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Higher-order ϵ terms in the triple-Regge region*

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Using the ϵ expansion of Reggeon field theory, we evaluate the exponent which governs the behavior of the inclusive differential cross section in the triple-Regge region at second order. It is found that the first-order result is modified by contributions coming from g^4 terms connecting three Pomeron lines, and that only diagrams with one or two Pomeron lines attached to the external particles are important at asymptotic energies.

I. INTRODUCTION

In the last few years Reggeon field theory has been studied by many authors[1] in order to explain the high-energy data as well as to know what the asymptotic behavior of the theory would be. In this direction the critical exponents of the theory have been studied and computed using several approximation schemes.[2-9] In particular, using ϵ-expansion techniques, the coefficients of the ϵ expansion of the critical exponents have been evaluated up to second order for the two-body scattering amplitude.[2-5] The results of Baker[4] and Bronzan and Dash[5] show that the second-order contribution is not much smaller than the first-order contribution.

A similar study has been done in the triple-Regge region of the inclusive cross section.[10-14] From the renormalization-group equation a scaling law has been obtained showing that the asymptotic energy behavior of the different contributions is governed by an exponent. The ϵ expansion of this exponent has been computed at first order in ϵ showing that the leading contribution is the one in which a single Pomeron is attached to each external particle. In this paper we compute the second term in the ϵ expansion, showing that the results obtained at first order in ϵ are not supported at second order in the sense that the diagrams with two Pomerons attached to each external particle are slightly more important than the diagrams with a single Pomeron.

In Sec. II we collect the results of Ref. 12, making the evaluations at second order in Sec. III (details of the evaluations are given in the Appendix). Finally in Sec. IV the results and conclusions are presented.

II. THE SCALING LAW FOR INCLUSIVE CROSS SECTION

Let us call, following the notation of Abarbanel *et al.*,[12] $I_{s_1s_2s_3}$ the contribution to the partial-wave amplitude $F(J_1,J_2,J_3,t)$ coming from the sum of all Reggeon diagrams with s_1 Pomerons attached to the particle where the energy E_1 enters the diagram and with s_2 and s_3 Pomerons connected to the particles in which the respective energies E_2 and E_3 leave the diagram (see Fig. 1). The renormalized contribution $I_{R,s_1s_2s_3}$ satisfies the renormalization-group equation

$$\left[E_N\frac{\partial}{\partial E_N}+\beta\frac{\partial}{\partial g}+\zeta\frac{\partial}{\partial\alpha'}+(s_1+s_2+s_3)\frac{\gamma}{2}-\gamma_{s_1}-\gamma_{s_2}-\gamma_{s_3}\right]\times I_{R,s_1s_2s_3}=0, \quad (1)$$

where

$$\beta=E_N\frac{\partial g}{\partial E_N},\quad \zeta=E_N\frac{\partial\alpha'}{\partial E_N},$$
$$\gamma=E_N\frac{\partial\ln Z_3}{\partial E_N},\quad \gamma_s=E_N\frac{\partial\ln Z_{s+3}}{\partial E_N}. \quad (2)$$

The renormalized and bare quantities are related by

$$r=Z_3^{3/2}Z_1^{-1}r_0,\quad N_s=Z_3^{3/2}Z_{s+3}^{-1}N_{0,s},\quad \alpha'=Z_2^{-1}\alpha_0' \quad (3)$$

for the triple-Pomeron coupling, the 2-particles-s-Pomerons vertex, and Pomeron slope, respectively. We have also used the dimensionless triple-Pomeron coupling

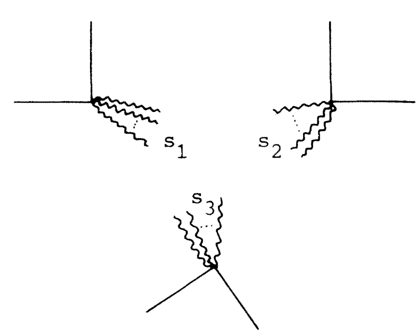

FIG. 1. Contribution to the triple-Regge amplitudes with s_1, s_2, and s_3 Pomerons attached to the external lines.

$$g=\frac{\gamma}{(\alpha')^{D/4}}(E_N)^{D/4-1}, \tag{4}$$

where D is the space dimension and E_N is the arbitrary energy chosen in the renormalization procedure.

Denoting the renormalized proper vertex for m incoming and n outgoing Reggeons by $\Gamma_R^{(n,m)}$, we choose the following normalization conditions to fix the Z's

$$Z_3^{-1}=\left.\frac{\partial i\Gamma^{(1,1)}}{\partial E}\right|_{E=-E_N,\vec{k}^2=0},$$

$$Z_2^{-1}=(-\alpha_0')^{-1}Z_3\left.\frac{\partial i\Gamma^{(1,1)}}{\partial \vec{k}^2}\right|_{E=-E_N,\vec{k}^2=0}, \tag{5}$$

$$Z_1^{-1}=(2\pi)^{(D+1)/2}(\gamma_0)^{-1}\Gamma^{(2,1)}\big|_{E_1=2E_2=2E_3=-E_N,\vec{k}_i^2=0}.$$

The other renormalization constant Z_{s+3} is determined from the normalization condition for the proper renormalized vertex of s Pomerons and the external particles $\Lambda_{R,s}$:

$$Z_{s+3}^{-1}=\left(\frac{(2\pi)^{(D+1)/2}}{i}\right)^{s-1}(N_{0,s})^{-1}\Lambda_s\big|_{E_i=-E_N,\vec{q}_i=0}. \tag{6}$$

The standard resolution of the renormalization-group equation[1] gives the following scaling law for the infrared behavior of I:

$$I_{R,s_1s_2s_3}(\lambda E_i,\lambda^{z(g_1)}\vec{q}^2)=\lambda^Q I_{R,s_1s_2s_3}(E_i,\vec{q}^2), \tag{7}$$

where

$$z(g_1)=1-\frac{\zeta(\alpha',g_1)}{\alpha'}, \tag{8}$$

g_1 is the infrared zero of $\beta(g)$, and Q is given by

$$Q=-2+z(g_1)\frac{D}{4}(s_1+s_2+s_3-4)+\frac{\gamma(g_1)}{2}(s_1+s_2+s_3)$$
$$-\gamma_{s_1}(g_1)-\gamma_{s_2}(g_1)-\gamma_{s_3}(g_1). \tag{9}$$

Such a law in terms of inclusive cross section in the triple-Regge region (M^2/m_0^2 and s/M^2 large) reads

$$\frac{d\sigma}{dt\,d\ln M^2}\left(\lambda\ln\frac{s}{M^2},\ \lambda\ln\frac{M^2}{m_0^2},\ t\right)$$
$$=\lambda^{-3-Q}\frac{d\sigma}{dt\,d\ln M^2}\left(\ln\frac{s}{M^2},\ \ln\frac{M^2}{m_0^2},\ \lambda^{z(g_1)}t\right), \tag{10}$$

which shows that the cross section shrinks and that the dominant contribution at high energy is the one corresponding to the lowest value of Q.

The ϵ-expansion computation of Q to first order in ϵ has been done in Ref. 12. One obtains ($\epsilon=4-D$)

$$\frac{g_1^2}{(2\pi)^{D/2}}=\frac{\epsilon}{6},\quad \gamma(g_1)=\frac{-\epsilon}{12},$$
$$z(g_1)=1+\frac{\epsilon}{24},\quad \gamma_s=\frac{-\epsilon}{6}s(s-1), \tag{11}$$

from where

$$Q=-3+\frac{\epsilon}{12}+\left(1-\frac{\epsilon}{4}\right)(s_1+s_2+s_3-3)$$
$$+\frac{\epsilon}{6}[s_1(s_1-1)+s_2(s_2-1)+s_3(s_3-1)], \tag{12}$$

which shows that the leading contribution at this order comes from diagrams with $s_1=s_2=s_3=1$.

III. SECOND-ORDER EVALUATION

In order to compute Q to second order in ϵ, we must calculate to the same order $\zeta(g_1)$, $\gamma(g_1)$, and $\gamma_s(g_1)$ and indirectly $\beta(g_1)$. Fortunately γ, β, ζ, and g_1 have been evaluated by Baker and Bronzan and Dash, and we have to compute γ_s. The result of these authors for γ, β, ζ, is ($\gamma_{\rm EM}$ is the Euler-Mascheroni constant)

$$\beta=-\tfrac{1}{4}\epsilon g+\frac{g^3}{(8\pi)^2}\left[\tfrac{3}{2}+\epsilon\left(\tfrac{15}{16}+\tfrac{5}{4}\ln 2+\tfrac{3}{4}\ln\pi-\tfrac{3}{4}\gamma_{\rm EM}\right)\right]$$
$$-\frac{g^5}{(8\pi)^4}\left(\tfrac{157}{32}+\tfrac{53}{16}\ln\tfrac{4}{3}\right), \tag{13}$$

$$\gamma=\left[-\frac{1}{2}+\frac{\epsilon}{4}(-3\ln 2-\ln\pi+\gamma_{\rm EM})\right]\frac{g^2}{(8\pi)^2}$$
$$+\left(-\tfrac{5}{2}\ln 2+\tfrac{9}{4}\ln 3-\tfrac{5}{8}\right)\frac{g^4}{(8\pi)^4}, \tag{14}$$

$$\frac{\zeta}{\alpha'}=\frac{1}{2}\left[-\frac{1}{2}+\frac{\epsilon}{4}(-3\ln 2-\ln\pi+\gamma_{\rm EM})\right]\frac{g^2}{(8\pi)^2}$$
$$+\left(\tfrac{7}{8}\ln 2+\tfrac{1}{16}\ln 3-\tfrac{17}{32}\right)\frac{g^4}{(8\pi)^4}, \tag{15}$$

and the calculation of the zero of the equation $\beta(g_1)=0$ to second order in ϵ gives

$$\frac{g_1^2}{(8\pi)^2}=\frac{\epsilon}{6}+\frac{\epsilon^2}{12}\left[\gamma_{\rm EM}-\ln\pi+\tfrac{1}{144}(-28\ln 2-106\ln 3-23)\right], \tag{16}$$

which substituted into (14) and (15) gives

$$-\gamma(g_1)=\frac{\epsilon}{12}+\left(\frac{\epsilon}{12}\right)^2\left(\tfrac{161}{12}\ln\tfrac{4}{3}+\tfrac{37}{24}\right), \tag{17}$$

$$z(g_1)=1-\frac{\zeta}{\alpha'}=1+\frac{\epsilon}{24}+\left(\frac{\epsilon}{12}\right)^2\left(\tfrac{59}{24}\ln\tfrac{4}{3}+\tfrac{79}{48}\right). \tag{18}$$

In order to compute the ϵ^2 contribution to γ_s we must compute all the diagrams with s Pomerons coupled to the external particles up to order $\epsilon^0 g^2$ and $\epsilon^{-1}g^4$. All the possible diagrams are illustrated

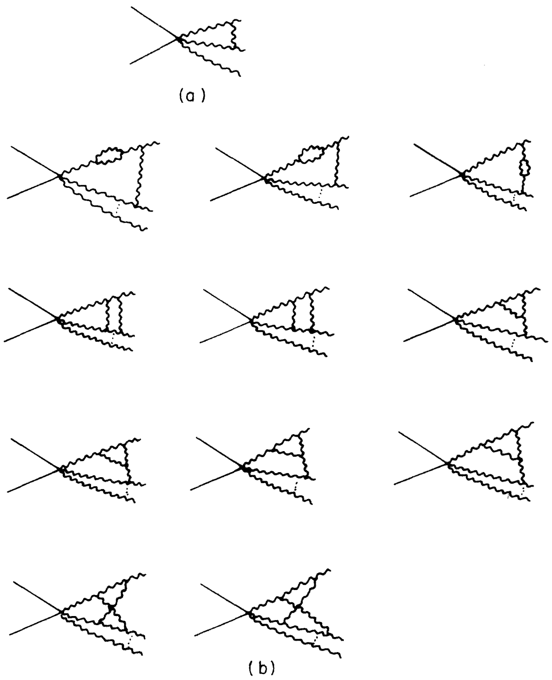

FIG. 2. (a) Diagram of order g^2 among two Pomeron lines. (b) Diagrams of order g^4 among two Pomeron lines.

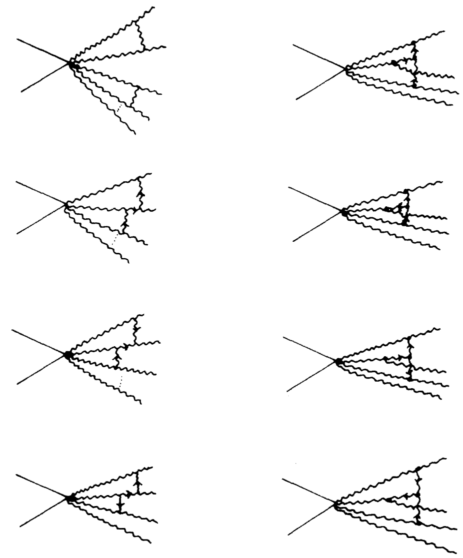

FIG. 3. Diagrams of order g^4 among three Pomeron lines.

in Fig. 2 and Fig. 3. Diagrams like the one of Fig. 2(a) have been calculated to order $\epsilon^{-1}g^2$ (Ref. 12), and we must calculate it to order $\epsilon^0 g^2$, which is done in a straightforward way. Diagrams similar to those of Fig. 2(b) have been evaluated before by Bronzan and Dash so that we have to compute only those of Fig. 3.

The sum of the contributions of the diagrams of Fig. 2(b) to the coupling function Λ_s is

$$\Lambda_s = \frac{i^{s-1}N_{0,s}}{(2\pi)^{(D+1)(s-1)/2}} \, \frac{s(s-1)}{2} \, \frac{g_0^{\,4}}{(8\pi)^4} \times\left[\frac{20}{\epsilon^2} + \frac{1}{\epsilon}(26 - 20\gamma_{\rm EM} + 20\ln\pi + 40\ln 2) + O(\epsilon^0)\right]. \tag{19}$$

The diagrams of Fig. 3 are in many cases quite difficult to compute; the details of their evaluation are given in the Appendix. Their contribution is the following:

$$\Lambda_s^a = K\frac{(s-3)}{2}\,\frac{1}{2^8\pi^4}\left(\frac{2}{\epsilon} + 1 - \gamma_{\rm EM} + \ln\pi\right)^2, \tag{20}$$

$$\Lambda_s^b = K\frac{\Gamma(\epsilon)}{(2\pi)^4}\,\frac{1+\epsilon\ln(2\pi)}{(1-\epsilon)(2-\epsilon)}\left(\frac{1}{2\epsilon} + \frac{10}{3} + \frac{1}{4}\ln 3 - \frac{3}{4}\ln 5 + \frac{9}{4}\ln 2\right), \tag{21}$$

$$\Lambda_s^c = -K\frac{\Gamma(\epsilon)}{(2\pi)^4}\,\frac{1+\epsilon\ln(2\pi)}{(1-\epsilon)(2-\epsilon)}\left(-\frac{1}{2\epsilon} - \frac{65}{18} - \frac{25}{4}\ln 2 + \frac{59}{8}\ln 3 - \frac{11}{8}\ln 5\right), \tag{22}$$

$$\Lambda_s^d = K\frac{\Gamma(\epsilon)}{(2\pi)^4}\,\frac{1+\epsilon\ln(2\pi)}{4}\left[-\frac{1}{2}\ln 2 + \frac{1}{2}\ln 3 + \frac{\sqrt{3}}{6}\ln\left(\frac{2-\sqrt{3}}{2+\sqrt{3}}\right) - \frac{\sqrt{3}}{12}\ln\left(\frac{4+\sqrt{3}}{4-\sqrt{3}}\right)\right], \tag{23}$$

$$\Lambda_s^e = K\frac{\Gamma(\epsilon)}{(2\pi)^4}\,\frac{1+\epsilon\ln(2\pi)}{(1-\epsilon)(2-\epsilon)}\left(\frac{1}{4\epsilon} - \frac{1}{8}\ln 3 + \frac{1}{2}\ln 2 - \frac{1}{12}\right), \tag{24}$$

$$\Lambda_s^f = K\frac{\Gamma(\epsilon)}{(2\pi)^4}[1+\epsilon\ln(2\pi)]\left[-\frac{3}{4\epsilon} - \frac{7}{8} + \frac{11}{8}\ln 2 - \frac{3}{2}\ln 3 - \frac{\pi}{2\sqrt{3}} + \frac{\sqrt{3}}{4}\ln\left(\frac{2+\sqrt{3}}{2-\sqrt{3}}\right) + \frac{\sqrt{5}}{8}\ln\left(\frac{3-\sqrt{5}}{2}\right)\right], \tag{25}$$

$$\Lambda_s^g = K\frac{\Gamma(\epsilon)}{(2\pi)^4}\,\frac{1+\epsilon\ln(2\pi)}{8}\left[\frac{1}{\epsilon} + \frac{3}{2}\ln 3 + \frac{3}{4}\ln 2 + \sqrt{3}\ln(2-\sqrt{3}) - \frac{\sqrt{3}}{4}\ln(2+\sqrt{3}) + \frac{\sqrt{5}}{2}\ln\left(\frac{3+\sqrt{5}}{2}\right)\right], \tag{26}$$

$$\Lambda_s^h = K\frac{\Gamma(\epsilon)}{(2\pi)^4}\,\frac{1+\epsilon\ln(2\pi)}{1-\epsilon}\,\frac{1}{16}\ln 3, \tag{27}$$

where

$$K \equiv (-2\alpha_0')^{-D} E_N^{D-4} s(s-1)(s-2) r_0{}^4 N_{0,s} \frac{i^{s-1}}{(2\pi)^{(D+1)(s-1)/2}} \,. \tag{28}$$

In conclusion, the contributions to Λ_s of order $\epsilon^0 g^2$ and $\epsilon^{-1} g^4$ are

$$\begin{aligned}
\Lambda_s = \frac{N_{0,s} i^{s-1}}{(2\pi)^{(D+1)(s-1)/2}} \\
\times \Bigg(1 - s(s-1) \frac{g_0{}^2}{(2\pi)^5} 2\pi^3 (2)^{\epsilon/2-2} \left(\frac{2}{\epsilon} + 1 - \gamma_{\rm EM} + \ln\pi \right) \\
+ g_0{}^4 s(s-1)(s-2)(2)^{\epsilon-4} \Bigg\{ \frac{s-3}{2^9\pi^4} \left(\frac{2}{\epsilon} + 1 - \gamma_{\rm EM} + \ln\pi \right)^2 + \frac{\Gamma(\epsilon)}{(2\pi)^4} \frac{1+\epsilon\ln(2\pi)}{(1-\epsilon)(2-\epsilon)} \left(\frac{5}{4\epsilon} + \frac{535}{144} + \frac{33}{4}\ln 2 - \frac{29}{4}\ln 3 - \frac{9}{8}\ln 5 \right) \\
+ \frac{\Gamma(\epsilon)}{(2\pi)^4} \frac{1+\epsilon\ln(2\pi)}{4} \Bigg[-\frac{5}{2\epsilon} - \frac{7}{2} + \frac{43}{8}\ln 2 - \frac{19}{4}\ln 3 - \frac{2\pi}{\sqrt{3}} - \frac{5\sqrt{3}}{6} \ln\left(\frac{2-\sqrt{3}}{2+\sqrt{3}} \right) \\
- \frac{\sqrt{3}}{12} \ln\left(\frac{4+\sqrt{3}}{4-\sqrt{3}} \right) + \frac{\sqrt{5}}{4} \ln\left(\frac{3-\sqrt{5}}{2} \right) + \frac{\sqrt{3}}{2} \ln(2-\sqrt{3}) \\
- \frac{\sqrt{3}}{8} \ln(2+\sqrt{3}) \Bigg] + \frac{\Gamma(\epsilon)}{(2\pi)^4} \frac{1+\epsilon\ln(2\pi)}{1-\epsilon} \frac{1}{16} \ln 3 \Bigg\} \\
+ \frac{g_0{}^4}{(8\pi)^4} \frac{s(s-1)}{2} \left[\frac{20}{\epsilon^2} + \frac{1}{\epsilon}(26 - 20\gamma_{\rm EM} + 20\ln\pi + 40\ln 2) \right] \Bigg) .
\end{aligned} \tag{29}$$

From here we have the following expression for $Z_{s+3}{}^{-1}$:

$$Z_{s+3}{}^{-1} = 1 + \left(\frac{g_0}{8\pi} \right)^2 \left(\frac{C_1}{\epsilon} + C_2 \right) + \left(\frac{g_0}{8\pi} \right)^4 \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon} \right), \tag{30}$$

where

$$\begin{aligned}
C_1 &= -2s(s-1), \quad C_2 = -s(s-1)[1 - \gamma_{\rm EM} + \ln(2\pi)], \\
A &= 4s(s-1)(s-2)(s-3)/2 + 10s(s-1), \\
B &= s(s-1) \Bigg\{ (s-2) \Bigg[4\frac{s-3}{2}[1 - \gamma_{\rm EM} + \ln(2\pi)] + \frac{553}{18} + \frac{175}{2}\ln 2 - 76\ln 3 - 9\ln 5 - \frac{10\sqrt{3}}{3} \ln\left(\frac{2-\sqrt{3}}{2+\sqrt{3}} \right) + 2\sqrt{3}\ln(2-\sqrt{3}) \\
&\quad - \frac{\sqrt{3}}{3} \ln\left(\frac{4+\sqrt{3}}{4-\sqrt{3}} \right) + \sqrt{5} \ln\left(\frac{3-\sqrt{5}}{2} \right) - \frac{8\pi}{\sqrt{3}} - \frac{\sqrt{3}}{2}\ln(2+\sqrt{3}) \Bigg] + 13 - 10\gamma_{\rm EM} + 10\ln\pi + 20\ln 2 \Bigg\},
\end{aligned} \tag{31}$$

and from (2) we can compute γ_s, which turns out to be

$$\gamma_s = \frac{1}{2} \left(\frac{g_0}{8\pi} \right)^2 (C_1 + \epsilon C_2) + \left(\frac{g_0}{8\pi} \right)^4 \left[\frac{1}{\epsilon} \left(A - \frac{C_1{}^2}{2} \right) + B - C_1 C_2 \right]. \tag{32}$$

From the power expansion of the bare coupling constant g_0 in terms of the renormalized one

$$\begin{aligned}
g_0 &= g - \frac{w}{(8\pi)^2} \frac{g^3}{\epsilon} , \\
w &\equiv -3 + \epsilon(-\tfrac{15}{8} - \tfrac{5}{2}\ln 2 - \tfrac{3}{2}\ln\pi + \tfrac{3}{2}\gamma_{\rm EM}),
\end{aligned} \tag{33}$$

we have finally

$$\gamma_s = \frac{1}{2} \left(\frac{g}{8\pi} \right)^2 (C_1 + \epsilon C_2) + \left(\frac{g}{8\pi} \right)^4 \left[\frac{1}{\epsilon} \left(A - \frac{C_1{}^2}{2} - wC_1 \right) + B - C_1 C_2 - wC_2 \right]. \tag{34}$$

Now we insert in this expression the value $g_1{}^2$ given in Eq. (16) to obtain γ_s to second order in ϵ (note the cancellation of the Euler constant and $\ln\pi$):

$$\gamma_s=\frac{\epsilon}{12}s(s-1)\left[-2+\frac{4}{3}+\frac{4}{3}\,\frac{(s-2)(s-3)}{2}-\frac{2}{3}s(s-1)\right]$$

$$+\frac{\epsilon^2}{12}s(s-1)\left\{\frac{445}{432}+\frac{317}{108}\ln 2-\frac{53}{216}\ln 3+\frac{s-2}{3}\left[4\frac{s-3}{2}\left(\frac{121}{144}+\frac{29}{36}\ln 2-\frac{106}{144}\ln 3\right)+\frac{503}{9}-\frac{8\pi\sqrt{3}}{3}+\frac{187}{2}\ln 2-76\ln 3\right.\right.$$

$$+5\ln 5-\frac{4\sqrt{3}}{3}\ln(2-\sqrt{3})+\frac{17\sqrt{3}}{6}\ln(2+\sqrt{3})-\frac{\sqrt{3}}{3}\ln\left(\frac{4+\sqrt{3}}{4-\sqrt{3}}\right)$$

$$\left.\left.+\sqrt{5}\ln\left(\frac{3-\sqrt{5}}{2}\right)\right]-\frac{2}{3}s(s-1)\left(\frac{121}{144}+\frac{29}{36}\ln 2-\frac{106}{144}\ln 3\right)\right\} \qquad (35)$$

$$=\frac{\epsilon}{12}s(s-1)[-2+\tfrac{4}{3}+\underset{3}{\tfrac{2}{3}}(6-4s)]+\frac{\epsilon^2}{12}s(s-1)(14.66-6.32\,s). \qquad (36)$$

Introducing now our value of γ_s and the values (17) and (18) of $-\gamma$ and z in (9) we have the scaling exponent to second order in ϵ,

$$Q=-2+\left[1-\frac{5\epsilon}{24}+\left(\frac{\epsilon}{12}\right)^2 0.85\right](s_1+s_2+s_3-4)-\frac{1}{2}(s_1+s_2+s_3)\left[\frac{\epsilon}{12}+\left(\frac{\epsilon}{12}\right)^2 5.40\right]-\gamma_{s_1}-\gamma_{s_2}-\gamma_{s_3}, \qquad (37)$$

and for $\epsilon=2$ we find

$$Q=-4.42+0.45(s_1+s_2+s_3)-\gamma_{s_1}-\gamma_{s_2}-\gamma_{s_3}. \qquad (38)$$

IV. RESULTS AND CONCLUSIONS

First of all we can see in formula (35) that the first-order value of γ_s is modified when one includes the g^4 diagrams. Nevertheless the value of γ_s when $s=2$ at first order is the same as the one obtained by Abarbanel. The difference comes from the inclusion of diagrams of Fig. 3 which have interactions among three Pomeron lines, interactions which cannot exist at order g^2. In this way our result for γ_s is correct for $s \leq 3$, but for $s>3$ it will be modified (at order ϵ and ϵ^2) when higher-order interactions among more than three Pomeron lines will be taken into account.

In Table I we present our results for γ_s and Q at first and second order in ϵ. We notice that the first-order results are largely modified by the second-order contribution even at $s=2$ where only the diagrams of Fig. 2(b), which have been computed by other authors, have a nonzero contribution.

What is more important is that at second order in ϵ it is not longer true that the diagram with one Pomeron attached to each particle line will be asymptotically dominant. We can see in Table I that the diagram with two Pomerons attached to each external line is slightly dominant (and this result will not be modified by inclusion of diagrams with more than three interacting Pomeron lines). The ϵ^2 Q value when any s is larger than 1 is very different from the corresponding ϵ value, which indicates that in this case we would have to go at higher ϵ order to obtain any reliable estimation. On the contrary the ϵ^2 estimation of the Q value, for $s_1=1$, $s_2=1$, $s_3=1$, does not appreciably differ from the ϵ^1 evaluation, indicating a reliable estimation.

It is also clear that diagrams with three or more lines will be negligible (at second order in ϵ) at asymptotic energies.

TABLE I. Our results for γ_s and Q at first and second order in ϵ.

	γ_s (order ϵ)	γ_s (order ϵ^2)
$s=1$	0	0
$s=2$	−0.66	−0.68
$s=3$	−4.66	−13.26
$s=4$	−14.66	−47.14
$s=5$	−33.33	−146.0
s_1, s_2, s_3	Q (order ϵ)	Q (order ϵ^2)
1 1 1	−2.83	−3.07
2 1 1	−1.67	−3.30
2 2 1	−0.51	−3.53
2 2 2	+0.66	−3.76
3 1 1	+2.83	+11.09
3 2 1	+3.99	+10.86

Note added in proof. The modification of the ϵ^1 order evaluation when computing the higher-order terms is due to the fact that the vacuum expectation value of more than three Pomeron fields at the same point needs some additive renormalization [E. Brézin *et al.*, in *Phase Transitions and Critical Phenomena*, Vol. 6, edited by C. Domb and M. S. Green (Academic, New York, 1976)]. If we make only the above multiplicative renormalization some infinities appear in the theory if $s>3$ which must be removed by subtractions. [In our paper use of dimensional regularization is the method responsible for the existence of the A/ϵ^2 term in formula (30) which in turn is responsible for the modification of the ϵ^1 term when making the ϵ expansion.] Such kinds of behavior appear when the external particles are introduced in Reggeon field theory connected to more than three Pomerons, and at this moment it is not clear to us how it modifies the results of our paper (for $s>3$) and of other papers which analyze the asymptotic behavior of the two-body cross section. We are grateful to J. Cardy and M. Moshe for pointing out to us this point and for many discussions about it.

APPENDIX

The calculation of the contribution to Λ_s coming from the diagrams shown in Fig. 3 are computed in a straightforward way using the Feynman rules given in Refs. 11 and 12,

After the integration over the energies of the internal lines by means of the Cauchy theorem and the introduction of the Feynman parameters, one can integrate over the momenta of the internal lines in D dimensions, using the formula

$$\int d^D q\, d^D q' (a\bar{q}^2 + b\bar{q}'^2 + c\bar{q}\bar{q}' + d)^{-n} = (2\pi)^D d^{D-n}\Gamma(n-D)\frac{(4ab-c^2)^{-D/2}}{\Gamma(n)} . \tag{A1}$$

One is left then with the integration on the Feynman parameters.

The diagram 3(a) can be calculated easily obtaining the result given in expression (20), with K given in formula (28) of the main text.

The amplitude of diagram 3(b) is

$$\Lambda_b = K\frac{\Gamma(\epsilon)}{(2\pi)^4}[1+\epsilon\ln(2\pi)]\int_0^1 dx \int_0^1 dy \int_0^1 dz (3-2x-y-z)^{-\epsilon}[-2(2x+2y-2)-(1-x-y)^2]^{\epsilon/2-2} . \tag{A2}$$

The integrals over x, y, and z can be done quite laboriously using the formulas (A27) given in the Appendix of Ref. 5 and we obtain formula (21).

Diagram 3(c) can be computed by an analogous calculation and one finds formula (22).

The contribution of diagram 3(d) can be calculated very easily when one realizes that the integral over the Feynman parameters is not singular at $\epsilon=0$. Then one finds formula (23). After a relatively easy computation one finds expression (24) for diagram 3(e).

Diagram 3(f) is more difficult to compute. After integration over the energies and momentum one is left with

$$\Lambda_f = K\Gamma(\epsilon)\frac{1+\epsilon\ln(2\pi)}{(2\pi)^4}\int_0^1 dx \int_0^1 dy \int_0^1 dz\{(2x+y+1)^{-\epsilon}[4(x+y+z)-(x+y+2z)^2]^{\epsilon/2-2} + (-1-x)^{-\epsilon}[4(1-x-y)z-(x+y-2z)^2]^{\epsilon/2-2}\} . \tag{A3}$$

The first integration can be done after introduction of the variables $(x,y,z)\to(v,y,z)$ with $v=x+y+2z$, integrating over y, and using the following general formula [which will be used also for the computation of diagram 3(g)]

$$\int_C^D du(\alpha u+\gamma)^{1-\epsilon}(\delta+\beta u)^{\epsilon/2-2} = \frac{1}{\beta(1-\epsilon/2)}\Big((\alpha C+\gamma)^{1-\epsilon}(\delta+\beta C)^{\epsilon/2-1} - (\alpha D+\gamma)^{1-\epsilon}(\delta+\beta D)^{\epsilon/2-1} + \frac{2\alpha(1-\epsilon)}{\epsilon\beta}\{(\alpha D+\gamma)^{-\epsilon}[-1+(\delta+\beta D)^{\epsilon/2}] - (\alpha C+\gamma)^{-\epsilon}[-1+(\delta+\beta C)^{\epsilon/2}]\}\Big) . \tag{A4}$$

In the second integral we change from (x,y,z) to $(x,u=x+y,z)$, and integrate over x. The integration over z of the function $(4z-4z^2-u^2)^{\epsilon/2-2}$ can be done using the general formula for increasing the exponent twice, and the remaining integral over $(4z - 4z^2 - u^2)^{\epsilon/2}$ has no pole at $\epsilon = 0$. Finally the integrations over u can be done in a straightforward way. In conclusion, the contribution of diagram 3(f) is given by expression (25).

The contribution of diagram (3g) can be calculated using the general formula (A9) and one gets the value given in (26). Finally, diagram 3(h) turns out to be given by expression (27).

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