General lepton structure with naturalness assumptions*

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Two classes of lepton models which have natural μe universality and μe-flavor-diagonal neutral currents are studied within the SU(2) × U(1) gauge-theory framework. One of them has both $V+A$ and $V-A$ decaying negatively charged heavy leptons, vanishing $\mu \rightarrow e\gamma$ and $\mu \rightarrow ee\bar{e}$ decay rates, and Weinberg-Salam–type neutral currents for known leptons. The other, which is vectorlike, has $V-A$ decaying (and probably V decaying) charged heavy leptons and nonvanishing $\mu \rightarrow ee\bar{e}$ decay rates at higher orders. Also an updated analysis of neutrino-electron scattering is presented.

I. INTRODUCTION

The recent discoveries of neutral currents,[1] ψ particles,[2] rising $R_C = \sigma(\bar{\nu}_\mu N \rightarrow \mu^+ \cdots)/\sigma(\nu_\mu N \rightarrow \mu^- \cdots)$ (Ref. 3), anomalous μe events[4] have led to the invention of so many quark and lepton models[5] that any theoretically appealing constraints are desirable to give criteria in selecting acceptable ones from the plethora of models. Most of these models are built such that the Adler-Bell-Jackiw (ABJ) anomalies are not present.

Recently, it has been emphasized that the *naturalness*[6] of the absence of strangeness-changing neutral current (SCNC) is a theoretically appealing constraint motivated by experimental observation of enormously small K_L-K_S mass difference and highly suppressed $K_L \rightarrow \mu\bar{\mu}$ decay rates. The merit of this naturalness assumption is that it selects a small group of models from a very large set. We say that a *conservation law is natural*[7] if it results from the group structure and representation content of the theory, while it is *artificial* if it is due to a specific tuning of parameters of the theory.

In this paper, we generalize this natural conservation law to the leptonic sector. The familiar gauge model based[8] on SU(2) ×U(1) is employed. The observed suppression of μe-flavor-changing neutral current (FCNC) in $\mu \rightarrow ee\bar{e}$ decay[9] is so dramatic numerically (the fraction for this decay mode is less than 6×10^{-9}) that it strongly suggests a natural mechanism for the absence of μe-FCNC. Another highly suppressed decay of muons,[9] $\mu \rightarrow e\gamma$ (the fraction for this decay is less than 2.2×10^{-8}), also suggests some secrets in the model building. Should any lepton model survive, the radiative decay amplitude for muons should not be of $G_F\alpha$ order.

We also note that μe universality is quite well satisfied; every model builder takes it for granted and makes it manifest through an artificial mechanism. However, we find that a natural mechanism for μe universality is more attractive.

Motivated by these observations we impose the following naturalness assumptions:

(*a*) *The absence of μe-FCNC is natural.*

(*b*) *Muon-electron universality is natural.*

For the SU(2) × U(1) gauge theory to be meaningful, it should be free of ABJ anomaly. In this connection we refer to the hadronic sector, which is assumed to have natural absence of strangeness-changing neutral currents (SCNC).

In Sec. II we present classes of leptonic models following from these assumptions, but without concerning ourselves with how the lepton masses are generated. It is discovered that the known charged leptons are of the Weinberg-Salam (WS) type (model A) or vectorlike (model B). In Sec. III, we compute the $\mu \rightarrow e\gamma$ decay rate for model B. It is found that the branching ratio for this decay is bounded from above by

$$\frac{25\alpha}{96\pi M_W^4}(m_{L^0}^{\max}+m_{L^0}^{\min})^2(m_{L^0}^{\max}-m_{L^0}^{\min})^2 .$$

In Sec. IV we discuss the phenomenology for the two classes of models. In particular, we find that the updated analysis of several leptonic neutral-current processes constrains the parameters $\sin^2\theta_W$ and z $(=M_Z^2/M_W^2 \sec^2\theta_W)$ in a small region. It is interesting to note that the WS-type model determines $0.22 \leq \sin^2\theta_W \leq 0.51$ and $0.7 \leq z \leq 3$.

II. LEPTON MODEL WITH NATURALNESS

There exist a few conservation laws in the leptonic world: muon-number conservation, electron-number conservation, and probably other heavy-lepton–number conservation.[9] This conservation law is strongly supported by the vanishing of the processes

$$\mu \rightarrow e + e^+ + e^- , \tag{1}$$

$$\mu \rightarrow e + \gamma , \tag{2}$$

$$\text{fast}(\pi^+ \text{ or } K^+) \rightarrow \mu^+ + \nu, \nu + n \rightarrow e + p . \tag{3}$$

The presently available upper bounds are[10,11]

$$\frac{\Gamma(\mu \to e e^+ e^-)}{\Gamma(\mu \to \text{all})} < 6 \times 10^{-9}, \quad (4)$$

$$\frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to \text{all})} < 2.2 \times 10^{-8}, \quad (5)$$

$$\frac{\Gamma(\text{fast } \pi^+ \to \mu^+\nu, \nu n \to ep)}{\Gamma(\text{fast } \pi^+ \to \mu^+\nu, \nu n \to \mu^- p)} = (1.1 \pm 0.5) \times 10^{-2}. \quad (6)$$

The other well established dynamics in the leptonic world is *μe universality*, which states that the $V-A$ charged current of μ and e interactions are exactly the same. This μe universality is tested by the ratio

$$\rho = \frac{\Gamma(\pi^+ \to e^+\nu) + \Gamma(\pi^+ \to e^+\nu\gamma)}{\Gamma(\pi^+ \to \mu + \nu)} \quad (7)$$

whose theoretical value[12,13] in $V-A$ theory is 1.233 (or 1.258) $\times 10^{-4}$ depending on the same cutoff Λ (or different cutoffs Λ_e and Λ_μ of the ratio $\Lambda_e/\Lambda_\mu \cong m_e/m_\mu$) for the two processes, while its experimental value is[13,14] $(1.274 \pm 0.024) \times 10^{-4}$.

Usually these two laws, lepton-number conservation and μe universality, are built *artificially* by assigning $V-A$ doublets

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \ldots . \quad (8)$$

However, one cannot distinguish (8) from the following:

$$\begin{pmatrix} \nu_e \cos\theta + E^0 \sin\theta \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \cos\theta \pm M^0 \sin\theta \\ \mu \end{pmatrix}_L, \ldots, \quad (8')$$

where E^0 and M^0 are heavy leptons. Certainly, the structures (8) or (8′) are sufficient for lepton-number conservation and μe universality.

In this section, the assumptions (a) *the absence of μe-FCNC is natural* and (b) *μe universality is natural* are studied to derive general *classes* of lepton structures. As will be clear in the subsequent discussions, the naturalness assumptions are so strong that we are permitted only one type of $V-A$ lepton structures. It should be noted that the condition (a) is a relaxed statement of lepton-number conservation applied only to neutral current between charged leptons.

Let us briefly review the FCNC in general. The Cabibbo formulation of the weak-interaction theory has been reproduced in the renormalizable gauge model of weak and electromagnetic interactions.[8] However, the existence of strangeness-changing neutral current in the original WS model with a $V-A$ doublet could not be resolved until Glashow-Iliopoulos-Maiani[8] (GIM) realized that at least two $V-A$ doublets are needed. The GIM model removes strangeness-changing neutral current *naturally*, irrespective of the choice of the Cabibbo angle θ_C. A recent study by Glashow and Weinberg[6] culminated this problem of neutral currents: In the SU(2) $\times$ U(1) gauge model *the equally charged fundamental fermions with the same helicity (leptons or quarks) should have the same values of $\vec{T}^2$ and T_3, where $\vec{T}$ is the weak isospin, to have natural absence of FCNC, otherwise the existence of FCNC is natural.*

We will consider the leptons which have charge $Q=1$, 0, and -1 and the SU(2) $\times$ U(1) gauge model for the weak and electromagnetic interactions. Because of the experimental existence of neutrino neutral current,[1] we assume only doublet and singlet representations.

First, let us consider $V-A$ representations. From assumption (a) then, all the $V-A$, $Q=-1$ leptons should belong to $T_3 = -\frac{1}{2}$ doublet representations since e_L and μ_L belong to them. (Subscripts L and R mean left-handed and right-handed, respectively). Hence, the number of $Q=-1$ leptons l^i ($l^1 = e$, $l^2 = \mu$, etc.) should not be more than the number of $Q=0$ leptons, otherwise we are left with singlet $V-A$ $Q=-1$ leptons. Then we can write down the leptonic structure of l_L^i leptons which contains e_L ($=l_L^1$) and μ_L ($=l_L^2$),

$$\begin{pmatrix} l_\nu^{i\prime} \\ l^i \end{pmatrix}_L i = 1, \ldots, m; \quad (l_\nu^{j\prime})_L \, j = m+1, \ldots, n, \quad (9)$$

where m is the number of $Q=-1$ leptons l^i and n is the number of $Q=0$ leptons l_ν^i with $n \geq m$, and the primes in $l_\nu^{i\prime}$ denote the mixed states of mass eigenstates l_ν^i, namely $l_\nu' = M l_\nu$, where M is an orthogonal $n \times n$ matrix. The form (9) is general. (If we had started with mixed $l^{i\prime}$, we could rearrange the doublet structures such that l^i leptons appear as mass eigenstates.) Now let us apply assumption (b) to (9). Suppose there are some neutral *heavy* leptons $H_1^0, H_2^0, \ldots$ in l_ν^i. Then $l_\nu^{1\prime}$ and $l_\nu^{2\prime}$ can be written as (subscript L is suppressed)

$$l_\nu^{1\prime} = a_\nu^1 \nu_1 + a_\nu^2 \nu_2 + \cdots + a_\nu^p \nu_p + a_H^1 H_1^0 + a_H^2 H_2^0 + \cdots,$$

$$l_\nu^{2\prime} = b_\nu^1 \nu_1 + b_\nu^2 \nu_2 + \cdots + b_\nu^p \nu_p + b_H^1 H_1^0 + b_H^2 H_2^0 + \cdots,$$

where $\nu_1, \nu_2, \ldots$ are massless and $H_1^0, H_2^0, \ldots$ are heavy neutral leptons and a_ν^i, a_H^i, b_ν^i, and b_H^i are elements of appropriate orthogonal matrices. We obtain μe universality for a particular set parameters satisfying[15]

$$|a_\nu^1|^2 + |a_\nu^2|^2 + \cdots + |a_\nu^p|^2 = |b_\nu^1|^2 + |b_\nu^2|^2 + \cdots + |b_\nu^p|^2,$$

which is not generally true if heavy neutral leptons are present. Hence, assumption (b) implies that all the *$V-A$ neutral leptons are massless*, namely, neutrinos [the subscript ν is used in (9) to denote l_ν^i as massless neutral particles]. A comment concerning muon- and electron-number conservation for the process (3) is in order. The measure of this law is usually given by[27]

$$\epsilon = \frac{\sum_{i=1}^{m} |a_\nu^i|^2 |b_\nu^i|^2}{\sum_{i=1}^{m} |b_\nu^i|^4}$$

since it is tested through neutrinos, which are produced together with muons and scattered off nuclei yielding final electrons. Theoretically, ϵ can vary from 0 (when $a_\nu^1 = 1$, $b_\nu^2 = 1$, all others = 0) to $m/2(m-1)^{1/2}$. Experimentally,[11] ϵ is very close to 0, which suggests that we should have almost pure ν_1 and ν_2,

$$\begin{pmatrix} \nu_1' \simeq \nu_e \\ l^1 = e \end{pmatrix}_L , \begin{pmatrix} \nu_2' \simeq \nu_\mu \\ l^2 = \mu \end{pmatrix}_L ,$$

though other mixtures of $l_\nu^{i\prime}$ $(i = 3, 4, \ldots, m)$ are not constrained yet. Determination of the angles of a_ν^i and b_ν^i by process (3) can be compared to determination of Cabibbo angle θ_C from strange-particle decays.

Secondly, let us use the requirement of the absence of the ABJ anomaly. There can be only two classes of anomaly-free models:

Case (i). The models where hadronic anomalies cancel among themselves and the leptonic anomalies cancel among themselves. One example is the familiar vectorlike model.[16] Another example is a vectorlike quark model with triplet representations for leptons, though it is not favored by the known weak-interaction phenomenology of $G_\beta \simeq G_\mu$ and observation of neutral currents.

Case (ii). The models where nonvanishing hadronic anomalies cancel nonvanishing leptonic anomalies.[17] For these kinds of theories, it is required that $\sum Q_{\text{leptons}} = \sum Q_{\text{quarks}}$, where the sums are taken over the same helicity multiplet members. Note that with lepton-hadron symmetry vectorlike models belong to both (i) and (ii).

In connection with the ABJ anomaly, we have to consider the quark representations which *are required* to have natural absence of SCNC. This requirement of natural absence of SCNC gives vectorlike quark representations or $V-A$ doublet and $V+A$ singlet quark representations, both of which do not have more $Q = -\frac{1}{3}$ quarks than $Q = \frac{2}{3}$ quarks. The former case leads to case-(i) anomaly-free models while the latter leads to case-(ii) anomaly-free models. Since we do not have less $Q = \frac{2}{3}$ quarks than $Q = -\frac{1}{3}$ quarks, the case-(i) anomaly-free models give $R_C \approx \frac{1}{4}(1 + 3\cos^2\beta)$ (sea-quark contribution is neglected) where $\cos^2\beta$ is the doublet contribution of the u_R quark, such as $(u\cos\beta - t\sin\beta, b)_R$ and $(u\sin\beta + t\cos\beta)_R$. Hence, case-(i) anomaly-free models can account for the rising trend of R_C up to the maximum value of 1, while case-(ii) anomaly-free models predict $R_C \approx \frac{1}{3}$ (in the absence of sea quarks). Therefore, we will consider only case-(i) anomaly-free models.

Excluding the possibility of triplet lepton representations, we have two models satisfying assumptions (a) and (b) of case-(i) anomaly-free type.

Model A.

$$\left.\begin{matrix} \begin{pmatrix} L^i \\ L_\nu^{i\prime} \end{pmatrix}_L , \; L_R^i \\ \begin{pmatrix} l_\nu^{i\prime} \\ l_i \end{pmatrix}_L , \; l_R^i \end{matrix}\right\} i = 1, \ldots, m , \qquad \left.\begin{matrix} L_{\nu L}^{j\prime} \\ l_{\nu L}^{j\prime} \end{matrix}\right\} j = m+1, \ldots . \tag{10}$$

Model B.

$$\begin{pmatrix} l_\nu^{i\prime} \\ l_i \end{pmatrix}_L , \begin{pmatrix} N^{i\prime} \\ l^i \end{pmatrix}_R , \; i = 1, \ldots, m \qquad l_{\nu L}^{j\prime}, N_R^{j\prime}, \; j = m+1, \ldots \tag{11}$$

where l^i are $Q = -1$ leptons, L^i are $Q = +1$ leptons, L_ν^i and l_ν^i are massless neutrinos, and N^i are neutral leptons. Some comments are in order for the models (10) and (11).

Model A. This is distinguished from the triplet-representation models in the contents of neutral current, overall running coupling constant, and the number of neutral leptons introduced. Any of neutrinos l_ν^i or L_ν^i cannot be massive, as discussed before. No leptonic anomalies are present. Lepton masses for l_i and L_i can be generated by Yukawa couplings to scalar particles by spontaneous symmetry breaking while model B cannot. Of course, the neutral current for known leptons (ν_e, ν_μ, e, μ) are of WS type,

$$\begin{aligned} J_\alpha^Z = [&\bar{\nu}_e \gamma_\alpha (1-\gamma_5)\nu_e + \bar{\nu}_\mu \gamma_\alpha (1-\gamma_5)\nu_\mu \\ &+ (-\tfrac{1}{2} + 2\sin^2\theta_W)(\bar{e}\gamma_\alpha e + \bar{\mu}\gamma_\alpha \mu) \\ &+ \tfrac{1}{2}(\bar{e}\gamma_\alpha\gamma_5 e + \mu\gamma_\alpha\gamma_5\mu)] . \end{aligned} \tag{12}$$

Model B. This model is the so-called vectorlike model. It has a problem in generating the lepton masses through spontaneous symmetry breaking since any attempt to introduce the mass for a neutral heavy lepton through Yukawa coupling to a scalar field requires the left-handed part for that neutral heavy lepton which was excluded by our assertion of natural μe universality. However, we will not be concerned about the detailed mechanism for the lepton mass generation. This model is also free of the ABJ anomaly as required. The neutral current for charged leptons are of pure vector. For known leptons, J_α^Z is

$$J_\alpha^Z = [\bar{\nu}_e\gamma_\alpha(1-\gamma_5)\nu_e + \bar{\nu}_\mu\gamma_\alpha(1-\gamma_5)\nu_\mu + (-1+2\sin^2\theta_W)(\bar{e}\gamma_\alpha e + \bar{\mu}\gamma_\alpha\mu)]\,. \tag{13}$$

III. $\mu \to e\gamma$ DECAY (REF. 18)

As noted in the previous section, the decay

$$\mu \to e\gamma \tag{14}$$

has a stringent upper bound

$$\frac{\Gamma(\mu\to e\gamma)}{\Gamma(\mu\to \text{all})} < 2.2\times 10^{-8}\,.$$

Both model A and model B are built such that (1) and (14) do not occur at the lowest order. However, at higher orders model B gives nonvanishing amplitudes for (1) and (14) while model A gives vanishing amplitudes. This is because the intermediate leptons can have different masses in model B while they all have the same zero masses in model A. Since presently available bounds (4) and (5) are comparable in magnitude, it is sufficient to test model B in the $\mu\to e\gamma$ decay case only. The reason is that at the next to second order the decay rate for $\mu\to ee\bar{e}$ is known to be smaller than the decay rate for $\mu\to e\gamma$ by an order of α/π.

For $\mu\to e\gamma$ decay, let us define the matrix elements of the electromagnetic current between μ and e states

$$\langle e|J_\mu^{\text{em}}|\mu\rangle = \bar{u}_e(p')\frac{1\pm\gamma_5}{2}[\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu}q^\nu F_2(q^2) + q_\mu F_3(q^2)]u_\mu(p)\,, \tag{15}$$

where $(1-\gamma_5)/2$ and $(1+\gamma_5)/2$ refers to left-handed or right-handed charged currents from which the third-order mixed matrix element (15) arises. It is easy to note that $F_1=0$ and the F_3 term does not contribute to $\mu\to e\gamma$ decay due to the current conservation and the conditions $\epsilon\cdot q=0$ and $q^2=0$ for a real photon.

Hence, we are interested only in the mixed magnetic-moment term, f,

$$\langle e|J_\mu^{\text{em}}|\mu\rangle = \bar{u}_e(p')\frac{1\pm\gamma_5}{2}i\sigma_{\mu\nu}q^\nu u_\mu(p)f\,. \tag{16}$$

The decay width for $\mu\to e\gamma$ can be easily computed for the case (16), neglecting the electron mass

$$\Gamma(\mu\to e\gamma) = \frac{f^2}{16\pi}m_\mu^{\ 3}\,, \tag{17}$$

which may be compared to the $\Gamma(\mu\to e\nu_\mu\bar{\nu}_e) = (G_F^{\ 2}/192\pi^3)m_\mu^{\ 5}$.

Now let us calculate the quantity f in Eq. (16). Model B has the doublet structure of the form

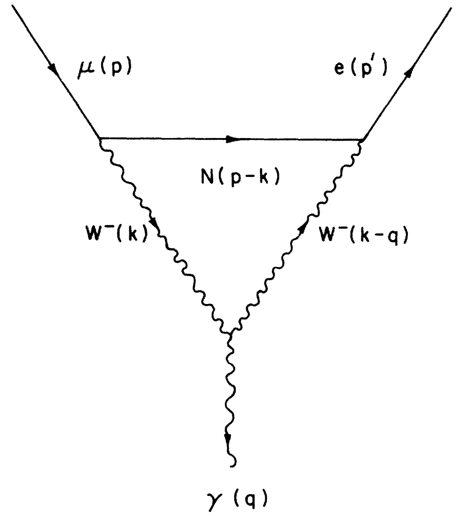

FIG. 1. The Feynman diagram which contributes to $\mu\to e\gamma$ decay in model B. The relevant momenta are defined in the figure.

$$\begin{pmatrix} aN \\ \mu \end{pmatrix}_R, \begin{pmatrix} bN \\ e \end{pmatrix}_R, \ldots\,, \tag{18}$$

which contribute to f through N intermediate state. Here a and b are appropriate elements of an orthogonal matrix. There may be other neutral-heavy-lepton contributions. Since we do not take the point of view that lepton masses arise through Yukawa couplings, it is not possible to carry out the calculation in the convenient $\xi=1$ gauge known as the 't Hooft-Feynman gauge. Instead we calculate in the unitary gauge ($\xi=0$), where it is known that the scalar-particle contributions are not present. The relevant Feynman diagram is shown in Fig. 1. In regularizing the divergent integrals, we adopted the n-dimensional regularization scheme of 't Hooft and Veltman.[19] The result is[20] (see Appendix)

$$f = \frac{eab}{8\sqrt{2}\,\pi^2}G_F m_\mu I(y)\,, \tag{19}$$

$$\begin{aligned} I(y) &= \frac{1}{(1-y)^2}\left(\tfrac{7}{6} - \tfrac{13}{3}y + \tfrac{31}{6}y^2 + \frac{y^2(1-3y)}{1-y}\ln\frac{1}{y}\right) \\ &\quad + \frac{1}{(1-y)^2}\left(\tfrac{1}{2} - \tfrac{2}{3}y + \tfrac{5}{3}y^2 + \frac{y^3}{1-y} - \frac{y}{(1-y)^2}\ln\frac{1}{y}\right) \\ &= \frac{1}{(1-y)^2}\left(\tfrac{5}{3} - 5y + \tfrac{41}{6}y^2 + \frac{y^3}{1-y} - \frac{y^3(4-3y)}{(1-y)^2}\ln\frac{1}{y}\right), \end{aligned} \tag{20}$$

where $y=(m_N/M_W)^2$ and $O(m_e/m_\mu)$ is neglected and the first and second terms in the first equation of Eq. (20) represent the contributions of g-g and g-k terms, respectively, from the product of two vector-boson propagators

$$-i\frac{g_{\mu\nu}-k_\mu k_\nu/M_W^2}{k^2-M_W^2+i\epsilon}\,.$$

From (17) and (19), we obtain

$$\Gamma(\mu\to e\gamma)=\frac{G_F^2}{192\pi^3}m_\mu^5\frac{3\alpha}{8\pi}\Big[\sum_i a_i b_i I(y_i)\Big]^2, \quad (21)$$

where all the heavy-lepton contributions are summed over. In the limit of $y_i \to 0$,

$$\sum_i a_i b_i I(y_i)\simeq\tfrac{5}{3}\sum_i\,[a_i b_i(1-\bar{y})-a_i b_i\Delta y_i]\,, \quad (22)$$

where $\bar{y}$ is the mean value of y_i

$$\bar{y}=\frac{\sum_i y_i}{\sum_i 1} \quad (23)$$

and Δy_i is

$$\Delta y_i = y_i-\bar{y}\,. \quad (24)$$

From the orthogonality condition $\sum_i a_i b_i=0$, the second term in (22) is the leading term and we obtain

$$\frac{\Gamma(\mu\to e\gamma)}{\Gamma(\mu\to e\nu_\mu\bar{\nu}_e)}=\frac{25\alpha}{24\pi}\Big(\sum_i a_i b_i\Delta y_i\Big)^2. \quad (25)$$

To get an idea how small (25) will be, let us take a particular case

$$\begin{pmatrix}\cos\theta N_1+\sin\theta N_2\\ \mu\end{pmatrix}_R \begin{pmatrix}-\sin\theta N_1+\cos\theta N_2\\ e\end{pmatrix}_R$$

and $y_1=(\frac{1}{25})^2$ and $y_2=(\frac{2}{25})^2$, e.g., $M_W=50$ GeV, $m_{N_1}=2$ GeV, and $m_{N_2}=4$ GeV will correspond to this. Then we estimate

$$\frac{\Gamma(\mu\to e\gamma)}{\Gamma(\mu\to e\nu_\mu\bar{\nu}_e)}=(2.4\times10^{-3})(y_2-y_1)^2\sin^2\theta\cos^2\theta \le 2.8\times10^{-8}\,, \quad (26)$$

which is consistent with the present bound. Generally, the branching ratio is bounded from above by

$$\frac{\Gamma(\mu\to e\gamma)}{\Gamma(\mu\to e\nu_\mu\bar{\nu}_e)}\le\frac{25\alpha}{96\pi M_W^4}(m_N^{\max}-m_N^{\min})^2(m_N^{\max}+m_N^{\min})^2\,, \quad (27)$$

where M_W is the W-boson mass, $m_N^{\max}$ and $m_N^{\min}$ are the largest and the smallest masses of neutral heavy leptons N_i.

IV. PHENOMENOLOGY

A. The unobserved decay modes of muons

Model A. The decay modes such as $\mu\to ee^+e^-$, $\mu\to e\gamma$, and $\mu\to e\gamma\gamma$ are strictly forbidden to all orders. The same is true for other charged leptons,

$$l^-\not\to e^-e^+e^-,\, e^-\gamma\,,$$
$$L^+\not\to e^+e^+e^-,\, e^+\gamma\,.$$

Any observation of the above decay *without missing neutrals* eliminates the model A.

Model B. There exist nonvanishing contributions to $\mu\to e\gamma$ and $\mu\to ee^+e^-$ decay modes which have been discussed in the previous section.

B. Heavy leptons

Model A. There are two types of charged heavy leptons, l type and L type, which should be heavier than the K meson. The classification of particle and antiparticle is a relative concept. It is important to note that l and $\bar{L}$, which are both negatively charged, have different weak interactions, namely l decays through $V-A$ coupling to the W boson while $\bar{L}$ decays through $V+A$ coupling to the W boson. The anomalous μe events in e^+e^- annihilation[4] are interpreted by several authors[21] as production of charged-heavy-lepton pairs subsequently decaying to $e\nu\bar{\nu}$ (or $\mu\nu\bar{\nu}$). The present experimental data are so meager that we cannot definitely say if $V+A$ or $V-A$ decaying heavy leptons are produced.

Model B. There is only one type of charged heavy leptons, l^i. This model has another complexity in that the mass parameters of neutral heavy leptons play a significant role in the charged-heavy-lepton decay. If the neutral heavy lepton M^0 is very light compared to the corresponding charged heavy lepton M^-, then M^- mimics the vector weak-interaction decay to the W boson. However, if M^0 is heavier than M^-, M^- decays only through $V-A$ weak interaction to W bosons. (If M^0 is slightly lighter than M^-, $V-A$ decay is approximate.)

Any definite observation of a $V+A$ decaying negatively charged lepton eliminates model B and observation of a V decaying heavy lepton eliminates model A, though the observation of a $V-A$ decaying heavy lepton would not select a class from model A and model B. In this respect, it is very important to find out the decaying properties of charged heavy leptons.

C. Leptonic neutral current

Though neutrino-electron scattering is difficult to measure, it offers immediate and unambiguous theoretical analysis. The study of this process is performed for the self-containment of the material and for the updated version of the analysis. We are interested in the ν_μ, $\bar{\nu}_\mu$, ν_e, and $\bar{\nu}_e$ scattering of the electron. The relevant neutral-current interaction is

$$\mathcal{L}_{\text{int}} = -\frac{G_F}{\sqrt{2}\,z}[\bar{\nu}_e\gamma_\alpha(1-\gamma_5)\nu_e + \bar{\nu}_\mu\gamma_\alpha(1-\gamma_5)\nu_\mu] \times \bar{e}\gamma^\alpha(zg_V - zg_A\gamma_5)e\,, \qquad (28)$$

where z is the ratio of the actual M_Z^2 to the WS value

$$z = \frac{M_Z^2}{M_W^2 \sec^2\theta_W} \qquad (29)$$

and

$$\left.\begin{aligned} g_V &= \frac{1}{z}(-\tfrac{1}{2} + 2\sin^2\theta_W) \\ g_A &= -\frac{1}{2z} \end{aligned}\right\} \text{ for model A}, \qquad (30)$$

$$\left.\begin{aligned} g_V &= \frac{1}{z}(-1 + 2\sin^2\theta_W) \\ g_A &= 0 \end{aligned}\right\} \text{ for model B}. \qquad (31)$$

Interaction (28) is the only contribution to ν_μ- ($\bar{\nu}_\mu$-) electron scattering while there is additional contribution from charged-current interactions to ν_e- ($\bar{\nu}_e$) electron scattering. The differential cross section can be written as[22]

$$\frac{d\sigma}{dE_e} = \frac{G_F^2}{2\pi} m_e \left[A + B\left(1 - \frac{E_e}{E_\nu}\right)^2 - C\frac{m_e E_e}{E_\nu^2}\right], \qquad (32)$$

where E_e is the final electron energy and A, B, and C are given in Table I.

TABLE I. Coefficient functions A, B, and C in Eq. (32).

	$\nu_\mu e \to \nu_\mu e$	$\bar{\nu}_\mu e \to \bar{\nu}_\mu e$	$\nu_e e \to \nu_e e$	$\bar{\nu}_e e \to \bar{\nu}_e e$
A	$(g_V + g_A)^2$	$(g_V - g_A)^2$	$(2 + g_V + g_A)^2$	$(g_V - g_A)^2$
B	$(g_V - g_A)^2$	$(g_V + g_A)^2$	$(g_V - g_A)^2$	$(2 + g_V + g_A)^2$
C	$g_V^2 - g_A^2$	$g_V^2 - g_A^2$	$(g_V - g_A) \times (2 + g_V + g_A)$	$(g_V - g_A) \times (2 + g_V + g_A)$

For a measured cross section, Eq. (32) gives an ellipse on the (g_V, g_A) plane. In Fig. 2 we present allowed domains in this plane from updated neutrino-electron scatterings.[23,24] The measured upper bound[23] on $\nu_\mu e$ scattering gives a convex domain. The measured cross section[23] of $\bar{\nu}_\mu e$ scattering gives an annular domain between two ellipses. Also the recent measurement[24] of the $\bar{\nu}_e e$ cross section gives a very narrow domain. The

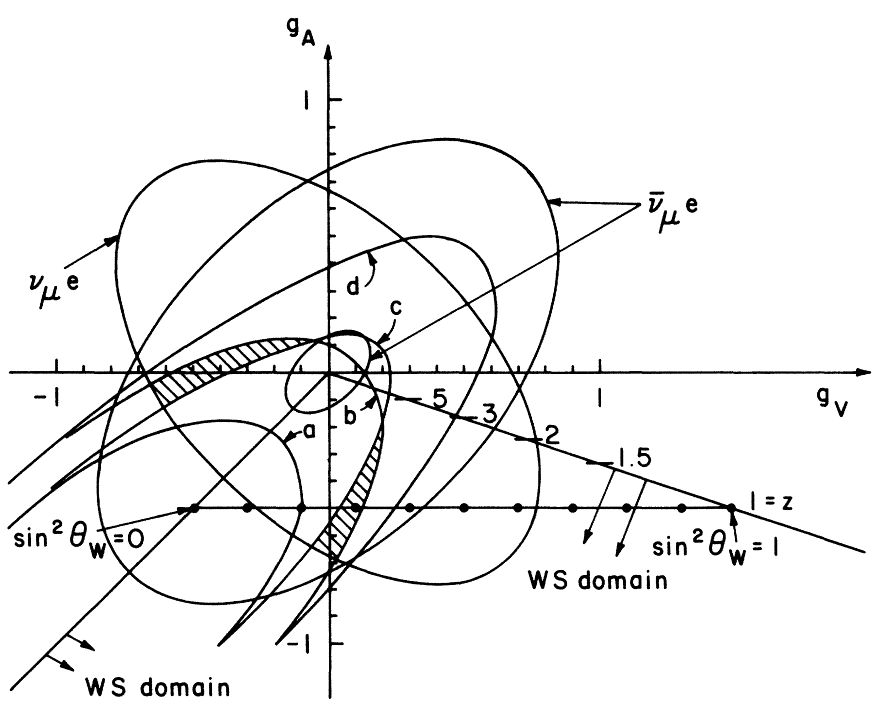

FIG. 2. Experimental bounds on $\nu_\mu, \bar{\nu}_\mu, \bar{\nu}_e$-electron scatterings plotted on the g_A-g_V plane. The WS domain is shown on the figure. The theoretical vectorlike domain is $-1 \leq g_V \leq 1$. The measured $\nu_\mu e$ scattering gives a convex domain while the measured $\bar{\nu}_\mu e$ scattering gives an annular domain bounded by two ellipses. The corresponding ellipses are 90% C.L. boundaries. Ellipses (a), (b), (c), and (d) are one-standard-deviation boundaries of $\bar{\nu}_e e$ scattering for two different energy regimes of the scattered electron (Ref. 24).

allowed region from all these scatterings are shaded.

Model A. This model (or WS model) allows the region given by the infinite lower triangle constrained by the lower bound of the Z-boson mass which is taken as about 10 GeV (otherwise we should have seen the Z boson already). However, the lower boundary

$$z > \left(\frac{10}{37.3}\right)^2 \sin^2\theta_W \cos^2\theta_W$$

cannot be shown in Fig. 2 since it is located too far below from the figure. One of two experimentally allowed domains is located inside this triangle and also crosses the WS line ($z=1$). This domain gives

$$0.22 \le \sin^2\theta_W \le 0.51\,, \quad 0.7 \le z \le 3\,. \tag{33}$$

We note that all points on the same ray $g_A = a g_V$ give the same value of r defined by

$$r = \frac{\sigma(\bar{\nu}_\mu e)}{\sigma(\nu_\mu e)}\,, \tag{34}$$

which is $(1-a+a^2)/(1+a+a^2)$ without an experimental cut. For the best value in (33), r is about 2.

Model B. This vectorlike model allows $g_A = 0$ and $-1 \le g_V \le 1$. The other experimentally allowed domain crosses this line and its value is

$$g_V = -0.47 \pm 0.10\,, \tag{35}$$

which alone cannot determine $\sin^2\theta_W$ and z except its combination given by Eq. (31).

The determination of $\sin^2\theta_W$ and z from neutrino-electron scattering is the most reliable one since it is not obscured by strong-interaction effects. This information can be used for studying neutrino interactions with hadrons.[25] Typically, the best $\sin^2\theta_W = 0.33$ in (33) implies sizable charm-changing neutral-current effects and

$$R_C = \frac{\sigma(\bar{\nu}_\mu N \to \mu^+ \cdots)}{\sigma(\nu_\mu N \to \mu^- \cdots)} \cong 0.5 \pm 0.3$$

at very high energies.

V. CONCLUSION

With the naturalness assumptions on μe universality, absences of μe-FCNC and SCNC, we were able to draw two types of lepton models given by (10) and (11). In model A, no massive neutral leptons are allowed, both $V+A$ and $V-A$ decaying negatively charged heavy leptons are present, $\mu \to e\gamma$ and $\mu \to ee^+e^-$ are completely forbidden, and neutral currents for known leptons are of pure WS type. The updated analysis of neutrino-electron scattering gives 0.33 as the best $\sin^2\theta_W$, which may be compared to the values obtained from inclusive neutral-current experiments (the analysis of inclusive neutral-current processes should be modified such that $Q=\frac{2}{3}$ FCNC may be included). In model B, $V-A$ or approximate V decaying negatively charged heavy leptons are allowed depending on the mass of neutral heavy leptons. $\mu \to e\gamma$ and $\mu \to ee^+e^-$ decay rates fall within the presently known experimental upper bounds. The neutral currents are of pure vector type and the analysis of ν-e and $\bar{\nu}$-e elastic scatterings determine $g_V = -0.47 \pm 0.1$, which alone cannot determine $\sin^2\theta_W$.

The naturalness assumptions (a) and (b) are so stringent that we are not left with any extra parameters except $\sin^2\theta_W$ and z in the leptonic neutral current, which simplified our phenomenological analysis. However, the basic question on the validity of the naturalness assumptions cannot be tested at the present time.

After completion of this work, we were informed of the rumors that $\mu \to e\gamma$ decay is observed with the branching ratio of 10^{-9} order. If this is confirmed by future experiment, model B survives with predictions of $V-A$ decaying or approximate V decaying charged heavy leptons, while model A should be changed such that it admits both doublets and quartets as its representations.

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APPENDIX

The relevant integral for mixed μe magnetic-moment calculation in the unitary gauge is[26]

$$\int \frac{d^n k}{(2\pi)^n} \bar{u}(p') \frac{1-\gamma_5}{2}\left(-\frac{ibg}{\sqrt{2}}\right)\gamma_\beta i(\not{p}-\not{k}+m)\left(-\frac{iag}{\sqrt{2}}\right)\gamma_\alpha \frac{1+\gamma_5}{2} u(p)[-iP^{\alpha\rho}(k)][-iP^{\beta\sigma}(k-g)](ieV_{\mu\rho\sigma})/D\,, \tag{A1}$$

where relevant momenta are defined in Fig. 1 and

$$P^{\alpha\beta}(k) = g^{\alpha\beta} - k^\alpha k^\beta / M_W^{\,2}\,, \tag{A2}$$

$$V_{\mu\rho\sigma} = g_{\rho\sigma}(2k-q)_\mu - g_{\mu\rho}(k+q)_\sigma + g_{\mu\sigma}(2q-k)_\rho\,, \tag{A3}$$

$$D=(k^2-M_W{}^2)[(k-q)^2-M_W{}^2][(k-p)^2-m^2]\,, \tag{A4}$$

and M_W, m, and m_μ are the charged vector boson, heavy lepton N, and muon masses, respectively. The integral (A1) will be evaluated for an arbitrary n which will approach 4 in the final result. The product of $P^{\alpha\rho}$ and $P^{\beta\sigma}$ gives three terms among which the k-k term does not contribute to the mixed magnetic moment. The remaining g-g and g-k terms are denoted by J^1_μ and J^2_μ, respectively,

$$(\text{A1})=-\frac{eab}{32\pi^4}g^2\bar{u}(p')\frac{1-\gamma_5}{2}(J^1_\mu+J^2_\mu)u(p)\,, \tag{A5}$$

$$J^1_\mu=\frac{(2\pi)^4}{(2\pi)^n}\int d^nk\,\frac{1}{D}[(4\not k-2m_\mu)k_\mu-(2\not k+2m_\mu)p_\mu+\gamma_\mu(2m_\mu\not p'-2\not k\not q)+(m_\mu\not p+\not p\not k)\gamma_\mu]\,, \tag{A6}$$

$$J^2_\mu=-\frac{(2\pi)^4}{(2\pi)^nM_W{}^2}\int d^nk\,\frac{1}{D}(4k_\mu p\cdot k\not k-2k_\mu k^2\not k-2m_\mu q\cdot k\gamma_\mu\not k+m_\mu k^2\gamma_\mu\not k)\,, \tag{A7}$$

where $O(m_e/m_\mu)$ and $O(m_\mu/M_W)$ are neglected and the terms which would not contribute to f after shifting the internal momenta are already discarded. J^1_μ can be calculated without using the n-dimensional regularization,

$$J^1_\mu=-\pi^2 i\left(\frac{m_\mu}{M_W{}^2}\right)\frac{2p_\mu}{(1-y)^2}\left[\tfrac{7}{12}-\tfrac{13}{6}y+\tfrac{31}{12}y^2+\frac{y^2(\frac{1}{2}-\frac{3}{2}y)}{1-y}\ln\frac{1}{y}\right], \tag{A8}$$

where $y=(m_N/M_W)^2$. J^2_μ is calculated by shifting the internal momenta, performing the symmetric integration

$$k_\mu k_\nu\rightarrow\frac{1}{n}k^2g_{\mu\nu}\,, \tag{A9}$$

and taking the $n\rightarrow 4$ limit to give

$$J^2_\mu=-\pi^2 i\left(\frac{m_\mu}{M_W{}^2}\right)\frac{2p_\mu}{(1-y)^2}\left[\tfrac{1}{4}-\tfrac{1}{3}y+\tfrac{5}{6}y^2+\frac{\frac{1}{2}y^3}{1-y}-\frac{\frac{1}{2}y^2}{(1-y)^2}\ln\frac{1}{y}\right]. \tag{A10}$$

The result (19) is obtained from (A5), (A8), (A10), and the Gordon decomposition

$$(m_\mu+m_e)\bar{u}_e(p')\gamma_\mu u_\mu(p)=(p+p')_\mu\bar{u}_e(p')u_\mu(p)-i\bar{u}_e(p')\sigma_{\mu\nu}(p-p')^\nu u_\mu(p) \tag{A11}$$

along with $g^2/8M_W{}^2=G_F/\sqrt{2}$.

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