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Soft-photon analysis of pion-proton bremsstrahlung*

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We define the soft-photon approximation consistent in its treatment of both the kinematical and dynamical aspects of bremsstrahlung. Using this consistent soft-photon approximation, we calculate $\pi^{\pm}p\gamma$ cross sections and obtain excellent agreement with experiment. Our results suggest that the contributions from off-mass-shell effects and resonances are small.

Recently, the pion-proton bremsstrahlung processes ($\pi^{\pm}p\gamma$) have been discussed with great interest both experimentally [1-4] and theoretically.[5-18] The two primary reasons for studying these processes were to obtain information about off-mass-shell effects of the πN interactions and electromagnetic properties of πN resonances. However, owing to an unexpectedly large discrepancy between the UCLA data[3,4] and most of the theoretical predictions, attention has now been focused on a search for a fundamental theory which can be used to describe the experimental observations and resolve this discrepancy. In this note we describe just such a theory and demonstrate the excellent fit to the experimental data produced by it.

The UCLA group has measured $\pi^{\pm}p\gamma$ cross sections, $d^5\sigma/d\Omega_\pi d\Omega_\gamma dk$, at 298 and 269 MeV for various photon angles. [Here $d\Omega_\pi$ is an element of solid angle in the direction of $\vec{q}_f$, the momentum of the scattered pion, specified by the polar angles (θ_π, ϕ_π) and $d\Omega_\gamma$ is an element of solid angle in the direction of the photon momentum $\vec{k}$, specified by the polar angles $(\theta_\gamma, \phi_\gamma)$.] The cross sections decrease smoothly and continuously with increasing photon energy k. What surprised us most is not the fact that most of the sophisticated calculations have failed to adequately describe the experimentally observed bremsstrahlung spectra, but rather the report that the cross sections predicted by the "soft-photon approximation," which is based upon the most important and fundamental theory in the bremsstrahlung process,[19-21] rise steeply with increasing photon energy above $k=80$ MeV in complete disagreement with experimental results. Another mystery has been why the data can be described by the "external-emission dominance" approximation proposed by Nefkens and Sober.[5]

The purposes of this note are threefold: (1) To clarify the definition of the soft-photon approximation and to show that there exist universal characteristic curves for the cross section in this approximation. (These characteristic curves are hyperbolas when the bremsstrahlung cross section $d^5\sigma/d\Omega_\pi d\Omega_\gamma dk$ is plotted against k.[22] Those calculations which fail to produce these characteristic curves should not be classified as "soft-photon approximations".) (2) To present the consistent method for performing calculations using the soft-photon approximation. (3) To show that our soft-photon results are in excellent agreement with the UCLA data. In fact, we have obtained the best fits to date for the UCLA data. *Thus we have shown that the soft-photon theory works remarkably well for pion-proton bremsstrahlung at* 298 *MeV*. Our results suggest that the contributions from off-mass-shell effects and resonances are small.

SOFT-PHOTON APPROXIMATION

It is well known that the bremsstrahlung cross section can be expanded in powers of the photon energy k as

$$\sigma=\sigma_{-1}/k+\sigma_0+\sigma_1 k+\cdots, \tag{1}$$

where

$$\sigma_{-1}=\lim_{k\to 0}(k\sigma),\qquad \sigma_0=\lim_{k\to 0}\frac{\partial}{\partial k}(k\sigma)_{x_i}. \tag{2}$$

Here the x_i refer to the set of observables which are held constant in carrying out the partial differentiation. This expansion is the soft-photon expansion. We should emphasize that all coefficients in this expansion are *independent* of k. In terms of this expansion, the low-energy theorem (or the soft-photon theorem or Low's theorem), first established by Low[19-21] states that σ_{-1} and σ_0 are independent of the off-mass-shell effects and that they can be evaluated from the knowledge of the two-body elastic scattering amplitude and its derivatives. This theorem thus provides us with an approximate mehtod for calculating bremsstrahlung cross sections. In order to distinguish this approximation from all other types of calculations which are also based on low-energy theorem, we *define* the soft-photon approximation (SPA) for the bremsstrahlung cross section as

$$\sigma_{\rm SPA} = \frac{\sigma_{-1}}{k} + \sigma_0. \tag{3}$$

When $\sigma_{\rm SPA}$ is plotted against k, Eq. (3) yields a family of hyperbolas characterized by two constants σ_{-1} and σ_0. For a given σ_{-1}, the shape of the hyperbola is determined and the constant σ_0 will shift this hyperbola up or down along the vertical axis without changing this shape. If we look at the UCLA data, we will find that every bremsstrahlung spectrum obtained by the UCLA group exhibits this shape (a hyperbola) in agreement with the characteristic curve of SPA. Thus without performing any calculations, we can conclude that the *shape* of the bremsstrahlung spectra observed by the UCLA group is predicted by SPA.

Unfortunately, Eq. (3) has not been used by previous investigators in doing the "soft-photon approximation" calculations. Instead, they have used the following expression:

$$\bar\sigma = \frac{\bar\sigma_{-1}(k)}{k} + \bar\sigma_0(k), \tag{4}$$

where $\bar\sigma_{-1}(k)$ and $\bar\sigma_0(k)$ are independent of off-mass-shell effects *but* are still functions of k. Unlike Eq. (3), Eq. (4) is not obtained from a true soft-photon expansion but uses Low's prescription to remove the off-mass-shell effects. In this prescription the bremsstrahlung *amplitude* is obtained from the πp elastic scattering amplitudes and its derivatives evaluated at the average total energy squared $\bar s$ and the momentum transfer squared t. Since $\bar s$ and t are functions of k (i.e., three-body kinematics should be used to calculate $\bar s$ and t) the first two terms in the expression for the cross section constructed using this prescription will still be functions of k. That is, as noted above, $\bar\sigma_{-1}(k)$ and $\bar\sigma_0(k)$ of Eq. (4) are not independent of k. Now, it is obvious that Eq. (4) does not represent a family of hyperbolas. As a consequence, when Eq. (4) was used by the UCLA group to predict bremsstrahlung spectra, they found that the cross sections rise steeply with increasing photon energy above $k=80$ MeV in total disagreement with experimental observations. This disagreement led the UCLA group to conclude that the fault lay with the soft-photon approximation. The problem does not arise from SPA, but rather from Eq. (4) which is not the soft-photon approximation.

Having defined SPA and described its relationship to other so-called soft-photon approximations, we next discuss the derivation of σ_{-1} and σ_0. To do this we must modify Low's prescription for obtaining the cross section. In particular we must address ourselves to the consistent expansion of the three-body kinematics around $k=0$.

METHOD OF CALCULATION

In all present bremsstrahlung experiments, there are three outgoing particles with nine degrees of freedom. This number is reduced to five by the four equations of energy-momentum conservation. The choice of these five independent kinematical variables is strongly influenced by experimental considerations. In the UCLA experiments, the variables θ_π, ϕ_π, θ_γ, ϕ_γ, and k were chosen to be independent, and the cross section was expressed in the form $d^5\sigma/d\Omega_\pi d\Omega_\gamma dk$. All other kinematical variables q_f, p_f, θ_p, and ϕ_p (which can be obtained from the energy-momentum-conservation equations) are functions of θ_π, ϕ_π, θ_γ, ϕ_γ, and k. [Here $\vec p_f =(p_f,\theta_p,\phi_p)$ is the momentum of the scattered proton.]

Since the low-energy theorem is derived for small k and is valid only to zeroth order in k, the expansion performed by Low to demonstrate the cancellation of the off-mass-shell effects in the limit of small k, must be accompanied by an expansion of the kinematical aspects to a consistent order when calculating σ_{-1} and σ_0. This means that one must expand the solution for q_f, p_f, θ_p, and ϕ_p around $k=0$, retaining only the terms to first order in k. The expansions for these variables can be combined to obtain two important four-vectors, p_f^μ and q_f^μ, correct to the first order in k:

$$\begin{aligned} q_f^\mu &= \bar q_f^\mu + A^\mu, \\ p_f^\mu &= \bar p_f^\mu - A^\mu - k^\mu, \\ A^\mu &= [m^2 p_i^\mu - (p_i\cdot\bar q_f)\bar q_f^\mu](\bar p_f\cdot k)/N, \end{aligned} \tag{5}$$

where

$$\bar q_f^\mu = (q_f^\mu)_{k=0},\quad \bar p_f^\mu = (p_f^\mu)_{k=0},$$
$$p_i^\mu = (M,0),\quad N=(p_i\cdot\bar q_f)(\bar p_f\cdot\bar q_f) - m^2(p_i\cdot\bar p_f)$$

and m (M) is the pion (nucleon) mass.

We use these expressions for q_f^μ and p_f^μ to expand the bremsstrahlung amplitude, phase-space factor, proton projection operator, etc. When all expansions are performed consistently, we obtain σ_{-1} and σ_0 completely independent of k:

$$\sigma_{-1} = ckF^{(0)}\,\mathrm{Tr}(X)$$

and (6)

$$\sigma_0 = c[F^{(1)}\,\mathrm{Tr}(X) + F^{(0)}\,\mathrm{Tr}(Y)],$$

where

$$c = -\frac{e^2M^2k}{16(2\pi)^5[(p_i\cdot q_i)^2 - m^2M^2]^{1/2}},\quad F^{(0)} = Q^{(0)}/D^{(0)},\quad F^{(1)} = \left[Q^{(1)} - \frac{Q^{(0)}D^{(1)}}{D^{(0)}}\right]\Big/\,D^{(0)},$$

$$X=\gamma^0(\mathfrak{M}^X_\mu)^\dagger\gamma^0\left(\frac{\bar{\not p}_f+M}{2M}\right)(\mathfrak{M}^{X\mu})\left(\frac{\not p_i+M}{2M}\right),$$

$$Y=\gamma^0(\mathfrak{M}^X_\mu)^\dagger\gamma^0\left(\frac{\bar{\not p}_f+M}{2M}\right)(\mathfrak{M}^{Y\mu})\left(\frac{\not p_i+M}{2M}\right)+\gamma^0(\mathfrak{M}^X_\mu)^\dagger\gamma^0\left[\frac{(\bar p_f\cdot k)g_1\bar{\not q}_f-(\bar p_f\cdot k)g_2\not p_i-\not k}{2M}\right](\mathfrak{M}^{X\mu})\left(\frac{\not p_i+M}{2M}\right)$$
$$+\gamma^0(\mathfrak{M}^Y_\mu)^\dagger\gamma^0\left(\frac{\bar{\not p}_f+M}{2M}\right)(\mathfrak{M}^{X\mu})\left(\frac{\not p_i+M}{2M}\right),$$

$$\mathfrak{M}^X_\mu=\left(\frac{\bar p_{f\mu}}{\bar p_f\cdot k}-\frac{p_{i\mu}}{p_i\cdot k}+\frac{e_\pi\bar q_{f\mu}}{\bar q_f\cdot k}-\frac{e_\pi q_{i\mu}}{q_i\cdot k}\right)T(s,t),$$

$$\mathfrak{M}^Y_\mu=\left[\frac{g_1(\bar p_f\cdot k)\bar q_{f\mu}-g_1(\bar q_f\cdot k)\bar p_{f\mu}+g_2(p_i\cdot k)\bar p_{f\mu}-g_2(\bar p_f\cdot k)p_{i\mu}}{(\bar p_f\cdot k)}-\frac{e_\pi(\bar p_f\cdot k)g_2[(p_i\cdot k)\bar q_{f\mu}-(\bar q_f\cdot k)p_{i\mu}]}{(\bar q_f\cdot k)^2}\right]T(s,t)$$
$$+e_\pi\left[\left(\frac{\bar q_{f\mu}}{\bar q_f\cdot k}+\frac{q_{i\mu}}{q_i\cdot k}\right)\frac{\not k}{2}-\gamma_\mu\right]B(s,t)+\frac{1}{2}\left(\frac{\bar p_{f\mu}}{\bar p_f\cdot k}-\frac{p_{i\mu}}{p_i\cdot k}+\frac{e_\pi\bar q_{f\mu}}{\bar q_f\cdot k}-\frac{e_\pi q_{i\mu}}{q_i\cdot k}\right)(\bar p_f\cdot k)(g_2\not p_i-g_1\bar{\not q}_f)B(s,t)$$
$$+\frac{(\lambda+1)\gamma_\mu\not k}{2(\bar p_f\cdot k)}T(s,t)+T(s,t)\frac{(\lambda+1)\not k\gamma_\mu}{2(p_i\cdot k)}+\frac{\lambda[(\bar p_f\cdot k)\gamma_\mu-\bar p_{f\mu}\not k]}{2M(\bar p_f\cdot k)}T(s,t)+T(s,t)\frac{\lambda[(p_i\cdot k)\gamma_\mu-p_{i\mu}\not k]}{2M(p_i\cdot k)}$$
$$+\frac{\partial T}{\partial s}\left[-2(\bar p_f+\bar q_f)_\mu+2\frac{(\bar p_f+\bar q_f)\cdot k}{p_i\cdot k}p_{i\mu}+\frac{2e_\pi(\bar q_f+\bar p_f)\cdot kq_{i\mu}-2e_\pi q_i\cdot k(\bar q_f+\bar p_f)_\mu}{(q_i\cdot k)}\right]$$
$$+\frac{\partial T}{\partial t}\left[2(1-g_3)\left(\bar p_{f\mu}-\frac{\bar p_f\cdot k}{p_i\cdot k}p_{i\mu}\right)+(2p_i\cdot k-2g_3\bar p_f\cdot k)\left(\frac{e_\pi\bar q_{f\mu}}{\bar q_f\cdot k}-\frac{e_\pi q_{i\mu}}{q_i\cdot k}\right)\right],$$

$$Q^{(0)}=\frac{1}{M^3}[(p_i\cdot\bar q_f)^2-m^2M^2]^{3/2},$$

$$Q^{(1)}=-\frac{1}{M^3}[(p_i\cdot\bar q_f)^2-m^2M^2]^{1/2}[3g_1(\bar p_f\cdot k)(p_i\cdot\bar q_f)^2-3g_2M^2(\bar p_f\cdot k)(p_i\cdot\bar q_f)],$$

$$D^{(0)}=\frac{1}{M}\{(p_i\cdot q_i)[(p_i\cdot q_i)-(p_i\cdot\bar p_f)]+(p_i\cdot q_i)M^2-(p_i\cdot\bar p_f)m^2\},$$

$$D^{(1)}=\frac{1}{M}[m^2+(p_i\cdot q_i)][-g_1(p_i\cdot\bar q_f)(\bar p_f\cdot k)+g_2M^2(\bar p_f\cdot k)+(p_i\cdot k)]-\frac{1}{M}(p_i\cdot q_i)(p_i\cdot k)$$
$$-\frac{1}{M}(p_i+q_i)\cdot k[(p_i\cdot q_i)-(p_i\cdot\bar p_f)+M^2],$$

$$T(s,t)=-A(s,t)+\tfrac{1}{2}(\not q_i+\bar{\not q}_f)B(s,t),$$

$$g_1=\frac{(p_i\cdot\bar q_f)}{(p_i\cdot\bar q_f)(\bar p_f\cdot\bar q_f)-m^2(p_i\cdot\bar p_f)},\quad g_2=\frac{m^2}{(p_i\cdot\bar q_f)(\bar p_f\cdot\bar q_f)-m^2(p_i\cdot\bar p_f)},\quad g_3=\frac{(p_i\cdot\bar q_f)^2-m^2M^2}{(p_i\cdot\bar q_f)(\bar p_f\cdot\bar q_f)-m^2(p_i\cdot\bar p_f)},$$

$$\lambda=1.793,$$

and

$$e_\pi=1\text{ for }\pi^+$$
$$=-1\text{ for }\pi^-.$$

In Eq. (6), $q_i^\mu=[\omega_i\equiv(m^2+\vec q_i^{\,2})^{1/2},\vec q_i]$ is the four-momentum for the incoming pion, $s=(p_i+q_i)^2=(\bar p_f+\bar q_f)^2$, and $t=(\bar p_f-p_i)^2=(\bar q_f-q_i)^2$. In deriving σ_{-1} and σ_0, we have expanded the bremsstrahlung amplitude around two k-independent Mandelstam variables, s and t. Roughly speaking, this expansion is valid if $k<\frac{1}{2}\sqrt{s}$ and $k<\frac{1}{2}\sqrt{t}$, and these conditions are satisfied for the whole range k (from zero to its maximum value). It should be emphasized that all contributions to σ_0 and σ_{-1} have been included. This is to be contrasted with calculations in which the k dependence of Eq. (4) is suppressed. All of the dynamics of the problem now reside in the elastic $\pi^\pm p$ amplitudes, $T(s,t)=-A(s,t)+\frac{1}{2}(\not q_i+\bar{\not q}_f)B(s,t)$, and their derivatives with respect to s and t; $\partial A/\partial s$, $\partial A/\partial t$, $\partial B/\partial s$, and $\partial B/\partial t$. Our results are still relativistic even through the expansion in kinematics was introduced here. This

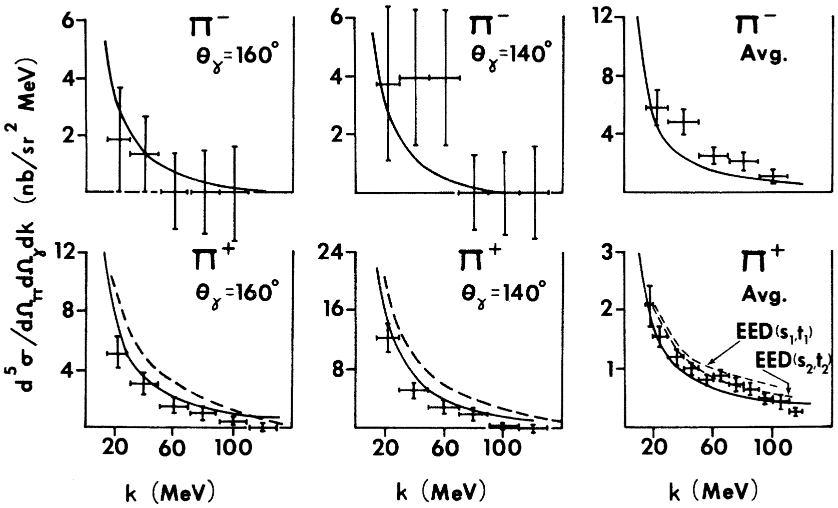

FIG. 1. A comparison of our theoretical predictions with $\pi^{\pm}p$ bremsstrahlung data at 298 MeV. The solid curves represent our theoretical prediction based upon the soft-photon approximation (see text). The data points are from Refs. 3 and 4. The dashed curves are the calculations of Nefkens and Sober, Ref. 5. The notations s_1, t_1, s_2, and t_2 used here are, respectively, those of $\bar{s}$, $\bar{t}$, s, and t defined in Ref. 5.

can be easily seen from the fact that the expressions for the expansion of p_f^μ and q_f^μ retain the four-vector forms. As for gauge invariance, it is satisfied automatically since the gauge condition has been used to determine the expression for the internal scattering terms. We can check the gauge invariance by noting that $(\mathfrak{M}^X+\mathfrak{M}^Y)_\mu k^\mu=0$.

RESULTS AND COMPARISON WITH EXPERIMENT

We have calculated $\pi^{\pm}p\gamma$ cross sections at 298 MeV for various θ_γ, using SPA as we have defined it in the previous sections. (We have considered only the coplanar case for simplicity.) We have used tabulated S and P partial wave πp phase shifts and inelasticities[23] to evaluate the πp elastic scattering amplitude and its derivatives, and we have included the proton anomalous magnetic moment in our calculation.

In general our results are in excellent agreement with the UCLA data. As a matter of fact, our fits are the best yet obtained. Some of our results and their comparison with the UCLA data are shown in Figs. 1 and 2. In Fig. 1, the $\pi^{\pm}p\gamma$ cross sections in the laboratory system $d^5\sigma/d\Omega_\pi d\Omega_\gamma dk$ are plotted as a function of photon energy k, for various photon angles θ_γ. In all cases, the incident pion energy in the laboratory system is 298 MeV and the angle for scattered pions is fixed at 50.5° (θ_π $=50.5°$). These bremsstrahlung spectra (solid curves) are all hyperbolas characterized by σ_{-1} and σ_0, which are independent of k but are functions of θ_γ. The first two solid curves in Fig. 1 represent, respectively, the $\pi^-p\gamma$ (top) and $\pi^+p\gamma$ (bottom) cross sections for $\theta_\gamma=160°$, and they are compared with the measurements obtained by photon detector G_{11} of Ref. 4 and with the external-emission-dominance (EED) calculations[5] (dotted curves). The next two solid curves represent those for $\theta_\gamma=140°$ compared with the data obtained by photon detector G_{12} of Ref. 4 and with EED calculations (dotted curves). Finally, the last two spectra in Fig. 1 are the average cross sections over $\theta_\gamma=-120°$, $-140°$, $-160°$, and 180°, and they are compared with the average data over the 10 photon detectors G_1 to

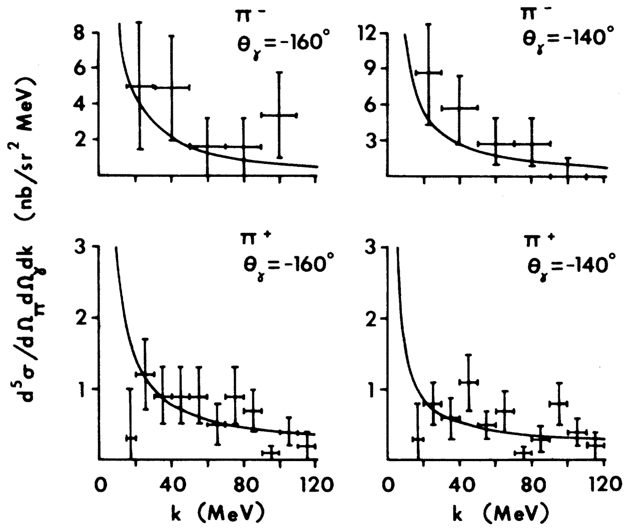

FIG. 2. A comparison of our theoretical predictions with $\pi^{\pm}p$ bremsstrahlung data at 298 MeV. The solid curves represent our theoretical prediction based upon the soft-photon approximation. The data points are from Ref. 3.

G_{10} of Ref. 3. In Fig. 2, the $\pi^{\pm}p\gamma$ cross sections are plotted as a function of k for $\theta_\gamma = -140$ and -160, and they are compared with the data obtained by photon detector G_4 and G_1 of Ref. 3. Our results show that the agreement between the predictions based on the soft-photon approximation, Eq. (3), and the experimental observations is excellent over the entire measured photon angle θ_γ. This excellent agreement suggests that the contributions from off-mass-shell effects and the resonance effects (if any) are small.

Based upon the arguments given in previous sections, we are now able to explain why the external emission dominance approximation proposed by Nefkens and Sober gives reasonable cross sections. The EED approximation is equivalent to the application of Eq.(4) without including $\overline{\sigma}_0(k)$, i.e., using $\overline{\sigma} = \overline{\sigma}_{-1}(k)/k$. This approximation would agree with our result if $\overline{\sigma}_{-1}(k)$ were well represented by $\overline{\sigma}_{-1}(0) + k(\partial\overline{\sigma}_{-1}/\partial k)_{k=0}$ and $(\partial\overline{\sigma}_{-1}/\partial k)_{k=0} \approx \sigma_0$. But the EED approximation is justified only if $\overline{\sigma}_0(k)$ is small. In fact, it is not small for large k. As we have previously noted, when $\overline{\sigma}_0(k)$ is included, this type of calculation [i.e., the use of Eq. (4)] gives results which are in total disagreement with data at large k.

In conclusion, we have calculated $\pi^{\pm}p$ bremsstrahlung in a consistent soft-photon approximation and obtained an extremely good fit to the data. The calculation is unambiguous, relativistic, gauge-invariant, model-independent, parameter-free, and based upon a fundamental theorem due to Low. There are two important implications of our results: (1) The off-mass-shell effects are small and (2) the effects of radiation from πp resonant states (when treated as a single particle) are small if they exist at all. Further improvements to our calculation, while requiring a model to generate the higher-order terms, must not affect the two leading terms which we have presented here.

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