Neutrino production of vector and axial-vector mesons at high energies

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A phenomenological study is made of the reactions $\nu N \to \mu^- V^+(A^+)N$ and $\nu N \to \nu V^0(A^0)N$ (and similar reactions induced by antineutrinos) at high energies. Particular attention is paid to a detailed kinematical analysis. The differential cross section contains nine independently measurable terms, each showing a different dependence on Φ and ϵ; four of these terms are vector–axial-vector interferences. An analogous separation holds for the spin density matrix elements of the decaying vector or axial-vector meson. Consequences resulting from s-channel helicity conservation and other properties of a diffraction mechanism have been worked out for the cross sections and the density matrix elements. Once the diffraction mechanism is known vector- and axial-vector-meson production can provide useful information about the structure of the weak neutral current. Problems of model calculations are discussed.

I. INTRODUCTION

During the last few years inclusive hadron production by neutrinos (antineutrinos) has been studied extensively. Presently there will also be increased interest in exclusive channels. Here the diffractive production of vector and axial-vector mesons will presumably play the most important role at high energies:

$$\nu(\bar{\nu})+N\to \mu^-(\mu^+)+V^+(V^-)+N, \qquad \text{(A)}$$

$$\nu(\bar{\nu})+N\to \mu^-(\mu^+)+A^+(A^-)+N, \qquad \text{(B)}$$

$$\nu(\bar{\nu})+N\to \nu(\bar{\nu})+V^0+N, \qquad \text{(C)}$$

$$\nu(\bar{\nu})+N\to \nu(\bar{\nu})+A^0+N, \qquad \text{(D)}$$

where V denotes a vector meson ($\rho, \omega, \phi, F^*, K^*, D^*, \psi, \ldots$) and A denotes an axial-vector meson ($A_1, \omega_A, \phi_A, F_A, K_A, D_A, \psi_A, \ldots$). Reactions (A) and (B) are induced by charged weak currents, whereas reactions (C) and (D) imply neutral ones. Some very preliminary data on ρ^+, A_1^+, ρ^0, and ϕ production already exist.[1]

Usually exclusive reactions can give more specific information on the dynamics involved than inclusive ones. In particular, this was the case in photoproduction and electroproduction of vector mesons. There much insight into the diffraction mechanism [e.g., s-channel helicity conservation (SCHC), magnitude of the diffractive slope, ...] as well as the nature of the photon (e.g., vector-meson dominance, shrinkage) and the structure of the vector mesons has been gained. In principle the weak reactions (A)–(D) could give even more information than electroproduction as the weak current contains both a vector and an axial-vector part and appears in both charged and neutral form. More specifically one can learn about the following problems:

(a) structure of the weak current (isospin and space-time properties of the neutral current),

(b) vector and axial-vector dominance of the weak current (generalized vector dominance, A_1 dominance?),

(c) properties of the produced vector and axial-vector mesons (Is the A_1 a resonance? Does the ψ_A exist? etc.),

(d) Diffractive mechanism (SCHC for the axial-vector current? Behavior of slope, shrinkage ...).

The importance of some of these points has already been stressed in Ref. 2. There the cross sections for ν production of $\rho^\pm$ and $A_1^\pm$ mesons were estimated by applying A_1 dominance of the weak axial-vector current and ρ dominance of the weak vector current. Gaillard *et al.*[3] extended these ideas to the reactions (C) and (D) induced by neutral currents including production of new particles. The useful role of exclusive diffractive processes for studying the Lorentz structure and the internal symmetry of the neutral current was further emphasized by Hung and Sakurai[4] and studied in more detail by Cho.[5] A further phenomenological study of diffractive vector-meson production in lepton-nucleon reactions has been given by Chen *et al.*,[6] using information from electroproduction. The contribution to charmed-meson production due to F^* dominance was also calculated in Ref. 7, and its implications for the high-

y anomaly and dimuon production in $\bar{\nu}N$ scattering were discussed in Ref. 8.

In the present paper we first give a detailed kinematical analysis of reactions (A) to (D) (in Sec. II). Generally the cross section for processes (A) to (D) depends on five kinematic variables: E, the incident neutrino energy, ν, the difference between the incoming and outcoming lepton energy in the lab system, Q^2, the momentum transfer squared between the incoming and outgoing lepton, t, the momentum transfer squared between the incoming and outgoing nucleon, and Φ, the angle between the lepton and hadron plane. In our kinematical analysis we shall follow closely the formalism already applied to electroproduction of vector mesons[9] where the dependence on the angle Φ allows a separation of the cross section into several distinct parts, each of which yields new dynamical information.

In the analysis of vector-meson production the observation of the vector-meson decay gives additional information. In particular, this will be useful for the neutral-current-induced reactions (C) and (D) where the decay products, but not the outgoing neutrino, can be detected directly. Questions such as SCHC and other properties of the reaction mechanism can be answered by studying the decay density matrix of the vector meson as has been the case in electroproduction.[10] We shall discuss the analogous consequences in neutrino production.

Since the structure of the charged weak currents is better known, the diffraction mechanism is best studied in the reactions (A) and (B). We elaborate more on this point in Sec. III. Once the features of diffractive scattering in neutrino production are known, the properties of the hadronic

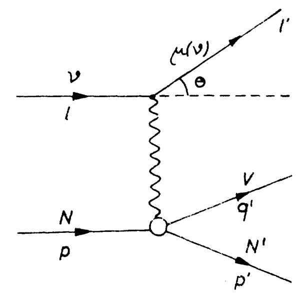

FIG. 1. Graph for the reaction $\nu + N \to \mu(\nu) + N + V$.

part of the weak neutral current can be determined with the help of processes (C) and (D). This is discussed in Sec. IV. In Sec. V we shall relate neutrino production to electroproduction and discuss the problems which arise in making numerical predictions. We finish with some concluding remarks in Sec. VI.

II. KINEMATICS

Considering reactions (A) to (D) we denote the four-momenta of the incoming neutrino, outgoing lepton, incoming and outgoing nucleons, and the vector (axial-vector) meson by l, l', p, p', q', respectively (Fig. 1).

We define

$$\nu = E - E' = \frac{(p \cdot q)}{M}, \tag{1}$$

$$q = l - l', \tag{2}$$

$$Q^2 = -q^2 = 4EE' \sin^2(\tfrac{1}{2}\theta), \tag{3}$$

where M is the nucleon mass, E, E' are the energies of the ingoing and outgoing leptons, and θ is the lepton scattering angle in the laboratory sys-

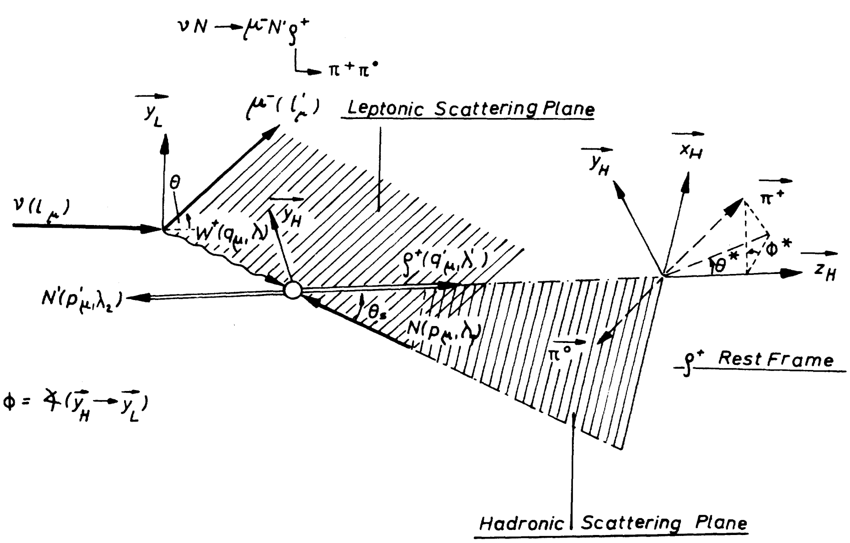

FIG. 2. Kinematics for $\nu N \to \mu^- N \rho^+$.

tem, respectively. We shall neglect the lepton mass throughout this paper and assume that $Q^2 \gg m^2$ and m is the charged lepton mass. Furthermore, we introduce

$$t=(p-p')^2, \tag{4}$$

$$s=W^2=(p+q)^2=M^2+q^2+2M\nu. \tag{5}$$

The angle Φ between the lepton and the hadron plane is defined in Fig. 2. The normals to the hadron and the lepton planes are given by

$$\vec{y}_H=\frac{\vec{q}\times\vec{q}\,'}{|\vec{q}\times\vec{q}\,'|} \quad\text{and}\quad \vec{y}_L=\frac{\vec{l}\times\vec{l}\,'}{|\vec{l}\times\vec{l}\,'|},$$

respectively. Φ is then the angle $(y_H\to y_L)$ and is positive for clockwise rotation around the direction of $\vec{q}$. Instead of Q^2 and ν it is convenient to use the scaling variables x and y:

$$x=\frac{Q^2}{2M\nu},\quad 0\le x,y\le 1 \tag{6}$$

$$y=\frac{\nu}{E}. \tag{7}$$

The matrix element for processes (A) to (D) is given by

$$T=\frac{G}{\sqrt{2}}\langle l'|j_\mu(0)|l\rangle\langle q,p'|J^\mu(0)|p\rangle, \tag{8}$$

where the leptonic current is

$$j_\mu=\bar{\mu}\gamma_\mu(1-\gamma_5)\nu+\text{H.c. for reactions (A) and (B)}$$
and (9)
$$j_\mu=\bar{\nu}\gamma_\mu(1-\gamma_5)\nu+\text{H.c. for reactions (C) and (D).}$$

Here the particle symbol is used to denote the corresponding field operator.

The hadronic current J^μ contains quite generally a vector and an axial-vector part:

$$J^\mu=V^\mu+A^\mu. \tag{10}$$

The cross section involves the quantity

$$\sum_{\text{spins}}|T|^2=L_{\mu\nu}T^{\mu\nu}\tfrac{1}{2}G^2, \tag{11}$$

with

$$L_{\mu\nu}=\sum_{\text{spins}}[\bar{u}(l')\gamma_\mu(1-\gamma_5)u(l)][\bar{u}(l')\gamma_\nu(1-\gamma_5)u(l)]^*, \tag{12}$$

$$T^{\mu\nu}=\sum_{\text{spins}}\langle q',p'|J^\mu(0)|p\rangle\langle q',p'|J^\nu(0)|p\rangle^*. \tag{13}$$

$L_{\mu\nu}$ has the form

$$L_{\mu\nu}=N[l_\mu l'_\nu+l'_\mu l_\nu-(l\cdot l')g_{\mu\nu}-\eta i\epsilon_{\mu\nu\rho\sigma}l^\rho l'^\sigma],$$
$$N=\begin{cases}4/m \text{ for reactions (A) and (B),}\\ 8 \text{ for reactions (C) and (D).}\end{cases} \tag{14}$$

$\eta=+1$ (-1) for incoming neutrinos (antineutrinos).

Notice that $L_{\mu\nu}$ contains a symmetric as well as an antisymmetric part: the latter is derived from the complete polarization of neutrinos. An analogous expression is also present in electroproduction with polarized electrons.[9] It is helpful, though not necessary, for us to imagine that processes (A) to (D) proceed via an intermediate vector boson $(W^\pm, Z^0)$. Then $L_{\mu\nu}$ is related to the spin density matrix of the intermediate vector boson

$$\rho^W_{ik}=\epsilon^{\mu\,*}(i)\epsilon^\nu(k)L_{\mu\nu}\frac{2(1-\epsilon)}{Q^2}\frac{1}{N}$$
$$=\rho^s_{ik}+\rho^{as}_{ik},\quad i,k=+,0,- \tag{15}$$

[$\epsilon_\mu(i)$ is the polarization vector of the intermediate vector boson], where ρ^s_{ik} and ρ^{as}_{ik} come from the symmetric and the antisymmetric part of $L_{\mu\nu}$, respectively:

$$\rho^s_{ik}=\begin{pmatrix}1 & [\epsilon(1+\epsilon)]^{1/2}e^{-i\phi} & -\epsilon e^{-2i\phi}\\ [\epsilon(1+\epsilon)]^{1/2}e^{i\phi} & 2\epsilon & -[\epsilon(1+\epsilon)]^{1/2}e^{-i\phi}\\ -\epsilon^{2i\phi} & -[\epsilon(1+\epsilon)]^{1/2}e^{i\phi} & 1\end{pmatrix}, \tag{16}$$

$$\rho^{as}_{ik}=\eta\begin{pmatrix}-(1-\epsilon^2)^{1/2} & -[\epsilon(1-\epsilon)]^{1/2}e^{-i\phi} & 0\\ -[\epsilon(1-\epsilon)]^{1/2}e^{i\phi} & 0 & -[\epsilon(1-\epsilon)]^{1/2}e^{-i\phi}\\ 0 & -[\epsilon(1-\epsilon)]^{1/2}e^{i\phi} & (1-\epsilon^2)^{1/2}\end{pmatrix}. \tag{17}$$

Here the polarization parameter ϵ is introduced in the same way as for electroproduction:

$$\epsilon=\left[1+\frac{2(Q^2+\nu^2)}{4E(E-\nu)-Q^2}\right]^{-1}. \tag{18}$$

Note that for vanishing lepton mass ρ_{ik} is a (3×3) matrix characterizing the polarization of a spin-1 particle since for $m\to 0$ the weak leptonic current j_μ is conserved. Again the presence of ρ^{as}_{ik} is due to the polarization of the incident neutrino.

According to known techniques[11,9] the cross section for reactions (A) to (D) reads

$$\frac{d\sigma}{dE'd\Omega_l d\Omega_h}=\Gamma\frac{d\sigma^W}{d\Omega_h},$$

$$\Gamma=\frac{G^2}{4\pi^3}\frac{E'}{E}\frac{Q^2}{(1-\epsilon)}K, \tag{19}$$

$$K=\frac{s-M^2}{2M}.$$

The cross section expressed in the convenient variables x and y is

$$\frac{d\sigma}{dx\,dy\,d\Omega_h}=\tilde{\Gamma}\,\frac{d\sigma^W}{d\Omega_h},$$

$$\tilde{\Gamma}=\frac{2G^2}{\pi^2}M^2E^2xy(1-x)\times\frac{\{1-y+\frac{1}{2}y^2[1+(M/E)(x/y)]\}}{[1+(2M/E)(x/y)]}, \tag{19'}$$

with

$$\epsilon=\frac{1-y-xyM/2E}{1-y+xyM/2E+\frac{1}{2}y^2}.$$

Γ and similarly $\tilde{\Gamma}$ can be interpreted as the flux factor of the intermediate vector boson. Here we have used the convention usually adopted in electroproduction[11] where the flux of equivalent real photons is taken. $d\sigma^W/d\Omega_h$ is then the differential cross section for scattering of a virtual (intermediate) vector boson on a nucleon:

$$\begin{aligned}\frac{d\sigma^W}{d\Omega_h}&=\sigma_U+\epsilon\sigma_L+\epsilon\cos 2\Phi\,\sigma_T+\epsilon\sin 2\phi\,\sigma_T'\\&\quad+[2\epsilon(1+\epsilon)]^{1/2}\cos\Phi\,\sigma_I+[2\epsilon(1+\epsilon)]^{1/2}\sin\phi\,\sigma_I'\\&\quad+\eta\{(1-\epsilon^2)^{1/2}\sigma_C+[2\epsilon(1-\epsilon)]^{1/2}\cos\phi\,\sigma_{CL}\\&\qquad+[2\epsilon(1-\epsilon)]^{1/2}\sin\phi\,\sigma_{CL}'\},\end{aligned} \tag{20}$$

where σ_U is a shorthand notation for $d\sigma_U/d\Omega_h$ etc. This particular structure of the cross section has also been derived by Cho,[5] who, furthermore, has shown that this characteristic Φ dependence of the cross section remains valid if one includes S, P, and T terms for the weak hadronic current in addition to V and A terms.

In electroproduction with unpolarized electrons only the cross sections $\sigma_U, \sigma_L, \sigma_T, \sigma_I$ appear. (σ_U, σ_T, σ_L, and σ_I are the transverse unpolarized, the transverse linearly polarized, the longitudinal and the transverse-longitudinal interference part, respectively, of the cross section.) The terms σ_T', σ_I', σ_C, and σ_{CL} are due to interference between the vector and axial-vector current.

σ_C is the contribution from the circular polarization of the intermediate vector boson; σ_{CL} and σ_{CL}' are circular-longitudinal interference terms. This can be seen more explicitly by expressing the various cross sections in terms of the center-of-mass-system (c.m.s.) helicity amplitudes. For this purpose we define the helicity amplitudes

$$V_{\lambda_1\lambda}^{\lambda_2\lambda'}=\langle\lambda_2,\lambda'|V^\mu(0)|\lambda_1\rangle\epsilon_\mu(\lambda)(-1)^\lambda, \tag{21}$$

$$A_{\lambda_1\lambda}^{\lambda_2\lambda'}=\langle\lambda_2,\lambda'|A^\mu(0)|\lambda_1\rangle\epsilon_\mu(\lambda)(-1)^\lambda, \tag{22}$$

where $\lambda_1,\lambda_2,\lambda,\lambda'$ denote the helicities of the incoming and outgoing nucleon, the intermediate vector boson, and the produced vector (axial-vector) meson, respectively. V^μ and A^μ are the vector and axial-vector current given in Eq. (10). The amplitudes defined in Eqs. (21) and (22) correspond to s-channel helicity amplitudes of the virtual process: $\binom{W}{Z}+N\to V(A)+N$. They obey the parity relations

$$\begin{aligned}V_{\lambda_1\lambda}^{\lambda_2\lambda'}&=(-1)^{(\lambda_2-\lambda_1)-(\lambda'-\lambda)}V_{-\lambda_1-\lambda}^{-\lambda_2-\lambda'},\\A_{\lambda_1\lambda}^{\lambda_2\lambda'}&=-(-1)^{(\lambda_2-\lambda_1)-(\lambda'-\lambda)}A_{-\lambda_1-\lambda}^{-\lambda_2-\lambda'}\end{aligned} \tag{23a}$$

for production of a vector particle, and

$$\begin{aligned}V_{\lambda_1\lambda}^{\lambda_2\lambda'}&=-(-1)^{(\lambda_2-\lambda_1)-(\lambda'-\lambda)}V_{-\lambda_1-\lambda}^{-\lambda_2-\lambda'},\\A_{\lambda_1\lambda}^{\lambda_2\lambda'}&=(-1)^{(\lambda_2-\lambda_1)-(\lambda'-\lambda)}A_{-\lambda_1-\lambda}^{-\lambda_2-\lambda'}\end{aligned} \tag{23b}$$

for production of an axial vector meson.

The various cross sections now have the following form (with q'^* as the c.m.s. three-momentum of the outgoing hadrons):

$$\sigma_U=\frac{q'^*\sqrt{s}}{MK}\,\frac{1}{4}\sum_{\lambda_1,\lambda_2,\lambda'}\left(|V_{\lambda_1+}^{\lambda_2\lambda'}|^2+|V_{\lambda_1-}^{\lambda_2\lambda'}|^2+|A_{\lambda_1+}^{\lambda_2\lambda'}|^2+|A_{\lambda_1-}^{\lambda_2\lambda'}|^2\right), \tag{24}$$

$$\sigma_L=\frac{q'^*\sqrt{s}}{MK}\,\frac{1}{2}\sum_{\lambda_1,\lambda_2,\lambda'}\left(|V_{\lambda_1 0}^{\lambda_2\lambda'}|^2+|A_{\lambda_1 0}^{\lambda_2\lambda'}|^2\right), \tag{25}$$

$$\sigma_T=\frac{q'^*\sqrt{s}}{MK}\left(-\tfrac{1}{2}\right)\times\sum_{\lambda_1,\lambda_2,\lambda'}\mathrm{Re}\left(V_{\lambda_1+}^{\lambda_2\lambda'}V_{\lambda_1-}^{\lambda_2\lambda'\,*}+A_{\lambda_1+}^{\lambda_2\lambda'}A_{\lambda_1-}^{\lambda_2\lambda'\,*}\right), \tag{26}$$

$$\sigma_I=\frac{q'^*\sqrt{s}}{MK}\,\frac{1}{\sqrt{2}}\sum_{\lambda_1,\lambda_2,\lambda'}\mathrm{Re}\left(V_{\lambda_1+}^{\lambda_2\lambda'}V_{\lambda_1 0}^{\lambda_2\lambda'\,*}+A_{\lambda_1+}^{\lambda_2\lambda'}A_{\lambda_1 0}^{\lambda_2\lambda'\,*}\right), \tag{27}$$

$$\sigma_{CL}'=\frac{q'^*\sqrt{s}}{MK}\left(-\frac{1}{\sqrt{2}}\right)\times\sum_{\lambda_1,\lambda_2,\lambda'}\mathrm{Im}\left(V_{\lambda_1+}^{\lambda_2\lambda'}V_{\lambda_1 0}^{\lambda_2\lambda'\,*}+A_{\lambda_1+}^{\lambda_2\lambda'}A_{\lambda_1 0}^{\lambda_2\lambda'\,*}\right). \tag{28}$$

These five cross sections do not contain vector–axial-vector interferences. The remaining parts contain only the following interference terms:

$$\sigma_T'=\frac{q'^*\sqrt{s}}{MK}(-1)\sum_{\lambda_1,\lambda_2,\lambda'}\mathrm{Im}\,V_{\lambda_1+}^{\lambda_2\lambda'}A_{\lambda_1-}^{\lambda_2\lambda'\,*}, \tag{29}$$

$$\sigma_I'=\frac{q'^*\sqrt{s}}{MK}\,\frac{1}{\sqrt{2}}\times\sum_{\lambda_1,\lambda_2,\lambda'}\mathrm{Im}\left(V_{\lambda_1+}^{\lambda_2\lambda'}A_{\lambda_1 0}^{\lambda_2\lambda'\,*}+A_{\lambda_1+}^{\lambda_2\lambda'}V_{\lambda_1 0}^{\lambda_2\lambda'\,*}\right), \tag{30}$$

$$\sigma_C=\frac{q'^*\sqrt{s}}{MK}(-1)\sum_{\lambda_1,\lambda_2,\lambda'}\mathrm{Re}\,V_{\lambda_1+}^{\lambda_2\lambda'}A_{\lambda_1+}^{\lambda_2\lambda'\,*}, \tag{31}$$

$$\sigma_{CL}=\frac{q'^*\sqrt{s}}{MK}\left(-\frac{1}{\sqrt{2}}\right)\times\sum_{\lambda_1,\lambda_2,\lambda'}\mathrm{Re}\left(V_{\lambda_1+}^{\lambda_2\lambda'}A_{\lambda_1 0}^{\lambda_2\lambda'\,*}+A_{\lambda_1+}^{\lambda_2\lambda'}V_{\lambda_1 0}^{\lambda_2\lambda'\,*}\right). \tag{32}$$

Comparing with Eq. (20) one observes that by varying the angle Φ the following five combinations of cross sections can be measured separately:

$$\sigma_U + \epsilon\sigma_L + \eta(1-\epsilon^2)^{1/2}\sigma_C,$$
$$[2\epsilon(1+\epsilon)]^{1/2}\sigma_I + \eta[2\epsilon(1-\epsilon)]^{1/2}\sigma_{CL},$$
$$\epsilon\sigma_T,$$
$$[2\epsilon(1+\epsilon)]^{1/2}\sigma_I' + \eta[2\epsilon(1-\epsilon)]^{1/2}\sigma_{CL}',$$
and σ_T'.

In principle all nine cross sections can be determined by varying ϵ as well. Since $d\Omega_h = (1/2q^*q'^*) \times d\Phi\, dt$ the Φ-integrated cross section is already very informative for studying diffraction scattering as it contains the t dependence. Integrating Eq. (20) over Φ gives

$$\frac{d\sigma^W}{dt} = 2\pi\left(\frac{d\sigma_U}{dt} + \epsilon\frac{d\sigma_L}{dt} + \eta(1-\epsilon^2)^{1/2}\frac{d\sigma_C}{dt}\right). \tag{33}$$

Notice that besides the combination $d\sigma_U/dt + \epsilon d\sigma_L/dt$ already present in electroproduction we have in addition the vector-axial-vector interference term $\eta(1-\epsilon^2)^{1/2}d\sigma_C/dt$.

Finally we want to mention that σ_U, σ_L, and σ_C can be related to the structure functions W_1, W_2, and W_3[12]:

$$W_1 = \frac{MK}{\pi}\int \sigma_U d\Omega_h,$$
$$W_2 = \frac{MK}{\pi}\frac{Q^2}{Q^2+\nu^2}\int(\sigma_U+\sigma_L)d\Omega_h,$$
$$W_3 = -\frac{2M^2K}{\pi(\nu^2+Q^2)^{1/2}}\int \sigma_C d\Omega_h.$$

We gain of course more information on the dynamics involved by studying in addition the angular distribution of the decay products of the outgoing vector (axial-vector) meson. In the case of $V \to \pi\pi$ the normalized distribution has the form[13]

$$\begin{aligned}W(\theta^*,\Phi^*) = \frac{3}{4\pi}\{&\tfrac{1}{2}(\rho^{++}+\rho^{--})\sin^2\theta^* + \rho^{00}\cos^2\theta^* \\ &- \sin^2\theta^*(\cos2\phi^*\,\mathrm{Re}\rho^{+-} \\ &- \sin2\Phi^*\,\mathrm{Im}\rho^{+-}) \\ &- \frac{\sin2\theta^*}{\sqrt{2}}[\cos\phi^*\,\mathrm{Re}(\rho^{+0}-\rho^{0-}) \\ &- \sin\phi^*\,\mathrm{Im}(\rho^{+0}-\rho^{0-})]\}.\end{aligned} \tag{34a}$$

The direction of one of the outgoing pions is described by the polar and azimuthal angles θ^* and Φ^* in the vector-meson rest system with the z axis opposite to the direction of the outgoing nucleon (see Fig. 2).

The formula (34a) for $W(\theta^*,\Phi^*)$ may also be used for the 3π decay[14] of a vector meson (e.g. ω). In this case θ^* and Φ^* have to be interpreted as the angles of the normal to the decay plane in the rest system of the vector meson. The quantities ρ^{mn} are the spin density matrix elements of the outgoing vector meson in the s-channel cms.

The decay angular distribution of an axial-vector particle (e.g. A_1) into three pions is given by[14]

$$\begin{aligned}W(\theta^*,\Phi^*) = \frac{3}{8\pi}(&\tfrac{1}{2}(\rho^{++}+\rho^{--})(1+\cos^2\theta^*) + \rho^{00}\sin^2\theta^* + \sin^2\theta^*(\mathrm{Re}\rho^{+-}\cos2\Phi^* - \mathrm{Im}\rho^{+-}\sin2\phi^*) \\ &+ \frac{1}{\sqrt{2}}\sin2\theta^*[\mathrm{Re}(\rho^{+0}-\rho^{0-})\cos\phi^* - \mathrm{Im}(\rho^{+0}-\rho^{0-})\sin\Phi^*] \\ &+ \lambda\{\cos\theta^*(\rho^{++}-\rho^{--}) + \sqrt{2}\sin\theta^*[\mathrm{Re}(\rho^{+0}+\rho^{0-})\cos\Phi^* - \mathrm{Im}(\rho^{+0}+\rho^{0-})\sin\Phi^*]\}).\end{aligned} \tag{34b}$$

λ is a parameter related to the ratio of the two possible couplings describing the $A \to 3\pi$ decay. The meaning of θ^*, Φ^* is the same as given above for the 3π decay of a vector particle.

The vector (axial-vector) density matrix ρ^{mn} is obtained in terms of the helicity amplitudes [Eqs. (21) and (22)] and the density matrix ρ^W of the intermediate vector boson [Eq. (15)]:

$$\begin{aligned}\rho^{mn} = \frac{1}{C}\sum_{\substack{i,j\\ \lambda_1,\lambda_2}}\rho^W_{ij}(&V^{\lambda_2 m}_{\lambda_1 i}V^{\lambda_2 n *}_{\lambda_1 j} + A^{\lambda_2 m}_{\lambda_1 i}A^{\lambda_2 n *}_{\lambda_1 j} \\ &+ V^{\lambda_2 m}_{\lambda_1 i}A^{\lambda_2 n *}_{\lambda_1 j} + A^{\lambda_2 m}_{\lambda_1 i}V^{\lambda_2 n *}_{\lambda_1 j}),\end{aligned}$$
$$C = \frac{2KM}{q'^*\sqrt{s}}\frac{d\sigma^W}{d\Omega_h}. \tag{35}$$

In the same way as for the differential cross section in Eq. (20), the density matrix elements contain nine terms having different Φ and ϵ dependence:

$$\begin{aligned}\rho = \rho_U &+ \epsilon\rho_L + \epsilon\cos2\Phi\,\rho_T + \epsilon\sin2\Phi\,\rho_T' \\ &+ [2\epsilon(1+\epsilon)]^{1/2}\cos\Phi\,\rho_I + [2\epsilon(1+\epsilon)]^{1/2}\sin\phi\,\rho_I' \\ &+ \eta\{(1-\epsilon^2)^{1/2}\rho_C + [2\epsilon(1-\epsilon)]^{1/2}\cos\Phi\,\rho_{CL} \\ &+ [2\epsilon(1-\epsilon)]^{1/2}\sin\Phi\,\rho_{CL}'\}.\end{aligned} \tag{36}$$

Owing to the parity relations (23a) and (23b), some of the density matrix elements contain only V-A interference terms, whereas the rest are free of interferences. The detailed expressions, which are the same for vector and axial-vector meson production, are given in Tables I(a) and I(b). For axial-vector mesons two further combinations of density matrix elements appear in the decay angular distribution, Eq. (34b). These are given in Table I(c). Integrating over Φ only

TABLE I. (a) Noninterference part of the vector-meson (axial-vector-meson) density matrix in terms of the s-channel helicity amplitudes $V_{\lambda_1\lambda}^{\lambda_2\lambda'}$ ($A_{\lambda_1\lambda}^{\lambda_2\lambda'}$) as defined in Eqs. (21) and (22). Summation over the nucleon helicities is always understood and the nucleon helicities are omitted. (b) The same as in (a) for the interference part of the vector-meson (axial-vector-meson) density matrix. (c) The same as in (a) and (b) for the additional combinations of the axial-vector-meson density matrix.

(a)

	$\frac{1}{2}(\rho^{++}+\rho^{--})$	ρ^{00}	ρ^{+-}	$\rho^{+0}-\rho^{0-}$
ρ_U	$\frac{1}{2C}[\lvert V_+^+\rvert^2+\lvert V_-^+\rvert^2+(V\to A)]$	$\frac{1}{C}[\lvert V_+^0\rvert^2+(V\to A)]$	$\frac{1}{C}\mathrm{Re}[V_+^+V_+^{-*}+(V\to A)]$	$\frac{1}{C}\mathrm{Re}[(V_+^+-V_+^-)V_+^{0*}+(V\to A)]$
ρ_L	$\frac{1}{C}[\lvert V_0^+\rvert^2+(V\to A)]$	$\frac{1}{C}[\lvert V_0^0\rvert^2+(V\to A)]$	$\frac{1}{C}\mathrm{Re}[V_0^+V_0^{-*}+(V\to A)]$	$\frac{2}{C}\mathrm{Re}[V_0^+V_0^{0*}+(V\to A)]$
ρ_T	$-\frac{1}{C}\mathrm{Re}[V_+^+V_-^{+*}+(V\to A)]$	$-\frac{1}{C}\mathrm{Re}[V_+^0V_-^{0*}+(V\to A)]$	$-\frac{1}{2C}\mathrm{Re}[V_+^+V_-^{-*}+V_-^+V_+^{-*}+(V\to A)]$	$-\frac{1}{C}\mathrm{Re}[(V_+^+-V_+^-)V_-^{0*}+(V\to A)]$
ρ_T'	0	0	$\frac{i}{2C}\mathrm{Re}[V_+^+V_-^{-*}-V_-^+V_+^{-*}+(V\to A)]$	$\frac{i}{C}\mathrm{Re}[(V_+^++V_+^-)V_-^{0*}+(V\to A)]$
ρ_I	$\frac{1}{\sqrt{2}C}\mathrm{Re}[(V_+^+-V_-^+)V_0^{+*}+(V\to A)]$	$\frac{\sqrt{2}}{C}\mathrm{Re}[V_+^0V_0^{0*}+(V\to A)]$	$\frac{1}{\sqrt{2}C}\mathrm{Re}[(V_+^+-V_-^+)V_0^{-*}+(V\to A)]$	$\frac{1}{\sqrt{2}C}\mathrm{Re}[(V_+^0-V_-^0)V_0^{+*}$ $+(V_+^+-V_-^+)V_0^{0*}+(V\to A)]$
ρ_I'	0	0	$-\frac{i}{\sqrt{2}C}\mathrm{Re}[(V_+^++V_-^+)V_0^{-*}+(V\to A)]$	$\frac{i}{\sqrt{2}C}\mathrm{Re}[(V_+^0+V_-^0)V_0^{+*}$ $-(V_+^++V_-^+)V_0^{0*}+(V\to A)]$
ρ_C	0	0	$-\frac{i}{C}\mathrm{Im}[V_+^+V_+^{-*}+(V\to A)]$	$-\frac{i}{C}\mathrm{Im}[(V_+^++V_+^-)V_+^{0*}+(V\to A)]$
ρ_{CL}	0	0	$-\frac{i}{\sqrt{2}C}\mathrm{Im}[(V_+^++V_-^+)V_0^{-*}+(V\to A)]$	$\frac{i}{\sqrt{2}C}\mathrm{Im}[(V_+^0+V_-^0)V_0^{+*}$ $-(V_+^++V_-^+)V_0^{0*}+(V\to A)]$
ρ_{CL}'	$-\frac{1}{\sqrt{2}C}\mathrm{Im}[(V_+^+-V_-^+)V_0^{+*}$ $+(V\to A)]$	$-\frac{1}{\sqrt{2}C}\mathrm{Im}[(V_+^0-V_-^0)V_0^{0*}$ $+(V\to A)]$	$-\frac{1}{\sqrt{2}C}\mathrm{Im}[(V_+^+-V_-^+)V_0^{-*}+(V\to A)]$	$-\frac{1}{\sqrt{2}C}\mathrm{Im}[(V_+^0-V_-^0)V_0^{+*}$ $+(V_+^+-V_-^+)V_0^{0*}+(V\to A)]$

TABLE I. (*Continued*)

(b)

	$\frac{1}{2}(\rho^{++}+\rho^{--})$	ρ^{00}	ρ^{+-}	$\rho^{+0}-\rho^{0-}$
ρ_U	0	0	$\frac{i}{C}\mathrm{Im}[V_+^+A_+^{-*}+(V\longleftrightarrow A)]$	$\frac{i}{C}\mathrm{Im}[(V_+^+ + V_+^-)A_+^{0*}+(V\longleftrightarrow A)]$
ρ_L	0	0	$\frac{2i}{C}\mathrm{Im}V_0^+A_0^{-*}$	$\frac{2i}{C}\mathrm{Im}[V_0^+A_0^{0*}+(V\longleftrightarrow A)]$
ρ_T	0	0	$-\frac{i}{C}\mathrm{Im}(V_+^+A_-^{-*}+V_-^+A_+^{-*})$	$-\frac{i}{C}\mathrm{Im}[(V_+^+ + V_+^-)A_-^{0*}+(V\longleftrightarrow A)]$
ρ'_T	$-\frac{1}{C}\mathrm{Im}[V_+^+A_-^{+*}+(V\longleftrightarrow A)]$	$-\frac{2}{C}\mathrm{Im}V_+^0A_-^{0*}$	$-\frac{1}{C}\mathrm{Im}(V_+^+A_-^{-*}-V_-^+A_+^{-*})$	$-\frac{1}{C}\mathrm{Im}[(V_+^+ - V_+^-)A_-^{0*}+(V\longleftrightarrow A)]$
ρ_I	0	0	$\frac{i}{\sqrt{2}C}\mathrm{Im}[(V_+^+ - V_-^+)A_0^{-*}+(V\longleftrightarrow A)]$	$-\frac{i}{\sqrt{2}C}\mathrm{Im}[(V_+^0 - V_-^0)A_0^{+*}$ $-(V_+^+ - V_-^+)A_0^{0*}+(V\longleftrightarrow A)]$
ρ'_I	$\frac{1}{\sqrt{2}C}\mathrm{Im}[(V_+^+ + V_-^+)A_0^{+*}$ $+(V\longleftrightarrow A)]$	$\frac{\sqrt{2}}{C}\mathrm{Im}[V_+^0A_0^{0*}+(V\longleftrightarrow A)]$	$\frac{1}{\sqrt{2}C}\mathrm{Im}[(V_+^+ + V_-^+)A_0^{-*}+(V\longleftrightarrow A)]$	$\frac{1}{\sqrt{2}C}\mathrm{Im}[(V_+^0 + V_-^0)A_0^{+*}$ $+(V_+^+ + V_-^+)A_0^{0*}+(V\longleftrightarrow A)]$
ρ_C	$-\frac{1}{C}\mathrm{Re}(V_+^+A_+^{+*}-V_-^+A_-^{+*})$	$-\frac{2}{C}\mathrm{Re}V_+^0A_+^{0*}$	$-\frac{1}{C}\mathrm{Re}[V_+^+A_+^{-*}+(V\longleftrightarrow A)]$	$-\frac{1}{C}\mathrm{Re}[(V_+^+ - V_+^-)A_+^{0*}+(V\longleftrightarrow A)]$
ρ_{CL}	$-\frac{1}{\sqrt{2}C}\mathrm{Re}[(V_+^+ + V_-^+)A_0^{+*}$ $+(V\longleftrightarrow A)]$	$-\frac{\sqrt{2}}{C}\mathrm{Re}[V_+^0A_0^{0*}+(V\longleftrightarrow A)]$	$-\frac{1}{\sqrt{2}C}\mathrm{Re}[(V_+^+ + V_-^+)A_0^{-*}+(V\longleftrightarrow A)]$	$-\frac{1}{\sqrt{2}C}\mathrm{Re}[(V_+^0 + V_-^0)A_0^{+*}$ $+(V_+^+ + V_-^+)A_0^{0*}+(V\longleftrightarrow A)]$
ρ'_{CL}	0	0	$\frac{i}{\sqrt{2}C}\mathrm{Re}[(V_+^+ - V_-^+)A_0^{-*}+(V\longleftrightarrow A)]$	$-\frac{i}{\sqrt{2}C}\mathrm{Re}[(V_+^0 - V_-^0)A_0^{+*}$ $-(V_+^+ - V_-^+)A_0^{0*}+(V\longleftrightarrow A)]$

TABLE I. (*Continued*)

(c)

	$\frac{1}{2}(\rho^{++}-\rho^{--})$	$\rho^{+0}+\rho^{0-}$	$\frac{1}{2}(\rho^{++}-\rho^{--})$	$\rho^{+0}+\rho^{0-}$
ρ_U	0	$\frac{i}{C}\mathrm{Im}[(V_{+}^{+}-V_{+}^{-})V_{+}^{0*}+(V\to A)]$	$\frac{1}{C}\mathrm{Re}(V_{+}^{+}A_{+}^{+*}+V_{-}^{+}A_{-}^{+*})$	$\frac{1}{C}\mathrm{Re}[(V_{+}^{+}+V_{+}^{-})A_{+}^{0*}+(V\longleftrightarrow A)]$
ρ_L	0	$\frac{2i}{C}\mathrm{Im}[V_{0}^{+}V_{0}^{0*}+(V\to A)]$	$\frac{2}{C}\mathrm{Re}V_{0}^{+}A_{0}^{+*}$	$\frac{2}{C}\mathrm{Re}[V_{0}^{+}A_{0}^{0*}+(V\longleftrightarrow A)]$
ρ_T	0	$-\frac{i}{C}\mathrm{Im}[(V_{+}^{+}-V_{+}^{-})V_{-}^{0*}+(V\to A)]$	$-\frac{1}{C}\mathrm{Re}[V_{+}^{+}A_{-}^{+*}+(V\longleftrightarrow A)]$	$-\frac{1}{C}\mathrm{Re}[(V_{+}^{+}+V_{+}^{-})A_{-}^{0*}+(V\longleftrightarrow A)]$
ρ_T'	$-\frac{1}{C}\mathrm{Im}[V_{+}^{+}V_{-}^{+*}+(V\to A)]$	$-\frac{1}{C}\mathrm{Im}[(V_{+}^{+}+V_{+}^{-})V_{-}^{0*}+(V\to A)]$	0	$\frac{i}{C}\mathrm{Re}[(V_{+}^{+}-V_{+}^{-})A_{-}^{0*}+(V\longleftrightarrow A)]$
ρ_I	0	$-\frac{i}{\sqrt{2}C}\mathrm{Im}[(V_{+}^{0}-V_{-}^{0})V_{0}^{+*}$ $-(V_{+}^{+}-V_{-}^{+})V_{0}^{0*}+(V\to A)]$	$\frac{1}{\sqrt{2}C}\mathrm{Re}[(V_{+}^{+}-V_{-}^{+})A_{0}^{+*}$ $+(V\longleftrightarrow A)]$	$\frac{1}{\sqrt{2}C}\mathrm{Re}[(V_{+}^{0}-V_{-}^{0})A_{0}^{+*}$ $+(V_{+}^{+}-V_{-}^{+})A_{0}^{0*}+(V\longleftrightarrow A)]$
ρ_I'	$\frac{1}{\sqrt{2}C}\mathrm{Im}[(V_{+}^{+}+V_{-}^{+})V_{0}^{+*}$ $+(V\to A)]$	$\frac{1}{\sqrt{2}C}\mathrm{Im}[(V_{+}^{0}+V_{-}^{0})V_{0}^{+*}$ $+(V_{+}^{+}+V_{-}^{+})V_{0}^{0*}+(V\to A)]$	0	$\frac{i}{\sqrt{2}C}\mathrm{Re}[(V_{+}^{0}+V_{-}^{0})A_{0}^{+*}$ $-(V_{+}^{+}+V_{-}^{+})A_{0}^{0*}+(V\longleftrightarrow A)]$
ρ_C	$-\frac{1}{2C}[\lvert V_{+}^{+}\rvert^{2}-\lvert V_{-}^{+}\rvert^{2}+(V\to A)]$	$-\frac{1}{C}\mathrm{Re}[(V_{+}^{+}+V_{+}^{-})V_{+}^{0*}+(V\to A)]$	0	$-\frac{i}{C}\mathrm{Im}[(V_{+}^{+}-V_{+}^{-})A_{+}^{0*}+(V\longleftrightarrow A)]$
ρ_{CL}	$-\frac{1}{\sqrt{2}C}\mathrm{Re}[(V_{+}^{+}+V_{-}^{+})V_{0}^{+*}$ $+(V\to A)]$	$-\frac{1}{\sqrt{2}C}\mathrm{Re}[(V_{+}^{0}+V_{-}^{0})V_{0}^{+*}$ $+(V_{+}^{+}+V_{-}^{+})V_{0}^{0*}+(V\to A)]$	0	$\frac{i}{\sqrt{2}C}\mathrm{Im}[(V_{+}^{0}+V_{-}^{0})A_{0}^{+*}$ $-(V_{+}^{+}+V_{-}^{+})A_{0}^{0*}+(V\longleftrightarrow A)]$
ρ_{CL}'	0	$-\frac{i}{\sqrt{2}C}\mathrm{Re}[(V_{+}^{0}-V_{-}^{0})V_{0}^{+*}$ $-(V_{+}^{+}-V_{-}^{+})V_{0}^{0*}+(V\to A)]$	$-\frac{1}{\sqrt{2}C}\mathrm{Im}[(V_{+}^{+}-V_{-}^{+})A_{0}^{+*}$ $+(V\longleftrightarrow A)]$	$-\frac{1}{\sqrt{2}C}\mathrm{Im}[(V_{+}^{0}-V_{-}^{0})A_{0}^{+*}$ $+(V_{+}^{+}-V_{-}^{+})A_{0}^{0*}+(V\longleftrightarrow A)]$

the combination $\rho_U+\epsilon\rho_L+\eta(1-\epsilon^2)^{1/2}\rho_C$ remains [compare Eq. (33)].

At high energies ($s\to\infty$) scattering processes are conveniently analyzed in terms of t-channel exchange of natural or unnatural parity. For this purpose it is useful to split the helicity amplitudes into two parts:

$$V^{\lambda_2\lambda'}_{\lambda_1\lambda}=v^{\lambda_2\lambda'}_{\lambda_1\lambda}+\bar{v}^{\lambda_2\lambda}_{\lambda_1\lambda},\qquad A^{\lambda_2\lambda'}_{\lambda_1\lambda}=a^{\lambda_2\lambda'}_{\lambda_1\lambda}+\bar{a}^{\lambda_2\lambda'}_{\lambda_1\lambda},\tag{37}$$

where

$$v^{\lambda_2\lambda'}_{\lambda_1\lambda}=\tfrac{1}{2}[V^{\lambda_2\lambda'}_{\lambda_1\lambda}+(-1)^{\lambda_2-\lambda_1}V^{-\lambda_2\lambda'}_{-\lambda_1\lambda}],\qquad \bar{v}^{\lambda_2\lambda'}_{\lambda_1\lambda}=\tfrac{1}{2}[V^{\lambda_2\lambda'}_{\lambda_1\lambda}-(-1)^{\lambda_2-\lambda_1}V^{-\lambda_2\lambda'}_{-\lambda_1\lambda}]\tag{38}$$

and analogously for $a^{\lambda_2\lambda'}_{\lambda_1\lambda}$ and $\bar{a}^{\lambda_2\lambda'}_{\lambda_1\lambda}$.

The amplitudes $v^{\lambda_2\lambda'}_{\lambda_1\lambda}$ and $a^{\lambda_2\lambda'}_{\lambda_1\lambda}$ correspond asymptotically ($s\to\infty$) to natural-parity exchange in the t channel, whereas asymptotically $\bar{v}^{\lambda_2\lambda'}_{\lambda_1\lambda}$ and $\bar{a}^{\lambda_2\lambda'}_{\lambda_1\lambda}$ receive contributions only from unnatural-parity exchange.[15] Since we are not interested in the polarizations of the nucleons, also the density matrix can be written as a sum of natural- and unnatural-parity exchange contributions:

$$\rho=\rho^{\text{nat}}+\rho^{\text{unnat}}.\tag{39}$$

In the Appendix we give a list of the most important density matrix elements ρ_U and ρ_L showing explicitly the separation into exchange of definite naturality in the t channel.

It is well known that in vector-meson photoproduction with polarized photons it is possible to separate the contributions from natural- and unnatural-parity exchange by forming adequate combinations of density matrix elements.[16] The same holds for vector-meson electroproduction.[9] In weak production of vector and axial-vector mesons the isolation of contributions from an exchange of definite naturality is not so simple because of the presence both of a vector and an axial-vector current.

As an example let us discuss the noninterference part of the vector-meson density matrix (the same holds for production of an axial-vector meson). By measuring the angular decay distribution of $\rho\to 2\pi$ ($A_1\to 3\pi$) one can determine the density matrix elements [see Eq. (36)] and then, for instance, form the following combinations:

$$\tfrac{1}{2}(\rho_U^{++}+\rho_U^{--})-\text{Re}\rho_T^{+-}=2\sum_{\lambda_1}(|v^{++}_{\lambda_1+}|^2+|v^{++}_{\lambda_1-}|^2+|\bar{a}^{++}_{\lambda_1+}|^2+|\bar{a}^{++}_{\lambda_1-}|^2)\tag{40}$$

and

$$\tfrac{1}{2}(\rho_U^{++}+\rho_U^{--})+\text{Re}\rho_T^{+-}=2\sum_{\lambda_1}(|\bar{v}^{++}_{\lambda_1+}|^2+|\bar{v}^{++}_{\lambda_1-}|^2+|a^{++}_{\lambda_1+}|^2+|a^{++}_{\lambda_1-}|^2).\tag{41}$$

This, however, shows that in both expressions on the right-hand side a sum of a natural- *and* an unnatural-parity t-channel exchange appears which cannot be further separated on a purely kinematical grounds. (In principle a separation would be possible by using a polarized target and determining the helicity state of the outgoing nucleon.) The same statement applies to all other possible combinations of measurable density matrix elements. One always gets either a combination of a natural-parity contribution of the vector current and an unnatural-parity contribution of the axial-vector current or vice versa. In order to proceed further dynamical assumptions about the production mechanism are necessary.

So far our considerations have been completely general apart from neglecting the lepton mass and assuming that $Q^2\gg m^2$. In the following we shall treat the diffraction region in more detail. The formalism given above is, however, not restricted to this kinematic domain. It may also be applied to low energies or to the deep-inelastic region.

III. DISCUSSION OF THE DIFFRACTION MECHANISM

The study of reactions (A) to (D) allows us to learn much about the structure of weak interactions as well as strong interactions. In the following we shall discuss only the diffraction region. This case is comparable to the electroproduction of vector mesons which gave us much insight into the nature both of the photon and of the diffraction mechanism.

The kinematical region for diffractive production is $s\geq s_0\gg Q^2$, M^2, $m_V{}^2$, and $|t|$ small ($|t|/s\ll 1$). s_0 depends, of course, on the mass of the produced meson; its value is, for instance, $s_0\simeq 5$ GeV^2 for ρ production. From Eq. (5) and $s\geq s_0$ we then have

$$0\leq Q^2\leq 2M\nu-s_0+M^2\tag{42a}$$

and consequently

$$x\leq 1-\frac{(s_0-M^2)}{2MEy}.\tag{42b}$$

Since the point $x=0$ is in the physically allowed region (neglecting the lepton mass) this gives a lower bound for y:

$$y\geq\frac{s_0-M^2}{2ME}.\tag{43}$$

Moreover, the physical region is limited by the condition $0\leq\sin^2(\frac{1}{2}\theta)\leq 1$ which together with Eq. (3) leads to

$$y\leq\frac{2E}{Mx+2E}.\tag{44}$$

Thus for $y \to 1$, x must go to zero. Equations (42b) and (44) give a maximal value for x:

$$x_{\max} = \frac{2E(2ME - s_0 + M^2)}{M(4E^2 + s_0 - M^2)}. \tag{45}$$

The diffractive region ($2M\nu \gg Q^2$) further implies $x \ll 1$. In order to keep $|t|$ small enough, Q^2 must not be too large since

$$\begin{aligned} t_{\min} &\simeq -\frac{M^2}{s^2}(m_V{}^2 + Q^2)^2 \\ &= -\left[\frac{m_V{}^2 + 2MExy}{2Ey(1-x)}\right]^2. \end{aligned} \tag{46}$$

Consequently we restrict Q^2 to the region $0 \le Q^2 \le Q_0{}^2$. This further restricts the allowed y range to

$$y \le \frac{Q_0{}^2}{2MEx}. \tag{47}$$

The kinematic domain in the x-y plane for $E = 10$ GeV, $s_0 = 5$ GeV^2, $Q_0{}^2 = 2$ GeV^2 is shown in Fig. 3.

Let us first discuss the questions related to the *diffraction mechanism*. Only if these are understood to some extent can we learn more about the nature of weak interactions.

One expects that the vector current can diffractively produce vector particles and, quite analogously, the axial-vector current can produce axial-vector particles. An interesting question is whether a vector particle can be produced also by the axial-vector current and, vice versa, an axial-vector particle also by the vector current. For the production of nonstrange mesons diffractive V-A transitions are already excluded by G parity. This question can be answered more generally by considering the $V - A$ interference terms in the cross section, i.e., σ_T', σ_I', σ_C, and σ_{CL} measured by variation of Φ and ϵ. It might happen that σ_T', σ_I', and σ_{CL} vanish due to s-channel helicity conservation (see below). In this case it is perhaps experimentally easier to study both ν and $\bar{\nu}$ production of vector or axial-vector mesons and to measure the Φ-integrated cross section [see Eq. (33)]. Since $d\sigma_U/dt + \epsilon d\sigma_L/dt$ at finite s need not be equal in ν and $\bar{\nu}$ scattering, the separation of $d\sigma_C/dt$ is more complicated. In principle the simplest way to isolate σ_C is ν and $\bar{\nu}$ scattering with an isoscalar target. Any difference of the Φ-integrated cross sections come then from σ_C. An equivalent way would be to measure the difference

$$\frac{d\sigma}{dt}(\nu p \to \mu^- \rho^+ p) - \frac{d\sigma}{dt}(\bar{\nu} n \to \mu^+ \rho^- n) = 2(1-\epsilon^2)^{1/2}\frac{d\sigma_C}{dt}(\nu p \to \mu^- \rho^+ p).$$

Of course only the energy-independent component is due to diffraction. At low s appreciable V-A transitions are expected to be present due to nondiffractive mechanisms, for instance, t-channel π exchange. A quite analogous situation is known to occur in ω photoproduction and electroproduction.

It is clear that the t dependence of $d\sigma^W/d\Omega_k$ of Eqs. (19) and (19′) as well as the behavior of the slope as a function of s and Q^2 can provide important information about the diffraction mechanism. These are well-known questions which have been extensively studied in various strong and electromagnetic processes. One expects the typical diffraction pattern $d\sigma^W/dt \simeq Ce^{At}$ with C being rather independent of s and the slope A possibly showing shrinkage in s ($\sim \ln s$). Any Q^2 dependence of A would be of particular interest. This property has been discussed in electroproduction in connection with a possible shrinkage of the photon, and it would be interesting to also look for such a behavior of the intermediate vector boson.

In photoproduction and electroproduction[10] one further observed that *s-channel helicity conservation (SCHC)* holds rather well for the vector current. One should therefore investigate this property also in the weak production case. For the axial-vector current the corresponding situation is unknown. Diffractive production of axial-vector mesons by neutrinos offers a tool to study this question.

If helicity is conserved at the nucleon vertex as well as at the current–vector particle (axial-vector-particle) vertex, i.e., $\lambda_1 = \lambda_2$ and $\lambda = \lambda'$, we have only the following independent amplitudes:

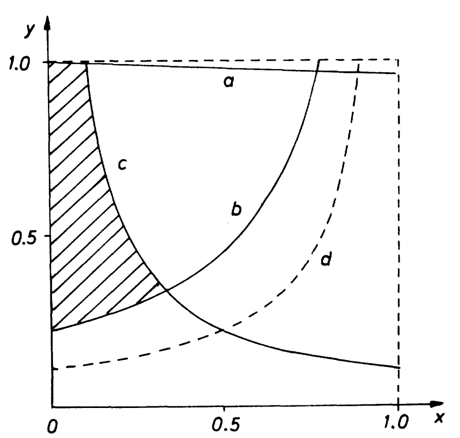

FIG. 3. Illustration of the kinematically allowed region in the x-y plane for production of ρ by neutrinos at $E = 10$ GeV. The shaded area is the diffractive region for $s \ge 5$ GeV^2 and $Q^2 \le 2$ GeV^2. Lines a, b, and c correspond to Eqs. (44), (42b), and (47), respectively. Line d shows the limit at the threshold $s = (M + m_\rho)^2$.

$$V^{(1/2)1}_{(1/2)1},\ V^{-(1/2)1}_{-(1/2)1},\ V^{(1/2)0}_{(1/2)0},\ A^{(1/2)1}_{(1/2)1},\ A^{-(1/2)1}_{-(1/2)1},\ A^{(1/2)0}_{(1/2)0}.$$

As a consequence all cross-section parts of (24)–(32) are zero except σ_U, σ_L, and σ_C. This implies that the cross section [Eq. (20)] exhibits no Φ dependence.

More detailed information about the helicity structure and the (t-channel) exchange mechanism can be obtained from the density matrix [Eqs. (35), (36), and (39)] by studying the angular distribution of the decay of the outgoing meson.

For instance, strict SCHC would imply that only the following measurable density matrix elements do not vanish:

(i) noninterference terms

$$(\rho_U^{++}+\rho_U^{--}),\ \rho_L^{00},\ \rho_T^{+-},\ \rho_T^{\prime +-},\ (\rho_I^{+0}-\rho_I^{0-}),$$

$$(\rho_I^{\prime +0}-\rho_I^{\prime 0-}),\ (\rho_{CL}^{+0}-\rho_{CL}^{0-}),\ (\rho_{CL}^{\prime +0}-\rho_{CL}^{\prime 0-}),$$

(ii) interference terms

$$(\rho_C^{++}+\rho_C^{--}),\ \rho_T^{+-},\ \rho_T^{\prime +-},\ (\rho_I^{+0}-\rho_I^{0-}),$$

$$(\rho_I^{\prime +0}-\rho_I^{\prime 0-}),\ (\rho_{CL}^{+0}-\rho_{CL}^{0-}),\ (\rho_{CL}^{\prime +0}-\rho_{CL}^{\prime 0-}).$$

In the case of A_1 production we have in addition the following nonvanishing matrix elements:

(iii) noninterference terms

$$(\rho_C^{++}-\rho_C^{--}),\ (\rho_I^{+0}+\rho_I^{0-}),\ (\rho_I^{\prime +0}+\rho_I^{\prime 0-}),$$

$$(\rho_{CL}^{+0}+\rho_{CL}^{0-}),\ (\rho_{CL}^{\prime +0}+\rho_{CL}^{\prime 0-}),$$

(iv) interference terms

$$(\rho_U^{++}-\rho_U^{--}),(\rho_I^{+0}+\rho_I^{0-}),(\rho_I^{\prime +0}+\rho_I^{\prime 0-}),$$

$$(\rho_{CL}^{+0}+\rho_{CL}^{0-}),(\rho_{CL}^{\prime +0}+\rho_{CL}^{\prime 0-}).$$

If, moreover, the V-A interference terms can be neglected, the density matrix is further simplified.

More explicitly in the case of SCHC the angular distribution for the decay $V\to\pi\pi$, Eq. (34a), reduces to the following simple form after integrating over Φ (i.e., the angle between the lepton and the hadron plane):

$$W(\theta^*,\Phi^*)=\frac{3}{4\pi}\{\tfrac{1}{2}[\rho_U^{++}+\rho_U^{--}+\eta(1-\epsilon^2)^{1/2}(\rho_C^{++}+\rho_C^{--})]\sin^2\theta^*+\epsilon\rho_L^{00}\cos^2\theta^*\}. \tag{48a}$$

Analogously we have for $A_1\to 3\pi$ [Eq. (34b)]

$$W(\theta^*,\Phi^*)=\frac{3}{8\pi}\{\tfrac{1}{2}[\rho_U^{++}+\rho_U^{--}+\eta(1-\epsilon^2)^{1/2}(\rho_C^{++}+\rho_C^{--})](1+\cos^2\theta^*)$$
$$+\epsilon\rho_L^{00}\sin^2\theta^*+\lambda\cos\theta^*[\rho_U^{++}-\rho_U^{--}+\eta(1-\epsilon^2)^{1/2}(\rho_C^{++}-\rho_C^{--})]\}. \tag{48b}$$

Note that also in this case no Φ^* dependence appears.

The V-A transitions are also contained in Eqs. (48a) and (48b). If they can be neglected, the terms $\eta(1-\epsilon^2)^{1/2}(\rho_C^{++}+\rho_C^{--})$ and $(\rho_U^{++}-\rho_U^{--})$ disappear; thus ν and $\bar{\nu}$ production are predicted to be the same for the case $V\to\pi\pi$ [Eq. (48a)], whereas there is a difference in the case of $A_1\to 3\pi$, Eq. (48b), due to the additional coupling λ. The remaining density matrix elements can then be written in the simple form

$$\rho_U^{++}+\rho_U^{--}=\frac{1}{1+\epsilon R}=-(\rho_C^{++}-\rho_C^{--}),$$

$$\rho_L^{00}=\frac{R}{1+\epsilon R}$$

with $R=\sigma_L/\sigma_U$. A measurement of R based on the ϵ dependence might be rather difficult; however, within the approximations made R can be determined by measuring the decay angular distribution, using Eqs. (48a) and (48b):

$$R=\frac{\rho_L^{00}}{\rho_U^{++}+\rho_U^{--}}.$$

This was already done in electroproduction.[10] Obviously, if, in addition to the Φ-integrated cross section

$$\frac{d\sigma^W}{dt}=2\pi\left(\frac{d\sigma_U}{dt}+\epsilon\,\frac{d\sigma_L}{dt}\right),$$

the density matrix elements ρ_L^{00} and $\rho_U^{++}+\rho_U^{--}$ can be measured, $d\sigma_U/dt$ and $d\sigma_L/dt$ are then given by

$$\frac{d\sigma_U}{dt}=\frac{1}{2\pi}\frac{d\sigma^W}{dt}(\rho_U^{++}+\rho_U^{--}), \tag{49a}$$

$$\frac{d\sigma_L}{dt}=\frac{1}{2\pi}\frac{d\sigma^W}{dt}\rho_L^{00}. \tag{49b}$$

IV. WHAT CAN WE LEARN ABOUT WEAK INTERACTIONS?

Let us distinguish between the better-known charged weak currents and the neutral ones, i.e., between the production of charged mesons [reactions (A) and (B)] and of neutral mesons [reactions (C) and (D)].

In analogy to electroproduction one of the most interesting questions concerns *vector- and axial-vector-meson dominance* of the weak current. One would expect that the charged vector current is dominated by $\rho^{\pm}, K^{*\pm}, F^{*\pm}, D^{*\pm}$ and the neutral vector current by $\rho^0, \omega, \phi, \psi$ and possibly related higher-mass states ($\rho', \psi', \ldots$ etc.). Furthermore, one would expect that the axial-vector current shows dominance of the corresponding axial partners of these particles, that is, $A_1, \pi, K_A^*, K, \omega_A$, ... This, however, is less clear as there exist models, for instance, the naive nonrelativistic quark model, where the A_1 does not couple to the axial-vector current. In the charged-current reactions (A) and (B) the search for A_1 production is therefore undoubtedly most interesting.

If the weak current is dominated by vector and axial-vector mesons, these particles *must* be produced diffractively and show the characteristics of the diffraction mechanism (see Sec. III) as has been the case for electroproduction of ρ and ω).

Owing to the absence (or strong suppression) of strangeness-changing neutral currents, the K^{*0} ($\overline{K}^{*0}$) will play no role in the dominance of the neutral current. Furthermore, in a quark model with u, d, c, s, states neutral currents are in general diagonal in these fields; thus they cannot be dominated by a $c\overline{u}$ state (D^{*0}) either.

Obviously, in order to learn more about the isospin and the space-time properties of the neutral current it is necessary to assume some properties of the diffraction mechanism. We shall assume that diffractive scattering with neutrinos proceeds in a way completely analogous to photoproduction and electroproduction. Furthermore, the assumption is made that elastic scattering on nucleons of an axial-vector particle is the same as of the corresponding vector particle.

The form of the weak neutral current using the quark-model notation is given in Ref. 4 as

$$\begin{aligned} J_\mu = &\tfrac{1}{2}[\overline{u}\gamma_\mu(\alpha+\beta\gamma_5)u - \overline{d}\gamma_\mu(\alpha+\beta\gamma_5)d] \\ &+\tfrac{1}{2}[\overline{u}\gamma_\mu(\gamma+\delta\gamma_5)u + \overline{d}\gamma_\mu(\gamma+\delta\gamma_5)d] \\ &+\overline{s}\gamma_\mu(\gamma_s+\delta_s\gamma_5)s + \overline{c}\gamma_\mu(\gamma_c+\delta_c\gamma_5)c + \cdots . \end{aligned} \tag{50}$$

α, β, γ, δ, etc. are parameters given by specific models.

In principle the inclusive processes $[\nu(\overline{\nu})+N \to \nu(\overline{\nu})+\text{hadrons}]$ on protons and neutrons separately, together with elastic νN scattering[17] or single-π production,[18] allow a complete determination of the couplings $\alpha, \beta, \gamma, \delta$. Inclusive hadron production by neutrinos supplemented by information on diffractive production of vector and axial-vector mesons provides a further possibility to measure them.[4] Indeed diffractive neutrino production of vector and axial-vector mesons by itself [see Eqs. (C) and (D)] can give an independent determination of all the coefficients appearing in Eq. (50), provided that the features of the diffraction mechanism are known from the charged-current reactions (A) and (B). As long as this is not the case, one has to resort to measuring appropriate ratios as illustrated in Refs. 4 and 5, for instance

$$\frac{\sigma_U(\nu+N\to\nu+\rho^0+N)}{\sigma_U(\nu+N\to\mu^-+\rho^++N)} = \tfrac{1}{2}\alpha^2, \tag{51}$$

$$\frac{\sigma_U(\nu+N\to\nu+A_1^0+N)}{\sigma_U(\nu+N\to\mu^-+A_1^++N)} = \tfrac{1}{2}\beta^2, \tag{52}$$

$$\frac{\sigma_U(\nu+N\to\nu+\omega^0+N)}{\sigma_U(\nu+N\to\nu+\rho^0+N)} = \left(\frac{\gamma}{\alpha}\right)^2, \tag{53}$$

where the diffractive part of the cross section σ_U at a fixed value of Q^2 is understood. If σ_U is not known from a measurement of the ϵ dependence a possible way to determine it can be obtained by using Eqs. (49a) and (49b). Instead of σ_U one can take $\sigma_U+\epsilon\sigma_L$ in Eqs. (51)–(53), but this is less certain due to the more complicated Q^2 behavior of σ_L. The relation (53) further implies equal Q^2 behavior for ω and ρ^0 production.

V. PROBLEMS IN MODEL CALCULATIONS

Previous analyses[2,3,6] have shown that numerical predictions for cross sections for the reactions considered are highly model dependent. This is the case for the absolute values as well as for the dependences on x and y or Q^2 and E. All these analyses are based on our knowledge of *electro*-production of vector mesons. At present only neutrino production cross sections integrated over Φ have been considered; hence only the behavior of $\sigma_U+\epsilon\sigma_L$ enters.

Besides the more general questions raised in Secs. III and IV, the further problem arises that even electroproduction of vector mesons has been measured experimentally only in a restricted kinematic domain ($Q^2 \le 1.5$ GeV^2, $W < 5$ GeV); see Refs. 10, 19, and 20 for the most recent data. As claimed in Ref. 10 the ρ^0 electroproduction data are well described by vector-meson dominance (VMD) in this region with $R=\sigma_L/\sigma_U = \xi_\rho^2 Q^2/m_\rho^2$, $\xi_\rho^2 = 0.4$. Since neutrino experiments involve larger incoming energies the range covered by the variables Q^2, s, and t will also be much larger than for electroproduction. The authors of Refs. 3 and 6 therefore had to make definite assumptions in particular about how the cross sections σ_U and σ_L behave at large Q^2. To some extent all these assumptions are rather speculative. While in Ref. 3 VMD was used also for higher Q^2, various Q^2 dependences for σ_U and σ_L disagreeing with simple VMD were studied in Ref. 6 leading to quite different numerical results. Moreover, a purely diffractive mechanism has been assumed for the whole kinematic range.

In order to extract useful information from the data about the weak coupling constants, the diffraction mechanism, and the nature of the particles produced, it is therefore necessary to restrict oneself to an appropriate kinematic region where the uncertainties mentioned above are minimized, i.e., a region where a diffractive mechanism really dominates and where the correspondence between electroproduction and neutrino production is expected to have the greatest validity. The ρ-electroproduction data suggest taking $Q^2 \leq Q_0^2 = 2\ \text{GeV}^2$, $|t| \leq 1\ \text{GeV}^2$, $s \geq s_0 = 5\ \text{GeV}^2$ (for production of mesons with higher masses a larger value for s_0 is necessary). At higher Q^2 one is already in a region where one expects nondiffractive mechanisms to become more important. In a parton picture, for instance, diffractive scattering is correlated with the sea quarks and for larger x the sea contribution becomes negligible. In particular, σ_L/σ_U will no longer follow the simple form $\xi_\rho^2 Q^2/m_\rho^2$ but will rather go to zero.

In the kinematic domain defined above the neutrino production cross section can be related to electroproduction by

$$\frac{d\sigma^W}{dt}\left[\begin{pmatrix}W^\pm\\ Z^0\end{pmatrix}N \to V^{\pm 0}N\right] = \frac{2d_V^2}{e^2}\,\frac{\Gamma_\rho}{\Gamma_V}\frac{d\sigma}{dt}(\gamma_V N \to V^0 N), \tag{54}$$

where we have assumed that V-A transitions are unimportant. Here Γ_V is the width of the leptonic decay $V^0 \to e^+e^-$; d_V measures the strength of the coupling of the intermediate boson to the vector meson and must be specified by some model. Equation (54) can in principle be used for a determination of d_V. The d_V's are of course related to the coefficients α, β, γ, etc. of Eq. (50). For K^*, F^*, D^*, invoking vector-meson dominance, we can write

$$\frac{d\sigma}{dt}(W^\pm N \to VN) = C_V^2\frac{m_V^4}{(m_V^2+Q^2)^2}\left(1+\xi_V^2\epsilon\frac{Q^2}{m_V^2}\right) \times \frac{q^*\sqrt{s}}{MK}\,\frac{d\sigma}{dt}(VN \to VN) \tag{55}$$

with

$$C_V = \frac{\sqrt{2}}{\gamma_\rho}\left(\frac{m_\rho}{m_V}\right)^{1/2} d_V \tag{56}$$

and $\gamma_\rho^2/4\pi \simeq 2.6$. The parameter ξ_V is the ratio of the amplitudes for elastic scattering of longitudinal and transverse polarized vector mesons on nucleons. A form completely analogous to Eq. (55) can be used for describing neutrino production of axial-vector mesons (charged and neutral). We want to stress again that the Q^2 dependence as given in Eq. (55) agrees with ρ^0-electroproduction data[10] in the kinematic region considered, although

TABLE II. The coefficient d_V and d_A according to Eqs. (56) and (57) as given in the Weinberg-Salam model.

	d_V		d_A
$\rho^\pm$	$\cos\theta_C$	$A_1^\pm$	$-\cos\theta_C$
$K^{*\pm}$	$\sin\theta_C$	$K_A^\pm$	$-\sin\theta_C$
$F^{*\pm}$	$\cos\theta_C$	$F_A^\pm$	$-\cos\theta_C$
$D^{*\pm}$	$-\sin\theta_C$	$D_A^\pm$	$\sin\theta_C$
ρ^0	$\frac{1-2\sin^2\theta_W}{\sqrt{2}}$	A_1^0	$-\frac{1}{\sqrt{2}}$
K^{*0}	0	φ_A	$\frac{1}{2}$
D^{*0}	0	ω_A	0
ω	$-\frac{\sqrt{2}\sin^2\theta_W}{3}$	ψ_A	$-\frac{1}{2}$
φ	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_W$		
ψ	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_W$		

the total inclusive hadronic electroproduction cross section σ_{tot} disagrees with simple VMD. Also, the ratio σ_L/σ_U is experimentally larger for ρ^0 production than for the total inclusive cross section.

In a picture where the weak current couples directly to a vector or axial-vector meson the most essential parameter is the corresponding coupling constant C_V or C_A, respectively. In Eq. (56) the mass factors enter because we have assumed that the decay widths for $V^+ \to \mu^+\nu$ and $V^0 \to \nu\bar{\nu}$ obey the relation $\Gamma_V/\Gamma_\rho = (d_V/d_\rho)^2$ as is suggested by the corresponding electromagnetic decay widths. The couplings d_V in the Salam-Weinberg model are given in Table II, where θ_C is the Cabibbo angle and θ_W is the Weinberg angle. Other models are discussed in Ref. 21.

For the axial-vector couplings C_A two possibilities have been proposed.[3,6] The first[3] is based on the assumption that the decay width of $A \to \mu\nu$ is the same as that of the corresponding vector particle decay $V \to \mu\nu$. This leads to

$$C_A = \frac{\sqrt{2}}{\gamma_\rho}\left(\frac{m_\rho}{m_A}\right)^{1/2} d_A, \tag{57}$$

where d_A is given in Table II. The second proposal[6] is to take $\gamma_{A_1}^2 = 4\gamma_\rho^2$ as suggested by the Weinberg sum rules leading to $\Gamma_{A_1} = (\sqrt{2}/4)\Gamma_\rho$ or equivalently

$$C_A = \frac{1}{2^{1/4}\gamma_\rho}\left(\frac{m_\rho}{m_A}\right)^{1/2} d_A. \tag{58}$$

A possible way to distinguish between these and further possibilities would be to determine $\Gamma(A_1 \rightarrow \mu\nu)$ by measuring the ratio

$$\frac{\sigma_U(\nu p \rightarrow \mu^- A_1^+ p)_{\mathrm{diffr}}}{\sigma_U(\nu p \rightarrow \mu^- \rho^+ p)_{\mathrm{diffr}}} = \frac{(d\sigma/dt)(A_1^+ p \rightarrow A_1^+ p)\Gamma(A_1 \rightarrow \mu\nu) m_{A_1}{}^3 (m_\rho{}^2 + Q^2)^2}{(d\sigma/dt)(\rho^+ p \rightarrow \rho^+ p)\Gamma(\rho \rightarrow \mu\nu) m_\rho{}^3 (m_{A_1}{}^2 + Q^2)^2}\,.$$

In order to give estimates for the rates to be expected in the restricted kinematic domain $Q^2 \leq 2\ \mathrm{GeV}^2$, $|t| \leq 1\ (\mathrm{GeV}/c)^2$, $s \leq s_0$, we have calculated the cross sections for production of several vector and axial-vector mesons [reactions (A) to (D)]. The parameters have been specified in the following way: $s_0 = 20\ \mathrm{GeV}^2$ for ψ_A production, $s_0 = 10\ \mathrm{GeV}^2$ for D^*, F^*, F_A, D_A production, and $s_0 = 5\ \mathrm{GeV}^2$ for the other particles, $\xi_V{}^2 = \xi_A{}^2 = \xi_\rho{}^2 = 0.4$. For all other quantities such as masses, hadronic cross sections, slopes, etc. we have taken the values used in Ref. 3. The results are shown in Figs. 4(a) and 4(b) with C_A as given in (57). The other possibility [Eq. (58)] would lead to cross sections for axial-vector mesons which are smaller by a factor $1/(2\sqrt{2})$. Note that in the Salam-Weinberg model (see Table II) no diffractively produced ω_A should be seen and for $\sin^2\theta_W = \frac{3}{8}$ also no ψ production should occur. The Q^2 dependence of $d\sigma/dQ^2$ for $\nu p \rightarrow \mu^- \rho^+ p$ at $E = 10$ GeV, shown in Fig. 5, is a test of our assumption about the Q^2 behavior as given in Eq. (55) which, moreover, contains implicitly the damping factor $e^{At_{\min}}$ which suppresses the cross section for large Q^2.[22]

Preliminary data on neutrino interactions in hydrogen at a mean neutrino energy of 38 GeV have been presented in Ref. 1. For ρ^+ production our predictions are consistent with the data. Regarding A_1^+ production the prediction, based on the coupling constant given in Eq. (57), is larger by a factor of approximately 3 than the experimental observation, which is, however, based only on three events. If this result should be confirmed by future data it would mean that a coupling to the axial-vector current according to Eq. (58) is favored. A clear experimental signal of A_1-neutrino production would, furthermore, be very helpful for a decision as to whether the A_1 is a genuine resonance or not. It might be possible that also here a Deck mechanism is present as was suggested for pure hadronic reactions. Most likely one will not be able to solve this question by looking only at the production cross section because a nonresonant $\pi\rho$ system might behave like an A_1 and also couple to the axial-vector current with a comparable strength. It will therefore be necessary to investigate in addition angular distributions as discussed in Secs. II and III as well

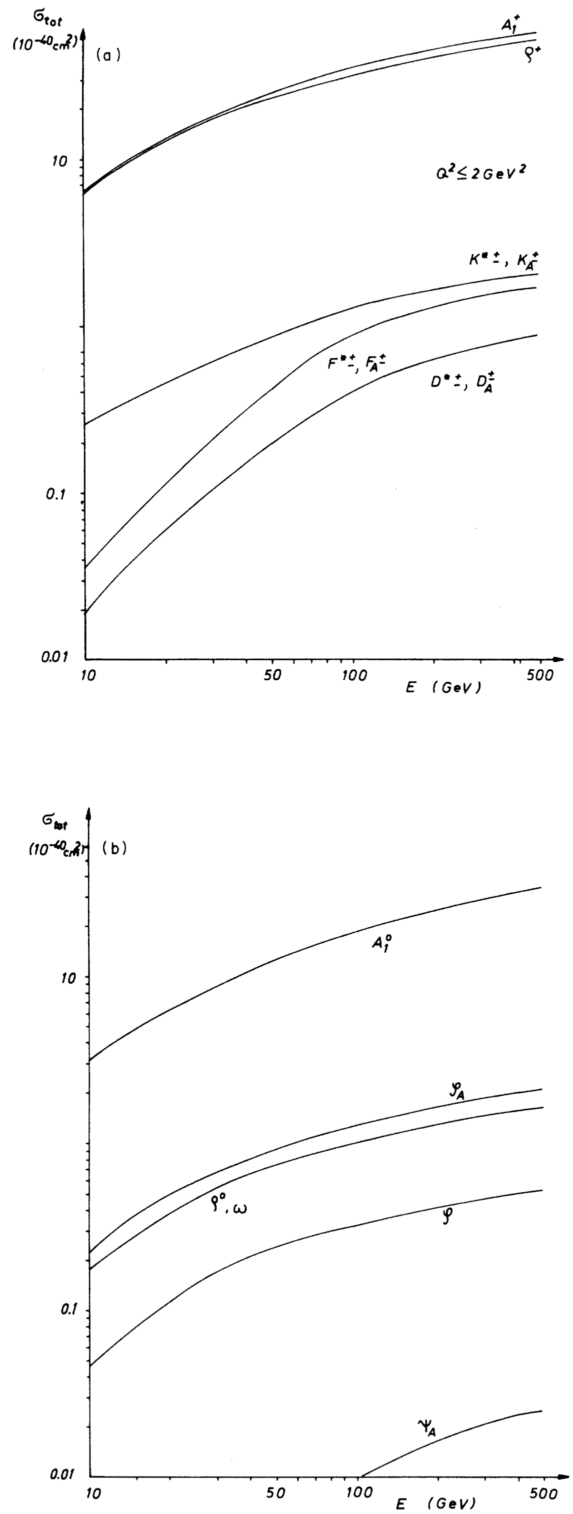

FIG. 4. Cross sections for diffractive neutrino production of vector and axial-vector mesons as a function of E for $Q^2 \leq 2\ \mathrm{GeV}^2$ (a) for charged-current reactions, (b) for neutral-current reactions.

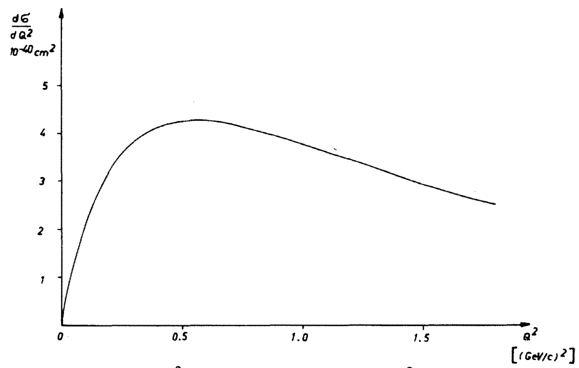

FIG. 5. The Q^2 dependence of $d\sigma/dQ^2$ for $\nu p \to \mu^- \rho^+ p$ at $E = 10$ GeV.

as the three-pion mass distribution.

In the case of ρ^0 production our prediction lies beneath the present data. In this case, however, one has to be aware that the diffractive component is rather small, approximately a factor $\frac{1}{30}$ smaller than for ρ^+ production if the Salam-Weinberg model is right. Consequently the nondiffractive amplitudes, in particular V-A transitions, play a more important role, since the coupling of the A_1^0 to the weak neutral current is much larger than that of the ρ^0 (see Table II). The nondiffractive component might even be of the same order of magnitude as the diffractive one, similarly, for instance, as for ω photoproduction and electroproduction.

It is known that ϕ photoproduction is a pure diffractive process if ϕ contains only strange quarks. In the same way neutrino production of ϕ and ϕ_A proceeds purely diffractively. This makes these processes very interesting for measuring these particular couplings of the weak neutral current as well as for proving the existence of axial-vector particles. If the ϕ_A exists, a prominent decay mode is expected to be $\phi_A \to K\bar{K}\pi$.

VI. CONCLUDING REMARKS

In the present paper we have carried out a detailed kinematical analysis of vector-meson and axial-vector-meson production by neutrinos. By measuring the Φ and ϵ dependence in principle nine independent cross sections can be evaluated, four of which are vector–axial-vector interferences. The decay distributions for $V \to \pi\pi$, $V \to 3\pi$, and $A \to 3\pi$ have also been discussed. The decay density matrices exhibit the same dependence on ϵ and Φ as the cross section. The cross-section contributions as well as the density matrix elements have been expressed in terms of s-channel helicity amplitudes. The density matrix can be written asymptotically as a sum of a natural- and unnatural-parity t-channel exchange contribution. In contrast to electroproduction these two parts cannot be separated purely by kinematics in the present case. If s-channel helicity conservation holds, and if V-A transitions are negligible, the expressions for the cross sections and the decay distributions become particularly simple. This will provide easy experimental tests of these assumptions for axial-vector as well as vector-meson production. At low s, however, appreciable nondiffractive contributions might be present which in turn could give rise to substantial V-A transitions.

If certain details of the reaction mechanism are understood, diffractive production of vector and axial-vector mesons can be used to determine the couplings of the charged axial-vector current as well as those of the weak neutral current.

In order to minimize the uncertainties which enter we have proposed that the analysis should be restricted to a kinematic domain ($Q^2 \le 2$ GeV^2, $|t| \le 1$ GeV^2, $s \ge 5$ GeV^2), where a diffractive mechanism is expected to be dominant and where information from electroproduction of vector mesons is available.

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APPENDIX

In the following we show the separation of the most important density matrix elements ρ_U and ρ_L into natural- and unnatural-parity t-channel exchange contributions expressing them in terms of the amplitudes defined in Eqs. (37) and (38).

(*i*) *Production of a vector meson.* We have

$$\tfrac{1}{2}(\rho_U^{++} + \rho_U^{--}) = \frac{1}{C}\sum_{\lambda_1} [\,|v^{++}_{\lambda_1+}|^2 + |v^{++}_{\lambda_1-}|^2 + |\bar{v}^{++}_{\lambda_1+}|^2 + |\bar{v}^{++}_{\lambda_1-}|^2 + (v \to a)],$$

$$\rho_U^{00} = \frac{2}{C}\sum_{\lambda_1} [\,|v^{+0}_{\lambda_1+}|^2 + |\bar{v}^{+0}_{\lambda_1+}|^2 + (v \to a)],$$

$$\rho_U^{+-} = \frac{2}{C}\left\{\sum_{\lambda_1} \mathrm{Re}[v^{++}_{\lambda_1+} v^{++\,*}_{\lambda_1-} - \bar{v}^{++}_{\lambda_1+}\bar{v}^{++\,*}_{\lambda_1-} - (v \to a)] - i\sum_{\lambda_1} \mathrm{Im}[v^{++}_{\lambda_1+} a^{++\,*}_{\lambda_1-} - \bar{v}^{++}_{\lambda_1+}\bar{a}^{++\,*}_{\lambda_1-} - (v \to a)]\right\},$$

$$(\rho_U^{+0}-\rho_U^{0-})=\frac{2}{C}\Big\{\sum_{\lambda_1}\mathrm{Re}[(v^{++}_{\lambda_1+}-v^{++}_{\lambda_1-})v^{+0*}_{\lambda_1+}+(\bar{v}^{++}_{\lambda_1+}+\bar{v}^{++}_{\lambda_1-})\bar{v}^{+0*}_{\lambda_1+}+(a^{++}_{\lambda_1+}+a^{++}_{\lambda_1-})a^{+0*}_{\lambda_1+}+(\bar{a}^{++}_{\lambda_1+}-\bar{a}^{++}_{\lambda_1-})\bar{a}^{+0*}_{\lambda_1+}]$$

$$+i\sum_{\lambda_1}\mathrm{Im}[(v^{++}_{\lambda_1+}+v^{++}_{\lambda_1-})a^{+0*}_{\lambda_1+}+(\bar{v}^{++}_{\lambda_1+}-\bar{v}^{++}_{\lambda_1-})\bar{a}^{+0*}_{\lambda_1+}+(a^{++}_{\lambda_1+}-a^{++}_{\lambda_1-})v^{+0*}_{\lambda_1+}+(\bar{a}^{++}_{\lambda_1+}+\bar{a}^{++}_{\lambda_1-})\bar{v}^{+0*}_{\lambda_1+}]\Big\},$$

$$\tfrac{1}{2}(\rho_L^{++}+\rho_L^{--})=\frac{2}{C}\sum_{\lambda_1}[|v^{++}_{\lambda_1 0}|^2+|\bar{v}^{++}_{\lambda_1 0}|^2+(v\to a)],$$

$$\rho_L^{00}=\frac{2}{C}\sum_{\lambda_1}(|v^{+0}_{\lambda_1 0}|^2+|\bar{a}^{+0}_{\lambda_1 0}|^2),$$

$$\rho_L^{+-}=\frac{2}{C}\Big[\sum_{\lambda_1}(-|v^{++}_{\lambda_1 0}|^2+|\bar{v}^{++}_{\lambda_1 0}|^2+|a^{++}_{\lambda_1 0}|^2-|\bar{a}^{++}_{\lambda_1 0}|^2)+2i\sum_{\lambda_1}\mathrm{Im}(v^{++}_{\lambda_1 0}a^{++*}_{\lambda_1 0}+\bar{a}^{++}_{\lambda_1 0}\bar{v}^{++*}_{\lambda_1 0})\Big],$$

$$\rho_L^{+0}-\rho_L^{0-}=\frac{4}{C}\Big[\sum_{\lambda_1}\mathrm{Re}(v^{++}_{\lambda_1 0}v^{+0*}_{\lambda_1 0}+\bar{a}^{++}_{\lambda_1 0}\bar{a}^{+0*}_{\lambda_1 0})+i\sum_{\lambda_1}\mathrm{Im}(\bar{v}^{++}_{\lambda_1 0}\bar{a}^{+0*}_{\lambda_1 0}+a^{++}_{\lambda_1 0}v^{+0*}_{\lambda_1 0})\Big].$$

(*ii*) *Production of an axial-vector meson.* This is as in the vector-meson case (i) but with $(v\leftrightarrow a)$. In addition, one has the following density matrix elements:

$$\tfrac{1}{2}(\rho_U^{++}-\rho_U^{--})=\frac{2}{C}\sum_{\lambda_1}\mathrm{Re}(v^{++}_{\lambda_1+}a^{++*}_{\lambda_1+}+v^{++}_{\lambda_1-}a^{++*}_{\lambda_1-}+\bar{a}^{++}_{\lambda_1+}\bar{v}^{++*}_{\lambda_1+}+\bar{a}^{++}_{\lambda_1-}\bar{v}^{++*}_{\lambda_1-}),$$

$$\rho_U^{+0}+\rho_U^{0-}=\frac{2}{C}\Big\{\sum_{\lambda_1}i\,\mathrm{Im}[(v^{++}_{\lambda_1+}+v^{++}_{\lambda_1-})v^{+0*}_{\lambda_1+}+(\bar{v}^{++}_{\lambda_1+}-\bar{v}^{++}_{\lambda_1-})\bar{v}^{+0*}_{\lambda_1+}+(a^{++}_{\lambda_1+}-a^{++}_{\lambda_1-})a^{+0*}_{\lambda_1+}+(\bar{a}^{++}_{\lambda_1+}+\bar{a}^{++}_{\lambda_1-})\bar{a}^{+0*}_{\lambda_1+}]$$

$$+\sum_{\lambda_1}\mathrm{Re}[(v^{++}_{\lambda_1+}-v^{++}_{\lambda_1-})a^{+0*}_{\lambda_1+}+(\bar{v}^{++}_{\lambda_1+}+\bar{v}^{++}_{\lambda_1-})\bar{a}^{+0*}_{\lambda_1+}+(a^{++}_{\lambda_1+}+a^{++}_{\lambda_1-})v^{+0*}_{\lambda_1+}+(\bar{a}^{++}_{\lambda_1+}-\bar{a}^{++}_{\lambda_1-})\bar{v}^{+0*}_{\lambda_1+}]\Big\},$$

$$\tfrac{1}{2}(\rho_L^{++}-\rho_L^{--})=\frac{4}{C}\sum_{\lambda_1}\mathrm{Re}(v^{++}_{\lambda_1 0}a^{++*}_{\lambda_1 0}+\bar{a}^{++}_{\lambda_1 0}\bar{v}^{++*}_{\lambda_1 0}),$$

$$(\rho_L^{+0}+\rho_L^{0-})=\frac{4}{C}\Big[\sum_{\lambda_1}i\,\mathrm{Im}(a^{++}_{\lambda_1 0}a^{+0*}_{\lambda_1 0}+\bar{v}^{++}_{\lambda_1 0}\bar{v}^{+0*}_{\lambda_1 0})+\sum_{\lambda_1}\mathrm{Re}(v^{++}_{\lambda_1 0}a^{+0*}_{\lambda_1 0}+\bar{a}^{++}_{\lambda_1 0}\bar{v}^{+0*}_{\lambda_1 0})\Big].$$

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