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Resonance photocouplings with a "mixed" nucleon and the even-wave harmonic-oscillator theory of baryonic states

M. Gupta, Sudhir K. Sood, and A. N. Mitra*
Department of Physics, University of Delhi, Delhi-7, India
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A model of the nucleon as a $|56\rangle\cos\theta + |70\rangle\sin\theta$ octet, with $\cot\theta \approx \pm\sqrt{2}$, facilitated by an even-wave harmonic-oscillator theory proposed by one of us (A.N.M.), together with a complementary set of selection rules for transitions to $|56\rangle$ and $|70\rangle$ states based on this theory, provides an almost parameter-free prediction of photocouplings in excellent accord with the latest data, including the so-far-unexplained cases of $D_{15}(p\gamma)$, $P_{11}(1470)$, and $F_{35}(1890)$.

Photocouplings of resonances probably represent a more reliable way of determining signs of amplitudes than do methods based on isobar analyses in view of unitarity correction uncertainties in the latter. Photoamplitudes are also governed by some quark-model selection rules,[1,2] experimental violations of which are quite sensitively reflected in the signs and magnitudes of these amplitudes and can therefore be linked in a fairly direct manner to appropriate theoretical assumptions designed to simulate such violations. Some striking examples of apparent violations are the photoproductions of (i) D_{15} on protons,[1] (ii) F_{15} on neutrons,[2] and (iii) $S'_{11}(1700)$ on the usual quark-model assignment of a quartet state.[3] In the simple single-quark-transition picture such violations are sought to be understood in terms of traditional interactions like direct (D) and "recoil" (R) terms based on the magnetic coupling,[4] and the "convective" (C) term based on the charge coupling.[2] Unfortunately, while the recoil interaction helps understand anomalies like $F_{15}(n\gamma)$, none of these interactions is adequate for the anomalies of the other two types without invoking much more involved couplings at the quark level (probably beyond single-quark transitions). On the other hand, in view of the otherwise impressive performance of the model—and this has a close parallel in the (alternative) Melosh language[5]—it is natural to look for simpler mechanisms within this basic framework.

It appears to us that anomalies such as $p\gamma \to D_{15}$ or $N\gamma \to S'_{11}(1700)$ are rather strongly indicative of more states in fairly low-lying regions than the usual classifications of $(\underline{70}, 1^-)$ states can provide. The main question is one of how to bring about such states to lie in the desired energy range so as to provide more candidates for photoproduction. Recently one of us had suggested a model based on a 50:50 mixture of direct and exchange q-q forces of the harmonic oscillator (h.o.) type, in which the $\underline{70}$ states of $L^P = 1^-$ exhibit an apparently dual spectrum (of l and u states) with considerable success for many observed splittings.[6] In this theory, the l states are the usual $(\underline{70}, 1^-)$ states of $N = 1$, but with a reduced excitation over the ground state ($\Delta M^2 \approx \frac{1}{2}$ GeV^2). The u states, on the other hand, which actually correspond to one particular class of $N = 3$ $\underline{70}$ states, are strongly depressed in mass by the action of the exchange force, so as to look like mere partners of l states at a modest excitation of $\Delta M^2 \approx \frac{1}{2}\sqrt{3}$ GeV^2 above the ground state (e.g., $D_{13}(1520)$ vs $D_{15}(1670)$, $S_{11}(1535)$ vs $S'_{11}(1700)$, etc.). The *ground* state $(\underline{70}, 0^+)$ now provides a natural candidate for Roper-like resonances. The selection rules for transitions from l, u states are complementary[7]:

$$(\underline{70}, 1^-)_u \not\to (\underline{56}, 0^+); \quad (\underline{70}, 1^-)_l \not\to (\underline{70}, 0^+) . \qquad (1)$$

A similar scheme holds for $(\underline{70}, 2^+)$ states and their transitions.

In this note we use this scheme not only to offer a natural explanation of the selection-rule anomalies, but also to provide an almost parameter-free fit to the recent data on γ couplings,[8-10] including some of the traditionally difficult cases like $P_{11}(1470)$ and $D_{15}(p\gamma)$. As to the essential ingredients, the strength of the charge coupling at the quark level is fixed by charge conservation, and that of the magnetic coupling by the neutron magnetic moment. For an "unmixed" nucleon, this last implies *equal* strengths of the charge ($m_\rho V_0$) and magnetic ($-i\sigma_{\mu\nu} k_\nu$) couplings at the quark level,[11] but for a "mixed" nucleon with angle θ, this necessitates multiplying the latter by $\sec^2\theta$. In the even-wave theory,[6] the proximity of a $(\underline{70}, 0^+)$ to $(\underline{56}, 0^+)$ suggests a natural candidate for the mixing of their respective $\underline{8}_d$ members, so that the physical nucleon (N) and the Roper P_{11}

are represented as

$$|56\rangle\cos\theta + |70\rangle\sin\theta$$

and

$$|70\rangle\cos\theta - |56\rangle\sin\theta,$$

respectively. A value of θ close to the ideal ($\cot\theta \approx \pm\sqrt{2}$) provides simultaneous fits to G_A/G_V, $\Delta \to N\pi$, and $G_{NN\pi}$.[7] We *assume* that the same mixing angle continues to hold for the Regge recurrences of N and P_{11}.

As to the "corrections" to the charge and magnetic interactions at the quark level, the strengths of the convective and recoil terms are given by the respective replacements ($\vec{\sigma}, \vec{P}$ = quark spin, momentum)

$$\begin{aligned} m_\rho V_0 &\to m_\rho V_0 - 2\vec{P}\cdot\vec{V}, \\ -i\vec{\sigma}\cdot\vec{V}\times\vec{k} &\to -i\vec{\sigma}\times\vec{V}\cdot(\vec{k}-\rho\vec{P}) , \end{aligned} \qquad (2)$$

the former following an argument based on Feynman *et al.*,[12] and the latter after Mitra and Ross.[13] The only parameter in this description is the strength ρ of the recoil coupling whose proportionality to the mass difference $(M - m)$ follows from the Gell-Mann–Oakes–Renner effect.[14] We have fixed the proportionality constant at 0.44 (GeV units) to agree with $D_{13}(1520) \to \Delta\pi$ decay. A positive sign for p implies the simultaneous signatures $\xi = \text{sign}(S/D) < 0$, $\xi' = \text{sign}(P/F) < 0$ for pionic transitions in the indicated waves,[15] since in this simple model there are no separate handles on even- and odd-wave transitions. Finally, the γ transitions are determined by h.o. wave functions[7] in the even-wave model,[6] so that no extra parameters accrue on this account. A general formalism given earlier for invariant γ amplitudes in the quark model[16] suffices for the determination of the necessary helicity amplitudes (with relativistic effects).

Before presenting the results a few words are in order on the assignments and transition patterns for the various states involved in this model. Typical doublet l states $D_{13}(1520)$, $S_{11}(1535)$, $D_{33}(1670)$, $S_{31}(1650)$ undergo transitions to the $|56\rangle$ component of $|N\rangle$, while a doublet u state like $S'_{11}(1700)$ couples to its $|70\rangle$ component without violating the quartet selection rule associated with low-J states.[3] The $p\gamma$ transition of D_{15} as a quartet u state now goes through the $|70\rangle$ of $|N\rangle$ without violating the Moorehouse rule which controls the corresponding l-state transition. (The smaller mass of the latter should be taken into account with the considerable mass spread of D_{15} in the Particle Data Group tables.[17]) Examples of transitions in which both $|56\rangle$ and $|70\rangle$ components are involved are $P_{11}(1470)$, $F_{15}(1690)$, and probably also $P_{13}(1860)$ as an $L=2$ ($J=\frac{3}{2}$) excitation of $(\underline{70}, 0^+)$ (the $J=\frac{5}{2}$ state has a feeble coupling to $N\gamma$).

Table I shows our predictions together with the data of BC.[10] For a compact presentation of the results with both sign alternatives for the recoil effect, we have listed the individual contributions (with signs) of the D, R, and C terms to each amplitude in the form $D \pm R + C$, where $\pm R$ indicates $\rho \gtrless 0$, respectively. (The totaling is left to the reader.) The data of BC are shown *below* the predicted values. For the less-established cases the data of other groups are also listed with the BC figures.

The general fits to the established cases, especially D_{13}, S_{11}, $D_{15}(n\gamma)$, P_{33}, D_{33}, S_{31}, and F_{37}, seem to leave little to be desired. We remind the reader that for these cases the fit is almost parameter-free since the R term is very small here. This success represents our internal-consistency check on the model with authorized strengths of the D and C terms. The more sensitive tests of the model now come about through cases like $D_{15}(p\gamma)$ and $S'_{11}(1700)$ which are predicted to be *zero* under conventional $(\underline{8}_q)$ assignments. In the present model these are doublet u states and the predicted amplitudes seem to be in rather good accord (magnitude and sign) with the data. Another notable success[18] is $P_{11}(1470)$ which has the wrong sign and magnitude in the usual theory.[12] The assignment of $P_{13}(1810)$ to an l-type $L=2$ excitation of $\underline{70}$ is also tolerable. Finally, the assignment of $F_{35}(1890)$ as a Δ state of $(\underline{70}, 2^+)$ with $l_x = l_y = 1$,[6] the supermultiplet which provides the natural Regge recurrence of the $(\underline{70}, 0^+)$ ground state,[6,7] is also well borne out by the data, in contrast to the usual $\underline{10}_q$ assignment which does not even give the right signs.[12]

As to the role of the R term, its smallness does not make it particularly sensitive to the data, but it plays a crucial role in $A^n_{3/2}$ of F_{15}, which would otherwise vanish.[2] The sign of R in this case is mildly indicated to be positive, in agreement with the conclusions of Ref. 15. The same sign also gives a vastly improved fit, e.g., to $S_{11}(1535)$, again in agreement with other findings for $L=1$ amplitudes.[5,15] (The other cases do not appear to be sensitive enough to the sign of R.)

To summarize, our model not only provides high-quality fits (almost without parameters) to *all* the established cases, but the suggested assignments seem to work surprisingly well for several difficult cases which have so far escaped explicit understanding in the conventional quark picture. Admittedly there are more states in the low-lying regions, but their detectability (especially the J

TABLE I. Experimental photocouplings of BC and others, together with predictions. For $I=\frac{3}{2}$ resonances the two "neutron" columns refer to the associated $J-1$ states corresponding to the main J states. (See text for other notations.) The units used are 10^{-3} GeV$^{-1/2}$.

Resonance	$A^p_{1/2}$	$A^p_{3/2}$	$A^n_{1/2}$	$A^n_{3/2}$
$P_{11}(1470)$	$-120\pm0+0$ $-53;-135$ [a]		$+114\pm0+0$ 58	
$D_{13}(1520)$	$-99\pm12+95$ -12	$0\pm22+165$ 158	$+33\pm4-95$ -56	$0\pm7-165$ -136
$S_{11}(1535)$	$-62\pm34+141$ 63		$+21\pm11-141$ -109	
$D_{15}(1670)$	$+32\pm0+0$ $8;19\pm7$ [b]	$+40\pm0+0$ $21;16\pm2$ [b]	$-54\pm0+0$ -58	$-69\pm0+0$ -80
$S'_{11}(1700)$	$+26\pm14+0$ 44		$-26\pm14+0$ -22	
$D'_{13}(1705)$	$-48\pm5+0$ -5	$0\pm8+0$ -9	$+48\pm5+0$ 17	$0\pm8+0$ 22
$F_{15}(1690)$	$-48\pm12+73$ -4	$0\pm21+126$ $132;147\pm6$ [b]	$+28\pm9+0$ $34;23\pm3$ [b]	$0\pm16+0$ $-28;-41\pm4$ [b]
$P_{13}(1810)$	$-16\pm16+10$ 86	$0\pm4+16$ -60	$6\pm5-10$ -20	$0\pm1-16$ $46;-83\pm90$ [c]
$P_{33}(1232)$	$-140\pm0+0$ -129	$-210\pm0+0$ -251		
$D_{33}(1670)$ $S_{31}(1650)$	$+40\pm3+78$ 120	$0\pm5+135$ 117	$+21\pm10+111$ $55;78\pm6$ [b]	
$F_{37}(1950)$ $F_{35}(1890)$	$-60\pm0+0$ $-76;-88\pm27$ [c]	$-64\pm0+0$ $-65,-80\pm21$ [c]	$+12\pm1+20$ $35;19\pm27$ [c]	$0\pm2+34$ $-13;78\pm20$ [c]

[a] Ref. 18. [b] Ref. 8. [c] Ref. 9.

satellites) could conceivably be an experimental problem.

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