Are hydrodynamic models of high-energy collisions credible?*

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The foundations of hydrodynamic models for high-energy collisions are questioned by pointing out that most of the energy in such collisions must be radiated off in electromagnetic and strong bremsstrahlung before the equilibrium initial conditions assumed in hydrodynamic models could be established.

A number of hydrodynamic models have been proposed and developed over the years for collisions of very-high-energy particles.[1] Although they differ in various details, one common and crucial element[2] is the assumption that, as the two particles collide, they come to a stop so that their material can thoroughly mingle and thus lose all memory of its origin. This is an assumption completely analogous to that in compound-nucleus formation which is used so successfully in nuclear physics at much lower energies.

The purpose of this note is to point out that this central assumption immediately leads to a major internal inconsistency that is quite independent of the details of the model, and which appears to be an insurmountable obstacle in taking any of these models seriously. In particular, we will demonstrate that the requirement of the two particles coming to a stop so as to intermingle would involve electromagnetic and strong bremsstrahlung on a scale that would dissipate most of the kinetic energy of the particles before the presumed hydrodynamic processes could even begin to function.

We begin with a calculation of the classical electromagnetic bremsstrahlung—electromagnetic since that is relatively calculable, classical because the hydrodynamic models are classical. The calculation is a simple one. Let us consider two disks approaching each other with the velocity of light. The disks represent the two particles, flattened by relativistic contraction. For the moment let us assume that the two particles do not interact until the two disks begin to overlap, and then decelerate so as to come to a stop by the time the two disks completely overlap. Since the thickness of the disk is γr_0, where r_0 is the "diameter" of the particle at rest (a quantity that is roughly 1 F), the time in question will be $r_0/c\gamma$ [here $\gamma = (1-\beta^2)^{1/2}$].

The bremsstrahlung energy loss per unit time, P, is given by

$$P = \frac{2}{3}\frac{e^3}{m^2c^3}\left(\frac{dE}{dx}\right)^2, \qquad (1)$$

where E is the total energy of the charged particle. We will make the most conservative assumption, namely that the variation of E is linear with distance (or with time). With this assumption the uncertainty principle is satisfied, because

$$\Delta E\,\Delta t \approx \Delta E\,\frac{\Delta x}{c} \approx \frac{\sqrt{s}}{2}\,\frac{10}{\sqrt{s}} > 1\,, \qquad (2)$$

where we use units of GeV with $\hbar = c = 1$. In contrast, any other assumption for the variation of E with distance is likely to violate the uncertainty principle (locally if not globally).

Thus the total energy loss W, in GeV's, is given by

$$W = \int \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dE}{dx}\right)^2 dt = \int \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dE}{dx}\right)^2 \frac{dx}{c}$$
$$\approx 5\times 10^{-3}\int_0^{10/\sqrt{s}}\left(\frac{dE}{dx}\right)^2 dx = 2.5\times 10^{-4} s^{3/2}\,. \qquad (3)$$

We see therefore that for energies such that $s \gtrsim 2000$ (and therefore energies including those at the CERN ISR) bremsstrahlung would carry away all of the kinetic energy before the whole presumed deceleration process could take place. In other words, such a deceleration simply cannot occur, and the two particles would be unable to stop each other by the process assumed in the hydrodynamic model. We note that even allowing the range of interaction (and hence the distance of deceleration) to be fixed 1 F in the center-of-mass system will not remove this problem. Indeed, there is another way to state these results. Because

$$dE > dW = P\,dx = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dE}{dx}\right)^2 dx\,, \qquad (4)$$

we require

$$\frac{dE}{dx} < 200\,. \qquad (5)$$

So, in general, for a classical particle with electronic charge, it is impossible to lose energy at a greater rate than about 1000 GeV/F. At sufficiently (but not astronomically) high energies, however, the hydrodynamic model would require exactly such an unreasonably high energy loss.

How would quantum-mechanical effects alter the above results? We want to include the notion of photons (so that the conservation of energy implies a maximum radiated frequency) and also the idea that there is a distribution of stopping distances (i.e., the photons have fuzzy edges). We can do this by taking the frequency spectrum of an instantaneously accelerated photon and cutting it off at $\omega_{max} \approx s^{1/2}/2$. We then find for the radiated energy, E_{rad},

$$\frac{E_{rad}}{E} \approx \frac{2}{\pi}\frac{e^2}{\hbar c}\left[\ln\left(\frac{2E}{mc^2}\right)-1\right] \approx 5\times 10^{-3}(\ln s^{1/2}-1)\,. \tag{6}$$

For $s^{1/2}=50$, this gives

$$\frac{E_{rad}}{E} \approx 1-5\%\,. \tag{7}$$

Thus the inclusion of quantum effects reduces the electromagnetic bremsstrahlung to "reasonable" proportions. At this point, however, we return to the question of the dynamics of hadronic particle production. We expect (although we cannot give an exact calculation of it) that there will also be a hadronic bremsstrahlung whose strength will be 10^3-10^4 times that of the electromagnetic bremsstrahlung as given in Eq. (6). Hence at $s^{1/2}=50$, for example, we expect, even with quantum effects taken into account, that most of the energy of the decelerating protons will be radiated away as hadronic bremsstrahlung *before* the hydrodynamic mechanism of production comes into operation. More succinctly, we expect that the fraction of events in which hydrodynamics can play a significant role is small and falls with increasing energy. As a result, we expect hydrodynamic processes to be only of marginal interest.

The above argument can be countered by the assertion that sure enough, strong bremsstrahlung will occur during the deceleration process, but then the radiated hadrons (moving predominately in the same direction as the radiating particles) will collide "head on", and this radiated material will then establish thermal equilibrium. However, such an argument cannot save the day. First of all we have shown that just by the uncertainty principle it is not possible to radiate off a substantial amount of the energy during the very short collision time. Furthermore, the mean free path of the radiated particles is highly unlikely to be short enough to produce the significant number of collisions needed for thermalization. On the energy shell, hadrons at high energies have a mean free path of several F in hadronic matter,[4] and, although one cannot be certain, it is quite implausible to assume that off the energy shell at the same energies the cross sections would increase by the order of magnitude to produce the requisite number of collisions for thermalization. Thus, trying to have the radiated material shoulder the responsibility of thermalization is not very promising either.

This conclusion is quite general. We know that in hadronic collision the initial particles retain, on the average, only half of their center-of-mass energy, and so by our previous arguments we expect most of the remaining energy to have been radiated away continuously during the process of deceleration. This tells us that immediately after the initial deceleration the produced hadronic material will have highly non-trivial velocity distribution, far from that of material in thermal equilibrium. Any credible hydrodynamic model must therefore include initial conditions that will realistically represent this velocity distribution. One may think of the hydrodynamic model as providing a description of the final-state interactions that occur after the initial "bremsstrahlung" within the two "blobs" of hadronic material flying off in directions and with velocities that are only little different from those of the two original colliding particles.[3] Whether the model will be tractable with such initial conditions remains to be seen, but it appears somewhat doubtful. In any case, before applying hydrodynamics one must generate some understanding of the quite separate *dynamics* ("bremsstrahlung") that will determine the initial conditions.

Finally it may be objected that we are taking a model too literally and seriously, and that as long as a model provides predictions which appear to be in agreement with experimental data, the foundations of such a model should not be questioned. However, there is a difference between a purely phenomenological fitting of data and a model. In the latter we not only summarize data in an economical way but also claim that the system in question functions in certain respects *as if it were* the system described in the model. If there are general laws that *a priori* prevent the system under study to behave at all as the model system, the plausibility of the model is certainly seriously compromised. Furthermore, recent work on the hydrodynamic models does take the analogy seriously enough to attempt a space-and-time description of the hydrodynamic processes. Under such circumstances, it is not only permissible, but mandatory to point out basic flaws in the conceptual foundations of the model.

Whereas the conclusions of this note are ours, we are indebted to a large number of people for stimulating, animated, and critical discussions, sometimes containing constructive expressions of disbelief. Among them are Charles Chiu, Paul Csonka, Rudolph Hwa, Peter Carruthers, Fred Cooper, and Hans Bethe.

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[1]For a sample of the old and new in the bibliography of hydrodynamic models see, e.g., L. D. Landau, Izv. Acad. Nauk SSSR Ser. Fiz. 17, 51 (1953); S. Z. Balankij and L. D. Landau, Usp. Fiz. Nauk 56, 309 (1956) [both reprinted in English in *Collected Papers of L. D. Landau*, edited by D. ter Haar (Gordon and Breach, New York, 1965)]; F. Cooper and E. Schonberg, Phys. Rev. Lett. 30, 880 (1973); Phys. Rev. D 8, 334 (1973); P. Carruthers and Minh Duong-van, Phys. Lett. 41B, 597 (1972); a review by P. Carruthers, Cornell Report No. CLNS-219, 1973 (unpublished); Suhonen *et al.*, Phys. Rev. Lett. 31, 1567 (1973); F. Cooper, G. Frye, and E. Schonberg, Phys. Rev. D 11, 192 (1975); R. C. Hwa, *ibid.* 10, 2260 (1974).

[2]To provide for leading-particle effects, more sophisticated versions of the model have the protons losing on the average only half of their center-of-mass energy. The remaining "material" is said to appear at rest initially in the center-of-mass system. All our conclusions continue to apply to these versions also, with appropriate insertions of factors of 2.

[3]This point of view has been stressed by Charles Chiu (private communication).

[4]Michael I. Sobel, Philip J. Siemens, Jakob P. Bondorf, and Hans A. Bethe, Nucl. Phys. A251, 502 (1975).