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Evaluation of effective potential in superspace

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We show that it is possible to calculate the effective potential in superspace without decomposing superfields into component fields. The most general renormalizable Lagrangian for chiral superfields is treated.

I. INTRODUCTION

The concept of superfields[1] is very useful in supersymmetric theories. Feynman rules have been written down,[2] Ward-Takahashi identities derived,[3] and a renormalization program developed[4] all using superfield notations exclusively, without resorting to decomposition into component fields. But so far all calculations of higher-order contributions to the effective potential have been made[5,6,9] using component fields. The purpose of this paper is to treat the problem using superfield notations only.

In calculating one-loop contributions to the effective potential, one goes over to momentum space[7] because the propagators in the shifted fields are diagonal there and evaluation of functional determinants becomes trivial. But this is not so in the case of superpropagators. Since the shifted fields depend on the anticommuting variable θ, the superpropagators are not diagonal even in momentum space. We show in this paper that the problem may be tackled using an ansatz by DeWitt.[8]

In Sec. II, we introduce the most general renormalizable Lagrangian for chiral superfields. We shift the superfields and find the superpropagators for the shifted fields not translationally invariant in θ space. In Sec. III, we calculate the functional determinant in superspace required for the one-loop contribution to the effective potential for the Lagrangian of Sec. II. Some important formulas are collected in the Appendix.

II. ONE-LOOP CONTRIBUTIONS TO EFFECTIVE POTENTIAL FOR CHIRAL SUPERFIELDS

We shall consider the positive-chirality superfields $\Phi_a(x,\theta)$, $a=1,2,\ldots,N$ with their negative-chirality counterparts $\Phi_a^\dagger(x,\theta)$. The most general action functional leading to a renormalizable theory is

$$S[\Phi]=\int d^4x(-\tfrac{1}{2}\bar{D}D)\{\Phi_a^\dagger(-\tfrac{1}{2}\bar{D}D)\Phi_a+(\lambda_a\Phi_a+\tfrac{1}{2}m_{ab}\Phi_a\Phi_b+\tfrac{1}{3}g_{abc}\Phi_a\Phi_b\Phi_c+\text{H.c.})\}\ , \tag{2.1}$$

where m_{ab} and g_{abc} are symmetric in their indices. If we do not impose any restrictions on m_{ab} and g_{abc}, Eq. (2.1) would cover all theories involving only chiral superfields (see Sec. III of Ref. 5). Let us now shift the superfield $\Phi_a(x,\theta)$ by

$$\Phi_a(x,\theta)\to\Phi_a(x,\theta)+\hat{\Phi}_a(\theta)\ , \tag{2.2}$$

where

$$\hat{\Phi}_a(\theta)=\hat{A}_a+\tfrac{1}{4}\bar{\theta}(1+i\gamma_5)\theta\hat{F}_a\ . \tag{2.3}$$

Because of the above shift, we shall pick up additional quadratic terms making the quadratic part of the action functional

$$S_{\text{quad}}[\Phi,\hat{\Phi}]=\int d^4x(-\tfrac{1}{2}\bar{D}D)\tfrac{1}{2}(\Phi_a\ \ \Phi_a^\dagger)\begin{pmatrix}\Phi^0_{ab} & -\tfrac{1}{2}\bar{D}D\delta_{ab}\\ -\tfrac{1}{2}\bar{D}D\delta_{ab} & \Phi^{0\dagger}_{ab}\end{pmatrix}\begin{pmatrix}\Phi_b\\ \Phi_b^\dagger\end{pmatrix}\ , \tag{2.4}$$

where

$$\Phi^0_{ab}=m_{ab}+2g_{abc}\hat{\Phi}_c(\theta)=M_{ab}+\tfrac{1}{4}\bar{\theta}(1+i\gamma_5)\theta f_{ab}\ , \tag{2.5}$$

with

$$M_{ab}=m_{ab}+2g_{abc}\hat{A}_c\ , \tag{2.6}$$

$$f_{ab}=2g_{abc}\hat{F}_c\ . \tag{2.7}$$

The inverse propagator

$$\mathfrak{D}^{-1}(\Phi^0)=\begin{pmatrix}\Phi^0 & -\tfrac{1}{2}\bar{D}D\\ -\tfrac{1}{2}\bar{D}D & \Phi^{0\dagger}\end{pmatrix}\ , \tag{2.8}$$

has the coordinate-space representation

$$\mathfrak{D}^{-1}{}_{ab}(x,\theta;y,\theta')=\begin{pmatrix}\Phi^0_{ab}(\theta)\delta_+(x,\theta;y,\theta') & \delta_{ab}(-\tfrac{1}{2}\bar{D}D)\delta_-(x,\theta;y,\theta')\\ \delta_{ab}(-\tfrac{1}{2}\bar{D}D)\delta_+(x,\theta;y,\theta') & \Phi^{0\dagger}_{ab}(\theta)\delta_-(x,\theta;y,\theta')\end{pmatrix} . \tag{2.9}$$

One can invert Eq. (2.9) to obtain the propagator $\mathfrak{D}$. The result is given in the Appendix.

The one-loop contribution to the effective potential is given by

$$V_1\int d^4x=-\frac{i}{2}\hbar\,\mathrm{Ln\,Det}\,\frac{\mathfrak{D}^{-1}(\Phi^0)}{\mathfrak{D}^{-1}(\hat{\Phi}^0)} , \tag{2.10}$$

where $\hat{\Phi}^0$ specifies the normalization point. Even if we go over to momentum space, the determinant in θ space would remain functional.

III. EVALUATION OF FUNCTIONAL DETERMINANT IN SUPERSPACE

First we observe that

$$\begin{aligned}\mathrm{Det}\mathfrak{D}^{-1}(\Phi^0)&=\begin{vmatrix}\Phi^0 & -\tfrac{1}{2}\bar{D}D\\ -\tfrac{1}{2}\bar{D}D & \Phi^{0\dagger}\end{vmatrix}\\ &=\mathrm{Det}\Phi^0\\ &\quad\times\mathrm{Det}\left(\Phi^{0\dagger}-(-\tfrac{1}{2}\bar{D}D)\frac{1}{\Phi^0}(-\tfrac{1}{2}\bar{D}D)\right).\end{aligned} \tag{3.1}$$

$\mathrm{Det}\Phi^0$ is to be evaluated in positive-chirality superspace, i.e., the space of x and $\frac{1}{2}(1+i\gamma_5)\theta$, while the second determinant is over negative-chirality superspace. Let us write

$$H(\Phi^0)=M\Phi^{0\dagger}-(-\tfrac{1}{2}\bar{D}D)M\frac{1}{\Phi^0}(-\tfrac{1}{2}\bar{D}D) . \tag{3.2}$$

Then we have

$$V_1\int d^4x=-\frac{i}{2}\hbar\,\mathrm{Ln\,Det}\,\frac{\Phi^0}{\hat{\Phi}^0}-\frac{i}{2}\hbar\,\mathrm{Ln\,Det}\,\frac{H(\Phi^0)}{H(\hat{\Phi}^0)} . \tag{3.3}$$

Using the formula Eq. (A7) for the inverse of a chiral superfield we have

$$\frac{1}{\Phi^0(\theta)}=M^{-1}-\tfrac{1}{4}\bar{\theta}(1+i\gamma_5)\theta M^{-1}fM^{-1} . \tag{3.4}$$

Now

$$\mathrm{Ln\,Det}\frac{\Phi^0}{\hat{\Phi}^0}=-\mathrm{Tr}\int_0^\infty\frac{ds}{s}(e^{-i\Phi^0 s}-e^{-i\hat{\Phi}^0 s}) \tag{3.5}$$

and

$$\begin{aligned}\mathrm{Tr}e^{-i\Phi^0 s}&=\mathrm{tr}\int d^4x(-\tfrac{1}{2}\bar{D}D)_+\langle x,\theta|e^{-i\Phi^0 s}|x,\theta\rangle\\ &=\mathrm{tr}\int d^4x(-\tfrac{1}{2}\bar{D}D)_+e^{-i\Phi^0(\theta)s}\\ &\qquad\times\delta_+(x,\theta;y,\theta')|_{x=y,\,\theta=\theta'}\\ &=0\times\int d^4x ,\end{aligned} \tag{3.6}$$

where tr indicates trace over internal indices.

In the above equations and everywhere below, the $i\epsilon$ prescription is used to make the integrations over s exist. Equation (3.6) tells us that the first term in Eq. (3.3) does not contribute to V_1. Next

$$\begin{aligned}\mathrm{Ln\,Det}\frac{H(\Phi^0)}{H(\hat{\Phi}^0)}&=-\mathrm{Tr}\int_0^\infty\frac{ds}{s}(e^{-iH(\Phi^0)s}-e^{-iH(\hat{\Phi}^0)s})\\ &=-\mathrm{tr}\int_0^\infty\frac{ds}{s}\int d^4x(-\tfrac{1}{2}\bar{D}D)_-(\langle x,\theta|e^{-iH(\Phi^0)s}|x,\theta\rangle-\langle x,\theta|e^{-iH(\hat{\Phi}^0)s}|x,\theta\rangle) .\end{aligned} \tag{3.7}$$

To calculate $\langle x,\theta|e^{-iH(\Phi^0)s}|x,\theta\rangle$ we use an ansatz by DeWitt.[8] Let us define

$$D(x,\theta;y,\theta')\equiv\langle x,\theta|1/H|y,\theta'\rangle=i\int_0^\infty ds\langle x,\theta|e^{-iHs}|y,\theta'\rangle\equiv i\int_0^\infty ds\langle x,\theta;y,\theta'|s\rangle . \tag{3.8}$$

Since

$$\left[M\Phi^{0\dagger}(\theta)-(-\tfrac{1}{2}\bar{D}D)M\frac{1}{\Phi^0(\theta)}(-\tfrac{1}{2}\bar{D}D)\right]D(x,\theta;y,\theta')=\delta_-(x,\theta;y,\theta') , \tag{3.9}$$

we find $\langle x,\theta;y,\theta'|s\rangle$ to satisfy the Schrödinger equation

$$\left[M\Phi^{0\dagger}(\theta)-(-\tfrac{1}{2}\bar{D}D)M\frac{1}{\Phi^0(\theta)}(-\tfrac{1}{2}\bar{D}D)\right]\langle x,\theta;y,\theta'|s\rangle=i\frac{\partial}{\partial s}\langle x,\theta;y,\theta'|s\rangle , \tag{3.10}$$

provided we use the normalization

$$\langle x,\theta;y,\theta'|s\rangle_{s=0}=\delta_-(x,\theta;y,\theta'). \tag{3.11}$$

Let us now make the ansatz

$$\langle x,\theta;y,\theta'|s\rangle=\frac{-i}{(4\pi s)^2}\exp(\tfrac{1}{4}\bar\theta\not\partial_x\gamma_5\theta)\exp(\tfrac{1}{4}\bar\theta'\not\partial_y\gamma_5\theta')$$
$$\times\sum_{n=0}^{\infty}(is)^n\left[a_n(x,y;\theta')+\bar\theta\frac{1-i\gamma_5}{2}\psi_n(x,y;\theta')-\tfrac{1}{4}\bar\theta(1-i\gamma_5)\theta Mf_n(x,y;\theta')\right]M^{-1}\exp\left[-i\frac{(x-y)^2}{4s}\right]. \tag{3.12}$$

Using the result

$$\lim_{s\to 0}\frac{-i}{(4\pi s)^2}\exp\left[-i\frac{(x-y)^2}{4s}\right]=\delta^4(x-y) \tag{3.13}$$

we find that the normalization Eq. (3.11) implies

$$a_0=\tfrac{1}{4}\bar\theta'(1-i\gamma_5)\theta'M,\quad \psi_0=-\frac{1-i\gamma_5}{2}\theta'M,\quad f_0=-1\ . \tag{3.14}$$

From Eq. (3.12) and the rules of multiplications for superfields[1] we find

$$-(-\tfrac{1}{2}\bar D D)M\frac{1}{\Phi^0(\theta)}(-\tfrac{1}{2}\bar D D)\langle x,\theta;y,\theta'|s\rangle$$
$$=\frac{-i}{(4\pi s)^2}\exp(\tfrac{1}{4}\bar\theta\not\partial_x\gamma_5\theta)\exp(\tfrac{1}{4}\bar\theta'\not\partial_y\gamma_5\theta')$$
$$\times\sum_{n=0}^{\infty}(is)^n\left[(\Box a_n-ff_n)+\bar\theta\frac{1-i\gamma_5}{2}\Box\psi_n+\tfrac{1}{4}\bar\theta(1-i\gamma_5)\theta(-M\Box f_n)\right]M^{-1}\exp\left[-i\frac{(x-y)^2}{4s}\right]. \tag{3.15}$$

$\Box$ acts on everything on the right. Equation (3.10) leads to

$$\frac{-i}{(4\pi s)^2}\exp(\tfrac{1}{4}\bar\theta\not\partial_x\gamma_5\theta)\exp(\tfrac{1}{4}\bar\theta'\not\partial_y\gamma_5\theta')\exp\left[-i\frac{(x-y)^2}{4s}\right]$$
$$\times\sum_{n=0}^{\infty}(is)^n\left\{\left[(\Box+MM^\dagger)a_n-i\frac{(x-y)^\mu}{s}\partial_\mu a_n-ff_n\right]+\bar\theta\frac{1-i\gamma_5}{2}\left[\Box+MM^\dagger)\psi_n-i\frac{(x-y)^\mu}{s}\partial_\mu\psi_n\right]\right.$$
$$\left.+\tfrac{1}{4}\bar\theta(1-i\gamma_5)\theta\left[-M(\Box+M^\dagger M)f_n+i\frac{(x-y)^\mu}{s}M\partial_\mu f_n+Mf^\dagger a_n\right]\right\}M^{-1}$$
$$=\frac{-i}{(4\pi s)^2}\exp(\tfrac{1}{4}\bar\theta\not\partial_x\gamma_5\theta)\exp(\tfrac{1}{4}\bar\theta'\not\partial_y\gamma_5\theta')\exp\left(-i\frac{(x-y)^2}{4s}\right)$$
$$\times\sum_{n=0}^{\infty}(is)^n\frac{ni}{s}\left[a_n+\bar\theta\frac{1-i\gamma_5}{2}\psi_n-\tfrac{1}{4}\bar\theta(1-i\gamma_5)\theta Mf_n\right]M^{-1}\ , \tag{3.16}$$

where we have canceled out factors in $-2i/s$ and $(x-y)^2/4s^2$ from both sides. Equating various terms, we are led to the following recursion relations:

$$(x-y)^\mu\partial_\mu a_0=(x-y)^\mu\partial_\mu\psi_0=(x-y)^\mu\partial_\mu f_0=0\ , \tag{3.17}$$

$$(n+1)a_{n+1}+(x-y)^\mu\partial_\mu a_{n+1}=-(\Box+MM^\dagger)a_n+ff_n\ , \tag{3.18}$$

$$(n+1)\psi_{n+1}+(x-y)^\mu\partial_\mu\psi_{n+1}=-(\Box+MM^\dagger)\psi_n\ , \tag{3.19}$$

$$(n+1)f_{n+1}+(x-y)^\mu\partial_\mu f_{n+1}=-(\Box+M^\dagger M)f_n+f^\dagger a_n\ . \tag{3.20}$$

From Eq. (3.14) we find that a_0, ψ_0, and f_0 are coordinate independent. Hence Eqs. (3.18)–(3.20) show that a_n, ψ_n, and f_n are also coordinate independent.

Iterating the recursion relations, we have

$$\begin{pmatrix}a_n\\ f_n\end{pmatrix}=\frac{(-1)^n}{n!}\begin{pmatrix}MM^\dagger & -f\\ -f^\dagger & M^\dagger M\end{pmatrix}^n\begin{pmatrix}a_0\\ f_0\end{pmatrix}\ , \tag{3.21}$$

$$\psi_n=\frac{(-1)^n}{n!}(MM^\dagger)^n\psi_0\ . \tag{3.22}$$

Using Eq. (3.14) we find

$$a_n=\frac{(-1)^n}{n!}[\tfrac{1}{4}\bar\theta'(1-i\gamma_5)\theta'X_{11}{}^{2(n)}M-X_{12}{}^{2(n)}]\ , \tag{3.23}$$

$$\psi_n = -\frac{(-1)^n}{n!}\frac{1-i\gamma_5}{2}\theta'(MM^\dagger)^n M\,, \tag{3.24}$$

$$f_n = \frac{(-1)^n}{n!}[\tfrac{1}{4}\bar{\theta}'(1-i\gamma_5)\theta' X_{21}{}^{2(n)}M - X_{22}{}^{2(n)}]\,, \tag{3.25}$$

where

$$X^{2(n)} = (X^2)^n = \begin{pmatrix} MM^\dagger & -f \\ -f^\dagger & M^\dagger M\end{pmatrix}^n \,. \tag{3.26}$$

This completes the determination of $\langle x,\theta;y,\theta'|s\rangle$ and hence $D(x,\theta;y,\theta')$. We are interested in the trace, so we put $x=y$ and $\theta=\theta'$ and obtain

$$\langle x,\theta;x,\theta|s\rangle = \frac{-i}{(4\pi s)^2}\sum_{n=0}^{\infty}(is)^n\frac{(-1)^n}{n!}\{\tfrac{1}{4}\bar{\theta}(1-i\gamma_5)\theta[X_{11}{}^{2(n)} + MX_{22}{}^{2(n)}M^{-1} - 2(MM^\dagger)^n] + \theta\text{-independent terms}\}\,, \tag{3.27}$$

where the θ-independent part would drop out when we integrate over superspace, i.e., $\int d^4x(-\frac{1}{2}\bar{D}D)_-$. Note that $X_{11}{}^{2(n)}, X_{22}{}^{2(n)}$ are $N\times N$ matrices in the space of the indices $a,b=1,2,\ldots,N$.

Using

$$\mathrm{tr}(MX_{22}{}^{2(n)}M^{-1}) = \mathrm{tr}X_{22}{}^{2(n)}\,, \tag{3.28}$$

$$\mathrm{tr}(MM^\dagger) = \tfrac{1}{2}\mathrm{tr}\begin{pmatrix} MM^\dagger & 0 \\ 0 & M^\dagger M\end{pmatrix}\,, \tag{3.29}$$

we find

$$\begin{aligned}\mathrm{tr}\int d^4x(-\tfrac{1}{2}\bar{D}D)_-\langle x,\theta;x,\theta|s\rangle &= \frac{-i}{(4\pi s)^2}\sum_{n=0}^{\infty}(is)^n\frac{(-1)^n}{n!}\mathrm{tr}[X_{11}{}^{2(n)} + X_{22}{}^{2(n)} - 2(MM^\dagger)^n]\int d^4x \\ &= \frac{-i}{(4\pi s)^2}\left\{\mathrm{tr}\exp\left[-i\begin{pmatrix} MM^\dagger & -f \\ -f^\dagger & M^\dagger M\end{pmatrix}s\right] - \mathrm{tr}\exp\left[-i\begin{pmatrix} MM^\dagger & 0 \\ 0 & M^\dagger M\end{pmatrix}s\right]\right\}\int d^4x\,.\end{aligned} \tag{3.30}$$

Observe that the minus sign for the fermion part comes out automatically. From Eq. (3.30) it is clear that if $f=0$, then the trace of $\langle x,\theta;x,\theta|s\rangle$ would disappear. Hence, by selecting $\hat{\Phi}^0$ such that $f=0$ for this, we shall have $\mathrm{Ln\,Det}\,H(\hat{\Phi}^0)=0$. Then

$$\mathrm{Ln\,Det}\frac{H(\Phi^0)}{H(\hat{\Phi}^0)} = -\mathrm{tr}\int_0^\infty\frac{ds}{s}\frac{-i}{(4\pi s)^2}[\exp(-iX^2s) - \exp(-iY^2s)]\,, \tag{3.31}$$

where X^2 is defined by Eq. (3.26) and

$$Y^2 = \begin{pmatrix} MM^\dagger & 0 \\ 0 & M^\dagger M\end{pmatrix} \tag{3.32}$$

is the fermion part of the (mass)^2 matrix.

By using

$$\frac{-i}{(4\pi s)^2} = \frac{1}{(2\pi)^4}\int d^4p\,e^{ip^2s} \tag{3.33}$$

we can put our result in the familiar form

$$\begin{aligned}V_1 &= \frac{i}{2}\hbar\frac{1}{(2\pi)^4}\mathrm{tr}\int d^4p\int_0^\infty\frac{ds}{s}\{\exp[-i(-p^2+X^2)s] - \exp[-i(-p^2+Y^2)s]\} \\ &= -\frac{i}{2}\hbar\frac{1}{(2\pi)^4}\int d^4p\,\mathrm{tr}[\ln(-p^2+X^2) - \ln(-p^2+Y^2)]\,.\end{aligned} \tag{3.34}$$

This result can also be obtained by the explicit method[5,6,9] using component fields. We include a brief derivation.

In terms of component fields the quadratic part of the shifted Lagrangian is

$$\mathcal{L}_{\rm quad} = -A_a^*\Box A_a + F_a^* F_a + \tfrac{1}{2}\bar\psi_a \frac{1-i\gamma_5}{2} i\not\partial\psi_a + \tfrac{1}{2}\bar\psi_a^c \frac{1+i\gamma_5}{2} i\not\partial\psi_a^c$$

$$+\tfrac{1}{2}M_{ab}\left(2A_aF_b - \bar\psi_a^c\frac{1+i\gamma_5}{2}\psi_b\right) + \tfrac{1}{2}M_{ab}^*\left(2A_a^*F_b^* - \bar\psi_a\frac{1-i\gamma_5}{2}\psi_b^c\right) + \tfrac{1}{2}f_{ab}A_aA_b + \tfrac{1}{2}f_{ab}^*A_a^*A_b^* \;,$$

$$= \tfrac{1}{2}(A_a^*\;\; F_a^*\;\; A_a\;\; F_a)\begin{pmatrix} f_{ab}^* & M_{ab}^* & -\delta_{ab}\Box & 0 \\ M_{ab}^* & 0 & 0 & \delta_{ab} \\ -\delta_{ab}\Box & 0 & f_{ab} & M_{ab} \\ 0 & \delta_{ab} & M_{ab} & 0 \end{pmatrix}\begin{pmatrix} A_b^* \\ F_b^* \\ A_b \\ F_b\end{pmatrix} + \tfrac{1}{2}(\bar\psi_a^c\;\; \bar\psi_a)\begin{pmatrix} i\sigma^\mu\hat\partial_\mu\delta_{ab} & -M_{ab} \\ -M_{ab}^* & i\sigma^\mu\partial_\mu\delta_{ab}\end{pmatrix}\begin{pmatrix}\psi_b^c \\ \psi_b\end{pmatrix} \;, \quad (3.35)$$

where for the fermion part we have used two-component notations and $\sigma^\mu = (1,\vec\sigma)$, $\hat\partial_\mu = (\partial_0, -\vec\nabla)$.

Then the one-loop contribution to the effective potential is given by[7]

$$V_1 = -\frac{i}{2}\hbar\frac{1}{(2\pi)^4}\int d^4p\left[\ln\begin{vmatrix} f^\dagger & M^\dagger & p^2 & 0 \\ M^\dagger & 0 & 0 & 1 \\ p^2 & 0 & f & M \\ 0 & 1 & M & 0\end{vmatrix} - \ln\begin{vmatrix}\sigma^\mu\hat p_\mu & -M \\ -M^\dagger & \sigma^\mu p_\mu\end{vmatrix}\right]$$

$$= -\frac{i}{2}\hbar\frac{1}{(2\pi)^4}\int d^4p\,{\rm tr}[\ln(-p^2+X^2) - \ln(-p^2+Y^2)] \;, \quad (3.36)$$

where $\hat p^\mu = (p^0, -\vec p)$.

Now returning to Eq. (3.34) let us discuss the normalization of the effective potential in the one-loop approximation. Including appropriate wave-function renormalization counterterms (no mass and coupling-constant renormalization counterterms required[2,4]), we have

$$V_{\rm eff}(\hat A, \hat F) = -\hat F_a^*\hat F_a + [-\hat F_a(\lambda_a + m_{ab}\hat A_b + g_{abc}\hat A_b\hat A_c) + {\rm H.c.}] + Z_{ab}\hat F_a^*\hat F_b$$

$$+ \frac{\hbar}{64\pi^2}[{\rm tr}(X^4\ln X^2] - {\rm tr}(Y^4\ln Y^2) - {\rm tr}(f^\dagger f + ff^\dagger)(\ln\Lambda^2 + \tfrac{1}{2})] + O(\hbar^2) \;. \quad (3.37)$$

We shall adopt the normalization

$$\left.\frac{\partial^2 V_{\rm eff}}{\partial\hat F_a^*\partial\hat F_b}\right|_{\hat F_a = e_a, \hat A_a = b_a} = -\delta_{ab} \;. \quad (3.38)$$

To retain generality, we leave e_a and b_a arbitrary. Then

$$Z_{ab} = \frac{\hbar}{8\pi^2}g_{acd}^* g_{bcd}(\ln\Lambda^2 + \tfrac{1}{2}) - W_{ab} \;, \quad (3.39)$$

where

$$W_{ab} = \frac{\hbar}{64\pi^2}\frac{\partial^2}{\partial\hat F_a^*\partial\hat F_b}[{\rm tr}(X^4\ln X^2)]_{\hat F_a = e_a, \hat A_a = b_a} \;. \quad (3.40)$$

All infinite terms in $O(\hbar)$ drop out and we have

$$V_{\rm eff}(\hat A, \hat F) = (-\delta_{ab} - W_{ab})\hat F_a^*\hat F_b + [-F_a(\lambda_a + m_{ab}\hat A_b + g_{abc}\hat A_b\hat A_c) + {\rm H.c.}] + \frac{\hbar}{64\pi^2}[{\rm tr}(X^4\ln X^2) - {\rm tr}(Y^4\ln Y^2)] + O(\hbar^2) \;. \quad (3.41)$$

If we define μ^2 by

$$\frac{\hbar}{64\pi^2}{\rm Tr}(f^\dagger f + ff^\dagger)\ln\mu^2 = W_{ab}\hat F_a^*\hat F_b \;, \quad (3.42)$$

then we may write

$$V_{\rm eff}(\hat A, \hat F) = -\hat F_a^*\hat F_a + [-\hat F_a(\lambda_a + m_{ab}\hat A_b + g_{abc}\hat A_b\hat A_c) + {\rm H.c.}] + \frac{\hbar}{64\pi^2}\left[{\rm tr}\left(X^4\ln\frac{X^2}{\mu^2}\right) - {\rm tr}\left(Y^4\ln\frac{Y^2}{\mu^2}\right)\right] + O(\hbar^2) \;. \quad (3.43)$$

The physical potential $V_{\rm eff}(\hat{A})$ is obtained by eliminating the auxiliary fields, $\hat{F}_a$, using

$$-\hat{F}_a^* = \lambda_a + m_{ab}\hat{A}_b + g_{abc}\hat{A}_b\hat{A}_c \tag{3.44}$$

in Eq. (3.43).[9,10]

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APPENDIX

The propagator $\mathfrak{D}$ satisfies the equation

$$\begin{pmatrix} \Phi^0_{ab}(\theta) & -\frac{1}{2}\bar{D}D\delta_{ab} \\ -\frac{1}{2}\bar{D}D\delta_{ab} & \Phi^{0\dagger}_{ab}(\theta) \end{pmatrix} \begin{pmatrix} \mathfrak{D}^{++}_{bc}(x,\theta;y,\theta') & \mathfrak{D}^{+-}_{bc}(x,\theta;y,\theta') \\ \mathfrak{D}^{-+}_{bc}(x,\theta;y,\theta') & \mathfrak{D}^{--}_{bc}(x,\theta;y,\theta') \end{pmatrix} = \begin{pmatrix} \delta_+(x,\theta;y,\theta')\delta_{ac} & 0 \\ 0 & \delta_-(x,\theta;y,\theta')\delta_{ac} \end{pmatrix} . \tag{A1}$$

Firstly, we find

$$\mathfrak{D}^{-+}_{ab}(x,\theta;y,\theta') = -\left(\frac{1}{\Phi^{0\dagger}}\right)_{ac}(-\tfrac{1}{2}\bar{D}D)\mathfrak{D}^{++}_{cb}(x,\theta;y,\theta') , \tag{A2}$$

$$\mathfrak{D}^{+-}_{ab}(x,\theta;y,\theta') = -\left(\frac{1}{\Phi^{0}}\right)_{ac}(-\tfrac{1}{2}\bar{D}D)\mathfrak{D}^{--}_{cb}(x,\theta;y,\theta') . \tag{A3}$$

Using these, we obtain

$$\left[\Phi^0(\theta) - (-\tfrac{1}{2}\bar{D}D)\frac{1}{\Phi^{0\dagger}(\theta)}(-\tfrac{1}{2}\bar{D}D)\right]_{ac}\mathfrak{D}^{++}_{cb}(x,\theta;y,\theta') = \delta_+(x,\theta;y,\theta')\delta_{ab} , \tag{A4}$$

$$\left[\Phi^{0\dagger}(\theta) - (-\tfrac{1}{2}\bar{D}D)\frac{1}{\Phi^{0}(\theta)}(-\tfrac{1}{2}\bar{D}D)\right]_{ac}\mathfrak{D}^{--}_{cb}(x,\theta;y,\theta') = \delta_-(x,\theta;y,\theta')\delta_{ab} . \tag{A5}$$

We define the inverse of the superfield $\Phi(x,\theta)$ (for generality, consider it to be a matrix in some internal space) by

$$\Phi^{-1}(x,\theta)\Phi(x,\theta) = 1 . \tag{A6}$$

Using the rule of multiplication for chiral superfields, we find

$$\Phi^{-1}(x,\theta) = \exp(-\tfrac{1}{4}\bar{\theta}\not{\partial}\gamma_5\theta)\{A^{-1}(x) - \bar{\theta}A^{-1}(x)\psi(x)A^{-1}(x) + \tfrac{1}{4}\bar{\theta}(1+i\gamma_5)\theta[-A^{-1}(x)F(x)A^{-1}(x) + A^{-1}(x)\bar{\psi}^c(x)A^{-1}(x)\psi(x)A^{-1}(x)]\} . \tag{A7}$$

Putting $\Phi(x,\theta) = \Phi^0(\theta)$ we obtain Eq. (3.4).

One can easily solve Eqs. (A4) and (A5) for $\mathfrak{D}^{++}$ and $\mathfrak{D}^{--}$. We just quote the results:

$$\begin{aligned}\mathfrak{D}^{++}(x,\theta;y,\theta') = {} & \exp(-\tfrac{1}{4}\bar{\theta}\not{\partial}_x\gamma_5\theta)\exp(-\tfrac{1}{4}\bar{\theta}'\not{\partial}_y\gamma_5\theta') \\ & \times\Big\{[D^{-1}M^\dagger\tfrac{1}{4}\bar{\theta}'(1+i\gamma_5)\theta' - D^{-1}f^\dagger(\Box+MM^\dagger)^{-1}] - \bar{\theta}\frac{1+i\gamma_5}{2}\theta'(\Box+M^\dagger M)^{-1}M^\dagger \\ & + \tfrac{1}{4}\bar{\theta}(1+i\gamma_5)\theta[-M^\dagger(\Box+MM^\dagger)^{-1}fD^{-1}M^\dagger\tfrac{1}{4}\bar{\theta}'(1+i\gamma_5)\theta' + M^\dagger D^{T\,-1}]\Big\}\,\delta^4(x-y) ,\end{aligned} \tag{A8}$$

$$\begin{aligned}\mathfrak{D}^{--}(x,\theta;y,\theta') = {} & \exp(\tfrac{1}{4}\bar{\theta}\not{\partial}_x\gamma_5\theta)\exp(\tfrac{1}{4}\bar{\theta}'\not{\partial}_y\gamma_5\theta') \\ & \times\Big\{[D^{T\,-1}M\tfrac{1}{4}\bar{\theta}'(1-i\gamma_5)\theta' - D^{T\,-1}f(\Box+M^\dagger M)^{-1}] - \bar{\theta}\frac{1-i\gamma_5}{2}\theta'(\Box+MM^\dagger)^{-1}M \\ & + \tfrac{1}{4}\bar{\theta}(1-i\gamma_5)\theta[-M(\Box+M^\dagger M)^{-1}f^\dagger D^{T-1}M\tfrac{1}{4}\bar{\theta}'(1-i\gamma_5)\theta' + MD^{-1}]\Big\}\,\delta^4(x-y) ,\end{aligned} \tag{A9}$$

where

$$D = (\Box+M^\dagger M) - f^\dagger(\Box+MM^\dagger)^{-1}f \tag{A10}$$

and D^T is its transpose.

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