Quark-parton model for the structure function W_2 of the proton and neutron in their rest systems

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(Received 1 March 1976; revised manuscript received 28 January 1977)

The deep-inelastic structure function W_2 is calculated using the impulse approximation for mass-shell partons in the target-nucleon rest system. Bjorken scaling and the approach to scaling are shown to follow from parton kinematics in the rest system. The partons are identified as quarks and a simple harmonic-oscillator quark model is used to fit the proton and neutron structure functions down to $x \sim 0.1$. The neutron structure function requires an inherently non-SU(6) quark model based on relative coordinates that we argue should replace shell-model SU(6) quark models that have been used extensively.

I. INTRODUCTION

The parton model[1,2] has had great success in explaining the general features of deep-inelastic lepton-nucleon scattering, especially the phenomenon of Bjorken scaling.[3] However, while the experiments point more and more to the identification of the partons as the three old-fashioned quarks,[4] the usual derivation of the parton model takes place in an infinite-momentum frame, which makes its relation to the low-energy quark model obscure, if not surprising.

In this paper, we treat the parton kinematics for deep-inelastic electron (or muon) scattering in the laboratory frame, the rest system of the initial nucleon.[5] Making the usual parton-model assumption of incoherent absorption of the virtual photon by a single parton, we derive scaling as a consequence of parton kinematics in the Bjorken limit. We can also, by this method, investigate the approach to scaling since we do not *start* at infinite momentum.

Because our treatment is in the rest system of the target nucleon, it makes sense to use a quark-model momentum distribution for the partons. As an example, we take a simple Gaussian distribution, consistent with the low-energy harmonic-oscillator quark model,[6] and find a good fit to the scaled proton structure function $F_2(x)$ for a large range of the scaling variable x. In order to fit the neutron and proton structure functions simultaneously, we consider an oscillator quark model with two different force constants, and find a good fit with parameters that are consistent with what might be expected from the quark model.

Our conclusion is that the same simple quark model gives a good description of nucleons both for deep-inelastic scattering and low-energy properties, while suggesting an explicit type of SU(6) breaking for the quark model.

In this paper, we concentrate on the electron-induced structure functions and then only on $F_2(x)$. Except for a few considerations, we leave the extension to neutrino scattering and details of $F_1(x)$ to later publications. It is well known that, for spin-$\frac{1}{2}$ partons, the electron-induced, scaled structure functions are related by[7] $F_1(x)=xF_2(x)$, corresponding to the ratio, R, of longitudinal to transverse photon absorption being zero. Experimentally, R is small ($20\pm10\%$) for most of the range of x (assuming that the experiments have reached the scaling region for R).[8] The model we discuss here has $R=0$ and only three spin-$\frac{1}{2}$ partons (quarks).

In Secs. II and III of this paper, we use the kinematics of virtual-photon absorption by partons in the laboratory system to derive scaling and the approach to scaling. In Secs. IV and V we put in quark dynamics in the form of a Gaussian momentum distribution for the partons and fit $F_2(x)$ for both proton and neutron with a reasonable choice of two different oscillator quark-model "force constants." In Sec. VI we compare our parameters with corresponding quark-model parameters. In Sec. VII we discuss our results and the conclusions we can draw from them.

II. PARTON KINEMATICS AND SCALING

The structure functions for electron-nucleon scattering are usually defined in terms of the two invariants, q^2 and

$$\nu = P\cdot q/M \ , \tag{1}$$

where q is the four-momentum transfer to the nucleon (or the four-momentum of the virtual photon exchanged) and P is the initial four-momentum of a nucleon of mass M. Another useful variable is the invariant squared mass of the produced hadrons, which is related to the other invariants by

$$W^2=(P+q)^2=M^2+2M\nu+q^2 \ . \tag{2}$$

In this framework, elastic scattering corresponds to virtual photon absorption with $W^2=M^2$ so that the invariants are related by

$$q^2=-2M\nu \quad \text{(elastic scattering)} . \tag{3}$$

This requires the form[2]

$$W_2(\nu, q^2)=\delta(\nu+q^2/2M) , \tag{4}$$

which results in the usual Mott formula for the elastic scattering cross section of an electron by a point particle of mass M.

The key assumption of the parton model considered here is that the virtual photon is absorbed by a single parton of charge Q_ie, mass m_i, and laboratory momentum $\vec{p}$. For such a parton we would have

$$W_2(\bar{\nu}_i, q^2)=Q_i^2\delta(\bar{\nu}_i+q^2/2m_i) , \tag{5}$$

where $\bar{\nu}_i$ is the parton-invariant variable $p\cdot q/m_i$. Evaluated in the laboratory system, $\bar{\nu}$ is given by

$$\bar{\nu}=p\cdot q/m=(\nu E-\vec{p}\cdot\vec{q})/m , \tag{6}$$

where $E=(\vec{p}^2+m^2)^{1/2}$. We take the partons to have (spherically symmetric) momentum distributions $\mathcal{P}_i(\vec{p})$. Then the nucleon structure function is given by

$$W_2(\nu, q^2)=\sum_i m_iQ_i^2\int d^3p\,\mathcal{P}_i(\vec{p})\delta(\nu E_i-\vec{p}\cdot\vec{q}+q^2/2) , \tag{7}$$

where the sum is over each parton in the nucleon.

We do the φ integration and then the z $(=\cos\theta_{pq})$ integration making use of the δ function. This leaves

$$|\vec{q}|\,W_2(\nu, q^2)=\sum_i 2\pi m_iQ_i^2\int_{p_m}^{\infty} dp\,p\,\mathcal{P}_i(p) , \tag{8}$$

where p_m is the minimum momentum for which $-1\leqslant z\leqslant 1$ is possible. Solution of the equation

$$\nu(\vec{p}^2+m^2)^{1/2}-|\vec{p}|\,|\vec{q}|z+\tfrac{1}{2}q^2=0 \tag{9}$$

for $z=\pm 1$ shows that p_m is given by

$$p_m=(\nu/2)\left|(1-4M^2x^2/q^2)^{1/2}-(1-4m^2/q^2)^{1/2}\right| , \tag{10}$$

with

$$x=-q^2/2M\nu . \tag{11}$$

The new variable x is the usual Bjorken scaling variable[2,3] and is limited by the kinematics to be $0\leqslant x\leqslant 1$.

In the deep-inelastic region of large ν, we can expand Eq. (10) resulting in

$$p_m=\tfrac{1}{2}M\left|x(1-Mx/2\nu)-(m^2/M^2x)(1-m^2/2\nu Mx)\right| + O(1/\nu^2) . \tag{12}$$

In the scaling limit, $\nu\to\infty$, p_m approaches a function of x alone,

$$\lim_{\nu\to\infty} p_m=\tfrac{1}{2}M\left|x-m^2/M^2x\right|\equiv p_m(x) . \tag{13}$$

Using this and the relation

$$|\vec{q}|=(\nu^2-q^2)^{1/2} = \nu(1+2Mx/\nu)^{1/2}\xrightarrow[\nu\to\infty]{}\nu , \tag{14}$$

we can rewrite Eq. (8) as

$$\lim_{\nu\to\infty}\nu W_2(\nu, q^2)=F_2(x) =2\pi\sum_i m_iQ_i^2\int_{p_m(x)}^{\infty} dp\,p\,\mathcal{P}_i(p) . \tag{15}$$

Equation (15) is our main result and shows that, in the scaling limit $\nu\to\infty$, the combination νW_2 is a function only of the scaling variable x. The form of $F_2(x)$ depends on the parton momentum distribution $\mathcal{P}_i(p)$ in the nucleon rest system with the x dependence coming only from the lower limit of the integral in Eq. (15). The partons are assumed to satisfy the mass-shell condition $p^2=m^2$ before and after absorbing the virtual photon and this leads to our starting point, Eq. (5). This mass-shell condition is not satisfied in field-theoretic treatments of the partons,[5] but is characteristic of quark-model treatments of resonance decays.[6] Our model is thus an extension of this type of quark model to deep-inelastic scattering. As far as the initial nucleon state is concerned, our model rests on the same basis as has been discussed for these quark models,[6] even though the virtual photon is very far off its mass shell. As $x\to 0$ or 1, the initial parton momentum becomes very large and the use of a mass-shell momentum distribution might be expected to break down so that our approach should only be valid for a middle range of x values where the mass-shell condition applies.

III. NEAR-SCALING

One of the features of deep-inelastic scattering has been that scaling seems to occur at surprisingly low values of q^2. Although the parton-model assumption of incoherent absorption of the virtual photon is too naive to allow much speculation on when it should be expected to break down, we can make a couple of observations.

We might expect (assuming equivalence of all partons) that the cross section for electron scattering would be proportional to $|\langle\beta|A\sum_iQ_i|\alpha\rangle|^2$, where A is the photon absorption operator and α and β are the initial and final hadron states. The coherent parton contribution would come from cross products and would be proportional to $\sum_{i\neq j}Q_iQ_j$. But, as first emphasized by Gottfried,[10] this double sum vanishes for the proton, given the quark charges $\frac{2}{3}$, $\frac{2}{3}$, and $-\frac{1}{3}$. [The vanishing of $\sum_{i\neq j} Q_i Q_j$ is equivalent to $(\sum_i Q_i)^2 = \sum_i Q_i^2$.] This would help explain early scaling for any model with these quark charges. The argument fails for the Han-Nambu[11] model with integral quark charges. However, the argument also fails for the neutron for which $(\sum_i Q_i)^2 = 0 \neq \sum_i Q_i^2$ for any combination of quark charges. Experimentally the neutron also seems to show early scaling, although the experiment is not as clear cut as for the proton.[12-14]

The early scaling in inelastic electron-proton scattering can also be correlated with the rapid falloff ($\sim 1/q^4$) of the proton elastic form factor. In each case, coherent effects seem to be disappearing more rapidly than anticipated.

Whatever the reason for the incoherence assumption working so well, if we believe it then it makes sense to use our equations to consider the approach to scaling in the "near-scaling" region $-q^2 \sim M^2$. The most direct way to do this is to use the exact equation (8) to define $(\nu^2 - q^2)^{1/2} W_2(\nu, q^2)$ as a function of the scaling variable $p_m(\nu, q^2)$ as given by Eq. (10). This requires having a parton momentum distribution such as we use for the specific model considered in Secs. IV and V.

Near-scaling can be related more closely to the usual scaling variable x in the following way. We define a near-scaling variable x' as the solution of the scaling-limit equation (13)

$$x' = [E_m(\nu, q^2) \pm p_m(\nu, q^2)]/M \quad (x \gtrless m/M) \tag{16}$$

but with $p_m(\nu, q^2)$ given by the exact equation (10). Then we can rewrite Eq. (8) as

$$\nu W_2(\nu, q^2) = (1 + 2Mx/\nu)^{-1/2} F_2(x') , \tag{17}$$

where $F_2(x')$ would be the scaling-limit function evaluated at the value x' calculated by the above procedure. The near-scaling variable x' approaches x in the scaling limit.

In the usual quark model with $m^2 \ll M^2$, near-scaling will result primarily from the first term in Eq. (10) and at large x. We can make use of this fact to find a somewhat simpler approximate form for x'. Taking the scaling limit for the second term of Eq. (10), but treating the first term exactly, our procedure leads to[15]

$$x' \simeq 2x/[1 + (1 + 2Mx/\nu)^{1/2}] , \tag{18}$$

as an effective near-scaling variable to be used in Eq. (17).

The near-scaling structure function has been fitted phenomenologically using the variable[8]

$$x'_{\rm expt} = x/(1 + M/2\nu) . \tag{19}$$

We can compare this with our x' by expanding Eq. (18) to find

$$x' \simeq x/(1 + Mx/2\nu) . \tag{20}$$

This approaches the phenomenological form (19) for large x, but tends to give less shift in x. The additional factor $(1 + 2Mx/\nu)^{-1/2}$ in Eq. (17) also tends to reduce the effect of near-scaling. In fact, because of the trend of $F_2(x)$, the two effects in Eq. (17) [the shift from x to x' and the factor $(1 + 2Mx/\nu)^{-1/2}$] tend to cancel and we find very little change in $F_2(x)$ in the near-scaling region.

This is demonstrated in Fig. 1 where the experimental $\nu W_2(\nu, q^2)$ is plotted in the near-scaling region along with our result using Eqs. (17) and (18) (dashed curve).[16] The phenomenological fit[8] using the form (19) is shown as the solid curve.

We see from Fig. 1 that our near-scaling result (which has no free parameters once the scaled structure function has been fitted) tends to fit νW_2 (until $-q^2 \ll 1$ GeV^2) leaving out the resonance contribution. This is appropriate for the parton model we are considering, where the process of elastic electron-parton scattering leads to Eq. (8) for the proton structure function. Then the final hadron state is arrived at in the following way. The quasi-free parton does not escape, but, on reaching whatever quark restraining force there is, radiates (like soft bremsstrahlung) one or more pions.[9] After the parton has lost energy to pions (and perhaps kaons) in this fashion, the hadron system falls back into resonant states which undergo further decay. In such a model, resonances cannot be produced via quasi-free parton scattering without at least one additional pion being radiated. The direct resonance production peaks seen in Fig. 1 then have to be the result of coherent production, not accounted for by our model. The phenomenological fit using Eq. (19) just happens to account for the falloff of this coherent contribution, but is not directly related to the parton model we are considering.

There are experimental indications of further deviations from scaling at high $-q^2$.[17] Any such deviations would not be related to the near-scaling discussed here, but could be manifestations of parton structure.

IV. A QUARK MODEL OF THE PROTON STRUCTURE FUNCTION

We now relate the scaled structure function of Eq. (15) to a simple quark model for the proton. For simplicity we start with the same normalized Gaussian momentum distribution for each parton,

$$\mathcal{P}(p) = [1/(\alpha\sqrt{\pi}\,)^3] e^{-p^2/\alpha^2} . \tag{21}$$

With this $\mathcal{P}(p)$, the integral in Eq. (15) can be evaluated exactly and we have

$$F_2(x)=\left(m\sum_i Q_i^2/\alpha\sqrt{\pi}\right)e^{-p_m^2/\alpha^2}\,, \tag{22}$$

with $p_m(x)$ given by Eq. (13). In the fractionally charged quark model of the proton with $\sum_i Q_i^2=1$, we obtain a good fit (Fig. 2)[18] to the proton structure function with $m=M/7$ and $\alpha^2=0.05M^2$ so that

$$F_2(x)=0.36e^{-5(x-1/49x)^2}\,. \tag{22'}$$

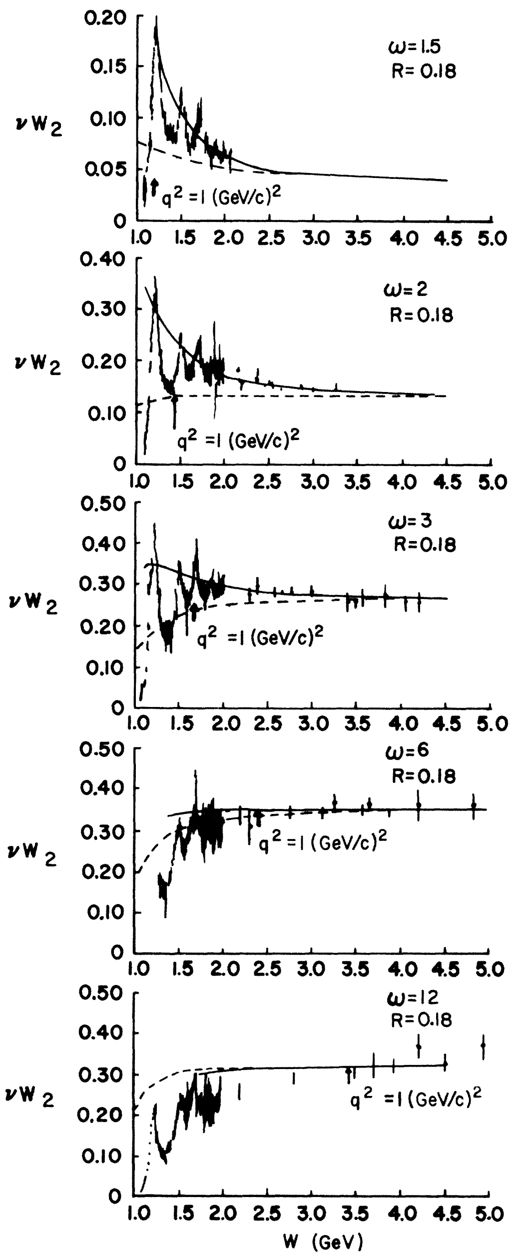

FIG. 1. The proton structure function $\nu W_2(\nu,q^2)$ plotted as a function of W for various values of $\omega=1/x$. The experimental points are from Ref. 8. The solid curve is the phenomenological fit of Ref. 8. The dashed curve is our near-scaling result using Eqs. (17) and (18).

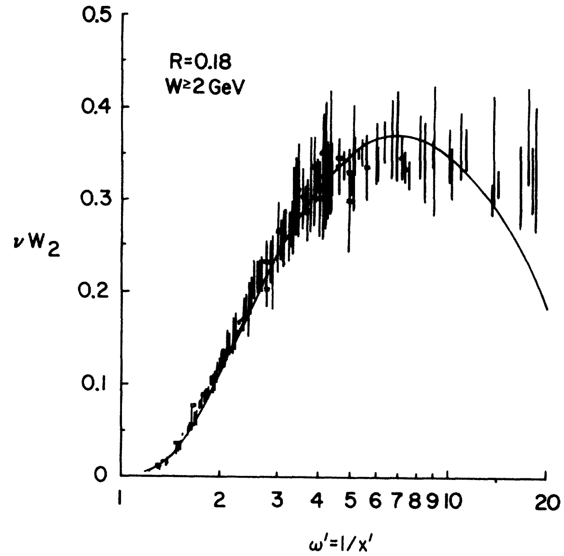

FIG. 2. The scaled proton structure function $F_2(x)$ of Eq. (30p) plotted as a function of $\omega'=1/x'$ (x' approaches x in the scaling limit). The experimental points are from Ref. 8.

Although $F_2(x)$ as given by Eq. (22′) has a maximum at $x=\frac{1}{7}$, we see from Fig. 2 that it gives a good fit down to about $x=0.1$. For smaller x, a more sophisticated model would have to be considered. (See Sec. V for a further discussion.)

We recall that we have put no final-state interaction into this naive parton model. As a consequence, we do not get the threshold condition $F_2(1)=0$. However, $F_2(1)$ as given by Eq. (22′) is so small that we still get a good fit to $F_2(x)$ up to large x.

V. A QUARK MODEL FOR BOTH NEUTRON AND PROTON STRUCTURE FUNCTIONS

If SU(6) [or SU(4)] symmetry were good, then the only difference between the neutron structure function W_2^n and the proton structure function W_2^p would be the $\sum_i Q_i^2$ factor in Eq. (22). This would predict $W_2^n(\nu,q^2)=\frac{2}{3}W_2^p(\nu,q^2)$ in contradiction with the measured ratio as shown in Fig. 3. Experimentally, the ratio $r(x)=F_2^n(x)/F_2^p(x)$ varies from about 0.9 for small x down to about 0.25 near $x=1$. This variation falls within the absolute quark-model limits[19] $\frac{1}{4}<r<4$ seeming to approach the lower limit as x approaches 1.

The variation of $r(x)$ with x and the approach of $r(x)$ to the quark-model lower limit of $\frac{1}{4}$ near $x=1$ can be understood in our simple quark-parton model as a dependence of the quark wave functions on the total spin states of each quark pair. This results in the required SU(6) breaking. We implement this by using quark wave functions with no

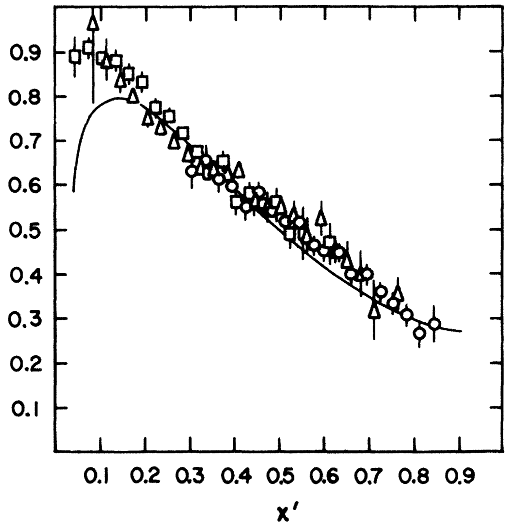

FIG. 3. The ratio $r(x)=F_2^n/F_2^p$ as determined by Eqs. (30n) and (30p) plotted as a function of x'. The experimental points are from Refs. 12 (△), 13 (□), and 14 (○).

built-in SU(6) symmetrization. The justification (and preference) for using these wave functions has been detailed in an earlier work.[20]

For nucleons the procedure is as follows. The nucleons, p and n, are constructed from the two types of nucleon quarks, u and d, which are considered to be distinct (no relation between them) and not to be permuted. If some form of statistics (Bose, Fermi, or parastatistics)[21] is assumed, then two identical quarks (e.g. u and u) must be in a pure spin state. (In the usual quark model this is spin 1.) We write the internal nucleon quark wave functions as $p(uud)$ and $n(ddu)$, in each case taking the first two quarks to be identical and thus in pure spin states. The only way for the three quark spins to add up to $J=\frac{1}{2}$ is for the unlike quarks u and d to be in a mixed spin state (a combination of spin 0 and spin 1 in the usual quark model). Given spin-dependent forces, the third (unlike) quark will have a different interaction with the first two (like) quarks than they will have with each other. In an harmonic-oscillator model this corresponds to two different force constants.

Although the quark momenta can be large, we guide our thinking by looking at a nonrelativistic oscillator with the Hamiltonian

$$H=(p_1^2+p_2^2+p_3^2)/2m+\tfrac{1}{2}kr_{12}^2+\tfrac{1}{2}k'(r_{23}^2+r_{31}^2)\ . \tag{23}$$

Assuming charge symmetry for the nucleon quarks, this Hamiltonian is the same for either the proton or the neutron. Charge independence need not be assumed here, but neither is it violated by this Hamiltonian since the ud quark pair is not in the same spin state as the uu or dd pairs.

We introduce the usual relative coordinates and momenta,

$$R=(r_1+r_2+r_3)/3,\quad P=p_1+p_2+p_3\ , \tag{24a}$$

$$r=r_1-r_2=r_{12},\quad p=\tfrac{1}{2}(p_1-p_2)\ , \tag{24b}$$

$$\rho=r_3-\tfrac{1}{2}(r_1+r_2),\quad \eta=\tfrac{2}{3}[\,p_3-\tfrac{1}{2}(p_1+p_2)]\ , \tag{24c}$$

in terms of which the Hamiltonian can be written

$$H=P^2/6m+p^2/m+\eta^2/(4m/3) + \tfrac{1}{2}(k+\tfrac{1}{2}k')r^2+k'\rho^2\ . \tag{25}$$

This Hamiltonian can be treated as two uncoupled oscillators. Solution of the Schrödinger equation in the momentum representation and the nucleon rest system leads to the momentum probability distribution

$$\mathcal{P}(\vec{P},\vec{p},\vec{\eta})=\delta(\vec{P})e^{-p^2/b^2}e^{-\eta^2/\beta^2}/(\pi b\beta)^3\ , \tag{26}$$

where

$$b^2=[\,\tfrac{1}{2}m(k+\tfrac{1}{2}k')]^{1/2} \tag{27}$$

and

$$\beta^2=(4mk'/3)^{1/2}\ . \tag{28}$$

We have considered a nonrelativistic oscillator model as our motivation for using the momentum distribution (26). But once we have arrived at it by this heuristic route, we treat it as a given momentum distribution to be used in a relativistic treatment of the electron-parton scattering. The momentum distribution is not covariant, but is to be used only in the rest system of the nucleon.

The appropriate generalization of our equation (7) for the structure function in terms of the probability distribution $\mathcal{P}(\vec{P},\vec{p},\vec{\eta})$ is

$$W_2(\nu,q^2)=m\int d^3p_1d^3p_2d^3p_3P(\vec{P},\vec{p},\vec{\eta}) \times\sum_i Q_i^2\delta(\nu E_i-\vec{p}_i\cdot\vec{q}+\tfrac{1}{2}q^2)\ . \tag{29}$$

After many integrations and passing to the scaling limit as in Sec. II, the scaled structure function is given by

$$F_2(x)=(m/\sqrt{\pi}\,)\{[\,(Q_1^2+Q_2^2)/\alpha\,]e^{-p_m^2(x)/\alpha^2} + (Q_3^2/\beta)e^{-p_m^2(x)/\beta^2}\}\ , \tag{30}$$

where

$$\alpha^2=b^2+\beta^2/4\ , \tag{31}$$

and $p_m(x)$ is given by Eq. (13). If all forces were equal we would have $\beta=\alpha$ and Eq. (30) would reduce to the SU(6)-symmetric result (22).

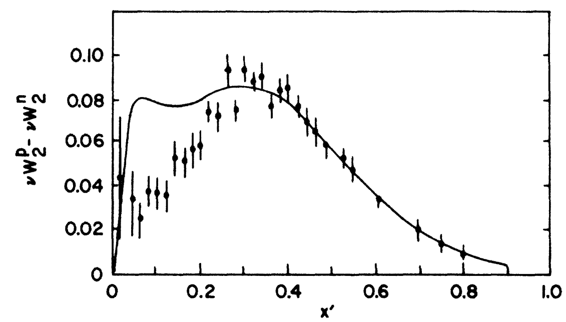

FIG. 4. The difference $F_2^p - F_2^n$ as determined by Eqs. (30p) and (30n) plotted as a function of x'. The experimental points are from Refs. 12 (high x) and 13 (low x).

A good fit can be found to both the neutron and proton scaled structure functions with the usual fractionally charged quarks $u(\frac{2}{3})$, $d(-\frac{1}{3})$, and the parameters

$$m = M/7, \quad \alpha^2 = M^2/20, \quad \beta^2 = M^2/36 \,. \tag{32}$$

This gives for each nucleon

$$F_2^p(x) = (1/7\sqrt{\pi}\,)[\tfrac{8}{9}\sqrt{20}\,e^{-5(x-1/49x)^2} + \tfrac{1}{9}\,6e^{-9(x-1/49x)^2}] \tag{30p}$$

and

$$F_2^n(x) = (1/7\sqrt{\pi}\,)[\tfrac{2}{9}\sqrt{20}\,e^{-5(x-1/49x)^2} + \tfrac{4}{9}\,6e^{-9(x-1/49x)^2}] \,. \tag{30n}$$

Equations (30p) and (30n) clearly illustrate the difference between the proton and neutron structure functions. The proton structure function is given predominantly by its two like quarks which have a relatively broad momentum distribution. But the major contribution to the neutron structure function is from the third (unlike) quark which has a sharper momentum distribution. Thus $F_2^n(x)$ approaches its minimum value of $\frac{1}{4}F_2^p(x)$ as $x \to 1$ corresponding to large $p_m(x)$ and the contribution of the third quark becomes negligible.[22]

In Figs. 2 and 3 we plot the fits of Eqs. (30) to $F_2^p(x)$ and $r(x)$. In each case, the fit is good down to $x \sim 0.1$.

One possibility for extending the model to smaller x would be by using a distribution in both energy and momentum $\mathcal{P}(\vec{p}, E)$. This would correspond to a variable mass with $m^2 = E^2 - p^2$ which could eliminate or shift downwards the predicted maximum in $F_2(x)$. Such distributions have been used in connection with relativistic and covariant oscillator quark models.[23]

The failure of the model at small x could also be related to deviations from the simple quark picture seen in comparison of neutrino- and antineutrino-nucleon scattering for $x \lesssim 0.1$.[24] To some extent this effect can be attributed to quark-antiquark pairs (somehow effective for $x \lesssim 0.1$)[24] which are not included in the simple model used here. For $x > 0.1$, the indications from ν and $\bar{\nu}$ scattering are that quark-antiquark pairs constitute less than 10%[24] of the parton probability distribution so that their neglect does seem appropriate for this range of x, where we find good agreement.

In Fig. 4, we compare the difference of Eqs. (30p) and (30n) with the experimental difference. The theoretical curve has a maximum at $x \sim 0.3$ in agreement with the data, but then the agreement becomes progressively worse.

Taking the difference $F_2^p - F_2^n$ should cancel out any contribution from quark-antiquark pairs. Looked at from this point of view, Fig. 4 might suggest a model with a quark mass $m \sim M/3$ and $q - \bar{q}$ pairs used to explain any resulting differences in Figs. 2 and 3. On the other hand, *any* model which fits Figs. 2 and 3 for all x should tend to fit Fig. 4. We put these speculations off for future work; we are satisfied for the present with a good overall fit down to $x \sim 0.1$ in a simple three-quark model with $q - \bar{q}$ pairs *completely* neglected.

VI. QUARK-MODEL COMPARISONS

It is of interest to compare the oscillator parameters (32) with corresponding parameters in other harmonic-oscillator quark models. These have all used SU(6)-symmetric, shell-model wave functions with a single force constant. Thus they cannot be compared directly with the non-SU(6) relative coordinate states we have used. But, for a rough check, we consider our parameter $1/b$ to represent (in an average sense) the distance between the two u quarks in a proton, while the shell-model parameter $1/\alpha_s$ represents the (shorter) distance from each quark to the center. If we compare $r_{12}{}^2$ for the shell-model states with what we get for our states, then we find that $\alpha_s{}^2$ should be compared with $2b^2 = 0.08$ GeV^2.

Faiman and Hendry used $\alpha^2 = 0.10$ GeV^2 to fit strong[6] and electromagnetic[25] resonance widths. Copley, Karl, and Obryk[26] used $\alpha^2 = 0.17$ GeV^2 to fit (by cancellation) the apparent absence of photoproduction of the $N^*(1688)$ in the forward and backward direction. Thornber[27] used $\alpha^2 = 0.063$ GeV^2 to fit electroproduction of resonances at low q^2. More recently, Berger and Feld[28] have used $\alpha^2 = 0.26$ GeV^2 to fit resonance photoproduction by polarized protons. However, they find that appreciable mixing between SU(6) states would be required to improve their fit.

We see that α^2 is not too well pinned down in the

SU(6) models but that our value of 0.08 is close to the range of variation. The two higher values (0.17 and 0.26) are the result of pushing α^2 up to try to fit peculiar angular distributions that do not seem natural to the SU(6) states considered.

Our fit of Eqs. (30) to the structure functions in fact tie down the oscillator parameters more than do the low-energy fits. The ratio $(m/\alpha\sqrt{\pi})$ is fairly well fixed by the maximum value of $F_2^{(p)}(x)$. The values of α and m are further constrained by the sharp rise in $F_2^{(p)}(x)$ as x decreases from 1 to ~ 0.3. The parameter β is then well determined by the trend of $r(x)$. By comparison, a wide range of range parameters have been used to fit low-energy data with SU(6) models.

It would be appropriate to redo the low-energy quark-model fits using the parameters m, α, β we have found here. This would be a different quark model than has been used previously, because it is inherently non-SU(6)-invariant.[29] It has the virtue of starting with parameters that are fixed by the deep-inelastic structure functions. Also, the method of coupling using only relative coordinates eliminates difficulties with center-of-mass motion that arise when shell-model states are used.[6] The model is a bit like a quark-diquark model,[30] but with the diquark dynamics built in.

The shell-model fits do not depend directly on the quark mass, but only on its magnetic moment. Our value of $m = M/7$ is about half the value $M/3$ suggested by the quark magnetic moment (if it is assumed to be a Dirac moment $Q_i/2m$). However, relativistic effects[31] make a considerable change from the Dirac moment and should be taken into account in photoproduction and in calculations of baryon magnetic moments.

Another question that might be asked is: What about all the SU(6) symmetry of the quark model? We have emphasized in an earlier paper that most of the apparent SU(6) symmetry of baryons is only apparent.[20] Once it is assumed that baryons are composed of three quarks, many of the static SU(6) results follow without assuming SU(6) symmetry and difficulties arise if too much SU(6) symmetry is assumed. Such difficulties arise in the higher multiplet structure[32] and in attempts at detailed fits. One of the clearest difficulties with SU(6) in the quark model is in fact the strong variation in $r(x)$.

VII. CONCLUSIONS

Our conclusions are summarized below:

(1) Scaling in deep-inelastic electron scattering can be understood as arising from mass-shell parton kinematics in the laboratory frame (nucleon rest system) [Eq. (15)].

(2) The approach to scaling [Eqs. (17) and (18)] also follows from the parton kinematics if the coherent resonance production is excluded, as it should be in this model.

(3) The proton structure function can be fitted [Eqs. (22) and (22′)] with a simple Gaussian momentum distribution (for quarks in the proton rest system) down to $x \sim 0.1$.

(4) The neutron and proton structure functions can be fitted simultaneously [Eqs. (30p) and (30n)] (down to $x \sim 0.1 - 0.2$) with harmonic-oscillator momentum distributions corresponding to two spin-dependent quark-quark force constants.

(5) The study of the structure functions, and particularly of $r(x) = F_2^n/F_2^p$, in the nucleon rest system strongly suggests a quark model for both high- and low-energy phenomena that uses relative coordinates [Eqs. (24)] without imposing any SU(6) constraints or permutation.

As is usual in parton models, our approach has been very naive and neglects, among other things, final-state interactions and initial-flux corrections while our quark model was chosen primarily for simplicity. So that, perhaps, the excellent fits we find to the experimental structure functions do not require the precise Gaussian momentum distribution we use. But this general features we deduce are probably characteristic of nucleon structure, at least down to $x \sim 0.1$. These features are as follows:

(1) The nucleon structure functions result from a reasonable rest-frame momentum distribution of point charges with the quark charge values.

(2) The behavior of $r(x)$ is determined by the two like quarks in a nucleon having a larger spread in momentum than the odd quark.

ACKNOWLEDGMENTS

Part of this work was developed while I was a visitor at the Technion. I would like to thank Arnon Dar and Paul Singer for their kind hospitality at the Technion.

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