PHYSICAL REVIEW D VOLUME 16, NUMBER 1 1 JULY 1977

$\Delta I = 1/2$ rule and Glashow-Iliopoulos-Maiani current with unconfined color

Mituaki Katuya and Yoshio Koide
Laboratory of Physics, Shizuoka Women's University, Yada 409, Shizuoka, Japan 422
(Received 4 January 1977)

We study a four-flavor model with unconfined color in the framework of the $SU(2)_L \otimes U(1)$ gauge model. The presence of a color-octet charged current leads to a violation of the universality of the hadronic and leptonic currents; therefore, heavy leptons are needed inevitably to compensate the violation. A condition for cancellation of $(\underline{84},\underline{1})$ representation in the combination of current products $(\underline{15},\underline{1}) \otimes (\underline{15},\underline{1}) + (\underline{15},\underline{8}) \otimes (\underline{15},\underline{8})$, which explains successfully the observed $\Delta I = 1/2$ rule, leads to the result that the slope of the linearly rising cross section $\sigma(\nu_\mu(\bar{\nu}_\mu)N \to \mu^-(\mu^+)X)$ must be more than twice as large at energies high enough to produce the color-octet states as it is at lower energies, even if the effects of the weak boson and heavy leptons are taken into account.

I. INTRODUCTION

The color quantum number was introduced in order to reconcile the symmetric three-quark model for baryons with quark-Fermi statistics.[1-3] It is a widely held view that $SU(3)^c$ (color group) is needed for hadron physics in order to explain the zero-triality rule and to help us to understand the hadronic e^+-e^- cross section and $\pi^0 \to 2\gamma$ decay.[4]

A variety of recent experimental and theoretical developments has led to the expectation that one or more new kinds of flavor quantum numbers (beyond the usual triplet u, d, s) are needed for hadron physics.[5] The case for at least one new quark, c, was first made in a four-quark $SU(2)_L \otimes U(1)$ gauge model[6] constructed in such a way as to lead to the suppression of the $\Delta S = 1$ neutral current, for which purpose the Glashow-Iliopoulos-Maiani (GIM) current[7] is so aptly designed. Here the four quarks are tripled with confined color.

It is our main purpose to study a four-flavor model[8] with unconfined color in the framework of the $SU(2)_L \otimes U(1)$ gauge model, and to pursue its theoretical consequences from minimal constraints which are imposed on all models in this framework.

In Sec. II, we discuss an $SU(2)_L \otimes U(1)$ gauge model with 12 quarks and find that no color-octet charged current can be introduced without violation of the universality of the hadronic and leptonic currents. If the color-octet components exist, therefore, the heavy leptons have to be introduced inevitably to compensate the violation.[9] The parameter κ $(0 \leq |\kappa| \leq 1)$ is defined so that the coefficients of the color-singlet $\Delta C = \Delta S = 0$ and $\Delta C = 0, \Delta S = 1$ currents are $\kappa \cos\theta_C$ and $\kappa \sin\theta_C$, respectively, where θ_C is the Cabibbo angle.

In Sec. III, we show that the parameter κ is useful in making a rough estimation of the ratio of the slope of the linearly rising cross section, $\sigma(\nu_\mu(\bar{\nu}_\mu)N \to \mu^-(\mu^+)X)$, well above both charm and color thresholds to that below both thresholds (the ratio is $1/\kappa^2$).

The approximate $\Delta I = \frac{1}{2}$ rule obeyed by strangeness-changing amplitudes is one of the major stumbling blocks for a weak Lagrangian made up of the product of Cabibbo currents. A solution to this problem was proposed by Kingsley,[10] and independently by Fujii *et al.*[11] Their idea, which is based on the three-triplet model, is to cancel the $(\underline{27}, \underline{1})$ part in the combination $(\underline{8}, \underline{1}) \otimes (\underline{8}, \underline{1}) + (\underline{8}, \underline{8}) \otimes (\underline{8}, \underline{8})$, where (a, b) denotes the dimensional representation of $SU(3)_F \otimes SU(3)_C$. Following their idea, we place a necessary condition for the absence of the $(\underline{84}, \underline{1})$ amplitude on the $\Delta C = 0, \Delta S = 1$ part. In Sec. IV we find that the condition leads to a rigid upper bound for κ^2, $\kappa_{\max}{}^2 = 0.37$. The parameter κ represents a proportion of magnitude of the color-singlet $\Delta C = 0$ current in the total charged current. Therefore, when the neutrino energy in the $\nu N(\bar{\nu}N)$ collision is high enough to produce color-octet states, a fairly large increase of the cross section must be observed.

II. COLOR-OCTET CURRENT

The 12 quarks have the following electric charges:

$$\begin{pmatrix} Q_{u_i} \\ Q_{d_i} \end{pmatrix} = \begin{pmatrix} Q_{c_i} \\ Q_{s_i} \end{pmatrix} = \begin{pmatrix} \frac{2}{3} + \alpha_i \\ -\frac{1}{3} + \alpha_i \end{pmatrix}, \tag{1}$$

where

$$\sum_i \alpha_i = 0 . \tag{2}$$

The condition (2) guarantees that the usual hadron spectroscopy is preserved among color-singlet states.[12] For example, in the Han-Nambu model[2] $\alpha_1 = -\frac{2}{3}$ and $\alpha_2 = \alpha_3 = \frac{1}{3}$, in the Fritzsch-Gell-Mann model[1] $\alpha_j = 0$, and in the Tati model[3] $\alpha_1 = -1$, $\alpha_2 = 0$, and $\alpha_3 = 1$. Below we study the case $\alpha_2 = \alpha_3$.

We assign these quark fields to doublets of the left-handed gauge group $SU(2)_L$ as follows:

$$\begin{pmatrix} u_1 \\ d_1' \end{pmatrix}, \begin{pmatrix} c_1 \\ s_1' \end{pmatrix}, \begin{pmatrix} u_2 \\ d_2' \end{pmatrix}, \begin{pmatrix} c_2 \\ s_2' \end{pmatrix}, \begin{pmatrix} u_3 \\ d_3' \end{pmatrix}, \begin{pmatrix} c_3 \\ s_3' \end{pmatrix}. \tag{3}$$

Here the fields (d_1', s_1') and (d_2', s_2', d_3', s_3') are given by the orthogonal transformations as

$$\begin{pmatrix} d_1' \\ s_1' \end{pmatrix} = \begin{pmatrix} \cos\phi_1 & \sin\phi_1 \\ -\sin\phi_1 & \cos\phi_1 \end{pmatrix} \begin{pmatrix} d_1 \\ s_1 \end{pmatrix}, \tag{4}$$

$$\begin{bmatrix} d_2' \\ s_2' \\ d_3' \\ s_3' \end{bmatrix} = \begin{bmatrix} \cos\psi_2 & \sin\psi_2 & 0 & 0 \\ -\sin\psi_2 & \cos\psi_2 & 0 & 0 \\ 0 & 0 & \cos\psi_3 & \sin\psi_3 \\ 0 & 0 & -\sin\psi_3 & \cos\psi_3 \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 & \sin\alpha & 0 \\ 0 & \cos\beta & 0 & \sin\beta \\ -\sin\alpha & 0 & \cos\alpha & 0 \\ 0 & -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\phi_2 & \sin\phi_2 & 0 & 0 \\ -\sin\phi_2 & \cos\phi_2 & 0 & 0 \\ 0 & 0 & \cos\phi_3 & \sin\phi_3 \\ 0 & 0 & -\sin\phi_3 & \cos\phi_3 \end{bmatrix} \begin{bmatrix} d_2 \\ s_2 \\ d_3 \\ s_3 \end{bmatrix}$$

$$\equiv A \begin{bmatrix} d_2 \\ s_2 \\ d_3 \\ s_3 \end{bmatrix}. \tag{5}$$

This mixing scheme is the most general one which will realize the GIM mechanism in the four-flavor model of the Han-Nambu type $(\alpha_2 = \alpha_3 \neq \alpha_1)$.[13]

We obtain the color-singlet part of the weak charged current as follows:

$$J_\rho^{(\underline{15},\underline{1})} = \tfrac{1}{3}(\cos\phi_1 + A_{11} + A_{33}) \sum_i \bar{u}_i\gamma_\rho(1-\gamma_5)d_i + \tfrac{1}{3}(\sin\phi_1 + A_{12} + A_{34}) \sum_i \bar{u}_i\gamma_\rho(1-\gamma_5)s_i$$
$$+ \tfrac{1}{3}(-\sin\phi_1 + A_{21} + A_{43}) \sum_i \bar{c}_i\gamma_\rho(1-\gamma_5)d_i + \tfrac{1}{3}(\cos\phi_1 + A_{22} + A_{44}) \sum_i \bar{c}_i\gamma_\rho(1-\gamma_5)s_i\,, \tag{6}$$

where the A_{ij}'s are the (i,j) elements of the orthogonal matrix A which are given in Appendix A. For the coefficients of $\sum \bar{u}_i\gamma_\rho(1-\gamma_5)d_i$ and $\sum \bar{u}_i\gamma_\rho(1-\gamma_5)s_i$, respectively, we must impose the restrictions

$$\tfrac{1}{3}(\cos\phi_1 + A_{11} + A_{33}) = \kappa\cos\theta_C\,,$$
$$\tfrac{1}{3}(\sin\phi_1 + A_{12} + A_{34}) = \kappa\sin\theta_C\,, \tag{7}$$

so that our theory may lead to results compatible with the semileptonic interactions of ordinary hadrons. Here κ is arbitrary at present.

We can express the above restrictions in terms of mixing angles defined in Eqs. (4) and (5),

$$\tfrac{1}{3}[\cos\phi_1 + a_2\cos(\phi_2+\delta_2) + a_3\cos(\phi_3+\delta_3)] = \kappa\cos\theta_C\,,$$
$$\tfrac{1}{3}[\sin\phi_1 + a_2\sin(\phi_2+\delta_2) + a_3\sin(\phi_3+\delta_3)] = \kappa\sin\theta_C\,, \tag{8}$$

where

$$a_i = (\cos^2\psi_i\cos^2\alpha + \sin^2\psi_i\cos^2\beta)^{1/2} \leq 1\,, \tag{9}$$

$$a_i\sin\delta_i = \sin\psi_i\cos\beta\,,$$
$$a_i\cos\delta_i = \cos\psi_i\cos\alpha\,. \tag{10}$$

From these we get

$$|\kappa| \leq 1\,. \tag{11}$$

The case $|\kappa| = 1$ holds only when

$$\sin\alpha = \sin\beta = 0\,,$$
$$A_{11} = A_{33} = \cos\phi_1 = \cos\theta_C\,, \tag{12}$$
$$A_{12} = A_{34} = \sin\phi_1 = \sin\theta_C\,.$$

This condition (12) is necessary and sufficient for the absence of color-octet components of the charged current (see Appendix A for the detailed forms of the charged current, the neutral current, and the electromagnetic current).

If the color-octet component of the charged current is involved in our theory, the absolute value of the parameter κ must be smaller than 1. The leptonic charged current then must be effectively[14]

$$J_\rho = \kappa[\bar{\nu}_e\gamma_\rho(1-\gamma_5)e + \bar{\nu}_\mu\gamma_\rho(1-\gamma_5)\mu + \cdots]\,, \tag{13}$$

in order to restore the universality of the semileptonic and pure leptonic interactions. We thus are forced to introduce at least two charged (neutral) heavy leptons which are mixed with e and μ (ν_e and ν_μ).[15]

III. DEEP-INELASTIC NEUTRINO REACTIONS AND PARAMETER κ

We calculate the cross section far above both charm and color thresholds and below both thresholds by using the naive quark-parton model, which is the simplest framework that will lead to scaling.

In the asymptotic region, far above both charm and color thresholds (denoted by "above"), we derive the cross sections

$$\left[\frac{d^2\sigma(\nu_\mu N \to \mu^- X)}{dxdy}\right]_{\text{above}} = \frac{G_F{}^2 M E_\nu}{\pi\kappa^2}\left[q_\nu(x) + (1-y)^2 \bar{q}_\nu(x)\right],$$
$$\left[\frac{d^2\sigma(\bar{\nu}_\mu N \to \mu^+ X)}{dxdy}\right]_{\text{above}} = \frac{G_F{}^2 M E_{\bar{\nu}}}{\pi\kappa^2}\left[\bar{q}_{\bar{\nu}}(x) + (1-y)^2 q_{\bar{\nu}}(x)\right], \tag{14}$$

where

$$q_\nu(x) = x[d(x) + u(x) + 2s(x)],$$
$$\bar{q}_\nu(x) = x[\bar{d}(x) + \bar{u}(x) + 2\bar{c}(x)],$$
$$q_{\bar{\nu}}(x) = x[d(x) + u(x) + 2c(x)],$$
$$\bar{q}_{\bar{\nu}}(x) = x[\bar{d}(x) + \bar{u}(x) + 2\bar{s}(x)]; \tag{15}$$

x and y are well-known scaling variables, $u(x) = 3u_i(x)$ $[\bar{u}(x) = 3\bar{u}_i(x)]$, and $u_i(x)$ $[\bar{u}_i(x)]$, etc. are the distribution functions of the u_i quark ($\bar{u}_i$ antiquark), etc. with a fraction x of the momentum of the proton. Note that the universal Fermi coupling constant is given by $G_F/\sqrt{2} = g^2\kappa^2/8M_W{}^2$, where g is the SU(2) gauge coupling constant and M_W is the weak-boson mass.[16]

The cross sections below both color and charm thresholds (denoted by "below") are

$$\left[\frac{d^2\sigma(\nu_\mu N \to \mu^- X)}{dxdy}\right]_{\text{below}} = \frac{G_F{}^2 M E_\nu}{\pi\kappa^2}\kappa^2 x\{\cos^2\theta_C[d(x)+u(x)] + 2\sin^2\theta_C s(x) + (1-y)^2[\bar{d}(x)+\bar{u}(x)]\},$$
$$\left[\frac{d^2\sigma(\bar{\nu}_\mu N \to \mu^+ X)}{dxdy}\right]_{\text{below}} = \frac{G_F{}^2 M E_{\bar{\nu}}}{\pi\kappa^2}\kappa^2 x\{\cos^2\theta_C[\bar{d}(x)+\bar{u}(x)] + 2\sin^2\theta_C \bar{s}(x) + (1-y)^2[d(x)+u(x)]\}. \tag{16}$$

A comparison between Eq. (14) and Eq. (16) leads to

$$\frac{(\alpha_\nu)_{\text{above}}}{(\alpha_\nu)_{\text{below}}} \simeq \frac{1}{\kappa^2},$$
$$\frac{(\alpha_{\bar{\nu}})_{\text{above}}}{(\alpha_{\bar{\nu}})_{\text{below}}} \simeq \frac{1}{\kappa^2}, \tag{17}$$

where α is the slope of the linearly rising cross section. This relation (17) is slightly modified even if the contribution of the weak boson is taken into account (see Appendix B).

When the contributions of the weak boson and heavy leptons are taken into account, we can derive the following lower bounds as shown in Appendix B:

$$\frac{(\alpha_\nu)_{\text{above}}}{(\alpha_\nu)_{\text{below}}} \gtrsim \frac{1}{(1+r)}\frac{1}{\kappa^2},$$
$$\frac{(\alpha_{\bar{\nu}})_{\text{above}}}{(\alpha_{\bar{\nu}})_{\text{below}}} \gtrsim \frac{1}{(1+r)}\frac{1}{\kappa^2}, \tag{18}$$

where $r \equiv 2ME_{\nu(\bar{\nu})}/M_W{}^2$, and M is the nucleon mass. If we take $E_\nu \simeq 200$ GeV, which is the available energy at present, and $M_W \simeq \kappa \times 70$ GeV, which comes from the popular value, $\sin^2\theta_W \simeq 0.3$, we obtain $r \simeq 0.1/\kappa^2$.

As shown in Eq. (17), the ratio $(\alpha)_{\text{above}}/(\alpha)_{\text{below}}$ is very sensitive to the parameter κ. The parameter κ thus is useful as a touchstone for models in the framework of the unconfined color gauge theory.

IV. $\Delta I = \frac{1}{2}$ RULE AND UPPER BOUND FOR κ^2

The most promising and simplest way to understand the $\Delta I = \frac{1}{2}$ rule is to take advantage of the color degrees of freedom as demonstrated by Kingsley and Fujii *et al.*,[10,11] Here we briefly recapitulate their idea, originally proposed in the three-triplet scheme, using terms in the three quartet scheme. The charged weak hadronic current is postulated to belong to the representation $(\underline{15},\underline{1}) + (\underline{1},\underline{8}) + (\underline{15},\underline{8})$ of $SU(4)_F \otimes SU(3)_C$, and all the ordinary hadronic states belong to the singlet representation of $SU(3)_C$. Note that $(\underline{15},\underline{8})$ and $(\underline{1},\underline{8})$ as well as $(\underline{15},\underline{1})$ components may contribute to the matrix elements of the nonleptonic interaction of ordinary hadrons. Therefore, the $\Delta I = \frac{1}{2}$ rule holds if a condition for the cancellation of the $(\underline{84},\underline{1})$ representation in the current-current interaction is satisfied in the combination $(\underline{15},\underline{1}) \otimes (\underline{15},\underline{1}) + (\underline{15},\underline{8}) \otimes (\underline{15},\underline{8})$.

Thus the models[10,11] along the line suggested by

Kingsley and Fujii *et al.*, explain successfully the $\Delta I=\frac{1}{2}$ rule, although they are not part of a gauge theory. Here we try to accommodate their idea to our theory. As a result of this accomodation, we show that a condition for canceling the $(\underline{84},\underline{1})$ part gives a rigid bound for κ^2. The reason is that we need at least a comparable magnitude of $(\underline{15},\underline{8})$ to that of $(\underline{15},\underline{1})$ for the cancellation of $(\underline{84},\underline{1})$.

A piece of the current product concerned with nonleptonic decays of ordinary hadrons is

$$\frac{1}{24} f_{84}[\bar{u}_i\gamma_\rho(1-\gamma_5)d_i\bar{s}_j\gamma^\rho(1-\gamma_5)u_j+\bar{u}_i\gamma_\rho(1-\gamma_5)d_j\bar{s}_j\gamma^\rho(1-\gamma_5)u_i] + \frac{1}{12} f_{20}[\bar{u}_i\gamma_\rho(1-\gamma_5)d_i\bar{s}_j\gamma^\rho(1-\gamma_5)u_j-\bar{u}_i\gamma_\rho(1-\gamma_5)d_j\bar{s}_j\gamma^\rho(1-\gamma_5)u_i], \quad (19)$$

where the coefficients are given by

$$f_{84(20)}=(\cos\phi_1+A_{11}+A_{33})(\sin\phi_1+A_{12}+A_{34})+(-)(\cos\phi_1\sin\phi_1+A_{11}A_{12}+A_{33}A_{34}+A_{13}A_{14}+A_{31}A_{32}). \quad (20)$$

See Appendix C. We require

$$f_{84}=0, \quad (21)$$

which is a condition that the $\Delta C=0$, $\Delta S=1$ part of the color-singlet nonleptonic interaction does not involve the $\Delta I=\frac{3}{2}$ term. The restriction (21) is written

$$9\kappa^2\sin 2\theta_C+\sin 2\phi_1+a_2^{\,2}\sin(2\phi_2+2\delta_2)+a_3^{\,2}\sin(2\phi_3+2\delta_3)+b_2^{\,2}\sin(2\phi_3+2\gamma_2)+b_3^{\,2}\sin(2\phi_2+2\gamma_3)=0, \quad (22)$$

where

$$b_i=(\cos^2\psi_i\sin^2\alpha+\sin^2\psi_i\sin^2\beta)^{1/2}\leq 1, \quad (23)$$

$$b_i\sin\gamma_i=\sin\psi_i\sin\beta, \qquad b_i\cos\gamma_i=\cos\psi_i\sin\alpha. \quad (24)$$

Equation (22) leads to an inequality

$$9\kappa^2\sin 2\theta_C\leq 3, \quad (25)$$

since $a_i^{\,2}+b_i^{\,2}=1$. Taking the popular value $\sin\theta_C=0.22$, one obtains

$$\kappa^2\leq 0.78. \quad (26)$$

Moreover, in addition to the condition (21), we must impose the restriction (7) on the model. Then we get a more rigid upper bound,[17]

$$\kappa_{\max}^{\,2}=0.37. \quad (27)$$

This means that the slope of the linearly rising cross section $\sigma(\nu_\mu(\bar{\nu}_\mu)N\rightarrow\mu^-(\mu^+)X)$ must be more than twice[18] as large at energies high enough to produce the color-octet states as it is at lower energies, even if the effects of the weak boson and heavy leptons are taken into account.

The data of the $\nu_\mu(\bar{\nu}_\mu)N$ reaction show no visible change of the slope at present.[19] This fact compels us either to accept that the color degrees of freedom are still not excited at present energies or to abandon the cancellation mechanism of the $(\underline{84},\underline{1})$ representation.

ACKNOWLEDGMENTS

The computational work for this paper was carried out at the Computer Center of Hiroshima University by Professor M. Yonezawa and Dr. T. Yanagida. We are both indebted to Dr. T. Yanagida also for helpful discussions.

APPENDIX A

Here we present the general structure of the weak charged, electromagnetic, and weak neutral currents in our theory. Below we take g to be associated with the $SU(2)_L$ gauge group and $g'/2$ with the U(1) gauge group.

Weak charged current

The charged current which couples to the charged weak boson W^c with the coupling strength $g/2\sqrt{2}$ is given by

$$J_\rho^{(c)}=\tfrac{1}{3}(\cos\phi_1+A_{11}+A_{33})\sum_i \bar{u}_i\gamma_\rho(1-\gamma_5)d_i+\tfrac{1}{3}(\sin\phi_1+A_{12}+A_{34})\sum_i \bar{u}_i\gamma_\rho(1-\gamma_5)s_i$$
$$+\tfrac{1}{3}(-\sin\phi_1+A_{21}+A_{43})\sum_i \bar{c}_i\gamma_\rho(1-\gamma_5)d_i+\tfrac{1}{3}(\cos\phi_1+A_{22}+A_{44})\sum_i \bar{c}_i\gamma_\rho(1-\gamma_5)s_i$$
$$+\tfrac{1}{2}(\cos\phi_1-A_{11})[\bar{u}_1\gamma_\rho(1-\gamma_5)d_1-\bar{u}_2\gamma_\rho(1-\gamma_5)d_2]+\tfrac{1}{2}(\sin\phi_1-A_{12})[\bar{u}_1\gamma_\rho(1-\gamma_5)s_1-\bar{u}_2\gamma_\rho(1-\gamma_5)s_2]$$
$$+\tfrac{1}{2}(-\sin\phi_1-A_{21})[\bar{c}_1\gamma_\rho(1-\gamma_5)d_1-\bar{c}_2\gamma_\rho(1-\gamma_5)d_2]+\tfrac{1}{2}(\cos\phi_1-A_{22})[\bar{c}_1\gamma_\rho(1-\gamma_5)s_1-\bar{c}_2\gamma_\rho(1-\gamma_5)s_2]$$
$$+\frac{1}{2\sqrt{3}}(\cos\phi_1+A_{11}-2A_{33})\frac{1}{\sqrt{3}}[\bar{u}_1\gamma_\rho(1-\gamma_5)d_1+\bar{u}_2\gamma_\rho(1-\gamma_5)d_2-2\bar{u}_3\gamma_\rho(1-\gamma_5)d_3]$$
$$+\frac{1}{2\sqrt{3}}(\sin\phi_1+A_{12}-2A_{34})\frac{1}{\sqrt{3}}[\bar{u}_1\gamma_\rho(1-\gamma_5)s_1+\bar{u}_2\gamma_\rho(1-\gamma_5)s_2-2\bar{u}_3\gamma_\rho(1-\gamma_5)s_3]$$
$$+\frac{1}{2\sqrt{3}}(-\sin\phi_1+A_{21}-2A_{43})\frac{1}{\sqrt{3}}[\bar{c}_1\gamma_\rho(1-\gamma_5)d_1+\bar{c}_2\gamma_\rho(1-\gamma_5)d_2-2\bar{c}_3\gamma_\rho(1-\gamma_5)d_3]$$
$$+\frac{1}{2\sqrt{3}}(\cos\phi_1+A_{22}-2A_{44})\frac{1}{\sqrt{3}}[\bar{c}_1\gamma_\rho(1-\gamma_5)s_1+\bar{c}_2\gamma_\rho(1-\gamma_5)s_2-2\bar{c}_3\gamma_\rho(1-\gamma_5)s_3]$$
$$+A_{13}\bar{u}_2\gamma_\rho(1-\gamma_5)d_3+A_{14}\bar{u}_2\gamma_\rho(1-\gamma_5)s_3+A_{23}\bar{c}_2\gamma_\rho(1-\gamma_5)d_3+A_{24}\bar{c}_2\gamma_\rho(1-\gamma_5)s_3$$
$$+A_{31}\bar{u}_3\gamma_\rho(1-\gamma_5)d_2+A_{32}\bar{u}_3\gamma_\rho(1-\gamma_5)s_2+A_{41}\bar{c}_3\gamma_\rho(1-\gamma_5)d_2+A_{42}\bar{c}_3\gamma_\rho(1-\gamma_5)s_2\,. \quad \text{(A1)}$$

Here the A_{ij} (i,j) elements of the matrix A defined in Eq. (5) are

$$A_{11}=\cos\psi_2\cos\alpha\cos\phi_2-\sin\psi_2\cos\beta\sin\phi_2\,, \quad \text{(A2a)}$$
$$A_{12}=\cos\psi_2\cos\alpha\sin\phi_2+\sin\psi_2\cos\beta\cos\phi_2\,, \quad \text{(A2b)}$$
$$A_{13}=\cos\psi_2\sin\alpha\cos\phi_3-\sin\psi_2\sin\beta\sin\phi_3\,, \quad \text{(A2c)}$$
$$A_{14}=\cos\psi_2\sin\alpha\sin\phi_3+\sin\psi_2\sin\beta\cos\phi_3\,, \quad \text{(A2d)}$$
$$A_{21}=-\sin\psi_2\cos\alpha\cos\phi_2-\cos\psi_2\cos\beta\sin\phi_2\,, \quad \text{(A2e)}$$
$$A_{22}=-\sin\psi_2\cos\alpha\sin\phi_2+\cos\psi_2\cos\beta\cos\phi_2\,, \quad \text{(A2f)}$$
$$A_{23}=-\sin\psi_2\sin\alpha\cos\phi_3-\cos\psi_2\sin\beta\sin\phi_3\,, \quad \text{(A2g)}$$
$$A_{24}=-\sin\psi_2\sin\alpha\sin\phi_3+\cos\psi_2\sin\beta\cos\phi_3\,, \quad \text{(A2h)}$$
$$A_{31}=-\cos\psi_3\sin\alpha\cos\phi_2+\sin\psi_3\sin\beta\sin\phi_2\,, \quad \text{(A2i)}$$
$$A_{32}=-\cos\psi_3\sin\alpha\sin\phi_2-\sin\psi_3\sin\beta\cos\phi_2\,, \quad \text{(A2j)}$$
$$A_{33}=\cos\psi_3\cos\alpha\cos\phi_3-\sin\psi_3\cos\beta\sin\phi_3\,, \quad \text{(A2k)}$$
$$A_{34}=\cos\psi_3\cos\alpha\sin\phi_3+\sin\psi_3\cos\beta\cos\phi_3\,, \quad \text{(A2l)}$$
$$A_{41}=\sin\psi_3\sin\alpha\cos\phi_2+\cos\psi_3\sin\beta\sin\phi_2\,, \quad \text{(A2m)}$$
$$A_{42}=\sin\psi_3\sin\alpha\sin\phi_2-\cos\psi_3\sin\beta\cos\phi_2, \quad \text{(A2n)}$$
$$A_{43}=-\sin\psi_3\cos\alpha\cos\phi_3-\cos\psi_3\cos\beta\sin\phi_3\,, \quad \text{(A2o)}$$
$$A_{44}=-\sin\psi_3\cos\alpha\sin\phi_3+\cos\psi_3\cos\beta\cos\phi_3\,. \quad \text{(A2p)}$$

Electromagnetic current

The electromagnetic current which couples to the photon with the coupling strength $gg'/(g^2+g'^2)^{1/2}=e$ is

$$J_\rho^{\rm em}=\tfrac{2}{3}\sum_i(\bar{u}_i\gamma_\rho u_i+\bar{c}_i\gamma_\rho c_i)-\tfrac{1}{3}\sum_i(\bar{d}_i\gamma_\rho d_i+\bar{s}_i\gamma_\rho s_i)+\tfrac{1}{2}(\alpha_1-\alpha_2)\sum_{q=u,d,s,c}(\bar{q}_1\gamma_\rho q_1-\bar{q}_2\gamma_\rho q_2)$$
$$+\frac{1}{2\sqrt{3}}(\alpha_1+\alpha_2-2\alpha_3)\sum_{q=u,d,s,c}\frac{1}{\sqrt{3}}(\bar{q}_1\gamma_\rho q_1+\bar{q}_2\gamma_\rho q_2-2\bar{q}_3\gamma_\rho q_3)\,, \quad \text{(A3)}$$

where the α_i's are defined in Eq. (1) with the condition (2).

Weak neutral current

The neutral current which couples to the neutral weak boson Z with coupling strength $(g^2+g'^2)^{1/2}/2$ has the form

$$J_\rho^{(0)}=\tfrac{1}{2}\sum_i[\bar{u}_i\gamma_\rho(1-\gamma_5)u_i-\bar{d}_i\gamma_\rho(1-\gamma_5)d_i]+\tfrac{1}{2}\sum_i[\bar{c}_i\gamma_\rho(1-\gamma_5)c_i-\bar{s}_i\gamma_\rho(1-\gamma_5)s_i]-2\sin^2\theta_W J_\rho^{\text{em}}, \tag{A4}$$

where $\sin\theta_W=g'/(g^2+g'^2)^{1/2}$.

The color-singlet parts of the electromagnetic and the neutral weak currents are identical to the currents in the standard four-quark model.

APPENDIX B

Weak-boson effect

The differential cross section for the inclusive neutrino reaction $\nu_\mu(\bar{\nu}_\mu)N\to\mu^-(\mu^+)X$ mediated by the weak boson with mass M_W is

$$\frac{d^2\sigma_{\nu(\bar{\nu})}}{dxdy}=\frac{G_F{}^2ME_{\nu(\bar{\nu})}}{\pi\kappa^2}\frac{1}{(1+rxy)^2}[xy^2F_1^{\nu(\bar{\nu})}+(1-y)F_2^{\nu(\bar{\nu})}-(+)(1-\tfrac{1}{2}y)xyF_3^{\nu(\bar{\nu})}], \tag{B1}$$

where

$$r\equiv 2ME_{\nu(\bar{\nu})}/M_W{}^2, \tag{B2}$$

and M is the nucleon mass. The naive quark-parton model allows us to write Eq. (B1) far above both charm and color thresholds as follows:

$$\left(\frac{d^2\sigma_\nu}{dxdy}\right)_{\text{above}}=\frac{G_F{}^2ME_\nu}{\pi\kappa^2}\frac{1}{(1+rxy)^2}[q_\nu(x)+(1-y)^2\bar{q}_\nu(x)], \tag{B3a}$$

$$\left(\frac{d^2\sigma_{\bar{\nu}}}{dxdy}\right)_{\text{above}}=\frac{G_F{}^2ME_{\bar{\nu}}}{\pi\kappa^2}\frac{1}{(1+rxy)^2}[\bar{q}_{\bar{\nu}}(x)+(1-y)^2q_{\bar{\nu}}(x)]. \tag{B3b}$$

By carrying out the y integral, Eqs. (B3a) and (B3b) become

$$(\sigma_\nu)_{\text{above}}=\frac{G_F{}^2ME_\nu}{\pi\kappa^2}\int_0^1 dx\left\{\frac{q_\nu(x)}{(1+rx)}+\bar{q}_\nu(x)\left[\frac{(2+rx)}{(rx)^2}-\frac{2(1+rx)}{(rx)^3}\ln(1+rx)\right]\right\}, \tag{B4a}$$

$$(\sigma_{\bar{\nu}})_{\text{above}}=\frac{G_F{}^2ME_{\bar{\nu}}}{\pi\kappa^2}\int_0^1 dx\left\{\frac{\bar{q}_{\bar{\nu}}(x)}{(1+rx)}+q_{\bar{\nu}}(x)\left[\frac{(2+rx)}{(rx)^2}-\frac{2(1+rx)}{(rx)^3}\ln(1+rx)\right]\right\}. \tag{B4b}$$

Since

$$\frac{1}{(1+rx)}\geq\frac{1}{(1+r)}, \tag{B5}$$

$$\frac{(2+rx)}{(rx)^2}-\frac{2(1+rx)}{(rx)^3}\ln(1+rx)\geq\frac{1}{3(1+r)}, \tag{B6}$$

for $1\geq x\geq 0$ and $r\geq 0$, the following inequalities hold:

$$(\sigma_\nu)_{\text{above}}>\frac{G_F{}^2ME_\nu}{\pi\kappa^2}\frac{1}{(1+r)}\int_0^1 dx[q_\nu(x)+\tfrac{1}{3}\bar{q}_\nu(x)], \tag{B7a}$$

$$(\sigma_{\bar{\nu}})_{\text{above}}>\frac{G_F{}^2ME_{\bar{\nu}}}{\pi\kappa^2}\frac{1}{(1+r)}\int_0^1 dx[\bar{q}_{\bar{\nu}}(x)+\tfrac{1}{3}q_{\bar{\nu}}(x)]. \tag{B7b}$$

We can safely neglect the weak-boson effect on the cross sections below color and charm thresholds. Therefore we can derive an inequality for the slope

$$\frac{(\alpha_{\nu(\bar{\nu})})_{\text{above}}}{(\alpha_{\nu(\bar{\nu})})_{\text{below}}}>\frac{1}{(1+r)}\frac{1}{\kappa^2}. \tag{B8}$$

Heavy-lepton effect

Well above the charm and color thresholds, heavy leptons may be produced and may decay into $\mu^-(\mu^+)$. Then the cross sections observed are equal to or larger than the cross sections (B4a) and (B4b). We can thus assert the inequality (B8) safely.

APPENDIX C

The charged currents consist of two parts, color-singlet and octet parts:

$$J_\mu=\sum_{a,b}\sum_{i,j}C^{bj}_{ai}J^{ai}_{\mu bj}=\tfrac{1}{3}\sum_{a,b}\mathrm{Tr}[C^b_a]\mathrm{Tr}[J^a_{\mu b}]+\sum_{a,b}\sum_{i,j}C^{bj}_{ai}(J^{ai}_{\mu bj}-\tfrac{1}{3}\delta^i_j\mathrm{Tr}[J^a_{\mu b}])\,, \tag{C1}$$

where $J^{ai}_{\mu bj}$ and C^{bj}_{ai} (a,b and i,j are flavor and color indices, respectively) mean the current $[\bar{q}_{ai}\gamma_\mu(1-\gamma_5)q_{bj}]$ and its coefficients, respectively, and the trace is carried out over all color indices. The color-singlet part of the current-current nonleptonic interaction is given by

$$(J^{\mu b}_a J^{\dagger d}_{\mu c})_{\text{color singlet}}=\tfrac{1}{12}f_{20}(\mathrm{Tr}[J^{\mu b}_a]\mathrm{Tr}[J^{\dagger d}_{\mu c}]-\mathrm{Tr}[J^{\mu b}_a J^{\dagger d}_{\mu c}])+\tfrac{1}{24}\,f_{84}(\mathrm{Tr}[J^{\mu b}_a]\mathrm{Tr}[J^{\dagger d}_{\mu c}]+\mathrm{Tr}[J^{\mu b}_a J^{\dagger d}_{\mu c}])\,, \tag{C2}$$

where

$$f_{20(84)}=\mathrm{Tr}[C^a_b]\mathrm{Tr}[C^c_d]-(+)\mathrm{Tr}[C^a_b C^c_d]\,. \tag{C3}$$

---

[1]H. Fritzsch and M. Gell-Mann, e.g., in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972*, edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 2, p. 135. See also O. W. Greenberg, Phys. Rev. Lett. 13, 598 (1964).

[2]M.-Y. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965), and Phys. Rev. D 10, 674 (1974); N. Cabibbo, L. Maiani, and G. Preparata, Phys. Lett. B25, 132 (1967); Y. Miyamoto, Prog. Theor. Phys. Suppl. Extra No. 187 (1965); S. Hori, Prog. Theor. Phys. 36, 131 (1966).

[3]T. Tati, Prog. Theor. Phys. 35, 126 (1966); 35, 973 (1966).

[4]H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. B47, 365 (1973).

[5]At the present stage the most direct evidence for charm comes from the observations of new particles, D^0(1865) [see G. Goldhaber *et al*., Phys. Rev. Lett. 37, 255 (1976)], D^*(1876) [see L. Peruzzi *et al*., *ibid*. 37, 569 (1976)], Σ_c^{++} (2426) [see E. G. Cazzoli *et al*., *ibid*. 34, 1125 (1975)], Λ_c^+(2260) [see B. Knapp *et al*., *ibid*. 37, 882 (1976)], and of neutrino-induced reactions in which a pair of oppositely charged muons appears in the final states [see A. Benvenuti *et al*., *ibid*. 34, 419 (1975), and 35, 1199 (1975); 35, 1203 (1975); 35, 1249 (1975); B. C. Barish *et al*., *ibid*. 36, 939 (1976)].

[6]For general reviews see, e.g., S. Weinberg, Rev. Mod. Phys. 46, 255 (1974); A. De Rújula, H. Georgi, S. L. Glashow, and H. R. Quinn, Rev. Mod. Phys. 46, 391 (1974); M. A. B. Bég and A. Sirlin, Annu. Rev. Nucl. Sci. 24, 379 (1974); M. K. Gaillard, B. W. Lee, and J. L. Rosner, Rev. Mod. Phys. 47, 277 (1975).

[7]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[8]C. A. Nelson, Lett. Nuovo Cimento 14, 369 (1975); Phys. Rev. D 15, 3256 (1977); Phys. Rev. D (to be published).

[9]M. Suzuki, Phys. Rev. Lett. 35, 1553 (1975).

[10]R. L. Kingsley, Phys. Lett. B40, 387 (1972).

[11]K. Fujii, M. Katuya, S. Okubo, and S. Tamura, Prog. Theor. Phys. 49, 995 (1973).

[12]A. I. Sanda and H. Terazawa, Phys. Rev. Lett. 34, 1403 (1975).

[13]In the four-quark model of the Fritzsch–Gell-Mann type ($\alpha_2=\alpha_3=\alpha_1=0$), ($d_1'$, s_1'), and (d_2', s_2', d_3', s_3') may be mixed generally.

[14]G. Feldman and P. T. Matthews, Phys. Lett. B63, 68 (1976); M. Katuya and Y. Koide, Lett. Nuovo Cimento 18, 21 (1977).

[15]For example, consider a model

$$\begin{pmatrix}\nu_e\cos\phi+N_e\sin\phi\\ e^-\end{pmatrix}_L,\ \begin{pmatrix}\nu_\mu\cos\phi+N_\mu\sin\phi\\ \mu^-\end{pmatrix}_L,\ \text{and singlets,}$$

where $\cos\phi=\kappa$. Note that in this model the four-fermion interaction describing low-energy phenomena is identical to that in the Weinberg-Salam model.

[16]In our model, the mass of a weak boson is represented by

$$M_W=\left(\frac{\pi\alpha}{\sqrt{2}\,G_F}\right)^{1/2}\frac{\kappa}{\sin\theta_W}\simeq\kappa\times\frac{38}{\sin\theta_W}\ \text{GeV},$$

so that too small a value of κ is unlikely.

[17]This maximum value of κ^2 was obtained by a Monte Carlo search under the condition

$$f_{84}^2+[k\cos\theta-(\cos\phi_1+A_{11}+A_{33})/3]^2+[k\sin\theta-(\sin\phi_1+A_{12}+A_{34})/3]^2=0.$$

Note that the parameters a_2, a_3, γ_2, γ_3 and δ_2, δ_3 are not independent in Eq. (22). The search was carried out by using the independent parameters ϕ_i, ψ_i, α, and β.

[18]This statement is correct below $E_{\nu(\bar\nu)}\simeq 300$ GeV ($\sin^2\theta_W\simeq 0.3$ is assumed).

[19]B. C. Barish *et al*., Phys. Rev. Lett. 35, 1316 (1975).