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Comments and Addenda

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Reggeization of scalars and dual model structure of Yang-Mills theory

M. T. Grisaru*
Department of Physics, Brandeis University, Waltham, Massachusetts 02154
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When the Regge amplitudes are required to satisfy the kinematical constraints on helicity amplitudes the scalars can Reggeize both in scalar QED (in agreement with findings of Cheng and Lo) and in non-Abelian gauge theories. For the SU(2) Yang-Mills model with spontaneously broken symmetry we exhibit a Regge-pole structure reminiscent of the dual model.

Since 1962, when the notion of Reggeization was introduced by Gell-Mann *et al.*,[1] it has generally been assumed that an elementary particle of Lagrangian field theory will lie on a Regge trajectory if for a process where the particle appears as an s-channel pole (1) there exist (two-particle) nonsense states and (2) the Born helicity amplitudes satisfy a factorization condition. This is the case for a fermion in massive QED[1] and for the massive vector mesons in a non-Abelian gauge theory with spontaneously broken symmetry,[2] but factorization does not hold for a scalar in massive QED[3] or the Higgs scalar in the gauge theory.[2,4] However, Cheng and Lo[5] have shown that through eighth order the high-energy behavior of the annihilation amplitude in scalar QED is consistent with the existence of three Regge trajectories, the scalar lying on one of them. They concluded that factorization is not a necessary condition for Reggeization.

The factorization condition arises when one solves the unitarity-analyticity equations at large J and demands that the continuation to low J of the solution without Castillejo-Dalitz-Dyson (CDD) poles agree to order g^2 with the Born amplitude computed there directly.[2,6] The results of Cheng and Lo seem to cast doubt on the validity of this procedure.

In this note we show that the unitarity-analyticity procedure properly handled can give results in agreement with those of Cheng and Lo for the scalar QED case. The same procedure shows that *the Higgs scalar in a non-Abelian SU(2) gauge theory may also Reggeize.* This theory, the zero-slope limit of a dual model,[7] exhibits then a rich Regge-pole structure and has no Kronecker-δ singularities anywhere. For simplicity we discuss here the scalar QED cases and only quote results for the Yang-Mills case.

Our observation is that for these cases the solution without CDD poles does not satisfy the kinematical constraints on helicity amplitudes. When CDD poles are introduced to satisfy the constraints solutions can be found which agree with field theory and Reggeize the scalars. A modified factorization condition holds but its role now is to fix the CDD parameters. For simplicity we discuss the case when vector and scalar have the same mass. We may ignore signature.

In scalar QED (the singular parts of) the parity-conserving scalar-vector Born amplitudes are,[3] near $J=0$,

$$f_{00}^{+}=g^2[6m^2(s-m^2)^{-1}-2m^2q^{-2}]\delta_{J0}, \tag{1a}$$

$$f_{10}^{+}=g^2m(s/2)^{1/2}q^{-2}J^{-1/2}, \tag{1b}$$

$$f_{11}^{+}=g^2(s/2-m^2)q^{-2}J^{-1}, \tag{1c}$$

$$f_{11}^{-}=2g^2J^{-1}. \tag{1d}$$

Away from $J=0$ we must solve the equations

$$T_{\mu\lambda}(s,J)=\frac{1}{q^2}V_{\mu\lambda}(s,J)+\frac{1}{\pi}\int_{4m^2}^{\infty}\frac{ds'}{s'-s}\rho(s')\sum_{\nu}T_{\mu\nu}T_{\lambda\nu}^{\dagger}, \tag{2}$$

$$V_{00}=0,\quad V_{10}=q^2f_{10},\quad V_{11}=q^2f_{11}. \tag{3}$$

We do an N/D calculation with $N=q^{-2}V$ and (see Ref. 2, Appendix A) subtract D so that

$$D=1-\frac{V}{\pi}\int\frac{ds'}{s'-s}\frac{\rho(s')}{q'^2}=1-VK(s)\ . \qquad (4)$$

We find the following (no CDD pole) solutions near $J=0$:

$$T_{11}^{-}(s,J)=2g^2(J-\alpha^-)^{-1}\ , \qquad (5a)$$

$$T_{00}^{+}(s,J)=g^4m^2sK[2q^2(J-\alpha^+)]^{-1}\ , \qquad (5b)$$

$$J^{-1/2}T_{10}^{+}(s,J)=g^2m(2s)^{1/2}[2q^2(J-\alpha^+)]^{-1}\ , \qquad (5c)$$

$$T_{11}^{+}(s,J)=g^2(s-2m^2)[2q^2(J-\alpha^+)]^{-1}\ , \qquad (5d)$$

$$\alpha^-(s)=g^2q^2K,\quad \alpha^+(s)=g^2(s/2-m^2)K\ . \qquad (6)$$

At $J=0$, T_{00}^{+} does not agree with f_{00}^{+}. This is equivalent to the statement that the Born approximation does not factorize.[2]

However, these solutions do not satisfy all the $s=0$ constraints and are not acceptable. These constraints read[8]

$$T_{10}^{+}(J)=0\ , \qquad (7a)$$

$$T_{00}^{+}(J)-T_{00}^{+}(J+2)=\frac{J}{J+1}T_{11}^{-}(J)-\frac{J+3}{J+2}T_{11}^{-}(J+2)+\frac{2J+3}{(J+1)(J+2)}T_{11}^{+}(J+1)\ . \qquad (7b)$$

Since $T_{00}^{+}(0,J)=0$ the second one is not satisfied in the vicinity of the (right-most) pole of T_{11}^{-} at $\alpha^-(0)$. Note that the vanishing of T_{00}^{+} is a consequence of the vanishing of V_{10} independent of any particular subtraction scheme for D.

We look at solutions with one CDD pole. This pole can be introduced in T_{11}^{-} to make it vanish at $s=0$ (evasion of constraints[9]) or in $T_{\mu\nu}^{+}$ to make $T_{00}^{+}(0,J)\neq 0$, in which case Regge poles in $T_{\mu\nu}^{+}$ must conspire[9] with the one in T_{11}^{-} to satisfy the constraints. Scalar QED chooses the second possibility[10] (Cheng and Lo[5] have also observed the conspiratorial nature of their results).

Although the CDD pole can be introduced directly in $D_{\mu\lambda}^{+}(s,J)$, the following equivalent procedure is more convenient. Introduce a fictitious nonsense channel at $J=0$ and couple it to the scalar-vector channel. Solve the new N/D equations for the scalar-vector amplitudes, let the fictitious channel threshold approach infinity to avoid inelastic effects in the physical amplitudes (while increasing its coupling strength), and adjust the fictitious Born amplitudes so that the physical amplitudes satisfy the constraints. The procedure is summarized in the following equations:

$$\mathfrak{N}=\frac{1}{q^2}\mathcal{V}=\frac{1}{q^2}\begin{pmatrix}0 & V_{01} & aJ^{-1/2}\\ V_{10} & V_{11} & b\sqrt{s}\,J^{-1}\\ aJ^{-1/2} & b\sqrt{s}\,J^{-1} & cJ^{-1}\end{pmatrix}\ , \qquad (8)$$

$$\mathfrak{D}=1-\frac{\mathcal{V}}{\pi}\int\frac{ds'}{s'-s}\frac{1}{q'^2}\mathcal{P}(s')\ , \qquad (9)$$

$$\mathcal{P}=\begin{pmatrix}\rho & 0 & 0\\ 0 & \rho & 0\\ 0 & 0 & \tilde{\rho}\end{pmatrix}\ . \qquad (10)$$

The three constants a, b, c, correspond to the three CDD pole parameters (two residues and one location) and the factor $\sqrt{s}$ multiplying b is required to satisfy (7a). The parameters a and c are determined by a conspiracy condition extracted from (7b): Near the pole at $\alpha^-(0)$ we must have

$$T_{00}^{+}=J(J+1)^{-1}T_{11}^{-}\ . \qquad (11)$$

However, b appears as a free parameter, consistent with the Mandelstam counting argument which indicates that at $J=0$ the physical and Regge solutions may differ by one parameter.[6,8] We can make the solutions agree by choosing b suitably and we find then that

$$T_{00}^{+}=\frac{g^4}{\Delta}\frac{m^2}{2q^2}[(8-sK)J-g^2(s+8m^2)K]\ , \qquad (12a)$$

$$\frac{1}{\sqrt{J}}T_{10}^{+}=\frac{g^2}{\Delta}\frac{m}{q^2}(\tfrac{1}{2}s)^{1/2}(J+3g^2)\ , \qquad (12b)$$

$$T_{11}^{+}=\frac{g^2}{\Delta}\frac{1}{2q^2}[(s-2m^2)J+2g^2(s-m^2)]\ , \qquad (12c)$$

$$\Delta=J^2+g^2[(s/2-m^2)K+1]J+g^4(s-m^2)K\ . \qquad (12d)$$

These results agree with those of Cheng and Lo[5] (but their residues are given in the annihilation channel). Two Regge poles are present with the scalar lying on one of the trajectories.

The condition that at $J=0$ the physical and Regge solutions agree is equivalent to the factorization condition

$$f_{00}^{+}=B_{sn}B^{-1}{}_{nn}B_{ns}\ , \qquad (13)$$

where B_{nn} is the lower right corner 2×2 submatrix of $\mathfrak{N}$ and B_{ns} is the upper right corner 1×2 submatrix of $\mathfrak{N}$. This factorization condition provides an easier way to determine the constants a, b, c, once the necessity of a CDD pole has been established. One can then check that the resulting Regge amplitudes satisfy the constraints (as might be expected of amplitudes which agree with the field theory amplitudes which automatically satisfy them).

We discuss now the $I=0$ sector of the non-Abelian SU(2) model studied in Refs. 2 and 4 containing an $I=1$ vector meson ρ and $I=0$ scalar σ. We assume that $m_\rho=m_\sigma$.

At $J=0$ the $I=0$ $\rho\rho$ Born amplitudes do not factorize.[4] The amplitudes must, however, satis-

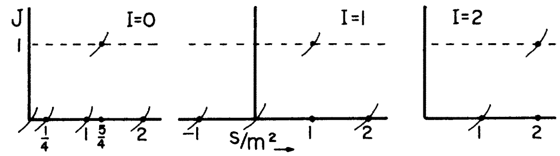

FIG. 1. Regge trajectories near $J=0$ and $J=1$ for the Yang-Mills model (Ref. 13).

fy a total of seven constraint equations at $s=0$. Two CDD poles must be introduced to satisfy all the constraints. If both poles are introduced in the positive-parity amplitudes we can find a Regge solution which agrees with field theory at $J=0$ and Reggeizes the scalar meson. It is thus possible that in this SU(2) Lagrangian field theory *both scalar and vector mesons lie on Regge trajectories and no Kronecker-δ singularities are present anywhere*.

We find that in this $I=0$ sector four Regge trajectories are present, the Higgs trajectory passing through $J=0$ at $s=m^2$ and three nonsense-choosing trajectories passing through $J=0$ at $s=0$, $s=m^2/4$, and $s=2m^2$, respectively. The model exhibits a spectrum of trajectories showing remarkable regularities. We show in Fig. 1 all the trajectories present in the weak-coupling approximation at non-negative J.[11-13] We have considered only natural-parity trajectories. These trajectories are reliable only in the vicinity of $J=0$ and $J=1$, but the temptation to continue them with parallel straight lines is almost overwhelming. We note that the $J=0$, $s=-m^2$ tachyon point has wrong signature so that no actual s-plane pole will be produced. The position of trajectories passing through integral values of s/m^2 seems to be insensitive to change from SU(2) to other groups. However, the trajectory passing through $s/m^2=\frac{1}{4},\frac{5}{4}$, and $J=0,1$ shifts as the group is changed.[14]

To conclude, we discuss our results in light of the Mandelstam counting argument. For scalar QED the counting shows that the physical and Regge solutions may differ by the value of one free parameter. In our calculations b appears to be this free parameter and its value cannot be determined by the kinematical constraints (either at $s=0$ or at threshold), consistent with the counting.

This is to be contrasted to the case of the Regge solution without the CDD pole of Eq. (5). The fact that this solution did not agree with field theory had been interpreted as being consistent with the counting. However, this is not the case: It is impossible to make this solution agree with field theory just by varying one parameter (which would have to be a subtraction constant in D).

Finally we observe that Reggeization of the scalar is not a necessity, yet in light of the results of Cheng and Lo[5] it takes place. Is there a criterion which forces the parameter b to have the right value for Reggeization? We suspect that gauge invariance provides such a criterion, as has been suggested elsewhere.[15] In S-matrix language, it may lead to better high-energy behavior than required by unitarity and this would modify the Mandelstam counting. For instance, in scalar QED unitarity only requires f^+_{00} to be bounded as $s\to\infty$. However, the seagull term $e^2A_\mu{}^2\phi^*\phi$ required by gauge invariance makes f^+_{00} vanish as $s\to\infty$ in the Born approximation. As another hint we point to the case of a fermion-meson model where again $s=0$ constraints require introducing a CDD pole and a free parameter is present, yet the meson does not Reggeize, as shown by explicit calculations.[16] The theory has no gauge invariance.

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