minimal rule,

$$\rho_1(x) = \sigma_1(x) + \tau_1(x) , \quad x > 0 ,$$

where

$$\sigma_1(x) = \text{const} \times \delta(1-x)$$

and

$$\tau_1(x) = \sum_{l=l_{\min}}^{l_{\max}} \rho_{1l}(x) . \qquad (3.2)$$

The $\sigma(x)$ comes from the fragmentation $a \xrightarrow{b} a$ which

TABLE III. Q_R^i versus $\sqrt{s}$, where Q_R^i are the fractions of $Q_R^{\pi^- p}$ for $\pi^- p \to c_i +$ (anything).

p_0^{lab} (GeV/c)	$\sqrt{s}$ (GeV)	π^- elastic	π^- inelastic	π^+	p	$Q_R^{\pi^- p}$
16	5.56	−0.170	−1.064	+0.576	+0.035	−0.62
40	8.72	−0.135	−1.422	+1.025		−0.53

is composed of elastic and single diffractive events, and the $\tau(x)$ is a summation over all exclusive reactions $a \xrightarrow{b} \sum_{i=1}^{l} a_i$. So

$$\rho_{1l}(x) = a_l \int \cdots \int \prod_{i=l_{\min}+1}^{l} dx_i |M_{fi}|^2 \times \delta\left(1 - \sum_{i=l_{\min}}^{l} x_i\right) \quad (x>0) , \tag{3.3}$$

where the δ function represents the minimal rule, and $|M_{fi}|^2$ is the matrix element squared for fragmentation into l particles. All transverse momentum dependence, plus all left-hemisphere ($x_1 < 0$) terms have been integrated out. The normalization[10] is

$$\int \rho_{1l}(x) dx = \sigma(l) n_c \tag{3.4}$$

for $a \xrightarrow{b} c + (l-1)$ other particles, where n_c is the number of particles of type c produced, and $\sigma(l)$ is the exclusive cross section for $a \xrightarrow{b} (l$ particles). We adopt a general polynomial form[8]

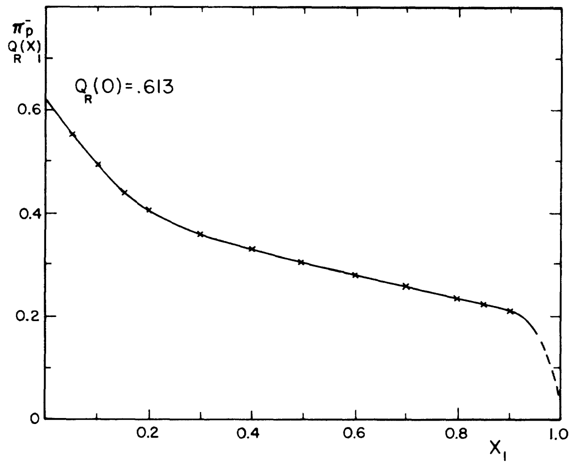

FIG. 3. $Q_R^{\pi^- p}(x_1)$ versus x_1 for $p_{\mathrm{lab}}^0 = 16$ GeV ($\sqrt{s} = 5.56$ GeV).

$$|M_{fi}|^2 = \sum_{\{\alpha\}} c_{\alpha_1 \cdots \alpha_l} x_1^{\beta_1} \cdots x_l^{\beta_l} , \tag{3.5}$$

where $\{c_{\alpha_1 \cdots \alpha_l}\}$ are some constants. Then the integral can be done exactly, and one has

$$\rho_{1l}(x) = a_l \sum_{\{\alpha\}} c_{\alpha_1 \cdots \alpha_l} d_{l-1} d_{l-2} \cdots d_2 x^{\beta_1} (1-x)^{\beta_2 + \cdots \beta_l + (l-2)} \tag{3.6}$$

with constants $d_{l-k} = d(\beta_{l-k}, \beta_l + \cdots + \beta_{l-k+1} + k - 1)$ and

$$d(a,b) = \sum_{k=0}^{b} \frac{(-1)^k b!}{(b-k)!k!(a+k+1)} , \tag{3.7}$$

which have the property $d(0,a) = 1/(a+1)$.

Then, from Eqs. (3.2), (3.4), (3.6) for $a \xrightarrow{b} c +$ (anything) the limiting distribution has the general form

$$\rho_1^c(x) = \text{const} \times \delta(1-x) + \sum_{l=l_{\min}}^{l_{\max}} \sigma(l) n_c \frac{\sum_{\{\alpha\}} c_{\alpha_1 \cdots \alpha_l} d_{l-1} \cdots d_2 x^{\beta_1}(1-x)^{\beta_2 + \cdots + \beta_l + l - 2}}{\sum_{\{\alpha\}} c_{\alpha_1 \cdots \alpha_l} d_{l-1} \cdots d_2 d_1} \quad (x>0) . \tag{3.8}$$

The simplest case is for $|M_{fi}|^2 = 1$, then the inclusive spectrum $\rho(x)$ reduces simply to

$$\rho^c(x) = \text{const} \times \delta(1-x) + \sum_{l=l_{\min}}^{l_{\max}} \sigma(l) n_c (l-1)(1-x)^{l-2} . \tag{3.9}$$

IV. CONTRIBUTION TO THE CENTRAL REGION FROM FRAGMENTATION

(i) It is interesting to note that although the fragmentation mechanism is well known and studied in the $x>0$ region,[4,8,9] it should also contribute, in some degree if not exclusively, to the central region. Here we investigate the fragmentation contribution to the central region at $x=0$.

For a general polynomial matrix element squared as in (3.5), (3.6) shows as $x \to 0$ that all terms in the matrix element sum with $\beta_1 \neq 0$ will go to zero as ϵ^{β_1}. So considering for simplicity only one term $c x_1^{\beta_1} \cdots x_l^{\beta_l}$, then the limiting single-particle distribution for the fragmentation process for $a + b \to c +$ (anything) is

$$\rho_1^c(x) \underset{x\to 0}{\longrightarrow} \begin{cases} 0, & \beta_1 \neq 0 \\ \sum_{l=l_{\min}}^{l_{\max}} \sigma(l) n_c (\beta_2 + \cdots + \beta_l + l - 1), & \beta_1 = 0 \end{cases} \tag{4.1}$$

and its slope is

$$\frac{d\rho_1^c(x)}{dx} \underset{x\to 0}{\longrightarrow} \begin{cases} 0, & \beta_1 \neq 0 \\ \displaystyle\sum_{l=l_{\min}}^{l_{\max}} \sigma(l) n_c(\beta_2 + \cdots + \beta_l + l - 1)(\beta_2 + \cdots \beta_l + l - 2), & \beta_1 = 0\,. \end{cases} \tag{4.2}$$

For example, $|M_{fi}|^2 \sim x_{\text{nucleon}}{}^3$ is found to be sufficient[2] to fit pion data $pp \to \pi^\pm +$ (anything) in the large-x region, so in this case the fragmentation contribution $d\sigma^F/dx$ at $x=0$ would be

$$\frac{d\sigma^F}{dx}\bigg|_{x=0}^{\pi^\pm} = \rho^{\pi^\pm}(x)\bigg|_{x=0} \sim \sum_{l=l_{\min}}^{l_{\max}} \sigma(l) n_{\pi^\pm}(l+2)\,, \tag{4.3}$$

which goes roughly as $\sigma_{\text{tot}}\langle l^2\rangle$ with $\sqrt{s}$.[11]

(ii) However, correlation measurements in the central region suggest that the fragmentation mechanism, which can provide long-range correlations, seems insufficient by itself[12] to account for the existence of short-range correlations. It is then natural to presume that there should exist some other mechanism in addition to the fragmentation mechanism which can contribute to the central region (for example, multiperipheral production, bremsstrahlung-type process, or specific cluster model) and produce an energy-dependent part of the inclusive cross sections centrally and which will give rising central pion distributions. As experimentally one finds that the minimal rule is well satisfied, it is an interesting question to ask what sort of upper bound it would place on the energy dependence of such a contribution to the cross section in the central region.

We split up $d\sigma/dx$ into a fragmentation contribution $d\sigma^F/dx$ which is to scale with $\sqrt{s}$ at infinite energies, plus a central contribution $d\sigma^c/dx$ which increases with $\sqrt{s}$, and then investigate what rate of increase is allowed for $d\sigma^c/dx$ without violating the minimal rule. That is,

$$\frac{d\sigma}{dx} = \frac{d\sigma^c}{dx} + \frac{d\sigma^F}{dx}\,, \tag{4.4}$$

where

$$\frac{d\sigma^c}{dx} = 0, \quad x > \epsilon\,. \tag{4.5}$$

ϵ is some point $0<\epsilon\ll 1$ beyond which the cross sections are observed to scale at high energy. Here F stands for the fragmentation mechanism and c stands for the contribution from other mechanisms in the central region. The minimal rule (2.3) can be rewritten in the form

$$\int_0^1 x\,\frac{d\sigma^i}{dx}\,dx = X_R^i \sigma_{\text{tot}}\,, \tag{4.6}$$

where X_R^i is the fractional contribution to X_R for the product particle c_i concerned such that $\sum_i X_R^i = 1$ from (2.3). (The sum is over all stable products with $x_i > 0$, as usual.)

In order to discover how the minimal rule (4.6) restricts $d\sigma^c/dx$, we expand $d\sigma^c/dx$ into a series:

$$\frac{d\sigma^c}{dx} = \sum_{i=1}^{\infty} \frac{c_{-i}}{(x+\alpha_i)^i} + c_0 + \sum_{j=1}^{\infty} c_j x^j\,, \tag{4.7}$$

with $\{c_i\} \geq 0$ and $\{\alpha_i\} \to 0$ asymptotically, so that all the terms in the first sum with nonzero coefficients will become singular at $x=0$. It is then easy to show that substitution of (4.7) in (4.6) at asymptotic energies requires that all $c_{-i}=0$ for $i \geq 2$. Therefore the most singular limiting behavior allowed by the (experimentally verified) minimal rule for distributions at $x \simeq 0$ is approximately $1/x$. This is consistent with the experimental trend for pion production. To be more precise, we calculate explicitly in the following.

At asymptotic energies substitution of (4.5) and (4.7) into the minimal-rule integral (4.6) (using $c_{-i}=0$ for $i\geq 2$) gives

$$c_{-1}\epsilon + c_0\frac{\epsilon^2}{2} + \sum_{j=1}^{\infty} c_j\,\frac{\epsilon^{j+2}}{j+2} = X_R^i\sigma_{\text{tot}} - F\,, \tag{4.8}$$

where $F = \int_0^1 x(d\sigma^F/dx)dx$ and X_R^i are almost independent of $\sqrt{s}$ at high energy. Therefore the fastest increase with $\sqrt{s}$ allowed for the coefficients c_j is

$$c_j \sim X_R^i\,\frac{\sigma_{\text{tot}}}{\epsilon^{j+2}}\,. \tag{4.9}$$

This means that the most rapid increase at high energy allowed by the minimal rule for the contribution by central mechanisms is

$$\frac{d\sigma^c}{dx} \sim X_R^i\,\frac{\sigma_{\text{tot}}}{\epsilon}\left[\frac{k_{-1}}{x+\alpha} + \frac{k_0}{\epsilon} + \sum_{j=1}^{\infty}\frac{k_j}{\epsilon}\left(\frac{x}{\epsilon}\right)^j\right], \tag{4.10}$$

where the $\{k_i\}$ are some constants (independent of both x and $\sqrt{s}$). This behavior depends crucially on the meaning of ϵ, as will be discussed in (a) and (b) below.

(a) We first consider the case where the energy-dependent central mechanism contributes to $d\sigma/dx$ over some small, but nevertheless finite region in x even at high energies, so that $d\sigma^c/dx$ contributes in (4.4) for $x \leq \epsilon$, where ϵ is a constant.[13] This would mean that inclusive distributions would always only scale over part of the full x range ($|x| > \epsilon$). For ϵ a constant, (4.10) gives the most rapid increase of $d\sigma^c/dx$ allowed by the minimal rule as

$$\frac{d\sigma^c}{dx} \sim X_R^i \sigma_{\rm tot}\left(\frac{k'_{-1}}{x+\alpha} + k'_0 + \sum_{j=1}^{\infty} k'_j x^j\right), \qquad (4.11)$$

where

$$k'_j = \frac{k_j}{\epsilon^{j+2}} \quad (\text{independent of } \sqrt{s})\ .$$

Therefore, in terms of rapidity,

$$\frac{d\sigma^c}{dy^*} \sim X_R^i \frac{\sigma_{\rm tot}}{\sqrt{s}}\left(\frac{k'_{-1}}{x+\alpha} + k'_0 + \sum_{j=1}^{\infty} k'_j x^j\right)\ . \qquad (4.12)$$

So in this case, $d\sigma^c/dx$ does increase with $\sqrt{s}$. However, remembering that $\alpha \to 0$ as $\sqrt{s} \to \infty$ probably at least as fast as $1/\sqrt{s}$, once X_R^i is near its limiting value $d\sigma/dy^*$ then decreases with $\sqrt{s}$ in the region $\frac{1}{2}\sqrt{s}\,\alpha \lesssim y^* \lesssim \frac{1}{2}\sqrt{s}\,\epsilon$ (and $\alpha \ll \epsilon$ at high energies). This disagrees with the experimentally observed behavior of inclusive pion distributions where, except in the fragmentation region ($y^* \gtrsim \frac{1}{2}\sqrt{s}\,\epsilon$), the $d\sigma/dy^*$ distributions are rising slowly with $\sqrt{s}$.

(b) We can alternatively demand only that any energy-dependent mechanism contribute to $d\sigma/dx$ over a finite region in y^* at high energies. That is, $d\sigma^c/dy^*$ contributes to $d\sigma/dy^*$ for $|y^*| \le L$, where L is a constant, so $d\sigma^c/dx$ in (4.4) is nonzero for $x \lesssim \epsilon = K/p_0^*$, and $\epsilon \sim 1/\sqrt{s}$ in this case (using $p_0^* \simeq \frac{1}{2}\sqrt{s}$). If we identify the energy-dependent central mechanism as a cluster contribution, then L is the correlation length and $L \approx 1$ unit of rapidity. For pion production, $\epsilon = K/p_0^*$ with $K \simeq 0.4$. So as $\sqrt{s} \to \infty$, $|x| \lesssim \epsilon \to 0$ and the analysis in (a) (for constant ϵ) which disagreed with experiment does not now apply. The most rapid increase allowed by the minimal rule is now

$$\frac{d\sigma^c}{dx} \sim X_R^i \sigma_{\rm tot}\, p_0^*\left[\frac{k''_{-1}}{x+\alpha} + k''_0 p_0^* + \sum_{j=1}^{\infty} p_0^* k''_j (x p_0^*)^j\right] \qquad (4.13)$$

from (4.10), where $\{k''_j\}$ are again independent of x and $\sqrt{s}$. In terms of rapidity,

$$\frac{d\sigma^c}{dy^*} \sim X_R^i \sigma_{\rm tot}\left[\frac{k''_{-1}}{x+\alpha} + k''_0 p_0^* + p_0^* \sum_{j=1}^{\infty} k''_j (x p_0^*)^j\right], \qquad (4.14)$$

where $(xp_0^*)^j \le (\epsilon p_0^*)^j = (0.4)^j$ so the sum in (4.14) converges. Equations (4.13), (4.14) show that $d\sigma^c/dx$, $d\sigma^c/dy^*$ can now both increase with $\sqrt{s}$; the fastest rates allowed are roughly

$$\frac{d\sigma^c}{dx} \sim X_R^i \sigma_{\rm tot}\,(p_0^*)^2$$

and (4.15)

$$\frac{d\sigma^c}{dy^*} \sim X_R^i \sigma_{\rm tot}\, p_0^*, \quad \sqrt{s} \simeq 2p_0^*\ .$$

In (a) and (b) above, we have analyzed the consequences of allowing an energy-dependent production mechanism in the central region in addition to the fragmentation contribution within the restriction of the minimal rule (4.6). We found that the experimentally observed rising behavior of $d\sigma/dy^*$ pion distributions in the central region is allowed by the minimal rule if the energy-dependent mechanism is restricted to a finite region in rapidity space as $\sqrt{s} \to \infty$. The mechanism cannot however, be allowed to apply asymptotically over a finite-x region.

(iii) It is interesting to note that the simple cluster model[12] or the like generally require that cross sections for production of particle and antiparticle be equal in the central region, e.g., $\rho_1(p \to c + \text{anything}) = \rho_1(p \to \bar{c} + \text{anything})$. Now the fragmentation process must contribute to the central region as well as any such cluster-type processes, so it is nice to look at the difference between the two cross sections from the fragmentation mechanism for specific products c and $\bar{c}$.

Experiment[14] shows that the invariant cross sections for $pp \to p + \cdots$ and $pp \to \bar{p} + \cdots$ are approaching each other at $x=0$ as $\sqrt{s}$ increases. To investigate what behavior the fragmentation model predicts for the fragmentation contribution at $x=0$, let us just look at terms $Cx_1^{\beta_1}\cdots x_l^{\beta_l}$ in $|M_{fi}|^2$ again, so that the model gives

$$\left.\frac{d\sigma_{c_1}^F}{dx}\right|_{x=0} = \rho_1^{c_1}(0) = \sum_{l=l_{\min}}^{l_{\max}} \sigma(l) n_{c_1}(\beta_2 + \cdots + \beta_l + l - 1) \qquad (4.16)$$

(if $\beta_1 = 0$) for $pp \to c_1 + (\text{anything})$ at $x=0$. We now investigate exclusive processes which will contribute to

$$\Delta_{p-\bar{p}}^F = \left.\left(\frac{d\sigma^{p\xrightarrow{p} p\cdots}}{dx} - \frac{d\sigma^{p\xrightarrow{p}\bar{p}\cdots}}{dx}\right)\right|_{x=0}\ . \qquad (4.17)$$

Now the exclusive processes

$$\begin{aligned} &p \to p\pi^0\ ,\\ &p \to p\pi^0\pi^0\\ &\cdot\\ &\cdot \qquad\qquad\qquad\qquad (4.18)\\ &\cdot\\ &p \to p + (l-1)\pi^0 \end{aligned}$$

all contribute to $d\sigma^{p\xrightarrow{p}p\cdots}/dx$ and are positive, but do not all to $d\sigma^{p\xrightarrow{p}\bar{p}\cdots}$, so they provide a positive and increasing (with $\sqrt{s}$) term in $\Delta_{p-\bar{p}}^F$. The exclusive processes

$$
\begin{aligned}
&p \to p(\bar{p}p)\ ,\\
&\quad\vdots \qquad\qquad\qquad\qquad\qquad\qquad (4.19)\\
&p \to p + \left(\frac{l-1}{2}\right)(\bar{p}p)
\end{aligned}
$$

contribute to both $d\sigma^{p \xrightarrow{p} p \cdots}/dx$ and $d\sigma^{p \xrightarrow{p} \bar{p} \cdots}/dx$, and their difference at $x=0$ is not zero because $n_p = n_{\bar{p}} + 1$. We have $n_p = \frac{1}{2}(l+1)$, $n_{\bar{p}} = \frac{1}{2}(l-1)$ from (4.19), so, if we substitute these in (4.16), we obtain a term

$$
\Delta^F_{p-\bar{p}} \sim \sum_{l=3,5,\ldots}^{l_{\max}} \sigma(l)(\beta_2 + \cdots + \beta_l + l - 1)\ , \tag{4.20}
$$

which increases with $l_{\max}$, i.e., increases as $\sqrt{s}$ increases. (The form of the energy dependence may be slightly modified by an overall $\sum_{\{\alpha\}} C_{\{\alpha\}}$ if $|M_{fi}|^2$ has more than one term.)

Another significant set of processes which will also contribute to $(d\sigma^{p \xrightarrow{p} \bar{p} \cdots}/dx)|_{x=0}$ is

$$
\begin{aligned}
&p \to p(\bar{p}p)\pi^0\ ,\\
&\quad\vdots \qquad\qquad\qquad\qquad\qquad\qquad (4.21)\\
&p \to p(\bar{p}p) + (l-3)\pi^0\text{'s}\ .
\end{aligned}
$$

Here $n_p = 2$, $n_{\bar{p}} = 1$, and these processes give a term

$$
\Delta^F_{p-\bar{p}} \sim \sum_{l=4,5,\ldots}^{l_{\max}} \sigma(l)(2\beta_{\bar{p}} + \beta_{\pi^0 3} + \cdots + \beta_{\pi^0 l} + l - 1)\ , \tag{4.22}
$$

which again increases with $\sqrt{s}$ in a rather similar form to (4.20). Processes such as

$$
\begin{aligned}
&p \to p(\bar{p}p)(\pi^+\pi^-)\ ,\\
&p \to p(\bar{p}p)(\pi^+\pi^-)(\pi^+\pi^-)\ ,\\
&\quad\vdots \qquad\qquad\qquad\qquad\qquad\qquad (4.23)\\
&p \to p(\bar{p}p) + \left(\frac{l-3}{2}\right)(\pi^+\pi^-)
\end{aligned}
$$

and the equivalent kaon processes give similar behavior to (4.20).

Therefore the fragmentation contribution at $x=0$ predicts that

$$
\Delta^{p-\bar{p}} \equiv \left(\frac{d\sigma^{p \xrightarrow{p} p \cdots}}{dx} - \frac{d\sigma^{p \xrightarrow{p} \bar{p} \cdots}}{dx}\right)\Bigg|_{x=0} \tag{4.17}
$$

should at least increase with $\sqrt{s}$. Processes like (4.18) with no $\bar{p}$'s produced contribute constructively to $d\sigma^{p \to p \cdots}/dx$ alone; those that contribute to both p and $\bar{p}$ inclusive distributions, such as (4.19) and (4.21), still give increasing functions of $\sqrt{s}$, as shown by (4.20) and (4.22), for example. Further, the dependence of (4.20) on $\sqrt{s}$ should give a lower bound on that for the inclusive $\Delta^F_{p-\bar{p}}$. In Fig. 4 we use the pure phase-space approximation $|M_{fi}|^2 = \text{const}$ (i.e., all $\beta_i = 0$) and $\sigma(l) \sim 1/l(l-1)$ to graph the energy dependence of (4.20) (dashed line) and compare it to the data.[14] The error bars are too large to be quantitative, but we suggest that the data is entirely consistent with our picture of a significant fragmentation contribution to the $d\sigma/dx$ distributions in the central region.

(iv) We also investigated the difference between the distributions for fragmentations $p \xrightarrow{p} \pi^{\pm} + \cdots$ in the central region. Considering the exclusive processes

$$
p \to p + n_1(\pi^+\pi^-\pi^0)\ , \tag{4.24a}
$$

$$
p \to n\pi^+ + n_1(\pi^+\pi^-\pi^0)\ , \tag{4.24b}
$$

$$
p \to p\pi^0 + n_1(\pi^+\pi^-\pi^0) \tag{4.24c}
$$

as candidates, only (4.24b) contributes to $\Delta(d\sigma/dx)|_{x=0}$ in the fragmentation model, because it is the only one where $n_{\pi^+} \neq n_{\pi^-}$ [see (4.16)]. As in Sec. IV (iii), we use (4.16) and $n_{\pi^+} = \frac{1}{3}(l+1)$, $n_{\pi^-} = \frac{1}{3}(l-2)$ to obtain

$$
\begin{aligned}
\Delta^F_{\pi^+-\pi^-} &\equiv \left(\frac{d\sigma^{p \to \pi^+ \cdots}}{dx} - \frac{d\sigma^{p \to \pi^- \cdots}}{dx}\right)\Bigg|_{x=0}\\
&= \sum_{l=5,8,\ldots}^{l_{\max}} \sigma(l)(\beta_2 + \cdots + \beta_l + l - 1)\\
&\qquad + (\beta_n + 1)\sigma(2)\ ,
\end{aligned} \tag{4.25}
$$

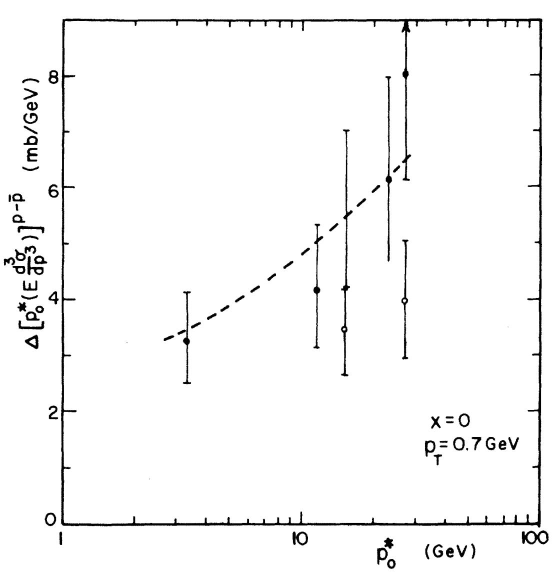

FIG. 4. This figure shows

$$
\left(p_0^* E \frac{d^3\sigma^{p \xrightarrow{p} p \cdots}}{dp^3} - p_0^* E \frac{d^3\sigma^{p \xrightarrow{p} \bar{p} \cdots}}{dp^3}\right)\Bigg|_{x=0,\, p_\perp = 0.7}
$$

versus p_0^*. Data are taken from Ref. 14. The closed and open circles refer to different experiments. The dashed line is the p_0^* dependence predicted in the text for a purely phase space $|m_{fi}|^2$.

which again is an increasing function of $\sqrt{s}$.

Some data on $pp \to \pi^{\pm} + \cdots$ at $x=0$ are presented in Ref. 15 in terms of

$$\frac{1}{\sigma_{\rm tot}} \int E^* \frac{d^2\sigma}{dp_L^* dp_\perp} dp_\perp \bigg|_{p_\perp^*=0} = \frac{1}{\sigma_{\rm tot}} \frac{d\sigma}{dy^*}\bigg|_{y^*=0} = \frac{E^*}{\sigma_{\rm tot}\, p_0^*} \frac{d\sigma}{dx}\bigg|_{x=0} ; \tag{4.26}$$

therefore at high energies, transformation of (4.25) (phase-space model) gives the prediction in terms of (4.26) that

$$\frac{1}{\sigma_{\rm tot}} \Delta \frac{d\sigma}{dy}\bigg|_{y=0}^{\pi^+-\pi^-} \simeq \frac{2(m^2+p_\perp^2)^{1/2}}{\sigma_{\rm tot}\sqrt{s}} \Delta \left(\frac{d\sigma}{dy}\right)\bigg|_{x=0}^{\pi^+-\pi^-} \propto \frac{\langle l+2\rangle}{\sqrt{s}} \to 0 \text{ as } \sqrt{s} \to \infty . \tag{4.27}$$

Not enough suitable data is available yet for both $pp \to \pi^+ + \cdots$ and $pp \to \pi^- + \cdots$ at various energies to investigate the importance of contribution (4.27) predicted by the fragmentation mechanism as against the double-Regge-predicted[16] $1/s^{1/4}$ behavior.

V. FACTORIZATION

In this section we wish to investigate the problem of factorization in the fragmentation picture. In principle, in each event fragmentation can occur with or without pionization in the central region. All our arguments in preceding sections hold for both possibilities. However, for simplicity in this section we shall consider only the case when the event is either a pure fragmentation or a pure pionization, and cannot be a mixture of both in the same event. Of course, we still allow fragmentation and pionization to occur in an overall interaction. In the following, all the cross sections and amplitudes will refer to events having fragmentation only.

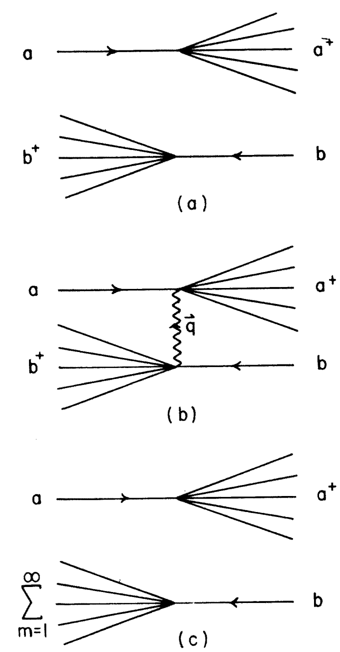

FIG. 5. (a) Fragmentation picture for $a+b \to a^+ + b^+$, where $a^+ = \sum_i^n a_i$, $b^+ = \sum_j^m b_j$; (b) as in (a), showing some transverse momentum transfer $\vec{q}$; and (c) diagram for $\sigma_n = \sum_{m=1}^{\infty} \sigma_{n,m}$, which is the process $a+b \to a^+ +$ (anything in the L hemisphere), where $a^+ = \sum_{i=1}^n a_i$.

The hadronic collision

$$a+b \to a^+ + b^+ , \tag{5.1}$$

where $a^+ = \sum_{i=1}^n a_i$ and $b^+ = \sum_{j=1}^m b_j$ in the c.m. frame [see Fig. 5(a)], requires only infinitesimal transfer of longitudinal momentum and energy, and only finite transverse momentum transfer. Therefore, it is interesting to see how these conditions modify the factorization of the cross section given that the scattering amplitude itself factorizes.

The invariant exclusive differential cross section for (5.1), with n particles to the right and m particles to the left is

$$\prod_{i=1}^n (2w_i) \prod_{j=1}^m (2w_j') \frac{d\sigma^{3(n,m)}}{d^3p_1 \cdots dp_n d^3k_1 \cdots d^3k_m} = |\langle n,m|T|a,b\rangle|^2 \delta^4\Big(p_a+p_b - \sum_i^n p_i - \sum_j^m k_j\Big) , \tag{5.2}$$

where $\bar{p}_i, w_i$ are the momentum and energy of the ith product in the R hemisphere and $\bar{k}_j, w_j'$ are the momentum and energy of the jth product in the L hemisphere.

We assume that the scattering amplitude factorizes, i.e.,

$$|\langle n,m|T|a,b\rangle|^2 = |\langle n|T|a\rangle\langle m|T|b\rangle|^2 . \tag{5.3}$$

The δ function can be broken up to give

$$\delta^4\Big(p_a+p_b - \sum_i^n p_i - \sum_j^m k_j\Big) = \delta\Big(2E_0 - \sum_i^n w_i - \sum_j^m w_j'\Big) \frac{\delta\Big(0 - \sum_i^n x_i - \sum_j^m x_j\Big)}{p_0^*} \delta\Big(\sum_i^n \vec{p}_{\perp i} + \sum_j^m \vec{p}_{\perp j}\Big) . \tag{5.4}$$

For pp collisions,

$$\delta\left(2E_0-\sum_i^n w_i-\sum_j^m w_j'\right)\simeq\frac{\delta\left(2-\sum_i|x_i|-\sum_j|x_j|\right)}{p_0^*}\text{ for } x\gg\frac{(m^2+p_\perp{}^2)^{1/2}}{p_0^*}\simeq\frac{\delta\left(2-\sum_i^n x_i+\sum_j^m x_j\right)}{p_0^*} \tag{5.5}$$

for negligible spill-over. Therefore

$$\begin{aligned}\delta^4\left(p_a+p_b-\sum^n p_i-\sum^m k_j\right)&\cong\frac{\delta(2-\sum x_i+\sum x_j)}{p_0^*}\,\frac{\delta(-\sum x_i-\sum x_j)}{p_0^*}\,\delta\left(\sum_i\vec{p}_{\perp i}+\sum_j\vec{p}_{\perp j}\right)\\&=\frac{\delta(1-\sum x_i)}{\sqrt{2}p_0^*}\,\frac{\delta(-1-\sum x_j)}{\sqrt{2}p_0^*}\,\delta\left(\sum_i\vec{p}_{\perp i}+\sum_j\vec{p}_{\perp j}\right)\,.\end{aligned} \tag{5.6}$$

Replacing (5.6) in (5.3) and writing the matrix elements $\langle|T|\rangle=\langle\|T\|\rangle|M_\perp|$, where $\langle\|T\|\rangle$ has only x dependence, then after some rearrangement

$$\frac{d^{3(n,m)}\sigma}{d^3p_1\cdots d^3p_n dk_1\cdots d^3k_m}\simeq\left[\frac{|\langle n\|T\|a\rangle|^2\delta\,(1-\sum^n x_i)}{\sqrt{2}p_0^*\prod_i^n(2|x_i|)(p_0^*)^n}\right]\left[\delta\left(\sum_i^n\vec{p}_{\perp i}+\sum_j^m\vec{p}_{\perp j}\right)|M_\perp|^2\right]\left[\frac{|\langle m\|T\|b\rangle|^2\delta\,(-1-\sum^m x_j)}{\sqrt{2}p_0^*\prod_j^m(2|x_j|)(p_0^*)^m}\right] \tag{5.7}$$

for $x\gg(m_+{}^2p_\perp{}^2)^{1/2}/p_0^*$. Factorization of the matrix element squared gives $|M_\perp|^2=|M_{\perp a^+}|^2|M_{\perp b^+}|^2$ and, integrating out the $p_\perp{}^2$ dependence, explicitly

$$\frac{d^{(n,m)}}{dx_1\cdots dx_n dx_1'\cdots dx_m'}\simeq\chi_n^{a^+}\chi_m^{b^+}p_\perp(m,n)\ , \tag{5.8}$$

where

$$\begin{aligned}\chi_n^{a^+}(x_1\cdots x_n)&=|\langle n\|T\|a\rangle|^2\delta\left(1-\sum_{i=1}^n x_i\right)\Big/\left[\sqrt{2}p_0^*\prod_i^n(2|x_j|)\right]\ ,\\\chi_m^{b^+}(x_1'\cdots x_m')&=|\langle m\|T\|b\rangle|^2\delta\left(-1-\sum_{j=1}^m x_j'\right)\Big/\left[\sqrt{2}p_0^*\prod_j^m(2|x_j'|)\right]\ ,\\p_\perp(m,n)&=\int\cdots\int\prod_i^n d^2\vec{p}_{\perp i}|m_{\perp a^+}|^2\int\cdots\int\prod_j^m d^2\vec{p}_{\perp j}|m_{\perp b^+}|^2\delta\left(\sum_i^n\vec{p}_{\perp i}+\sum_j^m\vec{p}_{\perp j}\right)\ .\end{aligned} \tag{5.9}$$

So, except for the $x<(m^2+p_\perp{}^2)^{1/2}/p_0^*$ region, the x-dependent part factorizes completely. The $p_\perp(m,n)$ remains to be investigated. Further, for weakly s-dependent average transverse momentum, (5.8) and (5.9) imply that scaling of the inclusive distributions gives

$$|\langle n\|T\|a\rangle\langle m\|T\|b\rangle|\sim p_0^*\ . \tag{5.10}$$

To investigate the transverse-momentum part of (5.8) we use the requirement that fragmentation only allows a finite amount of transverse momentum transfer as $p_0^*\to\infty$, and consider exchange of transverse momentum $\vec{q}$ as shown in Fig. 5(b). We assume a simple Gaussian form for the transverse-momentum matrix element

$$|m_\perp|^2=e^{-\alpha\sum_i^n\vec{p}_{\perp i}{}^2}e^{-\alpha\sum_j\vec{p}_{\perp j}{}^2}e^{-\beta\vec{q}^2}\ , \tag{5.11}$$

where α,β do not depend strongly on s, and write

$$p_\perp(m,n)=\int d^2\vec{q}e^{-\beta q^2}\int\cdots\int\prod_i^n d^2\vec{p}_{\perp i}\,e^{-\alpha\sum_i^n\vec{p}_{\perp i}{}^2}\delta\left(\sum_i^n\vec{p}_{\perp i}+\vec{q}\right)\int\cdots\int\prod_j^m d^2\vec{p}_{\perp j}\,e^{-\alpha\sum_j^m\vec{p}_{\perp j}{}^2}\delta\left(\sum_j^m\vec{p}_{\perp j}-\vec{q}\right). \tag{5.12}$$

It is then easy to show that

$$p_\perp(m,n)=\frac{1}{n}\left(\frac{\pi}{\alpha}\right)^{n-1}\frac{\pi}{2\beta+\alpha(1/m+1/n)}\,\frac{1}{m}\left(\frac{\pi}{\alpha}\right)^{m-1}\ , \tag{5.13}$$

and so

$$\frac{d\sigma^{(n,m)}}{dx_1\cdots dx_n dx_1'\cdots dx_m'}=\frac{1}{n}\left(\frac{\pi}{\alpha}\right)^{n-1}\chi_n^{a^+}(x_1\cdots x_n)\left[\frac{\pi}{2\beta+\alpha(1/m+1/n)}\right]\frac{1}{m}\left(\frac{\pi}{\alpha}\right)^{m-1}\chi_m^{b^+}(x_1'\cdots x_m') \tag{5.14}$$

for identical a, b.

Thus, in contrast to a two-body scattering process, one notes that a factorizable scattering amplitude in the fragmentation process does not automatically give a factorizable cross section [because of the term in large square brackets in (5.14)]. However, for a reasonable approximation to factorization of the cross section, one needs to have the term in large square brackets in (5.14) split up, e.g., $\beta \gg \frac{1}{2}\alpha$. This condition is satisfied as long as the average transverse momentum transfer is small at high energies, e.g., $\langle q \rangle \lesssim 0.3$.

However, the other restriction on factorization is the condition $x_i \gg (m_i{}^2 + p_\perp{}^2)^{1/2}/p_0^*$. For pion production at ISR at $\sqrt{s} = 53$ GeV, this gives roughly $x_i \gg 0.02$. Therefore even the fragmentation model will not factorize in the central region.

Assuming that we can make the rough $\beta \gg \frac{1}{2}\alpha$ approximation, then if we restrict ourselves to $x_i \gg (m_i{}^2 + p_\perp{}^2)^{1/2}/p_0^*$, i.e., to the fragmentation region then we can integrate up (5.14) to give various relations between the fragmentation cross sections. We denote

$$\sigma_{n,m} = \int_{x_{\min}}^{1} \frac{d\sigma^{(n,m)}}{dx_1 \cdots dx_n dx_1' \cdots dx_m'} dx_1 \cdots dx_n dx_1' \cdots dx_m'$$
$$= \chi_n^{a^+} \chi_m^{b^+} \tag{5.15}$$

and, on summing out the left hemisphere (that is, the m-dependent part), define

$$\sigma_n = \sum_{m=1}^{\infty} \sigma_{m,n} = \chi_n^{a^+} \sum_m \chi_m^{b^+} \tag{5.16}$$

for the process

$$a + b \to a^+ + (\text{anything in the } L \text{ hemisphere}) , \qquad a^+ = \sum_{i=1}^{n} a_i , \tag{5.17}$$

with n particles to the right and any particles to the left, as shown in Fig. 5(c). Using these definitions we obtain the relations

$$(\sigma_{k_1})^2 = \sigma_{k_1,k_1} \sigma_{\text{tot}}^D , \tag{5.18a}$$

$$\sigma_{k_1,k_2} \sigma_{k_2} = \sigma_{k_2,k_2} \sigma_{k_1} , \tag{5.18b}$$

$$\sigma_{k_1} \sigma_{k_2} = \sigma_{k_2,k_1} \sigma_{\text{tot}}^D . \tag{5.18c}$$

For $k_1 = 1$, it is easy to test the consequences of (5.18a). Then it gives

$$(\sigma_1)^2 = \sigma_{1,1} \sigma_{\text{tot}}^D \tag{5.19}$$

and we represent it as in Fig. 6, so that one identifies

$$\sigma_1 = \sigma_{\text{el}} + \tfrac{1}{2}\sigma_{\text{SD}} , \tag{5.20a}$$

$$\sigma_{1,1} = \sigma_{\text{el}} , \tag{5.20b}$$

$$\sigma_{\text{tot}}^D = \sigma_{\text{el}} + \sigma_{\text{SD}} + \sigma_{\text{DD}} , \tag{5.20c}$$

where

σ_{SD} is the single-diffractive cross section ,

σ_{DD} is the double-diffractive cross section ,

σ_{tot}^D is the total diffractive cross section .

Note that σ_{tot}^D does not refer to the central region, as we are using the condition $x_i \gg (m_i{}^2 + p_\perp{}^2)^{1/2}/p_0^*$.

If one uses the definition

$$K \equiv \frac{\sigma_{\text{SD}}{}^2}{\sigma_{\text{DD}} \sigma_{\text{el}}} , \tag{5.21}$$

then substitution of (5.21) into (5.19) after some rearrangement gives the result $K = 4$. This agrees with the experimental estimate[17] of the double-diffractive cross section

$$\sigma_{\text{DD}} \simeq \frac{\sigma_{\text{SD}}{}^2}{4\sigma_{\text{el}}} , \tag{5.22}$$

i.e., $K \simeq 4$, at the c.m. energy $\sqrt{s} = 55$ GeV. We conclude that factorization appears to work quite well in this framework for the fragmentation region (without any significant nondiffractive contribution). Now σ_{el} is well known at available ISR energies, and $\frac{1}{2}\sigma_{\text{SD}}$ is estimated by integrating the diffractive peak in the inelastic $pp \to p + \cdots$ spectrum—see Fig. 7, showing the inelastic peak (taken from Ref. 18). Combining these in (5.22) and (5.20c) one finds that the total diffractive cross section σ_{tot}^D is approximately 16 mb, leaving roughly 28 mb for the contribution from the central region.

For more general $k = n$, it is not so easy to com-

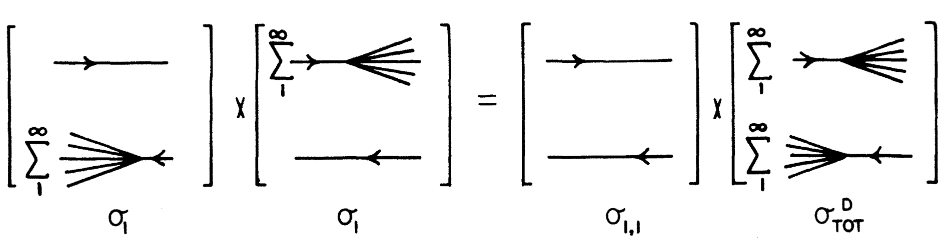

FIG. 6. Diagram representation of the formula $\sigma_1{}^2 = \sigma_{1,1}\sigma_{\text{tot}}^D$ from Sec. V.

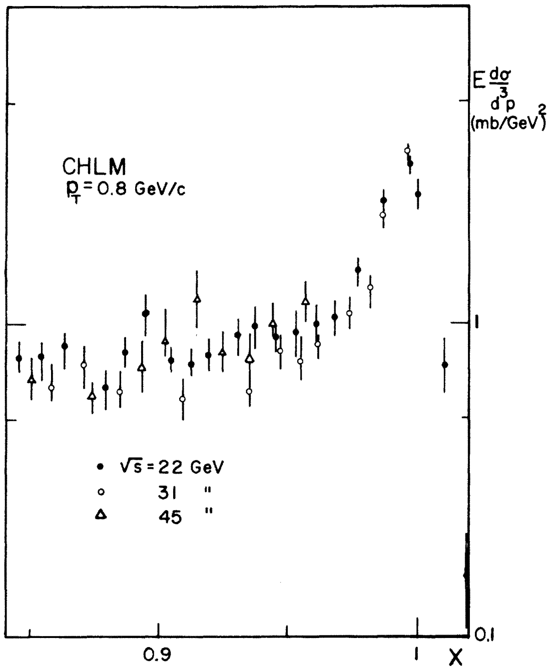

FIG. 7. Single-diffractive excitation as observed by the CERN-Holland-Lancaster-Manchester Collaboration. Data are taken from Ref. 16.

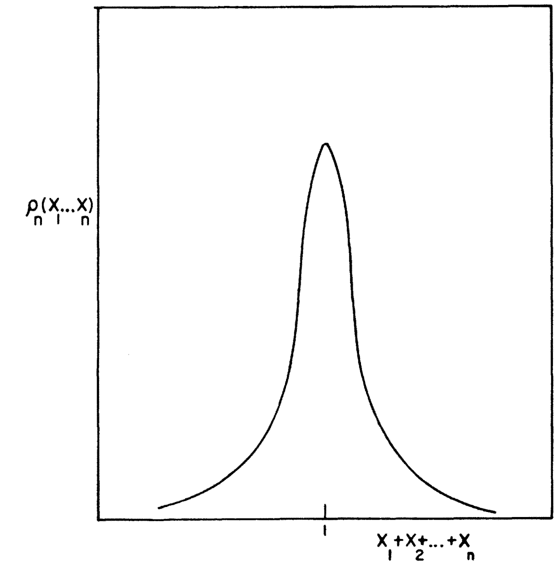

FIG. 8. Predicted form of $\rho_n(x_1 \cdots x_n)$ as a function of $(x_1+x_2+\cdots+x_n)$ at high energies.

pare (5.18) to experiment at present. It is interesting to note, however, that for the n-particle favored fragmentation, the minimal rule requires that the inclusive spectrum behave like a δ function

$$\rho_n(x_1 \cdots x_n) \sim \delta(x_1 + \cdots + x_n - 1)$$

as shown in Fig. 8. Other models do not expect this peaking.[12,19] We urge that experimental work be done to look for this striking behavior.

VI. CONCLUSIONS

Our calculations of the minimal rule sum for pp, π^-p collisions verify that it is an essential attribute of such high-energy hadronic collisions. The fragmentation model expresses this in a natural way. However, we found that charge exchange is still an important ingredient at the presently attainable π^-p energies, and so the asymptotic energy region predicted by the fragmentation model has certainly not yet been reached for π^-p. For pp collisions we investigated how well the model approximated factorization at the (much higher) pp ISR energies.

The central region is often regarded as a special case where the fragmentation process is not significant; however, we have shown that the fragmentation model does at least qualitatively provide a natural explanation for the behavior of some particle-antiparticle distributions at $x=0$, behavior which is not so obvious with purely cluster-type processes. Therefore we strongly suggest that the fragmentation process makes a significant contribution to the central region. Also, the verification of the minimal rule was shown to still allow any other energy-dependent central process also occurring to contribute to the pion $d\sigma/dy$ distributions in a small $-y^*$ region which can increase approximately as fast as $p_0^*\sigma_{\text{tot}}$.

ACKNOWLEDGMENT

One of us (M. M. M.) acknowledges the assistance of an Australian Postgraduate Research Award.

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