Comment on Weinberg's gauge theory of *CP* nonconservation*

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We show that if *CP* violation occurs through the exchange of *charged* Higgs bosons as recently proposed by Weinberg, then *P* (and hence *CP*) is also violated through the exchange of *neutral* Higgs bosons. Whereas $K_L \to 2\pi$ is not affected by this additional interaction, the neutron electric dipole moment is, and its value can be well within experimental limits.

Recently, Weinberg[1] proposed a gauge theory of *CP* nonconservation through the exchange of *charged* Higgs bosons. This interaction was shown to be "naturally" milliweak in strength, and obey the $\Delta I=\frac{1}{2}$ rule in $K_L\to 2\pi$. Its contribution to the neutron electric dipole moment D_n was estimated to be about $2.3\times 10^{-24}\, e$ cm, which is only somewhat bigger than the experimental result of $(0.4 \pm 1.1)\times 10^{-24}\, e$ cm. In this note, we point out that there is an additional contribution to D_n because parity (and therefore *CP*) is necessarily violated as well in such a scheme through the exchange of *neutral* Higgs bosons. Therefore, although D_n is still expected to be of the order of $10^{-24}\, e$ cm, its exact value cannot be determined from $K_L\to 2\pi$ data alone.

Consider first the most general gauge-invariant, renormalizable Higgs potential for *two* doublets, which is also invariant under the discrete transformation $\Phi_i\to -\Phi_i$:

$$\begin{aligned}V=&-\mu_1^2\Phi_1^\dagger\Phi_1-\mu_2^2\Phi_2^\dagger\Phi_2+h_1(\Phi_1^\dagger\Phi_1)^2+h_2(\Phi_2^\dagger\Phi_2)^2\\&+f_{12}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+g_{12}(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\\&+k_{12}(\Phi_1^\dagger\Phi_2)^2+k_{12}^*(\Phi_2^\dagger\Phi_1)^2\ ,\end{aligned}\tag{1}$$

where $\Phi_1=(\phi_1^+,\phi_1^0)$ and $\Phi_2=(\phi_2^+,\phi_2^0)$. Let ϕ_1^0, ϕ_2^0 acquire complex vacuum expectation values $v_1/\sqrt{2}$, $v_2/\sqrt{2}$, respectively, and define

$$\begin{aligned}\phi_1^0&\equiv\frac{v_1}{\sqrt{2}}\left(1+\frac{H_1+i\chi_1}{|v_1|}\right)\ ,\\ \phi_2^0&\equiv\frac{v_2}{\sqrt{2}}\left(1+\frac{H_2+i\chi_2}{|v_2|}\right)\ ,\end{aligned}\tag{2}$$

where $|v_1|$, $|v_2|$ and the relative phase between v_1 and v_2 are determined by the condition that the shifted potential has no terms linear in any of the fields H_1, H_2 or $|v_1|^{-1}\chi_1-|v_2|^{-1}\chi_2$. In the tree approximation, the three respective constraint equations are

$$-\mu_1^2|v_1|+h_1|v_1|^3+\tfrac{1}{2}(f_{12}+g_{12})|v_1||v_2|^2+\mathrm{Re}k_{12}(v_1^*v_2)^2|v_1|^{-1}=0\ ,\tag{3}$$

$$-\mu_2^2|v_2|+h_2|v_2|^3+\tfrac{1}{2}(f_{12}+g_{12})|v_1|^2|v_2|+\mathrm{Re}k_{12}(v_1^*v_2)^2|v_2|^{-1}=0\ ,\tag{4}$$

$$\mathrm{Im}k_{12}(v_1^*v_2)^2=0\ .\tag{5}$$

The mass matrix is then given by

$$\begin{aligned}&[-\tfrac{1}{2}g_{12}|v_1|^2|v_2|^2-\mathrm{Re}k_{12}(v_1^*v_2)^2]\left(\frac{\phi_1^-\phi_1^+}{|v_1|^2}+\frac{\phi_2^-\phi_2^+}{|v_2|^2}\right)\ ,\\&+[\tfrac{1}{2}g_{12}|v_1|^2|v_2|^2+k_{12}(v_1^*v_2)^2]\left(\frac{\phi_1^-\phi_2^+}{v_1^*v_2}+\frac{\phi_2^-\phi_1^+}{v_2^*v_1}\right)\ ,\\&+h_1|v_1|^2H_1^2+h_2|v_2|^2H_2^2+(f_{12}+g_{12})|v_1||v_2|H_1H_2\ ,\\&-\mathrm{Re}k_{12}(v_1^*v_2)^2\left(\frac{\chi_1}{|v_1|}-\frac{\chi_2}{|v_2|}\right)^2\\&\qquad+\mathrm{Im}k_{12}(v_1^*v_2)^2\left(\frac{H_1}{|v_1|}+\frac{H_2}{|v_2|}\right)\left(\frac{\chi_1}{|v_1|}-\frac{\chi_2}{|v_2|}\right)\ ,\end{aligned}\tag{6}$$

where we have only used the constraints (3) and (4). We see therefore that because of (5), the coefficient of $\phi_1^-\phi_2^+/v_1^*v_2$ has no imaginary part, and the state $|v_1|^{-1}\chi_1-|v_2|^{-1}\chi_2$ does not mix with H_1 or H_2. Together, they imply that *CP* is conserved by the Higgs propagators.[1,2] In fact, the first two terms of (6) become simply the mass-squared term for $v_1^{-1}\phi_1^+-v_2^{-1}\phi_2^+$, which is just the orthogonal state to the would-be charged Goldstone boson, as expected.

For *three* (or more) doublets, however, the situation is quite different. The Higgs potential (1) can be easily generalized by permutation of the indices 1, 2, 3. Let us define

$$\begin{aligned}A_3&\equiv-\tfrac{1}{2}g_{12}|v_1|^2|v_2|^2-\mathrm{Re}k_{12}(v_1^*v_2)^2\ ,\\B_3&\equiv\mathrm{Im}k_{12}(v_1^*v_2)^2\ ,\\C_3&\equiv-\tfrac{1}{2}(f_{12}+g_{12})|v_1|^2|v_2|^2-\mathrm{Re}k_{12}(v_1^*v_2)^2\ ,\\D_3&\equiv-\mathrm{Re}k_{12}(v_1^*v_2)^2\ ,\\h_1'&\equiv h_1|v_1|^4\ ,\end{aligned}\tag{7}$$

and similarly for the other permutations, then the constraint (5) is replaced by

$$B_1=B_2=B_3(\equiv B)\tag{8}$$

and the 3×3 mass-squared matrix for the charged states $v_1^{-1}\phi_1^+, v_2^{-1}\phi_2^+, v_3^{-1}\phi_3^+$ is given by[2]

$$\begin{pmatrix}A_2+A_3 & -A_3+iB & -A_2-iB\\ -A_3-iB & A_1+A_3 & -A_1+iB\\ -A_2+iB & -A_1-iB & A_1+A_2\end{pmatrix}\ ,\tag{9}$$

whereas the analogous 6×6 matrix for the neutral states $|v_1|^{-1}H_1, |v_2|^{-1}H_2, |v_3|^{-1}H_3, |v_1|^{-1}\chi_1, |v_2|^{-1}\chi_2, |v_3|^{-1}\chi_3$ has the following form:

$$\begin{bmatrix} h_1' & -C_3 & -C_2 & 0 & -B & B \\ -C_3 & h_2' & -C_1 & B & 0 & -B \\ -C_2 & -C_1 & h_3' & -B & B & 0 \\ 0 & B & -B & D_2+D_3 & -D_3 & -D_2 \\ -B & 0 & B & -D_3 & D_1+D_3 & -D_1 \\ B & -B & 0 & -D_2 & -D_1 & D_1+D_2 \end{bmatrix} . \quad (10)$$

Since the parameter B is not required to be zero, CP is in general violated by the *charged* Higgs propagators, as pointed out by Weinberg.[1] But since the same parameter also mixes H and χ states, P must be violated as well by the *neutral* Higgs propagators. Specifically, we can show[2] that, at zero momentum transfer,

$$\mathrm{Im}\frac{\langle\phi_1^-\phi_2^+\rangle}{v_1^* v_2} = -\frac{v_3^2}{v_0^2}\frac{B}{\Delta} , \quad (11)$$

where $v_0^2 = |v_1|^2 + |v_2|^2 + |v_3|^2$ and $\Delta = A_1A_2 + A_2A_3 + A_3A_1 - B^2$. The analogous expression for $H_1\chi_1$ mixing is then

$$\begin{aligned}\frac{\langle H_1\chi_1\rangle}{|v_1|^2} = {}& (m_1^{-2}\cdots m_5^{-2})|v_1|^{-4}|v_2|^{-4}|v_3|^{-4}B \\ & \times\{B^2|v_2|^2(C_1+C_2-h_3') - B^2|v_3|^2(C_1+C_3-h_2') + D_1(|v_2|^2+|v_3|^2)[C_1(C_2-C_3) - C_2h_2' + C_3h_3'] \\ & -D_2|v_2|^2[C_1(C_1+C_3) + h_2'(C_2-h_3')] + D_3|v_3|^2[C_1(C_1+C_2) + h_3'(C_3-h_2')]\} , \end{aligned} \quad (12)$$

where $m_1, \ldots, m_5$ are the masses of the five physical neutral Higgs bosons. We note that if $|v_1| = |v_2| = |v_3|$ and $D_1 = D_2 = D_3$, then

$$\langle H_1\chi_1\rangle + \langle H_2\chi_2\rangle + \langle H_3\chi_3\rangle = 0 . \quad (13)$$

Thus the sign of any particular $\langle H_i\chi_i\rangle$ is arbitrary.

In the standard four-quark gauge model,[3] the Yukawa interactions of the Higgs bosons with the quark fields can be chosen as follows[1]:

$$\begin{aligned} & \sqrt{2}\,(v_1^*)^{-1}\phi_1^-(m_s\sin\theta\bar{s}u_L + m_d\cos\theta\bar{d}u_L + m_s\cos\theta\bar{s}c_L - m_d\sin\theta\bar{d}c_L) , \\ & -\sqrt{2}\,v_2^{-1}\phi_2^+(m_u\cos\theta\bar{u}d_L + m_u\sin\theta\bar{u}s_L - m_c\sin\theta\bar{c}d_L + m_c\cos\theta\bar{c}s_L) + \text{H.c.} , \\ & +|v_1|^{-1}H_1(m_d\bar{d}d + m_s\bar{s}s) + i|v_1|^{-1}\chi_1(m_d\bar{d}\gamma_5 d + m_s\bar{s}\gamma_5 s) , \\ & +|v_2|^{-1}H_2(m_u\bar{u}u + m_c\bar{c}c) - i|v_2|^{-1}\chi_2(m_u\bar{u}\gamma_5 u + m_c\bar{c}\gamma_5 c) , \end{aligned} \quad (14)$$

where the subscript L denotes left-handed, and θ is the Cabibbo angle. Higgs exchange then leads to effective milliweak Fermi interactions which do not conserve CP. They are

$$\begin{aligned} & \frac{-2\langle\phi_1^-\phi_2^+\rangle}{v_1^* v_2}(m_s\sin\theta\bar{s}u_L + m_d\cos\theta\bar{d}u_L + m_s\cos\theta\bar{s}c_L - m_d\sin\theta\bar{d}c_L) \\ & \qquad \times(m_u\cos\theta\bar{u}d_L + m_u\sin\theta\bar{u}s_L - m_c\sin\theta\bar{c}d_L + m_c\cos\theta\bar{c}s_L) + \text{H.c.} , \end{aligned} \quad (15)$$

which was used in Ref. 1, and

$$\frac{i\langle H_1\chi_1\rangle}{|v_1|^2}(m_d\bar{d}d + m_s\bar{s}s)(m_d\bar{d}\gamma_5 d + m_s\bar{s}\gamma_5 s) - \frac{i\langle H_2\chi_2\rangle}{|v_2|^2}(m_u\bar{u}u + m_c\bar{c}c)(m_u\bar{u}\gamma_5 u + m_c\bar{c}\gamma_5 c) + \langle H_1\chi_2\rangle, \langle H_2\chi_1\rangle \text{ terms,}$$

which was not.

For $|\Delta S| = 1$ processes such as $K_L \to 2\pi$, only (15) contributes, and so the result of Ref. 1 is not altered. However, for $\Delta S = 0$ effects such as the neutron electric dipole moment D_n, both (15) and (16) will contribute. [We have shown explicitly in (11) and (12) that both CP-violating effects are proportional to a single nonzero parameter B.] Since the estimate given in Ref. 1 for D_n is based only on (15), the fact that it is somewhat larger than the experimental value should not be too distressing. Once (16) is also included, the electric dipole moments of the d and u quarks become

$$D_d = \mathrm{Im}\frac{\langle\phi_1^-\phi_2^+\rangle}{v_1^* v_2}\frac{em_d}{12\pi^2}\left(m_u^2\cos^2\theta\ln\frac{m_H^2}{m_u^2} + m_c^2\sin^2\theta\ln\frac{m_H^2}{m_c^2}\right) + \frac{\langle H_1\chi_1\rangle}{|v_1|^2}\frac{em_d}{24\pi^2}\left(m_d^2\ln\frac{m_G^2}{m_d^2}\right) \quad (17)$$

and

$$D_u = -\mathrm{Im}\frac{\langle\phi_1^-\phi_2^+\rangle}{v_1^* v_2}\frac{em_u}{24\pi^2}\left(m_d^2\cos^2\theta\ln\frac{m_H^2}{m_d^2} + m_s^2\sin^2\theta\ln\frac{m_H^2}{m_s^2}\right) + \frac{\langle H_2\chi_2\rangle}{|v_2|^2}\frac{em_u}{12\pi^2}\left(m_u^2\ln\frac{m_G^2}{m_u^2}\right) , \quad (18)$$

where m_H, m_G are typical masses for charged and neutral Higgs bosons, respectively. (The apparent factor-of-2 difference between our expressions and those of Ref. 1 is due to the $\sqrt{2}$ difference in our definitions of the vacuum expectation values.)

Without the extra terms introduced by (16), the neutron electric dipole moment $D_n = (4D_d - D_u)/3$ was estimated by Weinberg to be $D_n \simeq 2.3\times 10^{-24}$ e cm, whereas the most recent experimental value[4] is $(0.4 \pm 1.1)\times 10^{-24}$ e cm. However, with the inclusion of these extra terms in (17) and (18), such an estimate is not so easily done, because although the sign and magnitude of $\mathrm{Im}\langle\phi_1^-\phi_2^+\rangle/v_1^* v_2$ are determined from $K_L \to 2\pi$ data, those of $\langle H_1\chi_1\rangle$ and $\langle H_2\chi_2\rangle$ are not. But we do expect all terms in (17) and (18) to be comparable in magnitude, so barring accidental cancellations, D_n is still of the order of 10^{-24} e cm and m_G should be roughly equal to m_H, which is estimated to be about 15 GeV in Ref. 1.

In conclusion, we have shown in this note that the neutron electric dipole moment D_n in Weinberg's gauge theory of CP nonconservation is not derivable from $K_L \to 2\pi$ data alone. But its value is still expected to be of the order of 10^{-24} e cm and can very well be within experimental limits. We have also investigated elsewhere[2] the case of P and CP nonconservation through Higgs exchange in gauge models with right-handed charged currents, and found a bound on D_n which agrees with data. A substantial parity violation in $\psi' \to \psi\pi\pi$ is also predicted. Details are given in Ref. 2.

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