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Constraints imposed by *CP* conservation in the presence of pseudoparticles*

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We elaborate on an earlier discussion of *CP* conservation of strong interactions which includes the effect of pseudoparticles. We discuss what happens in theories of the quantum-chromodynamics type when we include weak and electromagnetic interactions. We find that strong *CP* conservation remains a natural symmetry if the full Lagrangian possesses a chiral U(1) invariance. We illustrate our results by considering in detail a recent model of (weak) *CP* nonconservation.

I. INTRODUCTION

In a recent letter[1] we have discussed the question of CP conservation of the strong interactions in theories of the quantum-chromodynamics (QCD) type. This paper will be devoted to a more detailed exposition of the arguments given in that note and to the examination of the constraints imposed by natural strong CP conservation on more realistic models. We illustrate the application of our results by discussing in detail a model of weak CP nonconservation due to Weinberg.[2]

In non-Abelian gauge theories the existence of Euclidean solutions labeled by a topological quantum number[3] has been shown to require a more complicated vacuum state[4,5,6] than is included in naive perturbation theory. There are in fact an infinity of possible vacuums which can be labeled by a parameter θ. The Hilbert space of the theory factors into the distinct subspaces of states built on each θ vacuum and there are no transitions between states in different subspaces.[7] The vacuum-to-vacuum transition amplitude in the θ vacuum for a theory with the Lagrangian $\mathcal{L}(A, \phi)$ is given by

$$\langle 0|0\rangle_\theta = \sum_{q=-\infty}^{\infty} \int (dA_\mu)_q \int d\phi \exp\left[\int d^4x\, \mathcal{L}_E(A,\phi)\right] \times \exp(iq\theta)\,, \qquad (1)$$

where A is the non-Abelian gauge field and ϕ represents all other fields occurring in $\mathcal{L}$.[8] $\int (dA_\mu)_q$ means integration over all configurations which satisfy the boundary condition

$$\frac{g^2}{32\pi^2}\int d^4x\, F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} = q\,. \qquad (2)$$

This defines the topologically distinct sectors of the theory. Naive perturbation theory includes only the $q=0$ term of the infinite sum in (1). The factor $\exp(iq\theta)$ appearing in (1) can be considered as a term in the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L} + i\theta\frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}\,. \qquad (3)$$

The appearance of this additional term shows the problem to which we address ourselves. It appears to be a P- and CP-violating term. Thus if $\mathcal{L}$ represents a non-Abelian gauge theory of the strong interactions this term may generate strong P and CP violations, even when $\mathcal{L}$ is chosen to exclude such effects. The pressing question is then to find under what conditions a theory like QCD describes a world such as the one in which we live, which has no strong P and CP violations. Our answer[1] is that $\mathcal{L}$ must possess a chiral U(1) invariance, such that changes in θ are equivalent to changes in the definitions of the various fields in $\mathcal{L}$ and have no physical consequences. Any such theory is equivalent to a $\theta=0$ theory and this has no strong P and CP violations. This property has previously been observed for theories where $\mathcal{L}$ represents a non-Abelian gauge field coupled only to massless fermions.[4] Our new contribution is that it remains true when some fermion masses are included in $\mathcal{L}$, or even when all strongly interacting fermions become massive, provided that at least one fermion gets its entire mass from a Yukawa coupling to a scalar field, so that the full $\mathcal{L}$ can possess at least a single chiral U(1) invariance.

The plan of our paper is as follows. In Sec. II we give a detailed demonstration of our result for a simple model with only one flavor of fermion coupled to a single complex color-singlet scalar field. Section III discusses how CP violations due to pseudoparticles are avoided in more general theories, in which weak and electromagnetic interactions are included. We choose to discuss, in particular, a model recently proposed by Weinberg,[2] in which the physically observed CP violations arise from scalar exchanges. Weinberg's version of the model does not have the requisite U(1) symmetry and would in fact suffer strong P and CP violations. However, one can readily modify his model to remove this problem, at the price of including some additional scalar multiplets. Section IV contains some concluding remarks.

II. SINGLE-FLAVOR MODEL

Let us begin by an examination of a simple single-flavor model.[1] The Lagrangian is

$$\mathcal{L} = -\tfrac{1}{4} F^a_{\mu\nu}F^a_{\mu\nu} + i\bar{\psi}D_\mu\gamma_\mu\psi + \bar{\psi}\left[G\phi\left(\frac{1+\gamma_5}{2}\right) + G^*\phi^*\left(\frac{1-\gamma_5}{2}\right)\right]\psi - |\partial_\mu\phi|^2 - \mu^2|\phi|^2 - h|\phi|^4 \tag{4}$$

with $\mu^2 < 0$. We note that (4) is formally invariant under the chiral rotation

$$\psi \to \exp(+i\sigma\gamma_5)\psi, \quad \phi \to \exp(-2i\sigma)\phi. \tag{5}$$

However, there is an anomaly in the chiral current[9]

$$\partial^\mu J^5_\mu = (g^2/16\pi^2)F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} \tag{6}$$

so that

$$\begin{aligned}\delta S_{\text{eff}} &= \delta \int d^4x\, \mathcal{L}_{\text{eff}} \\ &= -i \int d^4x (\partial_\mu J^5_\mu)\sigma \\ &= -i(2\sigma)(g^2/32\pi^2) \int d^4x\, F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}.\end{aligned} \tag{7}$$

Clearly then in this theory a chiral rotation redefines the θ parameter

$$\theta \to \theta - 2\sigma. \tag{8}$$

This demonstrates that for any θ value we obtain an equivalent theory. To show that these theories are CP conserving we need a further result:

$$\alpha = \arg(e^{i\theta}G\langle\phi\rangle) = 0. \tag{9}$$

This requirement is trivially satisfied if $\mu^2 > 0$ so that ϕ has a vanishing vacuum expectation value. In the more interesting case when $\langle\phi\rangle \neq 0$ it corresponds to requiring that the fermion mass $G\langle\phi\rangle$ be real when the fields are defined so that $\theta = 0$.[10]

In Ref. 1, we showed explicitly that (9) is satisfied, thereby guaranteeing the P and CP invariance of the model. We present here a more detailed version of our proof.[11]

To see that (9) is satisfied we examine the generating functional of the scalar Green's functions

$$\begin{aligned}Z_\theta(J,J^*) &= e^{iW(s)} \\ &= \sum_q e^{i\theta q}\int (dA_\mu)_q \int d\psi \int d\bar{\psi} \int d\phi \int d\phi^* \\ &\quad\times \exp\left[\int d^4x(\mathcal{L} + J\phi + J^*\phi^*)\right].\end{aligned} \tag{10}$$

The scalar vacuum expectation value is defined by

$$\frac{1}{Z_\theta}\left.\frac{\delta Z_\theta}{\delta J}\right|_{J=J^*=0} = \langle\phi\rangle = \lambda e^{i\beta}, \tag{11}$$

where λ and β are real constants. The argument proceeds as follows:

(1) We make the change of variables

$$\phi = e^{i\beta}(\lambda + \rho + i\sigma), \tag{12}$$

where ρ and σ are real scalar fields.

(2) We use the knowledge that only terms of chirality q contribute in each q sector[12] to allow us to formally integrate out vector and fermion fields and obtain an expression for Z in terms of nonlocal polynomials of the scalar fields.

(3) Using only known reality properties of the polynomials we can write the constraints $\langle\rho\rangle = \langle\sigma\rangle = 0$. We find that they require $\alpha = 0, \pi$. These are stationary points of the scalar potential. To find which is the true minimum we must examine the potential itself. This we cannot do with the full (all-order) generality of our previous argument. However, to leading order in G and h we can write the scalar potential, and we find that the minimum in fact occurs for $\alpha = 0$. We argue that there will always be some range of the parameters for which this is a correct result.

The generating functional (10) can be rewritten

$$\begin{aligned}Z_\theta(J,J^*) = \sum_{q=-\infty}^{\infty}\left(\int dA_\mu\right)_q e^{iq\theta}\int d\psi\int d\bar{\psi}\int d\phi\int d\phi^* \exp[\mathcal{L}(\phi,\phi^*)]\exp\left[+\int d^4x(-\tfrac{1}{4}FF + i\bar{\psi}\not{D}\psi)\right] \\ \times\sum_{n,m}\frac{1}{n!m!}\left[\int d^4x\,\bar{\psi}G\phi\left(\frac{1+\gamma_5}{1}\right)\psi\right]^n\int d^4x'\left[\bar{\psi}G^*\phi^*\left(\frac{1-\gamma_5}{2}\right)\psi\right]^m \exp(J\phi + J^*\phi^*).\end{aligned} \tag{13}$$

However in each q sector only the terms with $n - m = q$ contribute.[12] This can be seen as follows. Focus on the fermion and gauge field integration only. Terms in the path integral containing the structure $[\bar{\psi}(1+\gamma_5)\psi]^n[\bar{\psi}(1-\gamma_5)\psi]^m$ correspond to the expectation value of operators which change chirality by $2n - 2m$. If chirality were conserved then only $n - m = 0$ would contribute. However, the presence of pseudoparticles produces net changes in chirality.[4] It is not difficult to show[13] that in each q sector chirality changes by $\Delta Q_5 = 2q$, which establishes the above result.

We can formally perform the integration of the vector and fermion fields. We use the notation

$$\sum_n\left(\int dA_\mu\right)_q\int d\psi\int d\bar{\psi}\exp\left[\int d^4x(-\tfrac{1}{4}FF+i\bar{\psi}\not{D}\psi)\right]$$
$$\times\exp[\mathcal{L}(\phi\phi^*)]\frac{[\int d^4x\,\bar{\psi}G\phi\frac{1}{2}(1+\gamma_5)\psi]^{n+q}}{(n+q)!}\;\frac{[\int d^4x\,\bar{\psi}G^*\phi^*\frac{1}{2}(1-\gamma_5)\psi]^n}{n!}$$
$$=\sum_n\prod_{i=1}^{n+q}\prod_{j=1}^{n}\int d^4x_i[G\phi(x_i)]\int d^4y_j[G^*\phi^*(y_j)]\,\frac{c_n^q(x_i,y_j)}{n!(n+q)!}\exp[\mathcal{L}(\phi\phi^*)]$$
$$\equiv A_q(\phi\phi^*)(G\phi)^q. \tag{14}$$

We cannot of course write the specific form of the coefficients c_n^q but we note two relevant properties. Because the c_n^q are integrals over bilinear forms of fermion fields they are real. Furthermore, they satisfy the relationship

$$c_n^q(x_i,y_j)=c_n^{-q}(y_j,x_i)\,. \tag{15}$$

This result is simply a reflection of the symmetric role that ϕ and ϕ^* play in (14). Alternatively, it can be gleaned from Eq. (13) and the simple relationship that exists between chiral operators and pseudoparticle number. Since $q=n-m$ obviously $-q=m-n$ and (15) follows immediately. Using this result leads to the property

$$A_{-q}(\phi\phi^*)=A_q^*(\phi\phi^*)\,. \tag{16}$$

Using this notation we find

$$Z_\theta(J,J^*)=\int d\phi\int d\phi^*\left[A_0(\phi\phi^*)+\sum_{n=1}^{\infty}A_n(\phi\phi^*)(Ge^{i\theta}\phi)^n+A_n^*(\phi\phi^*)(G^*e^{-i\theta}\phi^*)^n\right]\exp(J\phi+J^*\phi^*)\,. \tag{17}$$

Furthermore, using (16) and the definitions (9) and (12) this implies

$$Z(J,J^*)=\int d\phi\int d\sigma\left\{A_0(\rho,\sigma^2)+\sum_n[F_n(\rho,\sigma^2)\cos n\alpha-\sigma G_n(\rho,\sigma^2)\sin n\alpha]\right\}\exp[Je^{i\beta}(\lambda+\rho+i\sigma)+J^*e^{-i\beta}(\lambda+\rho-i\sigma)]\,, \tag{18}$$

where F_n and G_n are the real and imaginary parts of $[|G|(\lambda+\rho+i\sigma)]^nA_n(\phi\phi^*)$, respectively. Now we impose the constraints that, by definition, the field ρ and σ have vanishing vacuum expectation values. This gives us

$$\langle\rho\rangle=\int d\rho\int d\sigma\,\rho\left(A_0+\sum_n F_n\cos n\alpha\right)=0\,, \tag{19}$$

$$\langle\sigma\rangle=\int d\rho\int\sigma^2\sum_n G_n\sin n\alpha=0\,, \tag{20}$$

where we have used the fact that any odd power of σ integrates to zero. For any value of α we can choose $\lambda=\lambda(\alpha)$ such that (19) is satisfied. The coefficients G_n are then functions of $\lambda(\alpha)$ and in general (20) will be satisfied only if $\alpha=0,\pi$. We have thus identified two stationary points of the scalar potential. To proceed further we must ask which of these is indeed a true minimum. This requires more knowledge of the details of the potential than we can obtain from the very general arguments given above. However, as we shall discuss below, there will be some range of the parameters G and h and λ for which the true minimum occurs at $\alpha=0$, which then gives a CP-conserving theory.

A more intuitive understanding of this result is perhaps gained by examining an explicit form for the scalar potential. This we can only do in the leading approximation for small G and h. As can be readily seen from (17) the terms with $|q|\geq 2$ contribute only in order G^2, so for the moment we can ignore them. This means we not only are making an approximation to leading order in G and h, but also in $G\lambda$, so the region of its validity may be very limited. (We will return to this point in more detail a little later.)

Let us examine, as usual,[14]

$$\Gamma(\Phi)=\ln Z-\int J\Phi\,,$$

where

$$\frac{\delta\ln Z}{\delta J(x)}=\Phi(x)\,.$$

When $\Phi(x)=\phi$, independent of x, then

$$\Gamma(\phi)=-(2\pi)^4\delta^4(0)V(\phi)\,, \tag{21}$$

where $V(\phi)$ is the scalar potential. In the approximation that all higher-order terms in G and h can be neglected we find that this gives

$$V_\theta(\phi) = U(\phi) - K|G\phi|\cos\alpha \tag{22}$$

with

$$U(\phi) = \mu^2\phi^*\phi + h(\phi^*\phi)^2 . \tag{23}$$

The constant K is real and positive. Formally we can write it as

$$K = \frac{\int (dA_\mu)_1 \int d\psi \int d\bar{\psi} \int d^4x\, \bar{\psi}(x)\tfrac{1}{2}(1+\gamma_5)\psi(x) \exp[\int d^4x'(-\tfrac{1}{4}FF + i\bar{\psi}\not{D}\psi)]}{\int (dA_\mu)_0 \int d\psi \int d\bar{\psi} \exp[\int (-\tfrac{1}{4}FF + i\bar{\psi}\not{D}\psi) d^4x]} . \tag{24}$$

From the form of (22) it is clear that the scalar potential in the θ vacuum does not possess the U(1) symmetry which we required in the original $\mathcal{L}$ and thus in $U(\phi)$. [This same lack of U(1) symmetry was noted by 't Hooft[4] and explains the absence of a Goldstone boson in this theory, even though naively one might have expected to find one. In a more physical model it explains the absence of a ninth light pseudoscalar meson.] However, as we remarked previously, (22) is in fact a rather crude approximation to the potential. Although G and h may be quite small in situations of real physical interest the combination $G\lambda$ is not so small, being in fact the fermion mass scale.

We cannot rigorously improve our estimate (22), but it is worth remarking that if the dilute-gas approximation[6] were valid in the multipseudoparticle sectors then we would in fact obtain (22) to all orders in $G\lambda$. To see this one must examine Eq. (14) in more detail. For constant ϕ and in the dilute-gas (independent-pseudoparticle) approximation the c_n^q factorize into a product of terms each of which corresponds to the integration over the zero mode of a fermion in the presence of a single pseudoparticle or antipseudoparticle at position z and with size λ:

$$c_q^n = \left[\int d^4x \int d^4z \int \frac{d\lambda}{\lambda^5}\,\omega(\lambda)\bar{\psi}_0\left(\frac{x-z}{\lambda}\right)\psi_0\left(\frac{x-z}{\lambda}\right)\right]^{2n+q} \cong K^{2n+q} , \tag{25}$$

where the weight $\omega(\lambda)$ represents the probability of such pseudoparticle configuration. Then the generating functional Z exponentiates and we obtain (22) to all orders in $G\lambda$. However, we are aware that this argument is very crude. It depends on the dilute-gas approximation, and furthermore on a semiclassical treatment of the individual pseudoparticles. Since we are dealing with a strongly interacting gauge theory this is a dangerous game to play. We present it here only because we feel it clarifies our result to some extent. The rigorous all-order arguments that gave us (19) and (20) are sufficient to reach our desired conclusion, that for some range of parameters $\alpha = 0$ is the true minimum of the scalar potential and thus the resultant theory is P and CP conserving.[15]

III. INCLUSION OF WEAK AND ELECTROMAGNETIC INTERACTIONS

So far our arguments have been made in the context of a very simple model. Clearly we really want to investigate a theory like QCD with weak and electromagnetic interactions included. We make the standard assumption that these latter interactions are also governed by a non-Abelian gauge theory, which is, however, spontaneously broken. Then, in addition to the strong non-Abelian group we have a weak non-Abelian group to worry about. In principle, we must also include the effects of the $q \neq 0$ sectors of the integrations over the weak gauge fields. However, 't Hooft[4] has argued that these contributions are cutoff because of the nonvanishing vacuum expectation value of the scalars, which are *not* singlets under this gauge group. He has estimated the strength of the $q_{\text{weak}} \neq 0$ sectors to be of the order of $e^{-2\pi/\alpha}$ and thus truly negligible.

The weak pseudoparticle effects are much too small to account for the observed CP violation in K decays. Thus to obtain physically relevant CP violations one must include this possibility in the weak Lagrangian. However, our work provides certain constraints on how one may incorporate these violations into the theory. Obviously, one must arrange things so that there is no possibility of obtaining strong CP violations arising through the (strong) pseudoparticle sectors. As we shall see, this requires that one violate CP in the weak Lagrangian in such a way that a chiral U(1) symmetry remains effective. We should note that this is a general requirement, even when the weak Lagrangian is CP conserving. We shall illustrate these points below by studying in detail a recent 4-quark model proposed by Weinberg,[2] in which physical CP violations can arise due to the exchange of scalar particles.

Before doing that a further general comment should be made. This concerns the question of whether we can avoid parity violations of order α

when we introduce weak isospin assignments which are not the same for the left- and right-handed quarks. In the context of a theory where all fermion masses arise from Yukawa couplings of scalars to fermions, all parity-violating terms of order α are in the form of complex higher-order corrections to these Yukawa couplings. Since our arguments allow a general (complex) Yukawa coupling such terms do not present a problem to us. The condition (9), or its generalization discussed below, will still apply, where G is now the Yukawa coupling as *corrected by the weak and electromagnetic interactions*.

Let us discuss these ideas in terms of a concrete model for the weak and electromagnetic interactions. A plausible model, with enough richness that it can incorporate weak CP violations, is provided by the 4-quark model discussed recently by Weinberg.[2] The weak group is $SU(2)\times U(1)$ and the quarks transform under the group as two left-handed doublets

$$\begin{pmatrix}\mathcal{P}_1\\ \mathfrak{N}_1\end{pmatrix}_L,\ \begin{pmatrix}\mathcal{P}_2\\ \mathfrak{N}_2\end{pmatrix}_L$$

and right-handed singlets. In order to ensure that the model conserve strangeness and charm naturally[16] there are at most two scalar doublets coupled to the quarks. The relevant terms to consider, in Weinberg's notation, are the Yukawa couplings

$$\mathcal{L}_Y=\sum_{i,j=1}^{2}\Gamma^1_{ij}\,\overline{\mathfrak{N}}_{iR}(\phi_1^{+*}\mathcal{P}_{jL}+\phi_1^{0*}\mathfrak{N}_{jL}) +\sum_{i,j=1}^{2}\Gamma^2_{ij}\,\overline{\mathcal{P}}_{iR}(\phi_2^{0}\mathcal{P}_{jL}-\phi_2^{+}\mathfrak{N}_{jL})+\text{H.c.} \tag{26}$$

and the scalar polynomial P

$$P(\phi)=\sum_r M_r^2(\phi_r^\dagger\varphi_r)+\sum_{r,s}a_{rs}(\phi_r^\dagger\varphi_r)(\phi_s^\dagger\varphi_s) +\sum_{rs}b_{rs}\,\phi_r^\dagger\vec{\tau}\varphi_r\cdot\phi_s^\dagger\vec{\tau}\phi_s+\sum_{rs}c_{rs}\,\varphi_r^\dagger\varphi_s\varphi_r^\dagger\phi_s\,. \tag{27}$$

Hermiticity requires that $c_{rs}=c^*_{sr}$ while the M_r^2, a_{rs}, and b_{rs} must all be real. In order to have a CP-violating theory Weinberg needs at least three different scalar doublets. Then the phases of the c_{rs} cannot simply be absorbed into a redefinition of the scalar fields and physical CP violations will in general occur.

As written by Weinberg the theory does have a U(1) symmetry which can be written, for the relevant fields,[17] as

$$\begin{aligned}&\mathfrak{N}_i\to\exp(i\alpha_{\mathfrak{N}}\gamma_5)\mathfrak{N}_i\,,\\ &\mathcal{P}_i\to\exp(i\alpha_{\mathcal{P}}\gamma_5)\mathcal{P}_i\,,\\ &\phi_1^+\to\exp[i(\alpha_{\mathfrak{N}}+\alpha_{\mathcal{P}})]\varphi_1^+\,,\quad \varphi_1^0\to\exp(2i\alpha_{\mathfrak{N}})\varphi_1^0\,,\\ &\phi_i^+\to\exp[-i(\alpha_{\mathfrak{N}}+\alpha_{\mathcal{P}})]\phi_i^+\,,\quad \phi_i^0\to\exp(-2i\alpha_{\mathcal{P}})\phi_i^0\\ &\qquad\qquad (i\neq 1)\end{aligned} \tag{28}$$

with the further identification

$$\alpha_{\mathfrak{N}}=-\alpha_{\mathcal{P}}\,. \tag{29}$$

Under such a redefinition, using Eq. (8) for each fermion flavor,[18] we have

$$\theta\to\theta-4\alpha_{\mathfrak{N}}-4\alpha_{\mathcal{P}}\,. \tag{30}$$

However, in view of the constraint (29) this gives

$$\theta\to\theta\,. \tag{31}$$

Clearly this U(1) symmetry is not of the type required. Because of the constraint (29) it does not allow us to change θ by a redefinition of fields in $\mathcal{L}$ since the net change in chirality from this rotation is zero. We will shortly see that this theory has strong CP violations. If however we remove the constraint (29) and require $\mathcal{L}$ to be formally invariant under (28) with arbitrary $\alpha_{\mathfrak{N}}$ and $\alpha_{\mathcal{P}}$ we do have a chiral symmetry such that we can change θ by redefining fields in $\mathcal{L}$ and the theory will not have strong CP violations. However, such a symmetry places an additional constraint on $P(\phi)$. This is satisfied provided all $c_{1j}=0$.[19]

We remark that Weinberg's original version, including the constraint (29), leaves open the possibility that ϕ_2 is in fact the same scalar multiplet as ϕ_1. However, if we impose the more general invariance with arbitrary $\alpha_{\mathfrak{N}}$ and $\alpha_{\mathcal{P}}$ then clearly ϕ_1 and ϕ_2 must be different multiplets since they have different transformation properties. *The theory with only a single scalar multiplet appearing in the Yukawa couplings does not have strong P and CP conservation because it also lacks a chiral U(1) symmetry that allows us to change θ.*

The simplest way to see that Weinberg's version of the theory is not P and CP conserving is to examine the (admittedly crude) leading approximation to the effective potential. The derivation proceeds as before except that in a theory with N fermion flavors the q-pseudoparticle sector gets nonvanishing contributions only from terms of chirality qN.[12] As discussed in the Appendix, we find

$$V_\theta(\phi)=\tilde{P}(\phi)-\tilde{K}\,\mathrm{Re}[(\det\tilde{\Gamma}^1)(\det\tilde{\Gamma}^2)(\phi_1^*\phi_2)^2e^{i\theta}]\,, \tag{32}$$

where $\tilde{P}$ and $\tilde{\Gamma}$ represent the renormalized scalar polynomial and Yukawa couplings, corrected to all orders in the weak and electromagnetic interactions and the constant $\tilde{K}$ [the multiflavor generalization of K in Eq. (24)] is real and positive and includes weak corrections. Now $\tilde{P}(\phi)$ already contains the term

$$\mathrm{Re}\tilde{c}_{12}(\phi_1^*\phi_2)^2 , \qquad (33)$$

so it is clear that the additional term from the $q \neq 0$ sectors is just a change in the effective $\tilde{c}_{12}$ parameter. Minimization of (32) corresponds to the condition

$$\arg\{[\tilde{c}_{12}+(\det\tilde{\Gamma}^1)(\det\tilde{\Gamma}^2)e^{i\theta}]e^{2i(\beta_2-\beta_1)}\}=0 , \qquad (34)$$

where β_1 and β_2 are the phases of ϕ_1 and ϕ_2 at the potential minimum. However, the generalization of Eq. (9), the requirement that fermion masses be real when the fields are defined so that $\theta=0$, is

$$\arg[(\det\tilde{\Gamma}^1)(\det\tilde{\Gamma}^2)e^{i\theta}e^{2i(\beta_2-\beta_1)}]=0 . \qquad (35)$$

It is quite clear that (34) and (35) are not compatible unless $\tilde{c}_{12}=0$. However, this is exactly what is achieved by imposing the more general invariance (28) with arbitrary $\alpha_{\mathcal{P}}$ and $\alpha_{\mathfrak{N}}$ on $\mathcal{L}$. With that constraint we can in fact repeat the general arguments which gave us (19) and (20) and find that the constraint (35) corresponds to a stationary point of V to all orders in perturbation theory. This stationary point will be the true minimum for some range of the parameters in the theory, and hence there will be no strong P and CP violations in this version of the theory.

In the Weinberg version of the theory physical CP violations occur due to scalar exchanges if there are at least three scalar doublets ϕ_i. However, this result depends on the fact that there are then three independent complex phases $\arg(c_{ij})$ in $\mathcal{L}$ only two of which can be absorbed by redefining the relative phases of the ϕ_i. By imposing the symmetry (28) we eliminate two of these c_{ij}'s and thereby eliminate the physical as well as the unphysical (strong) CP violations of Weinberg's model. The situation is, however, easily remedied. If there are four scalar doublets then even with invariance under (28), which guarantees strong CP conservation, $P(\phi)$ is once again rich enough in complex parameters c_{ij} to cause CP violations of the type discussed by Weinberg.

IV. CONCLUDING REMARKS

The Weinberg model discussed in the previous section serves as an illustration of a completely general result. We see explicitly that in the absence of a chiral U(1) symmetry at the Lagrangian level the theory cannot be written as one in which all fermion masses are real and $\theta=0$. Imposing a chiral U(1) symmetry remedies this situation and restores strong CP conservation. Physical CP-violating effects must still be introduced at the level of the Lagrangian, since weak pseudoparticle effects are too small to account for them. The Weinberg model discussed above is one possible way to achieve milliweak CP-violating effects. A class of alternative models has been discussed by Lee.[20] In these models the scalar Lagrangian is limited to two doublets and hence is automatically CP conserving. Microweak CP violations arise because of a richer quark structure allowing CP-violating phases to appear in the quark-gluon Lagrangian. These models also readily lend themselves to the imposition of a chiral U(1) symmetry and hence can be written so that strong P and CP violations are avoided naturally.

Finally we observe that although our detailed arguments rely on the inclusion of physical scalar fields in the Lagrangian we expect that the result is probably more general. If the Lagrangian has the requisite U(1) symmetry and fermion masses arise from some (as yet not understood) dynamical mechanism, then we would expect the resulting theory to exhibit strong P and CP conservation. The U(1) symmetry guarantees that the fields can be chosen such that $\theta_{\text{eff}}=0$, and thus has strong P and CP conservation before the dynamical mass generation. Our experience with scalar fields indicates that such strong-interaction symmetries are not spontaneously broken, at least for some range of parameters in the Lagrangian. Thus we believe that this would also be the case in the event that mass generation is dynamic.

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APPENDIX

The expression (32) for the effective potential to leading order in Yukawa couplings is obtained by examining the one-pseudoparticle sector, since all higher pseudoparticle numbers give contributions of higher order in Yukawa couplings. The nonvanishing terms in the $q=1$ sector are those which have $\Delta Q_5=2$ for *each* of the four fermion flavors. Hence we must look for terms in the expansion $\exp(\int d^4x\, \mathcal{L}Y)$ which have one right-handed antispinor and one left-handed spinor for each fermion flavor. Clearly this means we must examine the term $(1/4!)(\mathcal{L}Y)^4$ with $\mathcal{L}Y$ as given in Eq. (26). Each term contains four factors of the form $(\Gamma\overline{F}_R\phi F_L)$, where F is any fermion spinor. These factors commute with each other, so the 1/4!

simply cancels the 4! from counting all orderings of the factors. In order to get two $\bar{\mathcal{N}}_R$ factors and two $\bar{\mathcal{P}}_R$ we clearly must pick only terms quadratic in Γ^1's and Γ^2's. Furthermore only the combinations $\Gamma^i_{11}\Gamma^i_{22}$ and $\Gamma^i_{12}\Gamma^i_{21}$ will appear multiplying the desired set of spinors. Let us examine for example the term multiplying $(\phi_1^{+*})^2$. It is of the form $\Gamma^1_{11}\Gamma^1_{22}(\bar{\mathcal{N}}_{1R}\mathcal{P}_{1L})(\bar{\mathcal{N}}_{2R}\mathcal{P}_{2L})+I^1_{12}\Gamma^1_{21}(\bar{\mathcal{N}}_{1R}\mathcal{P}_{2L})(\bar{\mathcal{N}}_{2R}\mathcal{P}_{1L})$. By a Fierz rearrangement this can clearly be rewritten as $(\det\Gamma^1)\bar{\mathcal{N}}_{1R}\mathcal{P}_{1L}\bar{\mathcal{N}}_{2R}\mathcal{P}_{2L}$.

Rearranging all terms similarly we see that the relevant terms are

$$(\det\Gamma^1\det\Gamma^2)[(\bar{\mathcal{N}}_{1R}\mathcal{P}_{1L})(\bar{\mathcal{N}}_{2R}\mathcal{P}_{2L})(\varphi_1^{+*})^2(\bar{\mathcal{P}}_{1R}\mathcal{N}_{1L})(\bar{\mathcal{P}}_{2R}\mathcal{N}_{2L})(\varphi_2^{+})^2-(\bar{\mathcal{N}}_{1R}\mathcal{P}_{1R}\bar{\mathcal{N}}_{2R}\mathcal{N}_{2L})(\phi_1^{+*}\phi_1^{0*})(\bar{\mathcal{P}}_{1R}\mathcal{N}_{1L})(\bar{\mathcal{P}}_{2R}\mathcal{P}_{2L})(\varphi_2^{+}\phi_2^{0})$$
$$-(\bar{\mathcal{N}}_{1R}\mathcal{N}_{1L})(\bar{\mathcal{N}}_{2R}\mathcal{P}_{2L})(\phi_1^0\varphi_1^{+*})(\bar{\mathcal{P}}_{1R}\mathcal{P}_{1L})(\bar{\mathcal{P}}_{2R}\mathcal{N}_{1L})(\phi_2^0\phi_2^{+})+(\bar{\mathcal{N}}_{1R}\mathcal{N}_{1L})(\bar{\mathcal{N}}_{2R}\mathcal{N}_{2L})(\phi_1^0)^2(\bar{\mathcal{P}}_{1R}\mathcal{P}_{1L})(\bar{\mathcal{P}}_{2R}\mathcal{P}_{2L})(\varphi_2^0)^2].$$

Further Fierz rearrangements allow us to rewrite this in the form

$$\det\Gamma^1\det\Gamma^2(\phi_1^*\phi_2)^2(\bar{\mathcal{N}}_{1R}\mathcal{N}_{1L})(\bar{\mathcal{N}}_{2R}\mathcal{N}_{2L})(\bar{\mathcal{P}}_{1R}\mathcal{P}_{1L})(\bar{\mathcal{P}}_{2R}\mathcal{P}_{2L}).$$

We have written this expression in terms of the Yukawa couplings but we must first integrate out the weak vector-meson effects. This has the effect that $\Gamma\to\tilde{\Gamma}$ but does not otherwise alter our discussion. Performing the remaining integrations over vector and fermion fields leads to a result

$$Ke^{i\theta}\det\tilde{\Gamma}^1\det\tilde{\Gamma}^2(\phi_1^*\phi_2)^2$$

for the $q=1$ sector, where K is a real positive constant. For the $q=-1$ sector one obtains the complex conjugate of this expression by the same arguments. The sum of these gives the expression (32) for the effective scalar potential.

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[7]A similar phenomenon has been discussed in a different context by L. Schulman, Phys. Rev. 176, 1558 (1968).

[8]Here and throughout all fields are Euclidean fields.

[9]For a review, see, for example, S. L. Adler, in *Lectures on Elementary Particles and Quantum Field Theory*, 1970 Brandeis Summer Institute in Theoretical Physics, edited by S. Deser, M. Grisaru, and H. Pendleton (MIT Press, Cambridge, Mass., 1970), Vol. 1.

[10]One is used to assuming that a complex fermion mass can always be made real by a chiral rotation of the fermion fields. Note, however, that in the theory under consideration such a rotation also changes θ. In this case the requirement of P and CP invariance is that stated in Eq. (9).

[11]We acknowledge a conversation with Sidney Coleman which helped to formulate a more polished version of our proof.

[12]This fact is intimately related to the existence of $|q|$-independent zero eigenmodes with definite chirality for the fermion in this sector. It has been pointed out previously in Ref. 6.

[13]The simplest way to show that $\Delta Q_5=2q$ is to construct the conserved current $\tilde{J}_\mu=J_\mu^5-K_\mu$, where $\partial_\mu K_\mu=(g^2/16\pi^2)F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}$. Since the charge $\tilde{Q}=Q_5-\int d^3x\,K_0$ is time independent it follows that $\Delta Q_5=\int d^4x\,(g^2/16\pi^2)F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}=2q$.

[14]See, for example, J. Iliopoulos, C. Itzykson, and A. Martin, Rev. Mod. Phys. 47, 165 (1975).

[15]We are uncertain of the physical consequences of the choice $\alpha=\pi$. This corresponds to a theory with $\theta=0$ but some fermion mass negative though real. Such a theory may also be physically acceptable.

[16]S. L. Glashow and S. Weinberg, Phys.Rev. D 15, 1958 (1977).

[17]The charged weak vector fields also transform nontrivially, $W^\pm\to\exp[\pm i(\alpha_{\mathcal{P}}-\alpha_{\mathcal{N}})]\,W^\pm$.

[18]The weak pseudoparticle effects require the inclusion of a second θ angle which we call θ_w. This will also be redefined by the transformation (28). However, as commented previously all effects of the $q_{\text{weak}}\neq$ sectors, and hence of this angle, are physically negligible.

[19]It is clearly arbitrary whether we choose to make all remaining scalar fields transform like ϕ_1 or ϕ_2 or some like each of them. The Yukawa couplings dictate the form (28) for ϕ_1 and ϕ_2 and thus we eliminate the coupling c_{12} when we impose invariance under (28) with arbitrary $\alpha_{\mathcal{N}}$ and $\alpha_{\mathcal{P}}$. Certain other c_{ij} couplings will also be eliminated, depending on our assignment of transformation properties for the remaining scalar fields. These remaining assignments are irrelevant for our discussion. We use (28) as an example.

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