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Corrections to the Goldberger-Treiman relation in a unified gauge-field model*

B. de Wit
Instituut-Lorentz, University of Leiden, Leiden, The Netherlands

R. Maciejko†
Institut für Theoretische Physik E, Rheinisch-Westfälische Technische Hochschule, Aachen, Germany

J. Smith
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794

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The corrections to the Goldberger-Treiman relation are examined in a unified gauge-field model. The strong interactions are governed by the local chiral $SU(2)\otimes SU(2)$ gauge group, whereas the weak and electromagnetic interactions are based on $SU(2)\otimes U(1)$ gauge invariance. We find that the Goldberger-Treiman formula is a zeroth-order relation, so that its corrections are finite. We estimate the corrections in the case that the pion is a pseudo-Goldstone boson. The result, of which the gauge independence is explicitly verified, is proportional to the weak and electromagnetic coupling constants.

I. INTRODUCTION

In a previous publication[1] (henceforth referred to as I; equation numbers referring to this paper will be prefixed with a I) we presented the calculation of the corrections to several zeroth-order symmetry relations[2] in a unified gauge-field model of strong, weak, and electromagnetic interactions. Zeroth-order relations have two important aspects, as was pointed out in particular by Weinberg.[3] The first aspect is a technical one. Corrections to zeroth-order relations originate by definition from closed-loop contributions, and in a renormalizable theory these corrections must be free from ultraviolet divergences.[4] But secondly, the existence of zeroth-order symmetry relations may provide explanations for the various approximate symmetries that are found in nature.

The examples of zeroth-order relations that have been discussed in the literature concern mainly relations among masses or mass differences,[5] and the corrections calculated in I were to relations of that type. In this paper we establish the existence of a far-more-complicated zeroth-order relation, namely, the Goldberger-Treiman formula.[6] As is well known, this formula is a relation among several different physical quantities: the proton and neutron masses, the pion decay constant, the pion-nucleon coupling constant, and the axial-vector–current coupling constant as measured, for instance, in neutron β decay.

Both the pion mass and the corrections to the Goldberger-Treiman formula are considered to be a measure of the amount of chiral-symmetry breaking in hadron physics.[7] However, in the case that the pion is a pseudo-Goldstone boson that picks up its mass from higher-order weak and/or electromagnetic corrections, as is the case in our model, the effects of chiral-symmetry breaking are expected to be of weak and electromagnetic origin. In I we have calculated the mass of the pseudo-Goldstone pion, which indeed originated from weak and electromagnetic corrections, and we found the value of 37 MeV. Hence the corrections from weak and electromagnetic interactions can give rise to considerable chiral-symmetry-breaking effects in the hadronic sector.

In this paper we analyze the effect of chiral-symmetry breaking as it is measured by the deviations from the Goldberger-Treiman relation. This will be done by examining each of the quantities that are involved in this formula. Therefore we will first express the Goldberger-Treiman relation directly in physical quantities, which should be gauge independent according to the general theory, and we will explicitly establish this gauge independence in our calculation. Because of the complexity of our theory, and the variety of quantities that are calculated here, the cancellations among the gauge-dependent parts are highly nontrivial, so that it is fair to consider this calculation as yet another confirmation of the general results of the quantum theory of gauge fields.

We should mention here that previous discussions of the corrections to the Goldberger-Treiman relation have assumed a different approach to ours. Sirlin[8] has determined the second-order electromagnetic corrections to the quantities occuring in the Goldberger-Treiman relation. He treated the hadronic matrix elements in the corrections using partial conservation of the axial-vector current and operator-product expansion techniques and found an 8% deviation from an exact Goldberger-Treiman relation for neutron β decay. This conclusion is similar to that of Pagels.[7] Several authors have tried to explain this 8% deviation as a strong-interaction correction. Pagels[9] and Pagels and Zapeda[10] used dispersion relations to calculate the corrections while Domingues[11] used Veneziano-type form factors, Strubbe[12] and Braathen[13] have used the renormalizable σ model as a basis for calculating the hadronic corrections. Generally these calculations yield corrections of the order of magnitude of 1–2%.

With the advent of renormalizable gauge-field models of the strong, weak, and electromagnetic interactions it is possible to discuss *both* the electromagnetic and weak corrections to the Goldberger-Treiman relation in a more fundamental way. As was indicated above, it is crucial in this approach that the Goldberger-Treiman relation is a natural one. Apart from establishing this fact for the model discussed in this paper, we check by explicit calculation that the final answer is indeed free of ultraviolet divergences. We are then able to discuss some numerical aspects of our result. We find that there are no purely hadronic corrections so that the answer is proportional to the weak and electromagnetic coupling constants. The actual magnitude could be enhanced by the presence of terms involving the large vector-boson mass and/or the small (or vanishing)masses of the pion and the photon. A detailed investigation shows that there is no such enhancement in the limit of infinite vector-boson mass. Hence we are left with terms which have the typical magnitude of electromagnetic corrections and are numerically small. The final answer, however, still depends on the photon and pion mass parameters in an infrared divergent fashion. These effects are not substantially different from those discussed in previous work on the subject, so we have refrained from making any specific estimate of their magnitude.

This paper is organized as follows. In Sec. II we introduce the model and derive the Goldberger-Treiman relation. The next section (III) contains an introduction to the calculation. Then in Sec. IV we calculate the corrections to the pion-nucleon coupling constant and the pion-decay constant and show them to be gauge-independent. The next step is to discuss the gauge-dependent corrections to the axial-vector coupling constant, which is done in Sec. V. Section VI contains an analysis of the gauge-independent result for the corrections to the Goldberger-Treiman relation. Finally, Sec. VII contains our conclusions. We have given some details of our calculations in various Appendixes.

II. A GAUGE-FIELD MODEL AND THE GOLDBERGER-TREIMAN FORMULA

In this section we will first briefly introduce the model that we will be dealing with in this paper. This model is a unified gauge-field model of the strong, weak, and electromagnetic interactions.[14] It was previously used in I in a calculation of the pion mass, as well as of several other corrections to zeroth-order symmetry relations. Subsequently we will discuss the Goldberger-Treiman formula and the expected corrections to this formula.

The gauge symmetry that governs the strong interactions in our model is the chiral SU(2)⊗SU(2) group,[15] and we have corresponding gauge fields denoted by U_μ^a and V_μ^a $(a=1,2,3)$. The underlying group of the weak and electromagnetic interactions is the SU(2)⊗U(1) gauge group, and the leptonic interactions of the corresponding gauge fields, W_μ^a and A_μ, coincide with those of the Weinberg-Salam model.[16] The heavy gauge fields W_μ mediate the weak interactions and the massless photon field A_μ mediates the electromagnetic interactions.

The model contains four complex doublet fields, K_X, K_Y, K_Σ, and K_Z, of which the first three are hadronic, and the last one is the Higgs-Kibble field of the Weinberg-Salam model. All these fields have components that acquire vacuum expectation values such that all but one of the gauge fields are massive. As in I we decompose the spinless fields according to

$$\begin{aligned} K_X &= \tfrac{1}{2}(2\sqrt{2}g^{-1}M_U + \sigma_U + \sigma_V + 2i\psi_U + 2i\psi_V)\,,\\ K_Y &= \tfrac{1}{2}(2\sqrt{2}g^{-1}M_U + \sigma_U - \sigma_V + 2i\psi_U - 2i\psi_V)\,,\\ K_\Sigma &= \tfrac{1}{2}\sqrt{2}(\sqrt{2}g^{-1}\epsilon M_U + \sigma_\Sigma + 2i\psi_\Sigma)\,,\\ K_Z &= \tfrac{1}{2}\sqrt{2}(2g_W^{-1}M_Z + \sigma_Z + 2i\psi_Z)\,, \end{aligned} \qquad (1)$$

where we use the notation $\psi \equiv \frac{1}{2}\psi^a\tau_a$. The parameters M_U, M_Z, and ϵ are introduced to give vacuum expectation values to σ_U, σ_Σ, and σ_Z in the tree approximation. The Lagrangian contains terms which are linear in σ_U, σ_V, σ_Σ, and σ_Z, so M_U, M_Z, and ϵ are determined by the requirement that the terms linear in σ_U, σ_Σ, and σ_Z vanish. A complete discussion of the model is given in I. However, the physical content of the model is as follows. We have two isotriplets of hadronic gauge fields, presumably the ρ and A_1 vector mesons, and a triplet of pions, which are defined by a linear combination of the fields ψ_U, ψ_V, ψ_Σ, and ψ_Z. The remaining (independent) combinations of these fields are unphysical. Furthermore, we have a doublet of nucleons and three isosinglet spinless fields, σ_U, σ_Σ, and σ_V, the first two scalar and the last one pseudoscalar. The particles without strong interactions are the three intermediate vector bosons, the photon, a spinless field σ_Z, and the leptons l and ν_l, where l denotes electron or muon.

For further discussion we specify the most general, gauge-invariant Lagrangian of dimension less than or equal to four, as a sum of five terms:

$$\mathcal{L}_{\rm INV}=\mathcal{L}_S+\mathcal{L}_{\rm WEM}+\mathcal{L}_\lambda+\mathcal{L}_{\rm p.v.}+\mathcal{L}_b\,. \tag{2}$$

The explicit form of these terms can be found in I. The first term, $\mathcal{L}_S$, contains only the hadronic fields, U_μ, V_μ, K_X, K_Y, K_Σ, and N, together with interactions among themselves as well as with the weak and electromagnetic gauge fields W_μ and A_μ. $\mathcal{L}_{\rm WEM}$ is the Lagrangian of the fields W_μ, A_μ, K_z, and the leptons l and ν_l.

The remaining three terms are interactions among the spinless fields. $\mathcal{L}_\lambda$ gives the parity-conserving interactions of K_z with the hadronic fields, and $\mathcal{L}_{\rm p.v.}$ represents the parity-violating interactions among the various spinless fields. Finally, $\mathcal{L}_b$ is an interaction that is linear in each of the spinless fields, K_X, K_Y, K_Σ, and K_Z. This term can consistently be chosen equal to zero without disturbing the renormalizability of the Lagrangian, since it is the only interaction that is linear in each of the spinless fields. In that case the pion will be a pseudo-Goldstone boson, which implies that the pion mass originates from (finite) closed-loop corrections. The calculation of these corrections was presented in I. As we have explained in the previous section, we will be dealing with pseudo-Goldstone bosons in this paper, so that henceforth we will choose the interaction $\mathcal{L}_b$ equal to zero.

We will now introduce the Goldberger-Treiman (GT) formula. First, let us define the various physical quantities that are involved in this relation.

We will define the pion decay constant F_π by writing the invariant amplitude for the decay process in the form

$$M(\pi^-\to l^-+\bar\nu_l)=iF_\pi Q_\mu \bar u_l\gamma_\mu(1+\gamma_5)u_\nu\,, \tag{3}$$

where Q is the momentum of the incoming pion. Notice that we have not extracted the Fermi constant G_F of the weak interactions, which is still included in F_π. Notice also that the pion is not able to decay in lowest order since its mass is zero because of its pseudo-Goldstone character. However, this is merely a technical difficulty.

A similar problem exists in the definition of the pion-nucleon coupling constant, $G_{pn\pi}$. This constant is in principle not a directly observable quantity, because of kinematical reasons. In practice it has to be derived from a theoretical analysis of pion-nucleon scattering. However, in our calculation $G_{pn\pi}$ is kinematically accessible, because of the zero pion mass in lowest order. Hence, $G_{pn\pi}$ is directly calculable as an on-mass-shell quantity, and is defined by the invariant amplitude

$$M(n\to p+\pi^-)=i\bar u_p(G_{pn\pi}\gamma_5+G_{pn\pi}^{\rm p.v.})u_n\,. \tag{4}$$

We have introduced here a second parameter $G_{pn\pi}^{\rm p.v.}$ which represents the parity-breaking contributions in the pion-nucleon vertex. Such contributions are induced by radiative corrections from the weak interactions. In this particular case the effect of these corrections must be finite because of a zeroth-order symmetry relation: There is no corresponding counterterm available in the Lagrangian, and since the Lagrangian is renormalizable the correction must be finite. We will come back to these parity-violating contributions in Sec. IV.

We will now define the axial-vector coupling constant G_A from the invariant neutron-proton-lepton-neutrino amplitude. We abbreviate this to the $(npl\nu)$ amplitude. Most of the contributions to this amplitude are of the current-current form. By this we mean that they can be described by the exchange of some vector particle between a nucleonic and a leptonic vertex. Those vertices are then considered as the matrix elements of some current, which can be decomposed in three different terms: $\bar u\gamma_\mu u$, $\bar u[\gamma_\mu,\gamma_\nu]Q_\nu u$, and $\bar u Q_\mu u$, with corresponding form factors which are functions of Q^2 only. Here Q is the momentum that is exchanged between nucleons and leptons. A factor $(1+\gamma_5)$ should be added if the spinors $\bar u$ and u correspond to leptons. For nucleons there may be an additional γ_5, depending on the parity classification of the current that is involved. With this decomposition we can define G_A from the form factor in the scattering amplitude that is given by

$$G_A(Q^2)(\bar u_p\gamma_\mu\gamma_5 u_n)[\bar u_l\gamma_\mu(1+\gamma_5)u_\nu] \tag{5}$$

and

$$G_A\equiv G_A(0)\,.$$

However, we stress that this definition is not unique unless the remaining terms in the amplitude are fully specified. In particular there are terms coming from two-particle-exchange diagrams (box diagrams) in the one-loop approximation that may contribute to G_A. In fact, we will find that the gauge-dependent terms coming from such graphs will cancel against certain gauge dependent terms from the one-particle exchange diagrams. Therefore we will retain both the current-current terms of the form (5) and the contributions to the $(npl\nu)$ amplitude that are not of the current-current form.

We now define the quantity

$$\Delta_{\rm GT}=1-\left|\frac{G_A(m_p+m_n)}{G_{pn\pi}F_\pi}\right|, \tag{6a}$$

where m_p and m_n are the proton and neutron masses, respectively. The Goldberger-Treiman formula[6] then gives the result

$$\Delta_{\rm GT}=0\,. \tag{6b}$$

We wish to point out that we have defined Δ_{GT} directly in terms of physical, on-mass-shell amplitudes, and not in terms of matrix elements of currents. Consequently Δ_{GT} is a physical quantity which should be independent of the gauge in which we will perform our calculations. Notice that both F_π and G_A still contain the Fermi coupling constant of the weak interactions.

Let us now further discuss the Goldberger-Treiman formula in closer connection with our model. This relation is usually discussed in the framework of current algebra. In that case G_A and F_π are the axial-vector coupling constants of the hadronic axial-vector current (the Fermi constant G_F is usually not included in the definition.) The axial-vector current is related to the (approximate) internal chiral symmetry of the strong interactions, and according to the current-algebra hypothesis[17] it is identical to the hadronic axial-vector current that is measured in the weak interactions. This makes G_A and F_π observable in neutron β decay and pion decay, respectively.

In models of the type that we are dealing with, the hadronic chiral current does not coincide with the currents that describe the weak interactions. For example, the chiral currents are not gauge invariant with respect to the chiral gauge group of the strong interactions, contrary to the weak and electromagnetic currents which are gauge invariant with respect to this group because of the invariance under the combined strong, weak, and electromagnetic gauge transformations.[18] The main current-algebra results, however, can be derived from the Slavnov-Taylor identities[19] for the weak and electromagnetic gauge group, without any reference to the internal symmetry properties of the strong interactions. This was shown in Ref. 20 and we will briefly present a derivation of the Goldberger-Treiman formula along the same lines. From this derivation we will then be able to discuss the corrections to this formula.

We start by considering the charged gauge field W_μ, and the complex spinless field K_Z with charged component ψ_Z. Both W_μ and ψ_Z have interactions with hadrons and leptons, and W_μ couples with a universal coupling constant g_W. One of the neutral components σ_Z of K_Z has a vacuum expectation value which was given in lowest order in Eq. (1): $\langle\sigma_Z\rangle_0=\sqrt{2}g_W^{-1}M_Z$. Since we will consider perturbation theory in g_W, while keeping M_Z constant, interactions with the field ψ_Z will always be proportional to g_W. So is the coupling constant between leptons and ψ_Z, G_l, given by $G_l=\frac{1}{2}g_W m_l M_Z^{-1}$, as was shown in Eq. (I8). We wish to point out that such a perturbation expansion makes sense for two reasons: theoretically, because it is related to an expansion in the number of closed loops, and experimentally, because g_W is small, whereas M_Z is supposed to be large. In I we have already made the factors g_W explicit in the coupling constants of the Lagrangian where interactions with K_Z were involved.

We now quantize the gauge field, by defining a gauge-fixing term $C_W=\rho\,\partial_\mu W_\mu$ and adding to the invariant Lagrangian $-\frac{1}{2}C_W^2$. The quantization of the hadronic fields will be ignored here. In this gauge we find the following propagators in lowest order g_W and e:

$$\begin{aligned} &i(2\pi)^4\langle 0|(W_\mu W_\nu)_+|0\rangle \\ &\quad=(Q^2+M_Z^2)^{-1}\{\delta_{\mu\nu}+Q_\mu Q_\nu Q^{-4}\rho^{-2}[M_Z^2+(1-\rho^2)Q^2]\}. \\ &i(2\pi)^4\langle 0|(W_\mu\psi_Z)_+|0\rangle=iM_Z\rho^{-2}Q^{-4}Q_\mu\,, \qquad (7)\\ &i(2\pi)^4\langle 0|(\psi_Z\psi_Z)_+|0\rangle=Q^{-4}(Q^2+M_Z^2\rho^{-2})\,. \end{aligned}$$

Our starting point is now the Slavnov-Taylor identity for the matrix element between proton and neutron states

$$\langle p|C_W|n\rangle=0\,.$$

By using the propagators (7) we can write this as

$$\rho^{-1}Q^{-2}(iQ_\mu\langle p|J_\mu^W|n\rangle+M_Z\langle p|J_Z^\psi|n\rangle)=O(g_W^2,e^2)\,, \qquad (8)$$

where J_μ^W and J_Z^ψ are defined as the sources of the fields W_μ and ψ_Z, where in both cases we have extracted a factor g_W explicitly. The momentum associated with these sources is denoted by Q.

The second term of this identity (8) can be shown to be at least of order g_W^2, since the only interactions with the field ψ_Z involve either the gauge fields W_μ, the lepton fields, or additional components of K_Z. It is here that the pseudo-Goldstone character of the pion enters in an implicit way since the interaction term $\mathcal{L}_b$, which was taken to be equal to zero so that the pion became a pseudo-Goldstone boson, would give rise to a ψ_Z interaction without bringing in these additional fields that interact weakly. Hence we have found an approximate conserved-current identity:

$$Q_\mu\langle p|J_\mu^W|n\rangle=O(g_W^2,e^2)\,. \qquad (9)$$

We will now consider the contribution from the pion to Eq. (9). Suppose that in the limit of vanishing weak and electromagnetic interactions we have a pion field that we denote by π_S. In lowest order of g_W we then write down the Slavnov-Taylor identity

$$-i(2\pi)^4\langle 0|(C_W\pi_S)_+|0\rangle=\frac{1}{\rho Q^2}g_W f_\pi(Q^2)\,, \qquad (10)$$

where $f_\pi(Q^2)$ is in principle defined by the transformation character of π_S under the weak gauge group. However, $f_\pi(Q^2)$ will also contain the higher-order corrections from the strong interactions; in lowest order f_π would simply be a constant.

Hence, Eq. (10) shows that there is the following contribution from π_S to the current matrix element $\langle p|J_\mu^W|n\rangle$:

$$Q_\mu Q^{-2} f_\pi(Q^2)\langle p|J_\pi|n\rangle + O(g_W^2, e^2),$$

where J_π denotes the hadronic source of the field π_S. This result is found under the same assumptions as for the derivation of Eq. (9). Note that the Q^{-2} term indicates that the pion is massless up to order g_W^2 and e^2. If we then take relation (9) in the limit of vanishing Q for the axial-vector part of the current, we find

$$g_A(0)(m_p + m_n) - f_\pi(0)G_{pn\pi} = O(g_W^2, e^2), \tag{11}$$

where g_A is the axial-vector form factor of the current, and $G_{pn\pi}$ is the pion-nucleon coupling constant that was defined previously. g_A can, of course, be directly related to the quantity G_A by using the propagators defined in Eq. (7). However, to relate f_π (0) to the pion decay constant F_π we must specify what constitutes the physical pion. In first order of g_W the physical pion field will no longer be given by π_S, but by a linear combination of π_S and ψ_Z, which can also be defined off the mass shell

$$\pi = \pi_S + g_W M_Z^{-1} f_\pi(Q^2)\psi_Z, \tag{12}$$

which is determined by the general conditions on physical field components

$$\langle 0|(C_W\pi)_+|0\rangle = 0.$$

Using this definition for the pion one can then calculate the physical pion decay, making use of the propagators (7) and the fact that the coupling of ψ_Z to π_S is of higher order in g_W and can be neglected. As a result of our definition of the pion (12) the final result will no longer depend on the parameter ρ, and we find that the pion decay constant F_π is simply proportional to $f_\pi(0)$.

An explicit calculation then shows that Eq. (11) leads to the Goldberger-Treiman formula

$$\Delta_{GT} = O(g_W^2, e^2), \tag{13}$$

with the perturbation expansion as was specified before. As we have pointed out previously, there was indeed no need to refer to the internal-symmetry structure of the strong interactions. Our proof depended only on gauge invariance, and some simple information on the interactions of the field ψ_Z, namely that the interaction term $\mathcal{L}_b$, which was linear in ψ_Z, was absent. There is, however, a somewhat implicit relationship with the symmetry properties of the strong interactions, since these are indeed chirally invariant when $\mathcal{L}_b$ is absent.

Let us now consider the result for Δ_{GT} for our model in tree approximation. We introduce

$$\Delta = 1 + \epsilon^2 + \tfrac{1}{2}\epsilon^2 g_W^2 g^{-2} M_U^2 M_Z^{-2}, \tag{14a}$$

which enters our calculation in the definition of the physical pion field, and in the results for the gauge field propagators at $Q^2=0$. Using the results of I, we find

$$\begin{aligned} F_\pi &= \tfrac{1}{4} g_W^2 g^{-1}\epsilon M_U M_Z^{-2}\Delta^{-1/2}, \\ G_A &= \tfrac{1}{8} g_W^2 M_Z^{-2}\Delta^{-1}, \\ m^{-1}G_{pn\pi} &= g\epsilon^{-1}M_U^{-1}\Delta^{-1/2}, \end{aligned} \tag{14b}$$

so that $\Delta_{GT} = 0$.

In obtaining this result we have taken the two independent coupling constants of the chiral SU(2) $\otimes$ SU(2) gauge group of the strong interactions both equal to g, and, moreover, we chose the vacuum expectation value of the field σ_V equal to zero. In doing so, we have ignored part of the hadronic parity violations of this model, as we will further discuss in the next section. However, even if the contributions from these terms had been taken into account, we would still find the result that Δ_{GT} was equal to zero. Hence, although we expect in principle to find corrections to this result of order g_W^2 and e^2, we find that the Goldberger-Treiman formula is an *exact* relation in the tree approximation for arbitrary values of the parameters of the model. This implies that the Goldberger-Treiman formula is a zeroth-order symmetry relation: because Δ_{GT} is zero in the tree approximation there are no counterterms available in the Lagrangian to cancel possible ultraviolet divergences in the higher-order corrections to this relation, so that Δ_{GT} must be finite due to the renormalizability of the model. Hence the result of this section is that Δ_{GT} is both finite and of order g_W^2 or e^2.

Since we have not found a sufficiently elegant explanation for this zeroth-order symmetry relation, we will refrain from further elucidation on the generality of this result. However, it is worth mentioning that we have not found similar zeroth-order relations that involve the hadronic vector-current coupling constant. The presence of such relations could have been expected on the basis of the conserved-vector-current hypothesis or lepton-hadron universality. This shows that current-algebra results are in general no indication for the existence of corresponding zeroth-order theorems, as has been suggested elsewhere.[21]

III. INTRODUCTION TO THE CALCULATION

In order to determine the corrections to the Goldberger-Treiman formula Δ_{GT} in the one-loop calculation we will separately calculate G_A, m_p, m_n, $G_{pn\pi}$, and F_π as they were defined in the previous section. In this calculation we will first establish the gauge independence for each of these quantities in a way that will be described later. This will provide us with a first nontrivial check on the correctness of our results. All our calculations will be performed using the n-dimensional regularization method.[22]

The perturbation scheme that is used for these calculations was extensively discussed in I. Here we will briefly sketch the main ingredients.

To quantize the gauge model we have added gauge-fixing terms $-\frac{1}{2}C^2$ to the invariant Lagrangian given in Eq. (2) for each of the ten generators of the gauge group. We have made the following choice for these gauge-fixing terms C which depend on a set of parameters ξ_U, ξ_V, ξ_W, and ξ_A:

$$C_U^a = \xi_U \partial_\mu U_\mu^a - \xi_U^{-1} M_U \psi_U^a ,$$

$$C_V^a = \xi_V \partial_\mu V_\mu^a - \xi_V^{-1} M_U (\psi_U^a + \epsilon \psi_\Sigma^a) ,$$

$$C_W^a = \xi_W \partial_\mu W_\mu^a - \xi_W^{-1} M_Z [\psi_Z^a - \tfrac{1}{2}\sqrt{2} g_W g^{-1} M_U M_Z^{-1} (\psi_U^a + \psi_V^a)] ,$$

$$C_A = \xi_A \partial_\mu A_\mu .$$

The precise definition of the gauge fields can be found in I. In addition, we have corresponding Faddeev-Popov ghost fields, and their contribution to the effective Lagrangian is also explicitly given there. We have not diagonalized our propagators in lowest order, so that propagators are generally given as matrices. The gauge-field propagators are decomposed as follows:

$$D_{\mu\nu}(Q^2) = D_T(Q^2)(\delta_{\mu\nu} - Q_\mu Q_\nu Q^{-2}) + D_L(Q^2) Q_\mu Q_\nu Q^{-2} , \tag{15}$$

where D_T and D_L are 3×3 or 4×4 matrices, depending on the charge of the corresponding gauge fields. The propagators for the gauge fields D_T, D_L and the propagators for the ψ fields, D_ψ, are all listed in Appendix B in I.

Given a choice of the gauge one can write down the generalized Ward-Takahashi, or Slavnov-Taylor identities of the local gauge symmetry. These identities were given in I. In the tree approximation they provide us with relations among the gauge-dependent propagators: D_L, D_ψ, and $D_{\rm FP}$. $D_{\rm FP}$ is the propagator of the Faddeev-Popov ghost fields. In order to establish the gauge-independence of our results, we will express all the gauge-dependent parts in terms of the propagators D_L, such that the only dependence on the gauge-fixing parameters ξ_U, ξ_V, ξ_W, and ξ_A will be contained implicitly in D_L. This can be generally achieved by making use of the Ward-Takahashi identities. We will then systematically show that all the D_L-dependent terms cancel in the final results.

To establish these cancellations we will use several manipulations. For instance, we will frequently make use of the fact that all Feynman integrals are Lorentz-covariant in order to project quantities of interest. Special care is given to performing the limits $Q\to 0$ or $Q^2 \to 0$. (Q denotes the pion momentum in F_π and $G_{pn\pi}$, and the momentum transfer in G_A). Sometimes this limit can not be taken without using the explicit form of the propagators, in which case we have usually refrained from further evaluations.

We will also frequently change integration variables in order to show that certain terms are equal to zero. This is of course allowed in the context of the n-dimensional regularization method. Furthermore, we will express all $\sigma\psi\psi$ coupling constants in terms of the inverse propagators of the fields σ. Such inverse coupling constants are then usually multiplied by the σ propagators of the internal lines, which simplifies the expression considerably. A similar technique will sometimes be used for the D_T propagators. Since D_T^{-1} has a rather simple form in terms of coupling constants and masses, we can write the matrix identity $D_T D_T^{-1} = 1$ as a set of simple relations among the complicated matrix elements of D_T. We have collected some of these identities and techniques in Appendix A.

One more aspect of the calculational scheme deserves special attention, namely the occurrence of the factor Δ, which was introduced in Eq. (14a). The denominators of the gauge-field propagators are complicated polynomials, which at zero momentum are exactly given by $M_Z^2\Delta$ (see Appendix B of I). This means that after proper normalization with respect to the lowest-order amplitudes, each diagram will exhibit its own characteristic power in Δ. For instance, the box-diagram corrections to G_A are linear in Δ, propagator corrections are of order Δ^{-1}, whereas vertex corrections are of zeroth order in Δ. This implies, as we will see in subsequent sections, that cancellations are to be obtained in steps. One first combines the terms of order Δ^{-1}, which should yield a factor of Δ in order to be added to the next terms of order Δ^0, etc.

As we have mentioned in Sec. II, the fields ψ are mostly unphysical. The only physical components are the pion fields, defined as

$$\pi = \Delta^{-1/2}(-\epsilon\psi_V + \psi_\Sigma - \tfrac{1}{2}\sqrt{2} g_W g^{-1} M_U M_Z^{-1} \psi_Z) . \tag{16}$$

Since the pion is a pseudo-Goldstone boson, its mass is zero in lowest order. In the one loop approximation the charged pions pick up a mass proportional to e^2. This mass was calculated in I and we recall the result of that calculation:

$$M_\pi^2 = -6i(2\pi)^{-4} e^2 g^2 g_W^{-2} M_Z^2 \Delta \int d^4q\, \tilde{D}_T^{AA}(q) D_T^{UV}(q) . \tag{17}$$

This answer was obtained by choosing the two coupling constants of the chiral $SU(2)\otimes SU(2)$ gauge group of the strong interactions equal to a common constant g, and by taking the vacuum expectation value of the field ψ_V equal to zero in the tree approximation. This means that we are ignoring certain parity-violation effects in the strong interactions. Since we are always calculating corrections to zeroth-order symmetry relations, which hold by definition for a continuous range of the parameters of the model, this has no theoretical consequences. Also, since these parity-violating effects are experimentally of the size of the Fermi constant G_F, they can safely be ignored when comparing our final results with experiments. The corrections to the Goldberger-Treiman formula will be determined under these same conditions. For that case the tree-approximation results for the quantities that enter the Goldberger-Treiman relation have already been listed in Eq. (14).

We will make use of several results of the calculations in I that were not always listed there. We have the same decomposition of the tadpole diagrams T_i^{tot}, i.e., the diagrams with an external line σ_i vanishing into the vacuum,

$$T_i^{\text{tot}} = D_\sigma^{-1}(0)^{ij}t_j + T_i, \quad i,j = U, V, \Sigma, Z \tag{18}$$

where T_i and t_i are given in Eq. (I13). Note that t_i contains all the gauge-dependent contributions of these graphs.

From the self-energy diagrams of the ψ fields and the fermions of the model, which were mostly calculated in I, we have extracted the wave-function renormalization constants for the pion and the fermions, as well as for the fermion masses. These results are presented in Appendix B.

Although the corrections to the Goldberger-Treiman relation are ultraviolet finite, as we have shown in the previous section, they are in general not free from infrared divergences. Those divergences are related to the fact that the photon is massless, and that our calculations are performed for the case that the pion is a pseudo-Goldstone boson, which is by definition massless in the tree approximation. However, apart from that, we have often encountered unphysical infrared singularities in our calculation that are connected with spurious poles in our propagator decompositions, or, for instance, with the longitudinal photon propagator, D_L^{AA}. (In view of such infrared problems, the decomposition that we have used for the gauge-field propagators is obviously not the most convenient one.) Of course, such unphysical singularities should cancel in the final result. Since most of our calculations consist of performing algebraic manipulations on the integrand of Feynman integrals, which are dimensionally regulated, the presence of these singularities is usually irrelevant.

More precisely, the S-matrix elements are calculated for n, the continuous number of dimensions, different from 4. Only after the physically relevant quantities are obtained do we consider the limit $n\to 4$. However, the gauge dependence in those quantities will be absent for *all* values of n. Therefore we can freely perform algebraic manipulations on the integrands, such as taking external momenta to zero, in separate terms that exhibit unphysical infrared divergences at $n=4$, by simply chosing suitable values for n. This is not allowed if the infrared problems are of physical origin, unless special care has been taken to absorb the divergences at $n=4$. Therefore, we have kept the pion mass μ^2 different from zero in those cases where we encountered physical infrared problems to enable an identification of the origin of the singularity.

For consistency this should then be done for both internal and external pions. This makes sense for physical reasons also, since the infrared divergences due to $\mu^2=0$ are the result of our approximation scheme. The pions do acquire masses in higher orders of perturbation theory after all.[23]

In general, one should be careful to take limits and (n-dimensional) integrations in the correct order. In the presence of infrared divergences, limits and integrals sometimes cannot be interchanged.

IV. THE $(np\pi^-)$ AND $(\pi^- l\nu)$ AMPLITUDES; DETERMINATION OF $G_{pn\pi}^{\text{p.v.}}, G_{pn\pi}, F_\pi$

In this section we will describe the evaluation of the various diagrams which contribute to the $np\pi^-$ and $\pi^- l\nu$ amplitudes. An important part of the discussion will be to establish the gauge-independence of these quantities. We will also discuss the parity-violating term in the pion-nucleon amplitude, $G_{pn\pi}^{\text{p.v.}}$.

Since the gauge-dependent terms are only relevant insofar as one must show that they cancel among themselves, we will confine ourselves to a systematic description of the way in which these cancellations occur. In order to obtain the cancellations we have made extensive use of the techniques that were described in the previous section and in Appendix A.

We will distinguish three different contributions to the $np\pi^-$ and $\pi^- l\nu$ amplitudes. First, we have the lowest-order diagram with the contribution from the external line corrections, as shown in diagrams A of Fig. 1. These contributions follow from the wave-function renormalization constants, as were calculated in Appendix B. However, for

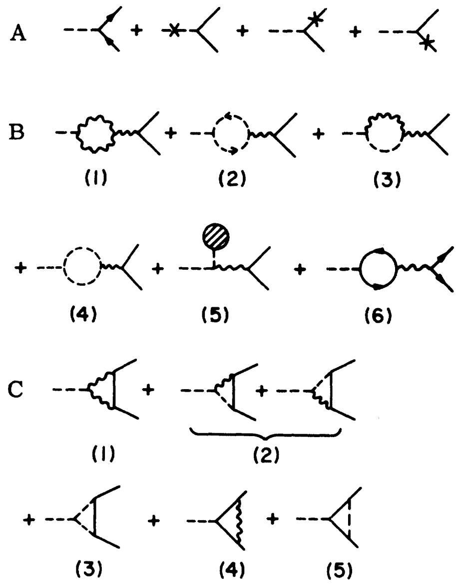

FIG. 1. The diagrams which contribute to the pion decay amplitude or to the pion-nucleon amplitude. In the first case the solid line represents the lepton spinors and in the second case the nucleon spinors. The wavy lines are gauge-field propagators and the dashed lines are spinless field propagators.

F_π there is an additional contribution, since the lowest-order diagram, which is proportional to $\bar{u}_l(1+\gamma_5)u_\nu$, must be written in the form (3) by applying the Dirac equation for the lepton and neutrino spinors. Through this the renormalized lepton mass M_l enters. Apart from that, the effect of the diagrams A of Fig. 1 is to multiply the lowest-order $np\pi^-$ vertex by $(Z_\pi Z_n Z_p)^{1/2}$, and the lowest-order value of F_π with $(Z_\pi Z_\nu Z_l)^{1/2} m_l M_l^{-1}$. Notice that the terms proportional to γ_5 in the lepton wave-function renormalization do not change the structure of the $\pi^- l\nu$ vertex, since those are proportional to $(1+\gamma_5)$. In the $np\pi^-$ vertex, however, they give rise to a new type of vertex, which is parity violating, i.e., it is proportional to $\bar{u}_p u_n$ instead of $\bar{u}_p\gamma_5 u_n$. This contribution to $G^{p.v.}_{pn\pi}$ follows straightforwardly from the nucleon wave-function renormalization constants, and is given by

$$\frac{G^{p.v.}_{pn\pi}}{G_{pn\pi}} = \frac{i}{(2\pi)^4}\frac{\sqrt{2}}{4}\, eg \int d^n q\, \tilde{D}^{VA}_T(q) \times [3-n-(n-2)p\cdot q\, m^{-2}] \times (q^2-2p\cdot q)^{-1} . \qquad (19)$$

where $p^2=-m^2$. The second contribution comes from the diagrams B of Fig. 1, where the pion interacts with the nucleons or leptons through a gauge-field line. The third contribution consists of the vertex corrections that are shown in C of Fig. 1.

In order to discuss schematically the cancellations among the various graphs we will use the following notation. We will denote the diagrams of B and C of Fig. 1 by $[B,i]$ and $[C,i]$, where i corresponds to the number of the diagram as given in Fig. 1. Moreover, we will denote by $T, L, \psi, \sigma, (\not{q}-im)^{-1}$ the transversal and longitudinal component of the gauge-field propagators, the propagators of the fields ψ and σ, and of the nucleons or leptons, respectively. Each (internal or external)momentum will be denoted generically by q_μ, and each quantity with the dimension of mass by M. The notation $(\not{q}-im)^{-1}$ will indicate the presence of a fermion propagator. If we then decompose the gauge-field propagators in their components L and T, and, moreover, use the substitutions for the propagators of the ψ fields as they were given in Appendix B of I, we find the following results:

$$[B,1]\to[q_\mu MLL]+[q_\mu MTL]+[q_\mu MTT] ,$$

$$[B,2]\to[q_\mu q^{-2}M^3LL]+[q_\mu q^{-2}ML] ,$$

$$[B,3]\to[q_\mu q^{-2}M^3LL]+[q_\mu q^{-2}ML]+[q_\mu q^{-2}M^3TL] +[q_\mu q^{-2}MT]+[q_\mu M\sigma L]+[q_\mu M\sigma T] ,$$

$$[B,4]\to[q_\mu q^{-2}M^3\sigma L]+[q_\mu q^{-2}M\sigma] ,$$

$$[B,5]\to[q_\mu q^{-2}ML]+[q_\mu q^{-2}M^{-1}]+[q_\mu(D_\sigma(0)T)] ,$$

$$[B,6]\to[(\not{q}-im)^{-1}(\not{q}-im)^{-1}] ,$$

$$[C,1]\to[MLL(\not{q}-im)^{-1}]+[MLL(\not{q}-im)^{-1}] +[MTT(\not{q}-im)^{-1}] ,$$

$$[C,2]\to[\not{q}(\not{q}-im)^{-1}L\sigma]+[q^{-2}\not{q}(\not{q}-im)^{-1}M^2LL] +[q^{-2}\not{q}(\not{q}-im)^{-1}L]+[\not{q}(\not{q}-im)^{-1}T\sigma] +[q^{-2}\not{q}(\not{q}-im)^{-1}M^2TL]+[q^{-2}\not{q}(\not{q}-im)^{-1}T] ,$$

$$[C,3]\to[q^{-2}(\not{q}-im)^{-1}M^3\sigma L]+[q^{-2}(\not{q}-im)^{-1}M\sigma] ,$$

$$[C,4]\to[(\not{q}-im)^{-1}T(\not{q}-im)^{-1}] +[(\not{q}-im)^{-1}L(\not{q}-im)^{-1}] ,$$

$$[C,5]\to[(\not{q}-im)^{-1}\sigma(\not{q}-im)^{-1}] +[(\not{q}-im)^{-1}q^{-2}M^2L(\not{q}-im)^{-1}] +[(\not{q}-im)^{-1}q^{-2}(\not{q}-im)^{-1}] .$$

In this notation we have not made explicit a factor $(M_Z{}^2\Delta)^{-1}$ in the diagrams B of Fig. 1, which originates from the gauge-field propagator at zero momentum squared, as well as the contraction of the vector index μ with that of the fermion vertex. It is obvious, as was explained in the previous section, that in order to have cancellations between the diagrams of B and C of Fig. 1 it is crucial that the terms from the diagrams B be proportional to a factor Δ, to cancel the effect of the gauge-field propagator, and that the terms from the diagrams C produce a factor to remove the extra internal-fermion propagator. Apart from the straightforward decomposition of gauge-field and ψ-field propagators, we used the expressions for the tadpole diagrams that were obtained in I [see Eq. (17)] in diagrams of type [B, 5]. In the diagrams of $[B,2]$ we first substituted the product of two Faddeev-Popov propagators by the product of a longitudinal gauge-field propagator and a ψ-field propagator, so that these diagrams were then free of explicit dependence on the gauge parameters ξ. These substitutions follow straightforwardly from the Ward-Takahashi identities which were given in Eq. (9) of I.

Obviously the schematic decomposition shows very clearly which diagrams are manifestly gauge-independent. The aim of the subsequent discussion is now to show how the gauge-dependent terms, i.e., the terms that contain the longitudinal components, cancel each other. The cancellations are established with the pion momentum Q satisfying $Q^2=0$, except when infrared divergences are encountered, and with the leptons or nucleons on the mass shell. As was mentioned previously, the limit $Q^2\to 0$ may give rise to difficulties, and one needs complicated algebraic manipulations to arrive at the final result. We now give a brief discussion of the various cancellations.

(a) The terms proportional to LL. The contribution from $[B,1]$ is absent. The terms from the diagrams $[B,2]$ $[B,3]$ can be summed in the $Q^2=0$ limit, and are indeed proportional to Δ. This then cancels, sometimes after involved algebraic manipulations similar to those described in Appendix A, against the contributions from $[C,1]$ and $[C,2]$.

(b) The terms proportional to σL. In the diagrams $[B,4]$ we express the $\sigma\psi\psi$ coupling constants, represented by a factor M in our decomposition, in terms of the inverse propagators of the σ fields. Schematically we have then $M\to(1/M)[D_\sigma^{-1}(q^2)-q^2]$ in those diagrams. Since these coupling constants are multiplied by D_σ, we can use that $D_\sigma^{-1}(q^2)D_\sigma(q^2)=1$, so that we arrive at the following decomposition for $[B,4]$:

$$[B,4]\to[q_\mu q^{-2}ML]+[q_\mu M\sigma L]+[q_\mu q^{-2}M^{-1}]+[q_\mu\sigma M^{-1}].$$

The terms proportional to σL can then be added to those of $[B,3]$. However, it is only after we combine these terms with similar terms from Z_π, the wave-function renormalization of the pion, that a factor Δ can be extracted. Then, finally, one can obtain a cancellation with the terms from $[C,2]$ and $[C,3]$.

(c) The terms proportional to TL. As is obvious from our decomposition the terms from $[B,1]$ and $[B,3]$ cannot be added directly since they differ by a relative "factor" q^2M^{-2}. In order to bring them on the same footing, we can use identities among the gauge-field propagators D_T. These identities are listed in Appendix A, and they have the following structure (schematically): $q^2T=M^2T+1$. After using these identities, the terms from $[B,1]$ and $[B,3]$ can be combined and turn out to be proportional to a factor Δ in the limit $Q^2\to 0$. However, through the use of the identity $q^2T=M^2T+1$, we generate additional terms of the form $[q_\mu q^{-2}ML]$ in the diagrams $[B,1]$. All the TL terms cancel when combined with the result from the diagrams C.

(d) The terms proportional to a single L. We have found many terms proportional to a single L, either directly in our schematic decomposition, or after subsequent manipulations similar to those described under (b) and (c) respectively, for the graphs $[B,4]$ and $[B,1]$. In order to obtain a complete cancellation we must add the contributions from the wave-function renormalization constants Z_π, Z_l, Z_ν and Z_N. After that, a factor Δ can be extracted from the diagrams $[B,1-5]$ and the result cancels with terms from the diagrams C of Fig. 1. Notice that the terms proportional to L and fermion propagators cancel separately.

We will now complete our discussion of the parity violation in the pion-nucleon form factor, $G_{pn\pi}^{\text{p.v.}}$. It turns out that the only contribution, beside the one from the wave-function renormalization constants given in (19), comes from a diagram of type $[C,5]$ in Fig. 1. It gives rise to the following result:

$$\frac{G_{pn\pi}^{\text{p.v.}}}{G_{pn\pi}}=\frac{i}{(2\pi)^4}\,\frac{1}{4}\,\sqrt{2}\,eg\,(n-1)\times\int d^nq\,\tilde{D}_T^{VA}(q)(q^2-2p\cdot q)^{-1}. \qquad (20)$$

If we consider the total answer (19) and (20) for M_Z very large, then we find that both contributions are of order $e^2M_Z^{-2}$, which is of the size of the Fermi constant G_F. This result supplements a discussion in I where it was argued that parity violations, although not naturally of order G_F in these types of models, are usually softer than could be expected on general grounds.

V. THE *(nplν)* AMPLITUDE; DETERMINATION OF G_A

We will now describe the evaluation of the diagrams which are relevant for the axial-vector form factor G_A. Again we will schematically indicate how the gauge-dependent terms cancel in the final answer. The diagrams of interest are given in Fig. 2. The diagrams A of Fig. 2 contain the effect of the wave-function renormalization constants of the external fermion legs. Again it is relevant that these constants, which are listed in Appendix B, contain terms proportional to γ_5. The diagrams B of Fig. 2 give the self-energy corrections to the exchanged vector boson. The diagrams C_N and C_l of Fig. 2 give the vertex corrections to the nucleon and lepton vertex, respectively. Finally, the diagrams D of Fig. 2 are the box-diagram contributions to the amplitude.

As we have already mentioned in Sec. II, the box-diagrams D are no longer of the current-current type. Nevertheless, these diagrams have gauge-dependent pieces, and, since those must cancel at the end, one should be able to cast at least the gauge-dependent parts in the current-current form. Several gauge-independent parts will also factorize directly in that form, but at the end we will still be left with terms that are of a different structure. We have listed these terms in Appendix C. Of all the current-current terms, we have kept only those that contribute to the axial-vector form factor G_A, evaluated at zero momentum transfer as we have explained in Sec. II.

We will now describe the decompositions of the diagrams A–D in Fig. 2 into gauge-dependent and gauge-independent terms, using the same conventions as in Sec. IV:

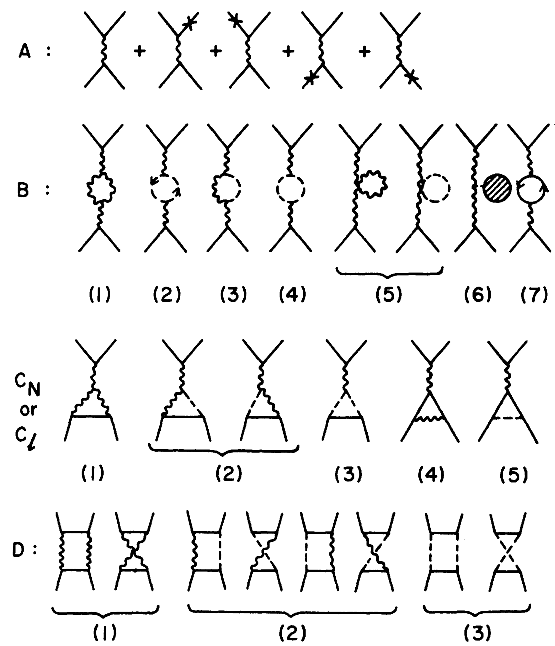

FIG. 2. The diagrams which contribute to the axial-vector coupling constant G_A. The notation is the same as in Fig. 1, and the dashed line with an arrow represents the Faddeev-Popov propagator.

$$[B,1]\to[q_\mu q_\nu M^{-2}TT]+[q_\mu q_\nu M^{-2}TL]+[q_\mu q_\nu M^{-2}LL]\,,$$

$$[B,2]\to[M^{-2}q^{-2}]+[q^{-2}L]+[M^2q^{-2}LL]\,,$$

$$[B,3]\to[q^{-2}L]+[q^{-2}T]+[M^2q^{-2}LL]+[\sigma L]+[\sigma T]\,,$$

$$[B,4]\to[q^{-2}L]+[M^2q^{-2}LL]+[M^{-2}\sigma]+[L\sigma]\,,$$

$$[B,5]\to[M^{-2}T]+[M^{-2}L]+[M^{-2}q^{-2}]+[q^{-2}L]+[M^{-2}\sigma]\,,$$

$$[B,6]\to[q^{-2}L]+[M^{-2}q^{-2}]+[M^{-1}(D_\sigma(0)T)]\,,$$

$$[B,7]\to[M^{-2}(\not{q}-im)^{-1}(\not{q}-im)^{-1}]\,,$$

$$[C,1]\to[Mq^{-2}L(\not{q}-im)^{-1}]+[q_\mu TL(\not{q}-im)^{-1}]+[q_\mu TT(\not{q}-im)^{-1}]\,,$$

$$[C,2]\to[Mq^{-2}L(\not{q}-im)^{-1}]+[Mq^{-2}T(\not{q}-im)^{-1}]+[M^3q^{-2}LL(\not{q}-im)^{-1}]$$
$$+[M^3q^{-2}LT(\not{q}-im)^{-1}]+[M\sigma L(\not{q}-im)^{-1}]+[M\sigma T(\not{q}-im)^{-1}]\,,$$

$$[C,3]\to[q_\mu q^{-4}(\not{q}-im)^{-1}]+[q_\mu q^{-4}M^2L(\not{q}-im)^{-1}]+[q_\mu q^{-4}M^4LL(\not{q}-im)^{-1}]$$
$$+[q_\mu q^{-2}\sigma(\not{q}-im)^{-1}]+[q_\mu q^{-2}M^2\sigma L(\not{q}-im)^{-1}]\,,$$

$$[C,4]\to[(\not{q}-im)^{-1}(\not{q}-im)^{-1}T]+[(\not{q}-im)^{-1}(\not{q}-im)^{-1}L)]$$
$$+[(\not{q}-im)^{-1}(\not{q}-im)^{-1}q^{-2}]+[(\not{q}-im)^{-1}(\not{q}-im)^{-1}M^2q^{-2}L]+[(\not{q}-im)^{-1}(\not{q}-im)^{-1}\sigma]\,,$$

$$[C,5]\to[(\not{q}-im)^{-1}(\not{q}-im)^{-1}q^{-2}]+[(\not{q}-im)^{-1}(\not{q}-im)^{-1}q^{-2}M^2L]\,,$$

$$[D,1] \to [M^2TT(\not{q}-im)^{-1}(\not{q}-im)^{-1}]+[M^2LL(\not{q}-im)^{-1}(\not{q}-im)^{-1}]+[M^2TL(\not{q}-im)^{-1}(\not{q}-im)^{-1}],$$

$$[D,2] \to [M^2q^{-2}T(\not{q}-im)^{-1}(\not{q}-im)^{-1}]+[q^{-2}TL(\not{q}-im)^{-1}(\not{q}-im)^{-1}]$$
$$+[M^2T\sigma(\not{q}-im)^{-1}(\not{q}-im)^{-1}]+[M^2L\sigma(\not{q}-im)^{-1}(\not{q}-im)^{-1}],$$

$$[D,3] \to [M^2q^{-2}\sigma(\not{q}-im)^{-1}(\not{q}-im)^{-1}]+[M^4q^{-2}L\sigma(\not{q}-im)^{-1}(\not{q}-im)^{-1}]$$
$$+[M^2q^{-4}(\not{q}-im)^{-1}(\not{q}-im)^{-1}]+[M^4q^{-4}L(\not{q}-im)^{-1}(\not{q}-im)^{-1}]+[M^6q^{-4}LL(\not{q}-im)^{-1}(\not{q}-im)^{-1}].$$

In making these decompositions we have written the gauge-field propagators as linear combinations of L and T, and we have made use of the expressions for the tadpole diagrams obtained in I [Eq. (13)], and the substitutions for the propagators of the Faddeev-Popov fields and the ψ fields, which were also given in I. The factors q^{-2} in these decompositions correspond to those in the ψ-field propagators. In the box graphs we have not yet taken the pion mass μ to zero, so that q^{-2} may stand for $(q^2+\mu^2)^{-1}$ also. In the other graphs no infrared problems are encountered when taking the limit $\mu^2 \to 0$. The decompositions are normalized dimensionally relative to the lowest-order amplitude.

It is again important that the diagrams A–D of Fig. 2 carry certain powers of the quantity Δ^{-1}, originating from the number of zero-momentum gauge fields that are exchanged. Normalized to the lowest-order amplitude [which has one such propagator, and carries thus Δ^{-1}; see Eq. (14)] the diagrams B of Fig. 2 have a factor Δ^{-1}, whereas the box graphs D of Fig. 2 have Δ. The others have no such factors. Hence, to establish the cancellation of gauge-dependent parts, the diagrams B of Fig. 2 have to yield a factor Δ. That result will then combine with the diagrams C of Fig. 2, and must then again lead to an answer which is proportional to a factor Δ in order to cancel with the terms from D of Fig. 2. It is rather tedious to do the algebra and extract these factors, but the general theory requires that the results are to be obtained along this line.

The aim of the subsequent discussion is now to describe systematically the cancellation of the gauge-dependent terms, i.e., the terms that contain the L propagators. In this discussion the limit $Q^2=0$ is generally understood.

(a) Terms proportional to LL. The contribution from $[B,1]$ is of order Q^2. The terms $[B,2,3,4]$, when added yield a factor Δ^2. The only contributions from the triangle graphs come from $[C_l,2]$ and $[C_l,3]$. They can be added and are then proportional to Δ. Subsequently this contribution and the previous one cancel completely against the contribution from diagrams $[D,1]$ and $[D,3]$.

(b) The terms proportional to σL. The contributions from $[B,3]$ and $[B,4]$, $[C_N,2]$ and $[C_N,3]$, $[C_l,2]$ and $[C_l,3]$, and $[D,2]$ and $[D,3]$ cancel among themselves.

(c) The terms proportional to TL. In order to add the terms from $[B,1]$ and $[B,3]$ one has to use the identities among the gauge-field propagators D_T, which are listed in Appendix A. If those identities are used twice we can add the two contributions, after which the quantity Δ factors out. In addition, the use of these identities will lead to terms proportional to single L's, which are discussed in (d). The terms proportional to LT are now combined with those from the diagrams $[C,1]$ and $[C,2]$, some of which exhibit a nucleon or lepton propagator together with an explicit factor Δ. The terms without such propagators will also yield a factor Δ after recombining them with the result of the self-energy graphs B of Fig. 2. The cancellation is then obtained after adding these terms to the box diagrams D of Fig. 2.

(d) Terms proportional to a single L. The diagrams A, B, and C of Fig. 2 give terms linear in L. If one also takes into account the terms that originate from the manipulations in (c) then the total result adds up to zero.

This, then, completes the discussion of the cancellation of the gauge-dependent terms. In the next section we will describe what happens to the gauge-independent terms in the calculation of Δ_{GT}, the deviation from the Goldberger-Treiman formula.

VI. CALCULATION OF Δ_{GT}

In the previous sections we have shown how the gauge independence of the quantities that enter into Δ_{GT} was obtained. The next step is to combine the answers for the various quantities and calculate Δ_{GT}. In doing so it will be of crucial importance to verify that Δ_{GT} is indeed free of ultraviolet divergences, and that the answer is of order $g_W{}^2$ or e^2, as we have argued in Sec. II. Since the box graphs which were listed in Appendix C already satisfy these two requirements, we will ignore those in the first part of this discussion.

We have again found a large number of cancellations which can be obtained by using the same techniques as were described in the previous sections. Again, the presence or absence of powers of the quantity Δ was usually relevant. First, we have calculated the effect of the various tadpole diagrams, and we have found that all these terms add up to zero. Concerning the remaining terms that still depend on the propagators of the fields σ, we have established by direct computation that all those terms cancel as well, except for

$$-\frac{i}{(2\pi)^4}\,\frac{n-1}{n}\,2g^2\,\frac{\Delta M_Z}{\epsilon M_U}\int d^nq\,D_\sigma^{\square Z}(q)D_T^{VW}(q)\,. \tag{21}$$

We have then considered the contributions coming from the fermion-loop diagrams, given in $[B,6]$ in Fig. 1 and $[B,7]$ in Fig. 2. A direct calculation of these quantities, when combined with the fermion-loop contributions of Z_π, leads to a result proportional to

$$\int d^np\left(\frac{1}{p^2+m^2}-\frac{2}{n}\,\frac{p^2}{(p^2+m^2)^2}\right),$$

which looks ultraviolet divergent at first sight. However, dimensional regularization ensures that this result vanishes. This can be seen either by substituting the explicit expressions for the various integrals, or by introducing $(1/n)[(\partial/\partial p_\mu)p_\mu]$ in the first integral, and performing a partial integration. In this context it is also worth mentioning that we have generally ignored integrals of the type $\int d^np\,p^{-2}$ throughout the calculation, since these integrals can be shown to vanish also within the method of dimensional regularization. Hence all the contributions from fermion-loop diagrams vanish. However, in establishing the gauge independence of our results, we have sometimes generated terms that are of a similar structure. Such terms, which do not contain the σ-field or gauge-field propagators are present in the wave-function renormalization factors, the corrections to the fermion masses, and in some of the triangle diagrams. When all these terms are combined, their contribution to $\Delta_{\rm GT}$ turns out to vanish also.

All these cancellations have been obtained by direct algebraic manipulations. However, to combine most of the triangle diagrams, like those given in $[C,1,5]$ in Fig. 1 and $[C,1]$ in Fig. 2, we will need a different approach. To introduce this, let us first consider the following example of a triangle graph with two internal fermion lines, as is shown in Fig. 3(a). The contribution of this diagram to G_A will be proportional to

$$\frac{1}{\not p'-im}\,\gamma_\mu\gamma_5\,\frac{1}{\not p-im}\,, \tag{22}$$

where p and p' are the momenta of the incoming and outgoing fermion line, respectively. Let us

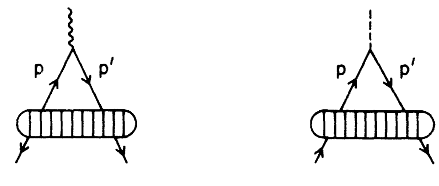

FIG. 3. Examples of triangle diagrams: (a) contributes to G_A and (b) contributes to $G_{pn\pi}$.

discuss the case that the fermions are the nucleons. We can then determine the contribution to G_A by taking the divergence with $Q_\mu = p'_\mu - p_\mu$ and dividing by $2im$. In the limit $Q^2 \to 0$ we will then obtain $G_A(\bar u_p\gamma_5 u_n)$. If we take the divergence of (22) we can now apply the well-known trick

$$\begin{aligned}\frac{1}{2im}\,Q_\mu\left(\frac{1}{\not p'-im}\,\gamma_\mu\gamma_5\,\frac{1}{\not p-im}\right)&=\frac{1}{2im}\,\frac{1}{\not p'-im}\,(\not p'\gamma_5+\gamma_5\not p)\,\frac{1}{\not p-im}\\&=\frac{1}{2im}\left(\gamma_5\,\frac{1}{\not p-im}+\frac{1}{\not p'-im}\,\gamma_5\right)\\&\quad+\frac{1}{\not p'-im}\,\gamma_5\,\frac{1}{\not p-im}\,.\end{aligned} \tag{23}$$

The last term will be proportional to a diagram that contributes to $G_{pn\pi}$, which is shown in Fig. 3(b). In this particular example, there is an exact correspondence to the diagram of $G_{pn\pi}$, so that by this method one can directly write the difference between the corrections to G_A and those to $G_{pn\pi}$ as an integral that involves only one, instead of the original two fermion propagators. Not only will this lead to simpler expressions, but this trick will enable us to treat larger blocks of diagrams at the same time. In this way we were then able to write the result for $\Delta_{\rm GT}$ from all the triangle graphs with two fermion propagators in a form with one fermion propagator and one gauge-field propagator. (The terms without any gauge-field propagator have been discussed before.)

A generalization of the same method can now be applied to the propagator and triangle diagrams with two gauge-field propagators, like the diagrams $[B,1]$, $[C,1]$ of Fig. 1 and $[B,1]$, $[C,1]$ of Fig. 2. The general structure of the diagrams that contribute to G_A is shown in Fig. 4(a), and leads to the following contribution:

$$D_1(p)D_2(q)[-(p+2q)_\alpha\delta_{\mu\beta}+(2p+q)_\beta\delta_{\mu\alpha}+(q-p)_\mu\delta_{\alpha\beta}]\times\left(\delta_{\alpha\nu}-\frac{p_\alpha p_\nu}{p^2}\right)\left(\delta_{\beta\rho}-\frac{q_\beta q_\rho}{q^2}\right),$$

where p and q are the momenta of the gauge-field

(a) (b)

FIG. 4. Examples of propagator or triangle diagrams: (a) contributes to G_A and (b) contributes to $G_{pn\pi}$ or F_π.

propagators, and $Q=p+q$. If we now again take the divergence with Q, we find

$$D_1(p)D_2(q)(q^2-p^2)\,\delta_{\alpha\beta}\left(\delta_{\alpha\nu}-\frac{p_\alpha p_\nu}{p^2}\right)\left(\delta_{\beta\rho}-\frac{q_\beta q_\rho}{q^2}\right).$$

We could now generalize the previous discussion and try to write q^2-p^2 as the difference of two inverse gauge-field propagators with possible correction terms. This can be done straightforwardly by using the identities (A3) and (A4) of Appendix A. Schematically we can then write

$$(q^2-p^2)D_1(p)D_2(q)\to D_2(q)-D_1(p)+M^2D_3(p)D_4(q). \tag{24}$$

Again the last term has the structure of a diagram that contributes to $G_{pn\pi}$ or F_π. The general structure of such graphs is shown in Fig. 4(b). However, in this case there is never an exact correspondence, and we are not able to completely eliminate the terms with two gauge-field propagators, as was previously found to be the case for diagrams with two fermion propagators. The remaining terms represent contributions to $\Delta_{\rm GT}$, which we have listed and further evaluated in Appendix D.

We will now discuss the terms that contain only one gauge-field propagator. We have already mentioned such terms, coming from the diagrams with two fermion poles after using the substitution (23). Those terms can be combined with similar terms from the fermion masses and wave-function renormalization factors. Now we have also generated contributions from the diagrams with two gauge-field propagators, after having used the substitution (24). In addition, we will find terms with one gauge-field propagator that originate from the pion wave-function renormalization factor Z_π, listed in Appendix B, as well as similar contributions from the diagrams $[B,3]$ and $[C,2]$ of Fig. 1 and $[B,3]$, $[B,5]$, and $[C,2]$ of Fig. 2. All these terms can be combined into the following relatively simple contribution to $\Delta_{\rm GT}$

$$\begin{aligned}&-\frac{i}{(2\pi)^4}\frac{1}{\Delta}\frac{n-1}{n}\frac{1}{p^2}\left[\frac{1}{2}g^2\left(D_T^{VV}+\tilde D_T^{VV}-D_T^{UU}-\tilde D_T^{UU}-\sqrt{2}\,\frac{e}{g}\,\tilde D_T^{UA}\right)-\frac{1}{4}\sqrt{2}gg_W\epsilon^2\left(D_T^{UW}+\tilde D_T^{UW}+\sqrt{2}\,\frac{e}{g}\,\tilde D_T^{WA}+D_T^{VW}+\tilde D_T^{VW}\right)\right.\\&\left.+\frac{1}{2}\frac{g_W^2}{g^2}\epsilon^2M_U^2M_Z^{-2}\left(\frac{1}{2}\sqrt{2}\,eg\tilde D_T^{VA}-eg_W\tilde D_T^{WA}\right)\right]-\frac{i}{(2\pi)^4}\frac{d}{ds}\left[\left(e^2\tilde D_T^{AA}+\frac{1}{2}\sqrt{2}\,eg\tilde D_T^{UA}\right)\frac{p^2-2q^2}{q^2+\mu^2}\right]_{s=(p+q)^2=-\mu^2}\\&+\frac{i}{(2\pi)^4}\frac{3}{n}\frac{1}{p^2}\left(e^2\tilde D_T^{AA}+\frac{1}{2}\sqrt{2}\,eg\tilde D_T^{UA}\right)-\frac{i}{(2\pi)^4}\left(e^2\tilde D_T^{AA}+\frac{1}{2}\sqrt{2}\,eg\tilde D_T^{UA}\right)\left(\frac{1}{q^2}-\frac{1}{q^2+\mu^2}\right)\frac{2m_l^2}{m_l^2+Q^2}\,\frac{2p_\nu\cdot q}{p^2-2p\cdot p_l}.\end{aligned}\tag{25}$$

In this result the argument of the gauge-field propagators is p. In the first and third terms the limit $Q^2\to 0$ has been taken and an integration $\int d^np$ has been suppressed. The second term originates from the pion wave-function renormalization Z_π. Here the integral $\int d^np\,d^nq\,\delta^n(p+q-Q)$ has been suppressed. In the fourth term we have ignored the same integral, and again we have refrained from taking the limit $Q^2=0$, because of infrared difficulties, as we have discussed in Sec. III.

Two remarkable properties of this result are worth mentioning. In the first place, one can easily verify [for instance, by using the identities (A3) and (A4) to extract the large momentum behavior] that the result (25) is ultraviolet finite. Secondly, the dependence on nucleon and lepton masses has cancelled.

We will now discuss the final result for $\Delta_{\rm GT}$. We distinguish the following five contributions:

(a) the term containing D_σ^{UZ}, given in Eq. (21);

(b) the box graphs that are not of the current-current form, given in Eq. (C2);

(c) terms proportional to $(n-1)/n$, given by the first term of Eq. (25) *and* by the following contributions from Z_π [see Eq. (B2)]:

$$\begin{aligned}-\frac{i}{(2\pi)^4}\frac{1}{\Delta}\frac{n-1}{n}\int d^np\,\frac{\epsilon^2M_U^2}{p^2}\,\{&\tfrac14 g_W^2(D_T^{UU}+D_T^{VV}+D_T^{UV})\tilde D_T^{WW}+\tfrac14 g_W^2(\tilde D_T^{UU}+\tilde D_T^{VV}+\tilde D_T^{UV})D_T^{WW}\\&-\tfrac12 g_W^2(D_T^{UW}+D_T^{VW})(\tilde D_T^{UW}+\tilde D_T^{VW})+\tfrac12 g^2(D_T^{UU}\tilde D_T^{VV}+D_T^{VV}\tilde D_T^{UU}-2D_T^{UV}\tilde D_T^{UV})\\&-\tfrac12\sqrt{2}gg_W[D_T^{UW}(\tilde D_T^{VV}+\tilde D_T^{UV})+(D_T^{VV}+D_T^{UV})\tilde D_T^{UW}-D_T^{VW}(\tilde D_T^{UU}+D_T^{UV})-(D_T^{UU}+D_T^{UV})D_T^{VW}]\}\end{aligned}\tag{26}$$

(the propagators all depend on the momentum p);

(d) the result of propagator diagrams, shown in Eq. (D3) combined with the remaining terms from Z_π:

$$\frac{-i}{(2\pi)^4}(n-1)M_Z{}^2\Delta\sqrt{2}\,\frac{g}{g_W}\,\frac{d}{dQ^2}\left(\int d^np\,d^nq\,\delta^n(p+q-Q)[(e^2\tilde{D}_T^{AA}+\tfrac{1}{2}\sqrt{2}\,eg\tilde{D}_T^{UA})D_T^{VW}-\tfrac{1}{2}\sqrt{2}\,eg\tilde{D}_T^{VA}D_T^{UW}]\right)_{s=Q^2=0}; \tag{27}$$

(e) the box graphs that were written in the current-current form, Eq. (C1), together with the terms originating from triangle diagrams that were given in Eq. (D1), and the remaining terms from Eq. (25).

All these contributions (a)–(e) are separately ultraviolet finite, which proves our claim that the Goldberger-Treiman formula is indeed a zeroth-order relation, from which the deviations are finite and calculable. The results (a)–(e) are also separately of order $g_W{}^2$ or e^2, as was required according to the arguments presented in Sec. II. The only terms which are not manifestly of this order are those given under (c). Namely, we have the following terms:

$$-\frac{i}{(2\pi)^4}\,\frac{n-1}{n}\,\frac{1}{\Delta}\,\frac{1}{2}g^2\int d^np\,\frac{1}{p^2}\left[\left(D_T^{VV}+\tilde{D}_T^{VV}-D_T^{UU}-\tilde{D}_T^{UU}-\sqrt{2}\,\frac{e}{g}\,\tilde{D}_T^{UA}\right)+\epsilon^2M_U{}^2(D_T^{UU}\tilde{D}_T^{VV}+D_T^{VV}\tilde{D}_T^{UU}-2D_T^{UV}\tilde{D}_T^{UV})\right]. \tag{28}$$

However, using the Eqs. (A3) and (A4) this can be rewritten in the form

$$-\frac{i}{(2\pi)^4}\,\frac{n-1}{n}\,\frac{1}{\Delta}\,\frac{1}{2}\,g^2\int d^np\,\frac{1}{p^2}\left(\frac{e}{g_W}M_Z{}^2(D_T^{UW}\tilde{D}_T^{VA}-D_T^{VW}\tilde{D}_T^{UA})\right.$$
$$\left.+\frac{1}{2}\sqrt{2}\,\frac{e}{g}\,M_U{}^2\{[D_T^{UV}-(1+2\epsilon^2)D_T^{VV}]\tilde{D}_T^{UA}-[D_T^{UU}-(1+2\epsilon^2)D_T^{UV}]\,\tilde{D}_T^{VA}\}\right), \tag{29}$$

which is of order e^2. In fact, one can show that the total contribution from (c) is of order e^2.

This concludes the calculation of Δ_{GT}. It is, however, worth mentioning that although Δ_{GT} is ultraviolet finite, it is still infrared divergent. The infrared divergences are contained in the conventional box-graph contributions, mentioned under (b), and in the contribution (e). Most of them are due to the presence of massless photons and pions (in the limit $\mu^2=0$). However, some of them do not seem related to the presence of massless physical particles. Of course, these divergences must be superficial and should cancel in the final result. Indeed, if we add all these terms, it turns out that they are all proportional to the term

$$\frac{1}{q^2}-\sqrt{2}\,\frac{g}{g_W}\,M_Z{}^2\Delta\,\frac{D_T^{VW}(q)}{q^2}.$$

This particular combination no longer exhibits a pole at $q^2=0$, so that the infrared divergence disappears.

VII. CONCLUSIONS

We have shown in the previous section that the result for Δ_{GT} is finite and of order e^2 or $g_W{}^2$. It is now straightforward to examine all the remaining terms for physical values of the parameters (i.e., $M_W\simeq M_Z\to\infty$, $e/g\ll 1$, $g_W/g\ll 1$). In this approximation the complicated gauge-field propagators in I reduce to much simpler forms, and the integrals can be evaluated. However, before we can obtain a final answer, there are several problems to be faced.

First of all, the numerical magnitudes of G_A and F_π which are commonly used in checking the validity of the Goldberger-Treiman relation have already been adjusted for radiative corrections. Taking G_A, for instance, this involves the computation of the virtual corrections through order e^2 and the infrared divergent bremsstrahlung corrections. The extraction of the virtual corrections[8,24] is usually done in the limit in which m_l and p_l can be neglected. This reduces a box graph, which is a function of two invariants, say Q^2 and $\omega=p_p\cdot p_l$, down to a triangle graph which is a function of Q^2 only. The infrared divergency is then cancelled by the corresponding bremsstrahlung term in the rate for the radiative decay. However, these standard considerations apply to the computation of various decay rates whereas our calculation concerns the ratio of amplitudes.

Since the same order of e^2 graphs are already included in our answer for G_A and F_π, these terms *would need to be subtracted out* before we can find the remaining corrections to Δ_{GT}.

Then there is the problem of infrared-divergent terms due to the presence of a massless pion. We have regulated this divergence by calculating these dangerous terms with a finite pion mass μ^2. Hence we are not strictly using a pseudo-Goldstone pion everywhere in the calculation. Clearly the regular infrared-divergent terms due to the presence of a massless photon now get mixed with these additional terms. Also the procedure used to extract the regular infrared-divergent terms, namely, setting m_l and p_l equal to zero, becomes much more subtle. For instance, what should one do with terms involving $\ln(m_l^2/\mu^2)$? Recognizing that we are dealing with a model and are only interested in the approximate magnitude of the numerical result, it does not seem worthwhile to get entangled with these problems. Hence we take the point of view that the remaining terms in our answer should only be checked to see whether there are any anomalously large terms. We have already shown that the corrections are of order e^2 and g_W^2. In the limit that $M_W\to\infty$ there could, for instance, be sizeable contributions from terms proportional to $e^2\ln M_W^2$. If such terms are absent, then we can safely conclude by saying that the corrections are of typical electromagnetic size, say 1% or less, and avoid giving an explicit number for the correction to the Goldberger-Treiman relation. An analysis of our final answer has indeed shown that the large value of M_W does not induce an enhancement of the corrections. Let us discuss some of the final terms and make a few additional comments.

First, consider the terms which are independent of nucleon or lepton propagators. These consist of the terms in $(n-1)/n$ in Eqs. (25), (26), and (29). Most of these terms do not contribute to order e^2. We find the result

$$\frac{-i}{(2\pi)^4}\frac{n-1}{n}\frac{e^2M_U^2}{\Delta}(1+\epsilon^2)\times\int d^nq\frac{M_W^2}{q^2(q^2+M_U^2)(q^2+M_V^2)(q^2+M_W^2)}=\frac{3e^2}{16\pi^2}\frac{M_U^2}{M_V^2-M_U^2}\ln\left(\frac{M_V^2}{M_U^2}\right),\quad(30)$$

which is proportional to the pion mass as calculated in I. The term in Eq. (21) is also proportional to $(n-1)/n$. However, it does not contribute to order e^2 because $D_\sigma^{\Sigma Z}$ is approximately given by

$$D_\sigma^{\Sigma Z}\cong g_Wg^{-1}M_U^3M_W^{-1}(q^2+m_\Sigma^2)^{-1}(q^2+m_Z^2)^{-1}.$$

The term in Eq. (27) which is proportional to $n-1$ can be evaluated by expanding the propagators in a Taylor series in Q^2. After isolating the terms which are finite in the limit $M_W^2\to\infty$ we find

$$\frac{-i}{(2\pi)^4}(n-1)M_Z^2\Delta\frac{\sqrt{2}g}{g_W}\frac{d}{dQ^2}\left\{\int d^np\int d^nq\,\delta^{(n)}(p+q-Q)\left[\left(e^2\tilde{D}_T^{AA}+\frac{\sqrt{2}}{2}eg\tilde{D}_T^{UA}\right)D_T^{VW}(q)-\frac{\sqrt{2}}{2}eg\tilde{D}_T^{VA}D_T^{UW}(q)\right]\right\}$$
$$=\frac{3e^2}{16\pi^2}\left\{\frac{M_U^2M_V^4}{(M_U^2-M_V^2)^3}\ln\left(\frac{M_V^2}{M_U^2}\right)+\frac{M_U^4}{(M_V^2-M_U^2)^2}+\frac{3}{2}\frac{M_U^2}{M_V^2-M_U^2}\right\}.\quad(31)$$

Clearly there is nothing surprising about these answers. All dependence upon M_W^2 has canceled completely. In this limit $(M_W\to\infty)$, we can identify M_U and M_V as the masses of the ρ meson and A_1 meson, respectively. Hence both results in Eqs. (30) and (31) are numerically small.

We now turn to the integrals with infrared problems and nucleon or lepton poles. The terms in Eq. (C1) and Eq. (D1) which are of order e^2 involve the propagator combinations $[e^2\tilde{D}_T^{AA}(p)+(\sqrt{2}/2)eg\tilde{D}_T^{UA}(p)]D_T^{VW}(q)$ and $(\sqrt{2}/2)eg\tilde{D}_T^{UA}(p)D_T^{VW}(q)$. All the other terms lack sufficient powers of M_W^2 to be finite in the limit where $M_W^2\to\infty$. Note that the former term has a factor of p^{-2} so it is more likely to be infrared divergent. The latter term has no such problems and is actually of order G_F. If we now collect together all the terms in $e^2\tilde{D}_T^{AA}+(\sqrt{2}/2)eg\tilde{D}_T^{UA}$, including those in Eq. (25), we find the result

$$\frac{-i}{(2\pi)^4}\int d^np\int d^nq\,\delta^{(n)}(p+q-Q)\left(e^2\tilde{D}_T^{AA}(p)+\frac{\sqrt{2}}{2}eg\tilde{D}_T^{UA}(p)\right)\left\{\frac{3}{n}\left(\frac{\sqrt{2}g}{g_W}M_Z^2\Delta\frac{D_T^{VW}(q)}{q^2}-\frac{1}{q^2}\right)-\frac{\sqrt{2}gM_Z^2\Delta}{g_W}\frac{4p\cdot q_\nu}{p^2-2p\cdot p_l}D(q)\right.$$
$$\left.+\frac{\sqrt{2}g}{g_W}M_Z^2\Delta D_T^{VW}(q)\left[\frac{2p\cdot Q}{p^2Q^2}+(n-2)\left(1+\frac{2q\cdot p_\nu}{m_l^2}\right)\frac{1}{p^2-2p\cdot p_l}-\frac{1}{q^2-2q\cdot p_n}+\frac{(n-2)p\cdot p_n}{m^2(p^2+2p\cdot p_n)}\right]\right\}$$
$$-\frac{i}{(2\pi)^4}\frac{d}{dQ^2}\left[\int d^np\int d^nq\,\delta^{(n)}(p+q-Q)\left(e^2\tilde{D}_T^{AA}(p)+\frac{\sqrt{2}}{2}eg\tilde{D}_T^{UA}(p)\right)\frac{p^2-2q^2}{q^2+\mu^2}\right]_{Q^2=-\mu^2},\quad(32)$$

where $D(q)$ is defined in Eq. (C3). As mentioned previously, some of the infrared divergences are superficial. The combination $\sqrt{2}\ gg_W^{-1}M_Z^{2}\Delta D_T^{VW}(q) - 1$ behaves like q^2 for small q^2, so the first term in Eq. (32) is infrared convergent. The second and fourth terms are infrared divergent in the limit that the pion is massless, so we have regulated the terms with a small mass μ^2. In the approximation that higher orders in e and g_W are neglected, $D(q)$ reduces to

$$D(q)=\frac{\sqrt{2}\,g_W}{2gM_Z^{2}\Delta}\left(\frac{1}{q^2+\mu^2}-\frac{1}{q^2+M_V^2}\right)\,. \tag{33}$$

Some of the integrals in Eq. (32) are trivial. The terms with no lepton or nucleon poles can be evaluated and lead to answers similar to those in Eqs. (30) and (31). The terms involving nucleon poles lead to more complicated expressions involving M_V^2, m^2, and m_l^2. Unfortunately, one of the lepton pole terms is infrared divergent and leads to a term involving $\ln(m_l^2/\mu^2)$. There are no terms containing $\ln M_W^2$ in this part of the answer.

The last part of the analysis concerns the contributions from the box graphs which cannot be written in current-current form, namely the terms in Eq. (C2). All these integrals are well behaved in the limit that $M_W^2\to\infty$ and never yield terms in $e^2\ln M_W^2$. Hence there are no anomalously large terms in our answer for $\Delta_{\rm GT}$.

We do not want to discuss the box graphs further because they have severe infrared divergence problems. The only way to extract the virtual photon terms is to take the limit that m_l and p_l tend to zero. Such a program runs into trouble due to the additional infrared singularities caused by zero-mass pions.

We conclude by briefly summarizing the main results of the paper. The corrections to the Goldberger-Treiman relation have been calculated in a unified gauge-field model. We have explicitly demonstrated that the corrections are finite and gauge independent, which confirms our general result that $\Delta_{\rm GT}=0$ is a zeroth-order relation. In the case that the pion is a pseudo-Goldstone boson, so that its mass is due to the electromagnetic and weak interactions, the corrections to $\Delta_{\rm GT}$ are also of order e^2 or g_W^2. In the physical limit ($M_W\to\infty$, $e/g\ll 1$, $g_W/g\ll 1$) the corrections are completely independent of M_W, so we expect the size of the correction to be approximately 1%.

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APPENDIX A: SOME USEFUL FORMULAS

In this appendix we have collected some results which have been used throughout the calculations. Some of those were already briefly sketched in Sec. III.

One can generally express a four-vector p_μ in terms of two other vectors P_1 and P_2 by means of the following formula:

$$p_\mu=\frac{(P_1\cdot P_2)(P_2\cdot p)-P_2^{2}(P_1\cdot p)}{(P_1\cdot P_2)^2-P_1^{2}P_2^{2}}\,(P_1)_\mu+\frac{(P_1\cdot P_2)(P_1\cdot p)-P_1^{2}(P_2\cdot p)}{(P_1\cdot P_2)^2-P_1^{2}P_2^{2}}\,(P_2)_\mu\,. \tag{A1}$$

This result, which can be generalized in various ways, allows one to write Feynman integrals of the form $\int d^n p\, p_\mu F(p,P_1,P_2)$ as a linear combination of the external momenta $P_{1\mu}$ and $P_{2\mu}$. The coefficients are integrals involving scalar functions of p, P_1, and P_2.

As was described in Sec. III, one can express the $\sigma\psi\psi$ coupling constants of the Lagrangian in terms of the inverse propagators of the fields σ. These relations are given by

$$\begin{aligned}
&64\mu_3 M_U^{2}=s-[D_\sigma^{-1}]^{UU}(s)\,,\\
&16\mu_4 M_U^{2}=s-[D_\sigma^{-1}]^{VV}(s)+8\sqrt{2}\,b\epsilon M_U M_Z\,,\\
&16\mu_5\epsilon^{2}M_U^{2}=s-[D_\sigma^{-1}]^{\Sigma\Sigma}(s)+8\sqrt{2}\,b\epsilon^{-1}M_U M_Z\,,\\
&16\mu_6\epsilon M_U^{2}=-[D_\sigma^{-1}]^{U\Sigma}(s)-8\sqrt{2}\,bM_U M_Z\,,\\
&32\rho_2 M_Z^{2}=s-[D_\sigma^{-1}]^{ZZ}(s)+4\sqrt{2}\,b\epsilon g_W^{2}g^{-2}M_U^{3}M_Z^{-1}\,,\\
&16\sqrt{2}\,g_W g^{-1}M_U M_Z\lambda_1=-[D_\sigma^{-1}]^{UZ}(s)-8b\epsilon g_W g^{-1}M_U^{2}\,,\\
&8\sqrt{2}\,\epsilon g_W g^{-1}M_U M_Z\lambda_2=[-D_\sigma^{-1}]^{\Sigma Z}(s)-8bg_W g^{-1}M_U^{2}\,,\\
&16g_W^{2}g^{-2}M_U^{2}\delta_2=-[D_\sigma^{-1}]^{UV}(s)\,,\\
&8g_W^{2}g^{-2}M_U^{2}\epsilon\delta_3=-[D_\sigma^{-1}]^{V\Sigma}(s)\,,\\
&8\sqrt{2}\,g_W g^{-1}M_U M_Z\delta_4=-[D_\sigma^{-1}]^{VZ}(s)\,.
\end{aligned} \tag{A2}$$

The argument of the propagator s is a free parameter in these relations. The coupling constant b has been taken equal to zero in most of the calculations reported in this paper.

Similarly one can derive relatively simple relations among the various transversal gauge-field propagators D_T, by substituting explicit expressions for $D_T^{-1}(s)$ into the relation $D_T(s)D_T^{-1}(s)=1$. We then find

$$\begin{aligned}
&sM_U^{-2}(D_T^{UW}+eg_W^{-1}D_T^{UA}) \\
&\qquad=\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}(D_T^{UU}+D_T^{UV})-M_W^2M_U^{-2}D_T^{UW}\,,\\
&sM_U^{-2}(D_T^{VW}+eg_W^{-1}D_T^{VA}) \\
&\qquad=\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}(D_T^{VV}+D_T^{UV})-M_W^2M_U^{-2}D_T^{VW}\,,\\
&sM_U^{-2}(D_T^{WW}+eg_W^{-1}D_T^{WA}) \\
&\qquad=\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}(D_T^{UW}+D_T^{VW})-M_W^2M_U^{-2}D_T^{WW}+M_U^{-2}\,,\\
&sM_U^{-2}(D_T^{UW}+eg^{-1}\sqrt{2}\,D_T^{WA})=\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}D_T^{WW}-D_T^{UW}\,,\\
&sM_U^{-2}D_T^{VW}=\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}D_T^{WW}-(1+\epsilon^2)D_T^{VW}\,, \qquad\qquad (A3)\\
&sM_U^{-2}(D_T^{UU}+eg^{-1}\sqrt{2}\,D_T^{UA})=\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}D_T^{UW}-D_T^{UU}+M_U^{-2}\,,\\
&sM_U^{-2}D_T^{VV}=\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}D_T^{VW}-(1+\epsilon^2)D_T^{VV}+M_U^{-2}\,,\\
&sM_U^{-2}(D_T^{UV}+eg^{-1}\sqrt{2}\,D_T^{VA})=\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}D_T^{VW}-D_T^{UV}\,,\\
&sM_U^{-2}D_T^{UV}=\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}D_T^{UW}-(1+\epsilon^2)D_T^{UV}\,.
\end{aligned}$$

For the charged propagators the terms that contain the photon field A must be deleted. For neutral propagators our notation requires the substitution of D by $\tilde{D}$. In addition, we have the following identities for neutral fields only:

$$\begin{aligned}
&sM_U^{-2}(eg_W^{-1}\tilde{D}_T^{AA}+\tilde{D}_T^{WA}) \\
&\qquad=\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}(\tilde{D}_T^{UA}+\tilde{D}_T^{VA})-M_W^2M_U^{-2}\tilde{D}_T^{WA}\,,\\
&sM_U^{-2}(eg^{-1}\tilde{D}_T^{AA}+\tfrac{1}{2}\sqrt{2}\,\tilde{D}_T^{UA})=\tfrac{1}{2}g_Wg^{-1}\tilde{D}_T^{WA}-\tfrac{1}{2}\sqrt{2}\,\tilde{D}_T^{UA}\,,\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (A4)\\
&sM_U^{-2}\tilde{D}_T^{VA}=\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}\tilde{D}_T^{WA}-(1+\epsilon^2)\tilde{D}_T^{VA}\,,\\
&sM_U^{-2}(\tilde{D}^{AA}+eg_W^{-1}\tilde{D}_T^{WA}+eg^{-1}\sqrt{2}\,\tilde{D}_T^{UA})=M_U^{-2}\,.
\end{aligned}$$

These relations (A3) and (A4) are convenient in rearranging some of our results in order to obtain the necessary cancellations discussed in the text. The square of the momentum of the propagators is $q^2=s$.

Finally, we will discuss a typical example of the algebraic manipulations that we have used. Consider the diagram $[B,1]$ of Fig. 1, with two different propagators D_1 and D_2. This diagram is proportional to

$$I=\int d^np\,d^nq\,\delta^n(p+q-Q)D_1(p)D_2(q)(p_\mu-q_\mu)\,. \qquad (A5)$$

If we now interchange p and q, and then use that this integral has to be proportional to Q_μ, we find the result

$$I=\tfrac{1}{2}Q_\mu Q^{-2}\int d^np\,d^nq\,\delta^n(p+q-Q) \times[D_1(p)D_2(q)-D_1(q)D_2(p)](p^2-q^2)\,.$$

In the limit $Q\to 0$, we know that $D_1(p)D_2(q)-D_1(q)D_2(p)$ can be written as $R(\frac{1}{2}(p-q))\cdot(p^2-q^2)$. If we then perform a symmetrical integration, we obtain the result

$$I=\frac{2}{n}\,Q_\mu\int d^np\,R(p)p^2\,. \qquad (A6)$$

Thus we have now evaluated the general structure of (A5), which is usually sufficient in order to obtain the various cancellations. In most cases it is not necessary to find the explicit form of $R(p)$.

APPENDIX B: THE WAVE-FUNCTION RENORMALIZATION CONSTANTS, AND THE FERMION MASSES

In order to calculate the correct expressions for $G_{pn\pi}$, F_π, and G_A, and to arrive at gauge-independent results for these quantities, we have to normalize the wave functions of the pion, the leptons, and the nucleons. The wave-function renormalization constant for the pion, Z_π, follows from the pion propagator D_π in the one-loop approximation, with the pion field as defined in Eq. (16). This propagator was calculated in I, where it was also found that the eigenstates of the mass matrix remain unchanged in this approximation. The definition of Z_π is then given by

$$Z_\pi=\lim_{s\to -M_\pi^2}(s+M_\pi^2)D_\pi(s)\,,$$

and as a byproduct of the calculations presented in I we find the following expression for Z_π:

$$Z_\pi=1-\frac{i}{(2\pi)^4}\frac{\epsilon^2}{\Delta}\int d^nq\,\Pi(q)\,,$$

where

$$\Pi(q)=\tfrac{1}{2}g^2\Big[D_\sigma^{VV}\Big(\frac{n-1}{n}D_T^{UU}-\tfrac{1}{2}D_L^{UU}\Big)+2(D_\sigma^{UV}-2\epsilon^{-1}D_\sigma^{V\Sigma})\Big(\frac{n-1}{n}D_T^{UV}-\tfrac{1}{2}D_L^{UV}\Big)$$

$$+(D_\sigma^{UU}+4\epsilon^{-2}D_\sigma^{\Sigma\Sigma}-4\epsilon^{-1}D_\sigma^{U\Sigma})\Big(\frac{n-1}{n}D_T^{VV}-\tfrac{1}{2}D_L^{VV}\Big)$$

$$+g_W^2g^{-2}[\tfrac{1}{2}D_\sigma^{UU}+\tfrac{1}{2}D_\sigma^{VV}+D_\sigma^{UV}+g_W^2g^{-2}M_U^2M_Z^{-2}D_\sigma^{ZZ}-\sqrt{2}\,g_Wg^{-1}M_UM_Z^{-1}(D_\sigma^{UZ}+D_\sigma^{VZ})]\Big(\frac{n-1}{n}D_T^{WW}-\tfrac{1}{2}D_L^{WW}\Big)$$

$$-g_Wg^{-1}(\sqrt{2}D_\sigma^{VV}+\sqrt{2}D_\sigma^{UV}-2g_Wg^{-1}M_UM_Z^{-1}D_\sigma^{VZ})\Big(\frac{n-1}{n}D_T^{UW}-\tfrac{1}{2}D_L^{UW}\Big)$$

$$-g_Wg^{-1}[\sqrt{2}D_\sigma^{UU}+\sqrt{2}D_\sigma^{UV}-2\sqrt{2}\epsilon^{-1}(D_\sigma^{U\Sigma}+D_\sigma^{V\Sigma}-\sqrt{2}g_Wg^{-1}M_UM_Z^{-1}D_\sigma^{\Sigma Z})$$

$$-2g_Wg^{-1}M_UM_Z^{-1}D_\sigma^{UZ}]\Big(\frac{n-1}{n}D_T^{VW}-\tfrac{1}{2}D_L^{VW}\Big)\Big]$$

$$-\tfrac{1}{8}g^2q^{-2}[(1+8\epsilon^{-2})D_L^{UU}+(1-4\epsilon^{-2})D_L^{VV}+3\sqrt{2}g_Wg^{-1}(D_L^{UW}+D_L^{VW})+g_W^2g^{-2}(1+g_W^2g^{-2}M_U^2M_Z^{-2})D_L^{WW}]$$

$$-e^2\Delta\epsilon^{-2}q^{-2}(D_L^{AA}-\tilde D_T^{AA})+\frac{1}{8}\frac{4-n}{n}g^2M_U^{-2}[D_\sigma^{UU}+D_\sigma^{VV}+4\epsilon^{-4}D_\sigma^{\Sigma\Sigma}+(g_Wg^{-1}M_UM_Z^{-1})^4D_\sigma^{ZZ}]$$

$$-4\epsilon^{-2}G_N^2(q^2+m^2)^{-2}+g_W^2g^{-2}M_U^2M_Z^{-2}G_l^2\frac{4}{n}(q^2+m_l^2)^{-2}$$

$$+\frac{n-1}{n}\frac{1}{2}g^2q^{-2}[(1+4\epsilon^{-2})(D_T^{UU}+\tilde D_T^{UU})+D_T^{VV}+\tilde D_T^{VV}+\sqrt{2}g_Wg^{-1}(D_T^{UW}+D_T^{VW}+\tilde D_T^{UW}+\tilde D_T^{VW})$$

$$+g_W^2g^{-2}(1+g_W^2g^{-2}M_U^2M_Z^{-2})(D_T^{WW}+\tilde D_T^{WW})+4\sqrt{2}eg^{-1}(1+2\epsilon^{-2})\tilde D_T^{UA}$$

$$+4eg_Wg^{-2}(1+g_W^2g^{-2}M_U^2M_Z^{-2})\tilde D_T^{WA}]+\frac{d}{ds}\Pi_1(q,k)\Big|_{s=0}\,. \qquad \text{(B1)}$$

$\Pi_1(q,k)$ is defined by

$$(2k^2+2p^2-q^2)(p^2+\mu^2)^{-1}\Delta\epsilon^{-2}e^2\tilde D_T^{AA}$$

$$+(n-1)\Big(1-\frac{1}{n}\frac{k^2}{p^2}\Big)M_U^2(\tfrac{1}{4}g_W^2\tilde D_T^{WW}(p)(D_T^{UU}+D_T^{VV}+2D_T^{UV})+\tfrac{1}{4}g_W^2[\tilde D_T^{UU}(p)+\tilde D_T^{VV}(p)+2\tilde D_T^{UV}(p)]D_T^{WW}$$

$$-\tfrac{1}{2}g_W^2\tilde D_T^{UW}(p)D_T^{UW}-\tfrac{1}{2}g_W^2\tilde D_T^{VW}(p)D_T^{VW}-\tfrac{1}{2}g_W^2\tilde D_T^{UW}(p)D_T^{VW}-\tfrac{1}{2}g_W^2\tilde D_T^{VW}(p)D^{UW}$$

$$+\tfrac{1}{2}g^2[\tilde D_T^{UU}(p)D_T^{VV}+\tilde D_T^{VV}(p)D_T^{UU}-2\tilde D_T^{UV}(p)D_T^{UV}]$$

$$-\tfrac{1}{2}\sqrt{2}gg_W\{\tilde D_T^{UW}(p)(D_T^{VV}+D_T^{UV})-D_T^{VW}(D_T^{UU}+D_T^{UV})$$

$$-[\tilde D_T^{UU}(p)+\tilde D_T^{UV}(p)]D_T^{VW}+[\tilde D_T^{VV}(p)+\tilde D_T^{UV}(p)]D_T^{UW}\})\,. \qquad \text{(B2)}$$

We have suppressed the argument q in the propagators. Furthermore, we used the definitions $s=k^2$, and $p=k-q$. Notice that there are infrared divergences contained in $\Pi_1(q,k)$. In such terms we have kept the lowest-order pion mass μ finite, in which case the derivative is evaluated at $s=-\mu^2$.

The definition of the fermion wave-function renormalization constant requires more care, because of terms proportional to $\not p\gamma_5$ in the fermion propagators. As far as the neutrino is concerned, only its left-handed chiral component has interactions. These give rise to a wave-function renormalization constant Z_ν (for the left-handed component only):

$$Z_\nu=1+\tfrac{1}{4}\frac{ig_W^2}{(2\pi)^4}\int d^nq\,D_L^{WW}(3q^2-m_l^2)q^{-2}\times(q^2+m_l^2)^{-1}+z_\nu \qquad \text{(B3)}$$

where z_ν is a gauge-independent contribution that we need not specify.

For the remaining fermions we first write the inverse propagator in the form

$$S^{-1}(p)=\exp[-a(p^2)\gamma_5]\bar{S}^{-1}(p)\exp[a(p^2)\gamma_5]\ ,$$

where $a(p^2)$ is chosen such that $\bar{S}$ is now free of terms proportional to γ_5. The wave-function renormalization constant of an incoming fermion is then given by $Z^{1/2}\exp[-a(-m^2)\gamma_5]$, and that of an outgoing fermion is given by $Z^{1/2}\exp[a(-m^2)\gamma_5]$, where m is the fermion mass, and Z is defined by

$$\bar{S}(p)\underset{\not p=-im}{\sim}\frac{Z}{\not p-im}\ .$$

Hence the total wave-function renormalization constant can be written in a form that includes a term proportional to γ_5, the sign of which depends on whether the fermion is incoming or outgoing.

For the (incoming) lepton we obtain in this way the following result:

$$\begin{aligned}Z_l=1+\frac{ig_W^2}{(2\pi)^4}\int d^nq\bigg\{&\frac{e^2}{g_W^2}D_L^{AA}\frac{1}{q^2}+\frac{1}{8}D_L^{WW}\left(\frac{1}{q^2-2p\cdot q}+2\frac{q^2+m_l^2}{q^2(p-q)^2}\right)(1+\gamma_5)\\&-\frac{1}{8}\gamma_5\bigg[\frac{m_l^2}{M_Z^2q^2(p-q)^2}+\left(\tilde{D}_T^{WW}+4\frac{e}{g_W}\tilde{D}_T^{WA}\right)\left(3-n+\frac{2-n}{2}\frac{q^2}{m_l^2}\right)(q^2-2p\cdot q)^{-1}\\&+D_T^{WW}[3-n+(2-n)q^2m_l^{-2}+m_l^2q^{-2}](p-q)^{-2}\\&+\frac{n-2}{2m_l^2}\left(2D_T^{WW}+\tilde{D}_T^{WW}+4\frac{e}{g_W}\tilde{D}_T^{WA}\right)+D_T^{WW}q^{-2}\bigg]\bigg\}+z_l\ ,\end{aligned}\tag{B4}$$

where we have suppressed the argument q^2 of the various propagators. z_l is again a gauge-independent constant, which contains a logarithmic infrared divergence. We have used that $p^2=-m_l^2$.

For a (incoming) nucleon we have

$$\begin{aligned}Z_N=1+\frac{ig^2}{(2\pi)^4}\int d^nq\bigg\{&\frac{1+\tau_3}{2}\frac{e^2}{g^2}D_L^{AA}q^{-2}+\tfrac{3}{8}(D_L^{UU}-D_L^{VV})q^{-2}+\tfrac{3}{4}D_L^{VV}\frac{1}{q^2-2p\cdot q}\\&-\tfrac{1}{4}\gamma_5\frac{1}{q^2-2p\cdot q}\left[3D_L^{UV}+\left(2D_T^{UV}+\tilde{D}_T^{UV}+\sqrt{2}\frac{e}{g}(1+\tau_3)\tilde{D}_T^{VA}\right)[3-n-(n-2)p\cdot qm^{-2}]\right]\bigg\}+z_N\ .\end{aligned}\tag{B5}$$

z_N is a gauge-independent constant, which is different for protons and neutrons. For the proton it contains the usual logarithmic infrared divergence. We have suppressed the argument q of the propagators and we have $p^2=-m^2$. Finally, we list here the expressions that were obtained for the fermion masses in the one-loop approximation:

$$M_\nu=0\ ,$$

$$\begin{aligned}\frac{M_l}{m_l}=1&-\tfrac{1}{2}\frac{i}{(2\pi)^4}\frac{g_W}{M_Z}D_\sigma^{Zi}(0)T_i\\&+\frac{1}{8}\frac{ig_W^2}{(2\pi)^4}\int d^nq\bigg\{\frac{1}{M_Z^2}\left(\frac{1}{q^2+m_l^2}+\frac{m_l^2}{q^2(p-q)^2}\right)-\left[\tilde{D}_T^{WW}\left(3-n+\frac{2-n}{2}\frac{q^2}{m_l^2}\right)-D_\sigma^{ZZ}\frac{q^2+4m_l^2}{M_Z^2}\right]\frac{1}{q^2-2p\cdot q}\\&-D_T^{WW}[3-n+(2-n)q^2m_l^{-2}+m_l^2q^{-2}](p-q)^{-2}+\tfrac{1}{2}(2-n)m_l^{-2}(2D_T^{WW}+\tilde{D}_T^{WW})\\&-D_T^{WW}q^{-2}-4\frac{e^2}{g_W^2}m_l^{-2}\left(\tilde{D}_T^{AA}+\tfrac{1}{2}\frac{g_W}{e}\tilde{D}_T^{WA}\right)\left(n-2+\frac{4m_l^2+(2-n)q^2}{q^2-2p\cdot q}\right)\bigg\}_{p^2=-m_l^2}\ ,\end{aligned}$$

$$\begin{aligned}\frac{m_N}{m}=1&-\frac{i}{(2\pi)^4}\tfrac{1}{2}\sqrt{2}g\epsilon^{-1}M_U^{-1}D_\sigma^{\Sigma i}(0)T_i\\&+\frac{i}{(2\pi)^4}\tfrac{1}{4}g^2\int d^nq(q^2-2p\cdot q)^{-1}\{\epsilon^{-2}M_U^{-2}[3+D_\sigma^{\Sigma\Sigma}(4m^2+q^2)]\\&\qquad-[D_T^{UU}+D_T^{VV}+\tfrac{1}{2}\tilde{D}_T^{UU}+\tfrac{1}{2}\tilde{D}_T^{VV}+2e^2g^{-2}(1+\tau_3)(\tilde{D}_T^{AA}+\tfrac{1}{2}\sqrt{2}ge^{-1}\tilde{D}_T^{UA})]\\&\qquad\times[2-(n-2)(p\cdot q)m^{-2}]+(n-1)(2D_T^{VV}+\tilde{D}_T^{VV})\}_{p^2=-m^2}\ .\end{aligned}\tag{B6}$$

APPENDIX C: THE BOX-DIAGRAM CONTRIBUTIONS TO G_A

In this appendix we will list the various gauge-independent contributions from the box graphs. These diagrams are all finite and of order $g_W{}^2$ or e^2. We distinguish two different contributions. The first one can be written in the current-current form. Here we have ignored all terms that do not have the axial-vector form. The second contribution cannot be written in the current-current form. In this case we have not yet taken the limit of vanishing momentum transfer. All the expressions have been normalized with respect to the lowest-order value of G_A.

We first give the current-current contribution:

$$\begin{aligned}
&e^2\tilde{D}_T^{AA}\sqrt{2}\,gg_W{}^{-1}\frac{D_T^{VW}}{q^2}\left(\frac{1}{n}-\frac{q^2}{p^2-2p\cdot p_l}-\frac{m_l{}^2}{m_l{}^2+Q^2}\,\frac{2p_\nu\cdot q}{p^2-2p\cdot p_l}-\frac{q^2}{q^2-2q\cdot p_n}\right)\\
&\quad-eg^2g_W{}^{-1}(D_T^{UW}\tilde{D}_T^{VA}+D_T^{VW}\tilde{D}_T^{UA})\frac{1}{p^2}\left(1-\frac{m_l{}^2}{p^2-2p\cdot p_l}\right)\\
&\quad\quad-g^2\left(\frac{1}{p^2+2p\cdot p_N}-\frac{1}{n}\,\frac{1}{p^2}\right)[D_T^{UW}\tilde{D}_T^{VW}+D_T^{VW}\tilde{D}_T^{UW}+eg_W{}^{-1}(D_T^{UW}\tilde{D}_T^{VA}+D_T^{VW}\tilde{D}_T^{UA})+\sqrt{2}\,eg^{-1}D_T^{VW}\tilde{D}_T^{WA}]\\
&\quad\quad-\tfrac{1}{2}g^2(D_T^{UW}\tilde{D}_T^{VW}+D_T^{VW}\tilde{D}_T^{UW}+\sqrt{2}\,eg^{-1}D_T^{VW}\tilde{D}_T^{WA})\left[\frac{2}{p^2}+\frac{1}{2}\,\frac{1}{p^2-2p\cdot p_l}\left(1-\frac{q^2}{p^2}-\frac{2m_l{}^2}{q^2}\right)-\frac{1}{q^2}\,\frac{m_l{}^2}{p^2+2p\cdot p_\nu}\right].
\end{aligned}\tag{C1}$$

In this result we have ignored the overall factor

$$-i(2\pi)^{-4}M_Z{}^2\Delta\int d^np\int d^rq\,\delta^{(n)}(p+q-Q)$$

and the momentum conventions are $p_l-p_\nu=Q=p_n-p_p$. We have generally taken the limit $Q^2=0$, except for the first term containing the photon propagator where we encounter infrared singularities. The momentum of the neutral gauge-field propagators is p, and that of the charged ones q.

We now give the contributions which cannot directly be written in current-current form. We have taken out the same factor as above and use the same momentum assignments:

$$\begin{aligned}
&\tfrac{1}{4}g^2[\gamma_\rho(\tilde{D}_T^{UW}+2\sqrt{2}\,eg^{-1}\tilde{D}_T^{WA}\tilde{D}_T^{VW}\gamma_5)S_p\gamma_\mu(D_T^{UW}+D_T^{VW}\gamma_5)\\
&\quad-\gamma_\mu(D_T^{UW}+D_T^{VW}\gamma_5)S_n\gamma_\rho(\tilde{D}_T^{UW}+\tilde{D}_T^{VW}\gamma_5)][\gamma_\mu S_\nu\gamma_\rho(1+\gamma_5)-\tfrac{1}{2}\gamma_\rho(1+\gamma_5)S_l\gamma_\mu(1+\gamma_5)]\\
&\quad+gg_W{}^{-1}[-\sqrt{2}\,e^2\tilde{D}_T^{AA}\gamma_\rho S_\mu(D_T^{UW}+D_T^{VW}\gamma_5)+\tfrac{1}{2}eg\gamma_\mu(D_T^{UW}+D_T^{VW}\gamma_5)S_n\gamma_\rho(\tilde{D}_T^{UA}+\tilde{D}_T^{VA}\gamma_5)\\
&\quad\quad-\tfrac{1}{2}eg\gamma_\rho(\tilde{D}_T^{UA}+\tilde{D}_T^{VA}\gamma_5)S_p\gamma_\mu(D_T^{UW}+D_T^{VW}\gamma_5)][\gamma_\rho S_l\gamma_\mu(1+\gamma_5)]\\
&\quad-\tfrac{1}{2}g^2mm_l\left(\frac{\not{q}\gamma_\rho(\tilde{D}_T^{UW}\gamma_5+\tilde{D}_T^{VW})}{q^2+2q\cdot p_p}+\frac{\gamma_\rho\not{q}(\tilde{D}_T^{UW}\gamma_5-\tilde{D}_T^{VW})}{q^2-2q\cdot p_n}\right)\left(\frac{(\not{q}-im_l)\gamma_\rho(1+\gamma_5)}{p^2+2p\cdot p_\nu}+\frac{im_l\gamma_\rho(1+\gamma_5)}{p^2-2p\cdot p_l}\right)D(q)\\
&\quad-g^2eg_W{}^{-1}mm_l\left(\frac{\not{q}\gamma_\rho(\tilde{D}_T^{UA}\gamma_5+\tilde{D}_T^{VA})}{q^2+2q\cdot p_p}+\frac{\gamma_\rho\not{q}(\tilde{D}_T^{UA}\gamma_5-\tilde{D}_T^{VA})}{q^2-2q\cdot p_n}\right)\left(\frac{\gamma_\rho(\not{q}+im_l)(1+\gamma_5)}{p^2-2p\cdot p_l}\right)D(q)\\
&\quad+egmm_l\left(\frac{\gamma_\rho\not{q}\gamma_5}{q^2-2q\cdot p_n}\right)\left(\frac{(\not{q}-im_l)\gamma_\rho(1+\gamma_5)}{p^2+2p\cdot p_\nu}+\frac{im_l\gamma_\rho(1+\gamma_5)}{p^2-2p\cdot p_l}\right)\tilde{D}_T^{WA}(p)D(q)\\
&\quad-\tfrac{1}{2}g^2mm_l\left(\frac{\not{p}\gamma_\mu(D_T^{UW}\gamma_5+D_T^{VW})}{p^2+2p\cdot p_p}+\frac{\gamma_\mu\not{p}(D_T^{UW}\gamma_5-D_T^{VW})}{p^2-2p\cdot p_n}\right)\left(\frac{\not{p}\gamma_\mu(1+\gamma_5)}{p^2-2p\cdot p_l}\right)\tilde{D}(p)\\
&\quad+g^2(mm_l)^2\left(\frac{\not{p}}{p^2+2p\cdot p_p}+\frac{\not{p}}{p^2-2p\cdot p_n}\right)\left(\frac{\not{p}(1+\gamma_5)}{p^2-2p\cdot p_l}\right)\tilde{D}(p)D(q)\\
&\quad-e^2gg_W{}^{-1}2\sqrt{2}\,mm_l\left(\frac{\gamma_\rho\not{q}\gamma_5}{q^2-2q\cdot p_n}\right)\left(\frac{\gamma_\rho(\not{q}+im_l)(1+\gamma_5)}{p^2-2p\cdot p_l}\right)\tilde{D}_T^{AA}D(q)\\
&\quad-\tfrac{1}{2}g^2\frac{mm_l}{\epsilon M_UM_Z}\left[D_T^{UW}\left(\frac{\gamma_\mu(2im-\not{p})}{p^2-2p\cdot p_n}+\frac{(2im+\not{p})\gamma_\mu}{p^2+2p\cdot p_p}\right)+D_T^{VW}\left(\frac{\gamma_\mu(2im+\not{p})}{p^2-2p\cdot p_n}+\frac{(2im+\not{p})\gamma_\mu}{p^2+2p\cdot p_p}\right)\gamma_5\right]\\
&\quad\quad\times\left(\frac{(-\not{p}+2im_l)\gamma_\mu(1+\gamma_5)}{p^2-2p\cdot p_l}\right)D_\sigma^{\Sigma Z}(p)-g^2\frac{m^2m_l{}^2}{\epsilon M_UM_Z}\left(\frac{\not{q}\gamma_5}{q^2+2q\cdot p_p}+\frac{\not{q}\gamma_5}{q^2-2q\cdot p_n}\right)\left(\frac{(-\not{p}+2im_l)(1+\gamma_5)}{p^2-2p\cdot p_l}\right)D_\sigma^{\Sigma Z}(p)D(q).
\end{aligned}\tag{C2}$$

We have used the following conventions. The first term in the large brackets is supposed to be sandwiched between the proton and neutron spinors $\bar{u}_p(p_p)$ and $u_n(p_n)$, while the second term is sandwiched between the electron and neutrino spinors $\bar{u}_l(p_l)$ and $u_\nu(p_\nu)$. All the spinors satisfy the Dirac equation $(\not{p} - im)u(p) = \bar{u}(p)(\not{p} - im) = 0$. Furthermore, we have introduced the following notation:

$$
\begin{aligned}
&D(q) = \frac{D_T^{VW}(q)}{q^2} - \frac{\sqrt{2}}{2}\,\frac{g_W}{g}\,\frac{1}{M_Z{}^2\Delta}\left(\frac{1}{q^2} - \frac{1}{q^2+\mu^2}\right), \quad \tilde{D}(p) = \frac{\tilde{D}_T^{VW}(p)}{p^2} - \frac{\sqrt{2}}{2}\,\frac{g_W}{g}\,\frac{1}{M_Z{}^2\Delta}\left(\frac{1}{p^2} - \frac{1}{p^2+\mu^2}\right),\\
&S_p = (\not{p}_n - \not{q} - im)^{-1} = (\not{p}_p + \not{p} - im)^{-1}, \quad S_n = (\not{p}_n - \not{p} - im)^{-1} = (\not{p}_p + \not{q} - im)^{-1},\\
&S_l = (\not{p}_\nu + \not{q} - im_l)^{-1} = (\not{p}_l - \not{p} - im_l)^{-1}, \quad S_\nu = (\not{p}_\nu + \not{p})^{-1} = (\not{p}_l - \not{q})^{-1}.
\end{aligned} \tag{C3}
$$

Notice that the functions D and $\tilde{D}$ will only exhibit a pole at zero momentum if μ^2, the lowest-order pion mass, is taken equal to zero.

APPENDIX D: SOME CONTRIBUTIONS TO Δ_{GT} QUADRATIC IN THE GAUGE-FIELD PROPAGATORS

In Sec. VI we have described how the triangle diagrams with two gauge-field propagators were reduced to expressions with only one such propagator. The effect of this reduction method was, however, not complete, and we were left with some terms. These terms will be listed in this appendix. We will distinguish two different contributions. The first one originates from the nucleon and lepton triangle diagrams:

$$
\begin{aligned}
&-\frac{i}{(2\pi)^4} M_Z{}^2 \Delta g\, g_W^{-1}\sqrt{2}\,\Bigg\{\left(e^2\tilde{D}_T^{AA} + \frac{\sqrt{2}}{2}\,eg\tilde{D}_T^{UA}\right)D_T^{VW}\left[\frac{2}{n}\,\frac{1}{q^2} + \frac{2p\cdot Q}{p^2Q^2} - \left(1 - n + \frac{(2-n)2q\cdot p_\nu}{m_l{}^2} + \frac{m_l{}^2}{m_l{}^2+Q^2}\,\frac{2q\cdot p_\nu}{q^2}\right)\right.\\
&\quad\left.\times\frac{1}{p^2 - 2p\cdot p_l} + \frac{n-2}{p^2+2p\cdot p_n}\,\frac{p\cdot p_n}{m^2}\right] - \frac{\sqrt{2}}{2}\,eg\tilde{D}_T^{VA}\,D_T^{UW}\left[\frac{1}{p^2} - \left(1 - n + (2-n)\frac{2q\cdot p_\nu}{m_l{}^2} + \frac{2q\cdot p_\nu}{q^2}\right)\frac{1}{p^2-2p\cdot p_l}\right.\\
&\quad\left.+\frac{n-2}{p^2+2p\cdot p_n}\,\frac{m^2 - p\cdot p_n}{m^2}\right]\Bigg\} - \frac{i}{(2\pi)^4} M_Z{}^2\frac{\Delta}{2}\,g^2\left(\tilde{D}_T^{VW}D_T^{UW} - \tilde{D}_T^{UW}D_T^{VW} - \sqrt{2}e\,g^{-1}\tilde{D}_T^{WA}D_T^{VW}\right)\\
&\quad\times\left[-\left((2-n)\frac{2p\cdot p_\nu}{m_l{}^2} - 1 - \frac{m_l{}^2}{q^2} - \frac{2p\cdot p_\nu}{q^2}\right)\frac{1}{p^2+2p\cdot p_\nu}\right.\\
&\quad\left.+\left((2-n)\frac{2q\cdot p_\nu}{m_l{}^2} - 1 + \frac{m_l{}^2p\cdot q}{p^2q^2} + \frac{Q\cdot p}{p^2}\,\frac{2q\cdot p_\nu}{q^2}\right)\frac{1}{p^2-2p\cdot p_l} + \frac{p\cdot q}{p^2q^2}\,\frac{p^2-q^2}{Q^2}\right].
\end{aligned} \tag{D1}
$$

The neutral gauge-field propagators carry momentum p, the charged ones q. The integration $\int d^np\,d^nq \times \delta^{(n)}(p+q-Q)$ has been suppressed. Notice that we have not always taken the limit $Q^2 \to 0$. However, it is crucial that the result (D1) is both ultraviolet convergent and of order e^2 or $g_W{}^2$.

The second contribution comes from the propagator diagrams. The reduction method goes along the same lines, and we have found the following result:

$$\tfrac{1}{2}g^2M_Z^2[D_T^{VV}(\tilde{D}_T^{UU}+\sqrt{2}eg^{-1}\tilde{D}_T^{UA})-D_T^{UU}\tilde{D}_T^{VV}]$$

$$+\tfrac{1}{2}g^2[M_Z^2(1+\epsilon^2)-\tfrac{1}{2}g_W^2g^{-2}M_U^2\epsilon^2][D_T^{UV}(\tilde{D}_T^{UU}+\sqrt{2}eg^{-1}\tilde{D}_T^{UA})-D_T^{UU}(\tilde{D}_T^{UV}+\sqrt{2}\ eg\tilde{D}_T^{VA})+D_T^{VV}\tilde{D}_T^{UV}-D_T^{UV}\tilde{D}_T^{VV}]$$

$$+g^2M_Z^2\Delta(1+\epsilon^2)[D_T^{VW}(\tilde{D}_T^{UW}+eg_W^{-1}\tilde{D}_T^{UA})-D_T^{UW}(\tilde{D}_T^{VW}+eg_W^{-1}\tilde{D}_T^{VA})]$$

$$+\frac{\sqrt{2}}{4}gg_W\left(M_Z^2(1+2\epsilon^2)+\frac{1}{2}\frac{g_W^2}{g^2}M_U^2(3+2\epsilon^2)\right)\{(D_T^{UU}+D_T^{UV})\tilde{D}_T^{VW}-D_T^{VW}[\tilde{D}_T^{UU}+\tilde{D}_T^{L}\sqrt{2}eg^{-1}(\tilde{D}_T^{UA}+\tilde{D}_T^{VA})]$$

$$+(D_T^{VV}+D_T^{UV})(D_T^{UW}+\sqrt{2}eg^{-1}\tilde{D}_T^{WA})-D_T^{UW}(\tilde{D}_T^{VV}+\tilde{D}_T^{UV})\}-\frac{\sqrt{2}}{4}gg_W\left(M_Z^2(1+\epsilon^2)-\frac{1}{2}\frac{g}{g}M_U^2(1+2\epsilon^2)\right)$$

$$\times\{(D_T^{UU}+D_T^{UV})(\tilde{D}_T^{UW}+\sqrt{2}eg^{-1}\tilde{D}_T^{WA})-D_T^{UW}[\tilde{D}_T^{UU}+\tilde{D}_T^{UV}+\sqrt{2}eg^{-1}(\tilde{D}_T^{UA}+\tilde{D}_T^{VA})]+(D_T^{VV}+D_T^{UV})_T^{VW}-D_T^{VW}(\tilde{D}_T^{VV}+\tilde{D}_T^{UV})\}$$

$$+\frac{\sqrt{2}}{2}g_W^3g^{-1}M_U^2(1+\epsilon^2)\{(D_T^{UW}+D_T^{VW})(\tilde{D}_T^{WW}+eg_W^{-1}\tilde{D}_T^{WA})-D_T^{WW}[\tilde{D}_T^{UW}+\tilde{D}_T^{VW}+eg_W^{-1}(_T^{UA}+\tilde{D}_T^{VA})]\}$$

$$+\left(e^2\tilde{D}_T^{AA}+\frac{\sqrt{2}}{2}eg\tilde{D}_T^{UA}\right)\Big[(M_Z^2+\tfrac{1}{2}g_W^2g^{-2}M_U^2)\left((1+\epsilon^2)D_T^{VV}-\frac{\sqrt{2}}{2}g_Wg^{-1}D_T^{VW}\right)$$

$$+\left(M_Z^2(1+\epsilon^2)+\tfrac{1}{2}g_W^2g^{-2}M_U^2(2+\epsilon^2)\right)\left(D_T^{UV}-\frac{\sqrt{2}}{2}g_Wg^{-1}D_T^{VW}\right)+\tfrac{1}{2}g_W^2g^{-2}M_U^2(1+\epsilon^2\ D_T^{UU}-\frac{\sqrt{2}}{2}g_Wg^{-1}D_T^{UW}\Big)\Big]$$

$$+\left(e^2\tilde{D}_T^{AA}+eg_W\tilde{D}_T^{WA}\right)\Big[-\frac{1}{2}g_W^2g^{-2}M_U^2(2+\epsilon^2)D_T^{WW}+\frac{\sqrt{2}}{2}g_Wg^{-1}M_U^2(1+\epsilon^2)D_T^{UW}$$

$$+(M_Z^2+\tfrac{1}{2}g_W^2g^{-2}M_U^2)(1+\epsilon^2)^2\sqrt{2}g\,g_W^{-1}D_T^{VW}\Big]$$

$$+\frac{\sqrt{2}}{2}eg\tilde{D}_T^{VA}\Big([M_Z^2(1+\epsilon^2)+\tfrac{1}{2}g_W^2g^{-2}M_U^2(2+\epsilon^2)]D_T^{UU}+[M_Z^2\epsilon^2+g_W^2g^{-2}M_U^2(1+\epsilon^2)]D_T^{L}$$

$$-[M_Z^2(2+\epsilon^2)+\tfrac{1}{2}g_W^2g^{-2}M_U^2(3+\epsilon^2)]\frac{\sqrt{2}}{2}g_Wg^{-1}D_T^{UW}-\frac{\sqrt{2}}{4}gg^{-3}M_U^2(1+\epsilon^2)D_T^{VW}\Big).\quad\text{(D2)}$$

We have ignored an overall factor and integration,

$$\frac{-i}{(2\pi)^4}\frac{1}{M_Z^2\Delta}(n-1)\int d^np\,d^nq\,\delta^{(n)}(p+q-Q)\frac{p^2-q^2}{Q^2},$$

and we have used the same momentum assignments as in the previous result. Alsche result (D2) is ultraviolet convergent, as can be deduced from the following argument. We can write a:rm of the form

$$\frac{p^2-q^2}{Q^2}D_1(p^2)D_2(q^2)\ \text{ at }Q^2=0\ \text{ as }\ \frac{d}{dQ^2}\{[p^2D_1(p^2)]D_2(q^2)-D_1(p^2)[q^2D_2(q^2)]\}.$$

We then use the Eqs. (A3) and (A4) to obtain terms of the form $(d/dQ^2)[D_3(p^2)D_4(q^2)]$, ιs well as $(d/dQ^2)[D_1(p^2)]$ and $(d/dQ^2)[D_2(q^2)]$. The first term is finite because of the differentiaon with respect to an external momentum. The other terms vanish, because their dependence on Q^2 is ιly superficial. If we use this technique, it turns out that Eq. (D2) can be written in a much simpler fon.

$$
\begin{aligned}
&-\frac{i}{(2\pi)^4}(n-1)\frac{\epsilon^2 M_U{}^2}{\Delta}\frac{d}{dQ^2}\Bigg\{\int d^n p\int d^n q\,\delta^{(n)}(p+q-Q)\Big(\tfrac{1}{4}g_W{}^2 D_T^{WW}(\tilde{D}_T^{UU}+\tilde{D}_T^{VV}+2\tilde{D}_T^{UV})+\tfrac{1}{4}g_W{}^2(D_T^{UU}+D_T^{VV}+2D_T^{UV})D_T^{WW}\\
&\quad-\tfrac{1}{2}g_W{}^2(D_T^{UW}+D_T^{VW})(\tilde{D}_T^{UW}+\tilde{D}_T^{VW})+\tfrac{1}{2}g^2(D_T^{VV}\tilde{D}_T^{UU}+D_T^{UU}\tilde{D}_T^{VV}-2D_T^{UV}\tilde{D}_T^{UV})\\
&\quad-\frac{\sqrt{2}}{2}gg_W[D_T^{UW}(\tilde{D}_T^{VV}+\tilde{D}_T^{UV})+(D_T^{VV}+D_T^{UV})\tilde{D}_T^{UW}-D_T^{VW}(\tilde{D}_T^{UU}+\tilde{D}_T^{UV})-(D_T^{UU}+D_T^{UV})\tilde{D}_T^{VW}]\Big)\Bigg\}_{s=Q^2=0}\\
&\quad-\frac{i}{(2\pi)^4}(n-1)M_Z{}^2\Delta\sqrt{2}\,gg_W{}^{-1}\frac{d}{dQ^2}\Bigg\{\int d^n p\int d^n q\,\delta^{(n)}(p+q-Q)\bigg[\Big(e^2\tilde{D}_T^{AA}+\frac{\sqrt{2}}{2}eg\tilde{D}_T^{UA}\Big)D_T^{VW}\\
&\qquad\qquad-\frac{\sqrt{2}}{2}eg\,\tilde{D}_T^{VA}D_T^{UW}\bigg]\Bigg\}_{s=Q^2=0}. \qquad \text{(D3)}
\end{aligned}
$$

This form can then be combined very easily with certain contributions from Z_π (see Appendix B), leading to a complete cancellation of the first block of terms.

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