PHYSICAL REVIEW D VOLUME 16, NUMBER 7 1 OCTOBER 1977

# Quark light-cone model and current conservation

M. A. Goñi and F. Ramirez-Cacho
*Departamento de Física Teórica, Facultad de Ciencias Físicas, Universidad Complutense, Madrid-3, Spain*
(Received 6 December 1976)

A formulation of the quark light-cone commutators is given that satisfies current conservation on the light cone. The results are applied to spin terms in deep-inelastic leptoproduction.

## INTRODUCTION

In the description of hadronic behavior the idea of constituents of the nucleons has played a very important role. These constituents have been frequently identified with the quarks of Gell-Mann and Zweig.[1] More recently, the quark idea has been encouraged by the experiments on deep-inelastic leptoproduction, where the matrix elements on the light cone of the commutators of hadronic currents are measured. These commutators can be abstracted[2] from a naive quark model to be used afterwards in the prediction of relations and sum rules for physical processes.

In the use of light-cone models of current commutators, current conservation must not be disregarded. In a previous paper[3] we have discussed, in a general situation, what current conservation on the light cone (CCLC) means, and the restrictions which have to be imposed on the light-cone expansions for CCLC to be satisfied.

The quark light-cone model (QLCM) of Fritzsch and Gell-Mann[2] does not include those restrictions, as we shall see below, and it has to be modified if we want to have a model giving suppressed scaling behavior for all nonconserved structure functions in deep-inelastic leptoproduction.

In this paper we shall study the inclusion of CCLC in the QLCM. Also some applications of the model thus corrected to spin-dependent leptoproduction in the deep-inelastic region will be worked out. Our more important results are the following:

(i) We have found one formulation of the QLCM which verifies CCLC without essential modifications of the model. Our correction only affects the nonconserved spin-dependent terms in leptoproduction. Therefore it invalidates only the results obtained by several authors for these terms. The spin-independent terms do not need to be corrected as they already satisfied CCLC.

(ii) We have applied the model thus corrected to the computation of the spin-dependent terms in neutrino and electron deep-inelastic scattering assuming the structure for the weak (charged and neutral) and electromagnetic currents given by a gauge model. We also find some relations for the scale functions of physical processes on proton and neutron targets, and some sum rules.

As far as we know, only Wray[4] has considered the problem of current conservation in the QLCM, but his treatment only affects electroproduction and predicts dominant scaling for some nonconserved spin-dependent $\nu$-production terms, in disagreement with our analysis. Hey and Mandula[5] have studied spin-dependent electroproduction starting from an explicitly conserved expansion of commutators on the light cone. Our treatment agrees with this kind of expansion only for spin-dependent electroproduction. Other workers on this field, such as Dicus[6] and Ward,[7] have disregarded the question.

Applications have been carried out by Wray,[4] Dicus,[6] and Ward[7] for electroproduction and $\nu$ (charged) production, by using standard SU(3) currents. Our results agree with (in general) and enlarge those found by these authors.

The paper has been divided into two sections. In Sec. I, we study the QLCM and its modification as imposed by CCLC. Section II is devoted to applications.

## I. THE QUARK MODEL

In the study of inclusive leptoproduction on nucleus, we are interested in the knowledge of the hadronic tensor $W_{\mu\nu}$:

$$W_{\mu\nu}=\int\frac{dx}{4\pi}e^{iq\cdot x}\langle\vec{p},s|[J_\mu^\dagger(x),J_\nu(0)]|\vec{p},s\rangle$$

$$=-g_{\mu\nu}^{\perp}W_1+p_\mu^{\perp}p_\nu^{\perp}\frac{W_2}{M^2}-i\epsilon_{\mu\nu\rho\sigma}p^\rho q^\sigma\frac{W_3}{2M^2}+q_\mu q_\nu\frac{W_4}{M^2}+(p_\mu q_\nu+p_\nu q_\mu)\frac{W_5}{M^2}$$

$$-i\epsilon_{\mu\nu\rho\sigma}q^\rho s^\sigma\frac{Y_1}{2M}-i(q\cdot s)\epsilon_{\mu\nu\rho\sigma}q^\rho p^\sigma\frac{Y_2}{2M^3}-i\epsilon_{\mu\nu\rho\sigma}p^\rho s^\sigma\frac{Y_3}{2M}-g^{\perp}_{\mu\nu}(q\cdot s)\frac{Y_4}{M}+(q\cdot s)p^{\perp}_\mu p^{\perp}_\nu\frac{Y_5}{M^3}$$

$$+(p^{\perp}_\mu s^{\perp}_\nu+p^{\perp}_\nu s^{\perp}_\mu)\frac{Y_6}{M}+q_\mu q_\nu(q\cdot s)\frac{Y_7}{M^3}+(q_\mu p_\nu+q_\nu p_\mu)(q\cdot s)\frac{Y_8}{M^3}+(q_\mu s_\nu+q_\nu s_\mu)\frac{Y_9}{M}\ , \qquad (1.1)$$

where $W_i$ $(i=1-5)$, $Y_i$ $(i=1-9)$ are dimensionless functions of $q^2$ and $\nu$, $\nu\equiv q\cdot p$, $g^{\perp}_{\mu\nu}\equiv g_{\mu\nu}-q_\mu q_\nu/q^2$, $a^{\perp}_\mu \equiv a_\mu - q_\mu(a\cdot q)/q^2$, and normal currents under time reversal have been assumed. For conserved ones $W_4, W_5, Y_3, Y_7, Y_8, Y_9$ vanish.

In the quark model, the physical currents are some linear combinations of $V^a_\mu(x)=\bar q(x)\gamma_\mu\frac{1}{2}\Lambda_a q(x)$, $A^a_\mu(x)=\bar q(x)\gamma_\mu\gamma_5\frac{1}{2}\Lambda_a q(x)$, where $\Lambda_a$, $a=0,1,\ldots,n^2-1$, are the generators of the Lie algebra associated to the group U$(n)$, and $q(x)$ is the supermultiplet of quark fields. The current commutators on the light cone are[2]

$$[V^a_\mu(x),V^b_\nu(0)]\simeq[A^a_\mu(x),A^b_\nu(0)]$$
$$\simeq\frac{i}{2\pi}\partial^\rho[\epsilon(x^0)\delta(x^2)]\{F^{abc}[s_{\mu\nu\rho\sigma}\mathcal{V}^\sigma_c(x|0)-\epsilon_{\mu\nu\rho\sigma}\bar{\mathcal{A}}^\sigma_c(x|0)]+D^{abc}[s_{\mu\nu\rho\sigma}\bar{\mathcal{V}}^\sigma_c(x|0)+\epsilon_{\mu\nu\rho\sigma}\mathcal{A}^\sigma_c(x|0)]\}\ , \qquad (1.2)$$

$$[A^a_\mu(x),V^b_\nu(0)]\simeq[V^a_\mu(x),A^b_\nu(0)]$$
$$\simeq\frac{i}{2\pi}\partial^\rho[\epsilon(x^0)\delta(x^2)]\{F^{abc}[s_{\mu\nu\rho\sigma}\mathcal{A}^\sigma_c(x|0)-\epsilon_{\mu\nu\rho\sigma}\bar{\mathcal{V}}^\sigma_c(x|0)]+D^{abc}[s_{\mu\nu\rho\sigma}\bar{\mathcal{A}}^\sigma_c(x|0)+\epsilon_{\mu\nu\rho\sigma}\mathcal{V}^\sigma_c(x|0)]\}\ , \qquad (1.3)$$

where the bilocal operators are defined as

$$\mathcal{A}^a_\mu(x|y)\equiv\tfrac{1}{2}[\bar q(x)\gamma_\mu\gamma_5\tfrac{1}{2}\Lambda_a q(y)+(y\leftrightarrow x)]\ ,$$

$$\bar{\mathcal{A}}^a_\mu(x|y)=\frac{1}{2i}[\bar q(x)\gamma_\mu\gamma_5\tfrac{1}{2}\Lambda_a q(y)-(y\leftrightarrow x)]\ ,$$

and similar definitions for $\mathcal{V}^a_\mu(x|y)$ and $\bar{\mathcal{V}}^a_\mu(x|y)$,

$$s_{\mu\nu\rho\sigma}\equiv g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho}-g_{\mu\nu}g_{\rho\sigma}\ ,$$

and $F^{abc}, D^{abc}$ are defined through

$$[\Lambda_a,\Lambda_b]=2iF^{abc}\Lambda_c,\quad \{\Lambda_a,\Lambda_b\}=2D^{abc}\Lambda_c\ .$$

In order to calculate the Bjorken limits of the structure functions $W_i$ $(i=1-5)$, $Y_i$ $(i=1-9)$

$$\lim_{\mathrm{Bj}}\nu^{\alpha_i}\,Y_i(\nu,q^2)\to G_i(\chi),\quad i=1-9$$
$$\lim_{\mathrm{Bj}}\nu^{\bar\alpha_i}\,W_i(\nu,q^2)\to F_i(\chi),\quad i=1-5\ , \qquad (1.4)$$

where $\alpha_i$, $\bar\alpha_i$ are the scale orders, and $\chi\equiv-q^2/2\nu$, one needs to define the expectation value on a proton state of the bilocal operators:

$$\langle p|\mathcal{V}^\mu_c(x|0)|p\rangle=p^\mu\tilde V^c_1(x^2x\cdot p)+x^\mu\tilde V^c_2(x^2,x\cdot p)\ , \qquad (1.5)$$

$$\langle p|\mathcal{A}^\mu_c(x|0)|p\rangle=s^\mu\tilde A^c_1(x^2,x\cdot p)+p^\mu(x\cdot s)\tilde A^c_2(x^2,x\cdot p)$$
$$+x^\mu(x,s)\tilde A^c_3(x^2,x\cdot p)\ ,$$

and similar definitions for $\langle\bar{\mathcal{A}}^\mu_c(x|0)\rangle$ and $\langle\bar{\mathcal{V}}^\mu_c(x|0)\rangle$. The spin-dependent scaling functions arise as combinations of

$$A^c_i(\chi)\equiv\frac{1}{2\pi}\int d\alpha\, e^{-i\chi\cdot\alpha}\tilde A^c_i(\alpha), \qquad (1.6)$$

for $i=1,2;$ $c=0,1,\ldots,n^2-1$, where

$$\tilde A^c_i(\alpha)\equiv\tilde A^c_i(0,\alpha)\ .$$

The use of the commutators given above leads to dominant scale order for the function $Y_3$ in electroproduction. The conservation of the electro-magnetic current invalidates this result.[4] The scale orders obtained from this model for $Y_3$, $Y_8$, and $Y_9$ in weak production are also dominant. On the other hand, the quark model gives suppressed scaling behavior for the nonconserved functions $W_4, W_5$ (spin independent) and $Y_7$ (spin dependent).

These results show that the QLCM given in (1.2), (1.3) does not accommodate the conservation of the current and, *a fortiori*, it violates the condition of CCLC[3] that predicts scaling suppressed for all the structure functions associated with nonconserved terms, independently of whether the current is conserved or not. CCLC must hold if the scale dimension of the current divergence is less than four, as happens in simple models where the current conservation is broken by mass terms in the Lagrangian.

To see how CCLC is violated by the QLCM, it is convenient to formulate the right-hand sides of (1.2), (1.3) in terms of irreducible local operators.

In Ref. 3 we have found the general expansion for the commutator of two currents giving any contribution to the spin-dependent terms in inclusive leptoproduction. The light-cone-dominant piece is (disregarding internal indices)

$$[J_\mu^\dagger(x),J_\nu(0)]\simeq \sum_n [E(x^2,\Delta)a_n x_\mu x_\nu x_{\lambda_1}\cdots x_{\lambda_n}+E(x^2,\Delta-1)c_n g_{\mu\nu}x_{\lambda_1}\cdots x_{\lambda_n}$$
$$+E(x^2,\Delta-2)d_n g_{\mu\lambda_1}g_{\nu\lambda_2}x_{\lambda_3}\cdots x_{\lambda_n}+E(x^2,\Delta-1)b_n(x_\mu g_{\nu\lambda_1}+x_\nu g_{\mu\lambda_1})x_{\lambda_2}\cdots x_{\lambda_n}$$
$$+E(x^2,\Delta-1)l_n\epsilon_{\mu\nu\lambda_1 k}x^k x_{\lambda_2}\cdots x_{\lambda_n}]\,O^{(\lambda_1\cdots\lambda_n)}(0)$$
$$+\sum_n [E(x^2,\Delta'-1)2g_n(g_{\mu[\rho}x_{\sigma]}g_{\nu\lambda_1}+\mu\leftrightarrow\nu)x_{\lambda_2}\cdots x_{\lambda_n}+E(x^2,\Delta')2h_n x_{[\rho}\epsilon_{\sigma]\mu\nu k}x^k x_{\lambda_1}\cdots x_{\lambda_n}$$
$$+E(x^2,\Delta')2k_n(x_\mu g_{\nu[\rho}x_{\sigma]}+\mu\leftrightarrow\nu)x_{\lambda_1}\cdots x_{\lambda_n}+E(x^2,\Delta')l_n(x_\mu\epsilon_{\nu\rho\sigma k}x^k-\mu\leftrightarrow\nu)x_{\lambda_1}\cdots x_{\lambda_n}]$$
$$\times O^{[\rho\sigma](\lambda_1\cdots\lambda_n)}(0)\,. \qquad (1.7)$$

The contribution to the spin-independent terms comes from a similar expansion[8] with only full symmetric operators and the corresponding set of coefficients $\bar a_n$, $\bar b_n$, $\bar c_n$, and $\bar d_n$.

After some straightforward algebra we can see that the QLCM (1.2), (1.3) corresponds to the following particular situation:

$$c_n+b_n=0,\quad a_n=0$$
$$\bar c_n+\bar b_n=0,\quad \bar a_n=0\,,$$
$$d_n=0,\quad l_n=0,\quad g_n=0,$$
$$\bar d_n=0\,,$$
$$\Delta-1=\Delta'=2$$

The coefficients of the fully symmetric operators satisfy the CCLC restrictions for $\Delta=3$:

$$(n-1)a_n+2(c_n+b_n)=0\,,\qquad n(c_n+b_n)+2d_n=0\,, \qquad (1.8)$$

and identical relations for the coefficient $\bar a_n,\bar b_n,\bar c_n,\bar d_n$. On the other hand, the coefficients of the mixed operators do not satisfy the CCLC relation for $\Delta'=2$:

$$(n+2)k_n+g_n=0\,,\qquad nl_n-(n+2)h_n=0\,. \qquad (1.9)$$

Therefore the spin-independent scaling functions satisfy CCLC. We see also that CCLC violation is the spin-dependent terms comes from the mixed operators. This fact explains the results quoted above, that CCLC is violated only by the spin terms in the QLCM.

Furthermore, we only need to add some new mixed-symmetry local operators to the commutator expansions to incorporate CCLC into this model. As a matter of fact it is this mechanism that works in the free model and restores current conservation on the light cone.

We are also interested in letting this modification be a minimal one. This will be accomplished if the addition of these new operators does not change the predictions from the unmodified quark model (1.2), (1.3) for the scale functions related to conserved terms in $W_{\mu\nu}$. The spin-independent functions are already free of suffering any modification as only mixed-symmetry operators are added to the expansions.

As for $G_3$, only $h_n$, $l_n$ mixed coefficients contribute; let us add to the commutator a new mixed piece with new characteristic coefficients $h'_n$, $l'_n$, in such a way that

$$h'_n=l'_n=-\left(\frac{n+2}{2}\right)h_n\,.$$

This addition does not modify $G_1, G_2$ (because $h'_n=l'_n$) and the resulting expression satisfies CCLC (1.9), i.e.,

$$nl'_n-(n+2)(h_n+h'_n)=0\,.$$

Therefore, the piece to be added to (1.2) is

$$\epsilon(x^0)\delta(x^2)\epsilon_{\mu\nu\rho\sigma}O_1^{[\rho\sigma]}(x|0)\,, \qquad (1.10)$$

where the bilocal operator is

$$O_1^{[\rho\sigma]}(x|0)=-\pi i\sum_n (n+2)h_n x_{\lambda_1}\cdots x_{\lambda_n}O^{[\rho\sigma](\lambda_1\cdots\lambda_n)}(0)\,.$$

Analogously, to correct $G_8$ and $G_9$ we shall add some suitable antisymmetric bilocal operators giving new coefficients $g'_n$, $k'_n$. The CCLC relations (1.9) require

$$2g'_n+(k_n+k'_n)(n+2)=0\,,$$

and $G_5, G_6$ do not alter if the relations below hold:

$$-2g'_n+nk'_n=0\,.$$

Note that mixed-symmetry operators do not give any dominant contribution to $G_4$ in this model, and dominant contributions to $G_7$ do not arise as $G_5$ and $G_6$ remain unchanged (see Ref. 3).

Therefore the new piece to be added to (1.3) is

$$[\epsilon(x^0)\delta(x^2)(g_{\mu[\rho}x_{\sigma]}\partial_\nu+\mu\leftrightarrow\nu)+2\epsilon(x^0)\delta'(x^2)(x_\mu g_{\nu[\rho}x_{\sigma]}+\mu\leftrightarrow\nu)]\,O_2^{[\rho\sigma]}(x|0)$$
$$=(\epsilon(x^0)\delta(x^2)(g_{\mu[\rho}x_{\sigma]}\partial_\nu+\mu\leftrightarrow\nu)+\{\partial_\mu[\epsilon(x^0)\delta(x^2)]g_{\nu[\rho}x_{\sigma]}+\mu\leftrightarrow\nu\})\,O_2^{[\rho\sigma]}(x|0)\,. \qquad (1.11)$$

The final result is that the QLCM must be modified in the following way:

$$[V_\mu^a(x), V_\nu^b(0)] \simeq [A_\mu^a(x), A_\nu^b(0)] \simeq \text{old terms} - \frac{i}{2\pi}\epsilon(x^0)\delta(x^2)\epsilon_{\mu\nu\rho\sigma}[D^{abc}O_{1c}^{[\rho\sigma]}(x|0) - F^{abc}\overline{O}_{1c}^{[\rho\sigma]}(x|0)], \tag{1.2$'$}$$

$$[A_\mu^a(x), V_\nu^b(0)] \simeq [V_\mu^a(x), A_\nu^b(0)] \simeq \text{old terms} + \frac{i}{2\pi}\partial_\mu\{[\epsilon(x^0)\delta(x^2)g_{\nu[\rho}x_{\sigma]} + (\mu \leftrightarrow \nu)][F^{abc}O_{2c}^{\rho\sigma}(x|0) + D^{abc}\overline{O}_{2c}^{\rho\sigma}(x|0)]\}, \tag{1.3$'$}$$

where the bilocals $O_{1,c}$, $O_{2,c}$ have matrix elements at $x^2=0$ defined through

$$\langle O_{i,c}^{[\rho\sigma]}(x|0)\rangle|_{x^2=0} = p^{[\rho}s^{\sigma]}\tilde{g}_i(p\cdot x) + \cdots, \quad i=1,2,$$

with $g_1^c(\chi) = g_2^c(\chi) = i[\chi A_1^2(\chi) + iA_2^c(\chi)]$ and similar definitions hold for $\overline{O}_{1,c}(x|0)$, $\overline{O}_{2,c}(x|0)$.

The right-hand sides of (1.2′), (1.3′) can be written in a more compact form by adding some new terms that do not give any dominant contribution to the scale functions. The resulting expressions for these commutators are the following:

$$\begin{aligned}[V_\mu^a(x), V_\nu^b(0)] &\simeq [A_\mu^a(x), A_\nu^b(0)] \\ &\simeq \frac{i}{2\pi}(\partial^\rho[\epsilon(x^0)\delta(x^2)]s_{\mu\nu\rho\sigma}[F^{abc}\mathcal{V}_c^\sigma(x|0) + D^{abc}\overline{\mathcal{V}}_c^\sigma(x|0)] \\ &\quad + \partial^\rho\{\epsilon(x^0)\delta(x^2)\epsilon_{\mu\nu\rho\sigma}[D^{abc}\mathcal{A}_c^\sigma(x|0) - F^{abc}\overline{\mathcal{A}}_c^\sigma(x|0)]\}),\end{aligned} \tag{1.2$''$}$$

$$\begin{aligned}[A_\mu^a(x), V_\nu^b(0)] &\simeq [V_\mu^a(x), A_\nu^b(0)] \\ &\simeq \frac{i}{2\pi}(\partial^\rho[\epsilon(x^0)\delta(x^2)]\{s_{\mu\nu\rho\sigma}[F^{abc}\mathcal{A}_c^\sigma(x|0) + D^{abc}\overline{\mathcal{A}}_c^\sigma(x|0)] + \epsilon_{\mu\nu\rho\sigma}[D^{abc}\mathcal{V}_c^\sigma(x|0) - F^{abc}\overline{\mathcal{V}}_c^\sigma(x|0)]\} \\ &\quad - \tfrac{1}{2}(g_{\mu[\rho}\partial_{\sigma]}\partial_\nu + \mu \leftrightarrow \nu)\epsilon(x^0)\theta(x^2)[F^{abc}O_{2c}^{\rho\sigma}(x|0) + D^{abc}\overline{O}_{2c}^{\rho\sigma}(x|0)]).\end{aligned} \tag{1.3$''$}$$

Our treatment of the quark model is not equivalent to writing an explicitly conserved expansion for the light-cone current commutators. The modified QLCM (1.2″), (1.3″) gives suppressed scaling behavior for $W_4, W_5, Y_3, Y_7, Y_8, Y_9$ as dictated by CCLC. For instance, the scaling behavior of the spin-dependent structure functions is

$$\begin{aligned}&\frac{\nu Y_1}{2M} \to G_1(\chi), \quad \frac{\nu^2 Y_2}{2M^3} \to G_2(\chi), \quad \frac{\nu Y_4}{M} \to G_4(\chi), \quad \frac{\nu^2 Y_5}{M^3} \to G_5(\chi), \quad \frac{\nu Y_6}{M} \to G_6(\chi), \\ &\frac{\nu^\alpha Y_3}{2M} \to G_3(\chi), \quad \frac{\nu^{\alpha+1} Y_7}{M^3} \to G_7(\chi), \\ &\frac{\nu^{\alpha+1} Y_8}{M^3} \to G_8(\chi), \quad \frac{\nu^\alpha Y_9}{M} \to G_9(\chi),\end{aligned} \tag{1.12}$$

where $\alpha > 1$, $G_3, G_7, G_8, G_9$ are unknown for weak processes and, of course, are zero for electroproduction.

## II. APPLICATIONS

In this section we shall use the modified QLCM in order to investigate the behavior and properties of the spin terms in electroproduction and weak (charged and neutral) production on nucleons. We have chosen physical currents suggested by gauge models of the kind first discussed by Salam[9] and Weinberg,[10] and incorporating the Glashow-Iliopoulos-Maiani[11] mechanism for the suppression of neutral currents that violate strangeness. Those currents are linear combinations of U(4) vector and axial-vector currents in the following way:

$$J_\mu^\gamma(x) = V_\mu^{(3)}(x) + \frac{1}{\sqrt{3}}V_\mu^{(8)}(x) - (\tfrac{2}{3})^{1/2}V_\mu^{(15)}(x) + \frac{\sqrt{2}}{3}V_\mu^{(0)}(x), \tag{2.1}$$

$$J_\mu^{W\pm}(x) = [V_\mu(x) - A_\mu(x)]^{[1\pm i2]+[13\mp i14]}\cos\theta_C + [V_\mu(x) - A_\mu(x)]^{[4\pm i5]-[11\mp i12]}\sin\theta_C, \tag{2.2}$$

$$J_\mu^Z(x) = [V_\mu^{(3)}(x) - A_\mu^{(3)}(x)] + \frac{1}{\sqrt{3}}[V_\mu^{(8)}(x) - A_\mu^{(8)}(x)] - (\tfrac{2}{3})^{1/2}[V_\mu^{(15)}(x) - A_\mu^{(15)}(x)] - \Omega J_\mu^\gamma(x), \tag{2.3}$$

where $\Omega \equiv 4\sin^2\theta_W$, $\theta_C$ is the Cabibbo angle and $\theta_W$ is the Weinberg angle.

Applications for the spin-independent terms have been worked out in a paper by Budny and Scharbach.[12]

Therefore we shall restrict ourselves to the spin-dependent terms. Taking into account the discussion of Sec. I and the procedure of Ref. 2, we have obtained for the scaling functions the expressions below:

(i) *electroproduction (disregarding weak effects)*

$$G_1^{ep}(\chi)=-\frac{1}{2}\left\{\frac{5\sqrt{2}}{9}A_1^{(0)}+\frac{1}{3}\left[A_1^{(3)}+\frac{1}{\sqrt{3}}A_1^{(8)}-(\tfrac{2}{3})^{1/2}A_1^{(15)}\right]\right\}, \tag{2.4}$$

$$G_2^{ep}(\chi)=-\frac{i}{2}\left\{\frac{5\sqrt{2}}{9}A_2^{(0)\prime}+\frac{1}{3}\left[A_2^{(3)\prime}+\frac{1}{\sqrt{3}}A_2^{(8)\prime}-(\tfrac{2}{3})^{1/2}A_2^{(15)\prime}\right]\right\}; \tag{2.5}$$

(ii) *production by charged weak current*

$$G_1^{\nu(\bar{\nu})p}(\chi)=\frac{1}{2}\left\{\pm 2i\left[\bar{A}_1^{(3)}+\frac{1}{\sqrt{3}}\bar{A}_1^{(8)}+(\tfrac{2}{3})^{1/2}\bar{A}_1^{(15)}\right]-2\sqrt{2}A_1^{(0)}\right\}, \tag{2.6}$$

$$G_2^{\nu(\bar{\nu})p}(\chi)=\frac{i}{2}\left\{\pm 2i\left[\bar{A}_2^{(3)\prime}+\frac{1}{\sqrt{3}}\bar{A}_2^{(8)\prime}+(\tfrac{2}{3})^{1/2}\bar{A}_2^{(15)\prime}\right]-2\sqrt{2}A_2^{(0)\prime}\right\}, \tag{2.7}$$

$$G_4^{\nu(\bar{\nu})p}(\chi)=-\frac{i}{2}\left\{2\sqrt{2}\,(\bar{A}_1^{(0)}+i\bar{A}_2^{(0)\prime})\pm 2i\left[(A_1^{(3)}+iA_2^{(3)\prime})+\frac{1}{\sqrt{3}}(A_1^{(8)}+iA_2^{(8)\prime})-(\tfrac{2}{3})^{1/2}(A_1^{(15)}+iA_2^{(15)\prime})\right]\right\}, \tag{2.8}$$

$$G_5^{\nu(\bar{\nu})p}(\chi)=\left\{2\sqrt{2}\,(\bar{A}_2^{(0)}+\chi\bar{A}_2^{(0)\prime})\pm\left[(A_2^{(3)}+\chi A_2^{(3)\prime})+\frac{1}{\sqrt{3}}(A_2^{(8)}+\chi A_2^{(8)\prime})-(\tfrac{2}{3})^{1/2}(A_2^{(15)}+\chi A_2^{(15)\prime})\right]\right\}; \tag{2.9}$$

(iii) *production by neutral weak currents*

$$G_1^{zp}(\chi)=\frac{1}{4}\left\{-\sqrt{2}\,(\tfrac{5}{18}\Omega^2-\Omega+2)A_1^{(0)}+\left(1-\frac{\Omega}{2}\right)\frac{\Omega}{3}\left[A_1^{(3)}+\frac{1}{\sqrt{3}}A_1^{(8)}-(\tfrac{2}{3})^{1/2}A_1^{(15)}\right]\right\}, \tag{2.10}$$

$$G_2^{zp}(\chi)=\frac{i}{4}\left\{-\sqrt{2}\,(\tfrac{5}{18}\Omega^2-\Omega+2)A_2^{(0)\prime}+\left(1-\frac{\Omega}{2}\right)\frac{\Omega}{3}\left[A_2^{(3)\prime}+\frac{1}{\sqrt{3}}A_2^{(8)\prime}-(\tfrac{2}{3})^{1/2}A_2^{(15)\prime}\right]\right\}, \tag{2.11}$$

$$G_4^{zp}(\chi)=\frac{i}{4}\left\{-2\sqrt{2}\left(1-\frac{\Omega}{2}\right)(\bar{A}_1^{(0)}+i\bar{A}_2^{(0)\prime})+\frac{\Omega}{3}\left[(\bar{A}_1^{(3)}+i\bar{A}_2^{(3)\prime})+\frac{1}{\sqrt{3}}(\bar{A}_1^{(8)}+i\bar{A}_2^{(8)\prime})-(\tfrac{2}{3})^{1/2}(\bar{A}_1^{(15)}+i\bar{A}_2^{(15)\prime})\right]\right\}, \tag{2.12}$$

$$G_5^{zp}(\chi)=\frac{1}{2}\left\{-2\sqrt{2}\left(1-\frac{\Omega}{2}\right)(\bar{A}_2^{(0)}+\chi\bar{A}_2^{(0)\prime})+\frac{\Omega}{3}\left[(\bar{A}_2^{(3)}+\chi\bar{A}_2^{(3)\prime})+\frac{1}{\sqrt{3}}(\bar{A}_2^{(8)}+\chi\bar{A}_2^{(8)\prime})-(\tfrac{2}{3})^{1/2}(\bar{A}_2^{(15)}+\chi\bar{A}_2^{(15)\prime})\right]\right\}. \tag{2.13}$$

The scale functions $G_6$ in weak production can be obtained from the general relation

$$2\chi G_4(\chi)=G_5(\chi)+2G_6(\chi)\,, \tag{2.14}$$

which is a relation characteristic of the QLCM that holds, whatever the structure chosen for the weak currents. Therefore it also arises for $\nu(\bar{\nu})$ production from standard SU(3) currents.[7]

Note that all the formulas are independent of the Cabibbo angle. If the target was a neutron instead of a proton, the scaling functions would be obtained by changing the sign in front of the $A^{(3)}$ terms.

The formulas above imply the relations

$$G_i^{\nu p+\bar{\nu}p}=G_i^{\nu n+\bar{\nu}n}\equiv G_i^{\nu+\bar{\nu}}\quad (i=1,2,4,5,6)\,, \tag{2.15}$$

$$12(G_1+G_2)^{ep(n)}=\tfrac{5}{3}\,(G_1+G_2)^{\nu+\bar{\nu}}-G_4^{\nu p(n)-\bar{\nu}p(n)}\,, \tag{2.16}$$

$$2G_{1,2}^{zp(n)}=\left(\frac{1}{2}-\frac{\Omega}{9}\right)G_{1,2}^{\nu+\bar{\nu}}-\Omega\left(1-\frac{\Omega}{2}\right)G_{1,2}^{ep(n)}\,, \tag{2.17}$$

$$4G_4^{zp(n)}=\left(1-\frac{\Omega}{2}\right)G_4^{\nu+\bar{\nu}}+\frac{\Omega}{6}\,(G_1+G_2)^{\nu p(n)-\bar{\nu}p(n)}\,, \tag{2.18}$$

and therefore

$$G_4^{\nu p-\nu n}=-6(G_1+G_2)^{ep-en}\,, \tag{2.19}$$

$$G_{1,2}^{zp-zn}=-\left(1-\frac{\Omega}{2}\right)\frac{\Omega}{2}\,G_{1,2}^{ep-en}\,, \tag{2.20}$$

$$G_4^{zp-zn}=\frac{\Omega}{12}\,(G_1+G_2)^{\nu p-\nu n}. \tag{2.21}$$

As for sum rules (SR), we have the following:

$$\int_0^1 G_2^{ep(n)}d\chi=0\,, \tag{2.22}$$

$$\int_0^1 G_2^{\nu+\bar{\nu}}d\chi=0\,, \tag{2.23}$$

$$\int_0^1 G_2^{zp(n)}d\chi=0\,, \tag{2.24}$$

which are independent of the structure of the currents. (2.22) was derived by Hey and Mandula[5] and by Dicus, Jackiw, and Teplitz.[13] (2.23) has been obtained by Dicus[6] and Ward.[7]

Another set of SR connect integrals over scaling functions with measureable axial-vector form factors at zero momentum transfer in weak elastic processes:

$$\int_0^1 (\tfrac{5}{6} G_1^{\nu+\bar\nu} - 6G_1^{ep(n)})d\chi = \tfrac{1}{2}\,\Gamma_A^{zp(n)}\,, \tag{2.25}$$

$$\int_0^1 G_4^{\nu p(n)-\bar\nu p(n)}d\chi = \Gamma_A^{zp(n)}\,, \tag{2.26}$$

where

$$s_\mu \Gamma_A^{zp(n)} = \langle p(n)(\vec p, s)\,|A_\mu^z(0)|\,p(n)(\vec p, s)\rangle\,.$$

SR (2.26), when $G_4$ is put in terms of $G_{5,6}$ through (2.14), are the scaling limit of the generalized Adler SR,[14] and they depend only on the [0, 0] current-algebra commutators. The general fixed-$q^2$ SR has been derived by Lane and Suzuki[15] from the standard weak current; in this case the right-hand side is not directly related to any physical processes.

From (2.25), (2.26), the Bjorken[16] SR arise:

$$\int_0^1 G_1^{ep-en}d\chi = -\frac{1}{12}\frac{G_A}{G_V} \tag{2.27}$$

and its weak analogous

$$\int_0^1 G_4^{\nu p-\nu n}d\chi = \frac{1}{2}\frac{G_A}{G_V}. \tag{2.28}$$

Moreover, if we assume that the weak axial-vector currents are a good SU(4) 15-plet and that the nucleons belong to the 20′ representation,[17] one obtains $\langle A_\mu^{(8)}(0)\rangle = \sqrt{2}\langle A_\mu^{(15)}(0)\rangle$. This relation gives the right-hand sides of (2.25), (2.26) in terms of $G_A/G_V$, so one has

$$\int_0^1 (\tfrac{5}{6}G_1^{\nu+\bar\nu} - 6G_1^{ep(n)})d\chi = \pm\frac{1}{4}\frac{G_A}{G_V}\,, \tag{2.29}$$

$$\int_0^1 G_4^{\nu p(n)-\bar\nu p(n)}d\chi = \pm\frac{1}{2}\frac{G_A}{G_V}. \tag{2.30}$$

From (2.29) we have

$$\int_0^1 G_1^{ep+en}d\chi = \tfrac{5}{18}\int_0^1 G_1^{\nu+\bar\nu}d\chi\,, \tag{2.31}$$

and from (2.30) and the relation (2.17), it follows that

$$\int_0^1 G_1^{zp+zn}d\chi = \tfrac{1}{2}(\tfrac{5}{18}\,\Omega^2 - \Omega + 2)\int_0^1 G_1^{\nu+\bar\nu}d\chi\,, \tag{2.32}$$

$$\int_0^1 G_1^{zp+zn}d\chi = (\tfrac{1}{2}\Omega^2 - \tfrac{9}{5}\Omega + \tfrac{18}{5})\int_0^1 G_1^{ep+en}d\chi. \tag{2.33}$$

Further SR beyond the scope of SU(4) symmetry can be obtained by estimating $A_1^{(0)}(0)$ in the way pointed out by Ellis and Jaffe.[18] These approximate SR are

$$\int_0^1 G_1^{ep(n)}d\chi \simeq -\frac{1}{24}\frac{G_A}{G_V}\left[\pm 1 + \frac{5}{3}\,\frac{3(f/d)-1}{f/d-1}\right], \tag{2.34}$$

$$\int_0^1 G_1^{\nu+\bar\nu}d\chi \simeq -\frac{1}{2}\frac{G_A}{G_V}\left[\frac{3(f/d)-1}{f/d+1}\right], \tag{2.35}$$

$$\int_0^1 G_1^{zp(n)}d\chi \simeq \frac{1}{16}\left[\pm\left(1-\frac{\Omega}{2}\right)\frac{\Omega}{3} - \left(\frac{5\Omega^2}{16} - \Omega + 2\right)\frac{3(f/d)-1}{f/d+1}\right], \tag{2.36}$$

where $f, d$ are the usual axial-vector SU(3) couplings $f + d = \frac{1}{2}G_A/G_V$, $f/d \simeq \frac{2}{3}$.

Heretofore we have considered the effect of the whole weak charged current (2.2), but in the kinematical region below the charm threshold, the charm-changing piece is ineffective. It is worthwhile to consider the scaling functions associated to this truncated weak current. In that case, and in the $\theta_C = 0$ approximation, only the relations (2.19)–(2.21) can be obtained, and (2.15) is replaced by

$$G_i^{\nu p} = G_i^{\bar\nu n} \quad (i = 1, 2, 4, 5, 6)\,. \tag{2.15′}$$

As for the SR (1.25)–(1.28), only the two last ones can be obtained. (2.29), (2.31) do not arise and we get instead

$$\int_0^1 (\tfrac{5}{6}G_1^{\nu+\bar\nu} - 3G_1^{ep(n)})d\chi = -\tfrac{5}{4}(f - d/3) \pm \tfrac{1}{4}(f+d) \tag{2.29′}$$

and

$$\int_0^1 (\tfrac{5}{3}G_1^{\nu+\bar\nu} - 3G_1^{ep+en})d\chi = -\tfrac{5}{2}(f - d/3)\,. \tag{2.31′}$$

(2.30) is now equivalent to (2.28). (2.33) is obviously still valid and a new (2.32′) could be obtained from (2.33) and (2.31′). The last three approximate SR (2.34)–(2.36) are still arising, and from them (2.31), (2.32) appear as approximated SR.

It is also interesting to carry out a study of the spin-dependent terms under the assumption of standard SU(3) electromagnetic and weak charged currents, instead of the ones in (2.1), (2.2). The results, in the case $\theta_C = 0$, can be summarized as follows: relations (2.15′), (2.19) and the general sum rules (2.22), (2.23), (2.27), (2.28) can be obtained. These conclusions were derived by a number of workers in this field (see Ward[7] and references therein). By assuming $A_\mu^i$ ($i = 1-8$) are a good octet, one gets

$$\int_0^1 (\tfrac{2}{3}G_1^{\nu+\bar\nu} - 4G_1^{ep(n)})d\chi = -\tfrac{1}{3}(f-d/3) \pm \tfrac{1}{3}(f+d), \qquad (2.29'')$$

$$\int_0^1 (G_1^{ep+en} - \tfrac{1}{6}G_1^{\nu+\bar\nu})d\chi = +\tfrac{1}{6}(f-d/3). \qquad (2.31'')$$

SR (2.29″), (2.31″) were first derived by Wray.[4] Approximate SR (2.34), (2.35) also follow, and from them (2.31) appears now as an approximate SR. (2.37) was derived by Ellis and Jaffe,[18] by assuming a low contribution of the quarks that carry strangeness in the nucleon.

We can summarize our results in this section.

(1) Relations (2.15)–(2.21) are new; (2.15), (2.19) are also valid for the $\Delta S = \Delta C = 0$ piece of the weak currents, and they were derived in this situation by Wray,[4] Dicus,[6] Ward.[7]

(2) Sum rules (2.25), (2.26) are new; their consequences (2.27), (2.28) were found by Bjoken,[16] Wray,[4] and Ward.[7] The sum rules (2.29)–(2.36) and (2.29′), (2.31′) are also new.


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