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$L = 0$ charmonium spectrum from a generalization of Todorov's quasipotential equation for scalar interactions

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The spectrum resulting from a relativistic treatment of a linear confinement potential is presented using a generalization of Todorov's quasipotential equation. The potential used is a linear potential and is regarded as a Lorentz scalar. Excellent agreement is obtained with the psion mass spectrum (3.1,3.7,4.1,4.4 GeV). The values for the charm-quark mass and constant of the linear potential are significantly different from those in the nonrelativistic and other relativistic approaches. The fourth and fifth radial excitations are very close to the values predicted by a three-parameter Regge model proposed recently.

It is generally accepted that the ψ/J particle at 3.095 GeV and the ψ' at 3.684 GeV are the ground- and first-excited states of a quark-antiquark system called charmonium. It has been assumed that the broad structure around 4.1–4.2 GeV is or contains a second radial excitation of the system. This has given support to the linear-nonrelativistic-potential model for the confinement of quarks. That is, assuming

$$H\psi = E\psi \ , \tag{1}$$

with

$$H = -\frac{1}{m_c}\nabla^2 + kV \ , \tag{2}$$

it is found that[1] $m_c = 1.16$ GeV and $k = 0.211$ GeV^2 fit the ψ/J and ψ' particles and predict a second radial excitation at 4.18 GeV and a third at 4.61 GeV. The early data did seem to support this nonrelativistic model. However, more recent data seem to indicate that a third radial excitation[2,3] $\psi'''(4.415)$ has been found, and this lies about 200 MeV below that predicted by the nonrelativistic linear-potential model.

Several relativistic calculations of this same spectrum have been performed. The two mentioned here are ones similar to the approach used in this paper. Jhung, Chung, and Willey[4] have used the Kadyshevskii version of the quasipotential approach. Their results do not show an improvement in the position of this third radial excitation. Gunion and Li[5,6] employ a modified Klein-Gordon equation regarding the linear potential as a Lorentz scalar. That is, the potential appears as an addition to the mass terms. They obtain the spectrum 3.1, 3.71, 4.17, 4.54 for the psion family. Although this does represent an improvement over the nonrelativistic calculations, it is still more than 100 MeV above the excitation at 4.415 GeV.

The modified Klein-Gordon equation used in Refs. 5 and 6 for equal-mass two-body bound states is not a two-body relativistic formulation like the quasipotential approach. In particular, it is not made clear in these papers how the case of unequal masses is treated. On the other hand, their method allows an explicit commitment to the choice of the Lorentz-transformation properties of the potential. Lorentz-scalar potentials are included in association with a mass, and vector potentials are included in association with the four-momentum (as a time component). The quadratic terms in the potential that result from this modify some of the higher levels.[5] This commitment to a definite Lorentz-transformation property is not made in the quasipotential approach of Ref. 4, where the potential appears only linearly. Usually, the quasipotential is derived from field theory and this field theory can provide by way of higher-order terms the appropriate quadratic contributions. At least this has been shown to be so in the case of electromagnetic interactions.[7] One cannot rely entirely on a field theory for the potential in a phenomenological approach. An ansatz of some sort must be employed. This introduces some ambiguity, however, which can only be resolved by comparison with the experimental results.

I intend to incorporate the linear potential kr as a Lorentz scalar in a quasipotential formulation. The one I shall use is a modification of Todorov's[8] relativistic two-body Schrödinger equation as presented in two recent articles.[9,10] I will not go into any details of Todorov's approach and its philosophy as they are adequately commented on in Refs. 8–10. For spinless particles, Todorov's equation resembles the Schrödinger equation. Its form in the c.m. frame is

$$\left(\frac{-\vec{\nabla}^2}{2m_w} + V\right)\psi = \frac{b^2}{2m_w}\psi \ . \tag{3}$$

Instead of the ordinary reduced mass, one has the

TABLE I. Model calculations of the psion spectrum. Model I is the nonrelativistic approach in Ref. 1. Model II is the quasipotential approach of Ref. 4. Model III is the modified Klein-Gordon approach of Refs. 5 and 6. Model IV is based on (10) and model V is based on (11). Model VI is the three-parameter Regge-type model of Ref. 11. Models I–V are two-parameter models. The values of those parameters, (m_c, k), are listed in units of GeV and GeV^2, respectively, at the column headings. The masses given are in GeV.

n	I (1.16, 0.211)	II (1.16, 0.205)	III (1.12, 0.137)	IV (1.15, 0.218)	V (0.942, 0.717)	VI (3-parameter model)
1	3.105	3.1	3.1	3.095	3.095	3.095
2	3.695	3.7	3.71	3.684	3.684	3.684
3	4.18	4.2	4.17	4.161	4.088	4.103
4	4.61	4.7	4.54	4.578	4.408	4.415
5	5.00	···	4.89	4.954	4.676	4.654
6	···	···	5.19	5.304	4.912	4.841

relativistic reduced mass

$$m_w = \frac{m_1 m_2}{w} , \tag{4}$$

where w is the total energy in the c.m. system. The variable $b^2/2m_w$ reduces to the binding energy in the nonrelativistic limit. In general,

$$\frac{b^2}{2m_w} = \frac{w}{8m_1 m_2}\left[w^2 - 2(m_1{}^2 + m_2{}^2) + \frac{(m_1{}^2 - m_2{}^2)^2}{w^2}\right] . \tag{5}$$

The original form (3) of Todorov's quasipotential equation was modified for scalar interactions by Crater and Naft[9] and by Crater and Palmer.[10] The motivation behind these modifications is not related to the problem under consideration here. In those papers, a method was given for adding a potential for scalar interactions to mass terms in the two-body quasipotential equation analogous to that which is done in the one-body Klein-Gordon equation. Two methods were given, indicating that the procedure is not unique. For comparison, (3) will be rewritten here as

$$(\vec{p}^{\,2} + 2m_w V)\psi = (E^2 - m_w{}^2)\psi , \tag{6}$$

where

$$E^2 = m_w{}^2 + b^2 . \tag{7}$$

Equation (6) can also be written in the covariant form

$$(\tilde{p}_\mu \tilde{p}^\mu + m_w{}^2 + 2m_w V)\psi = 0 , \tag{8}$$

where $\tilde{p}^\mu = (E, \vec{p})$ in the c.m. frame. The first method consists of replacing (8) by

$$[\tilde{p}_\mu \tilde{p}^\mu + (m_w + V)^2]\psi = 0 . \tag{9}$$

In the Schrödinger representation this becomes

$$(-\nabla^2 + 2m_w V + V^2)\psi = b^2\psi . \tag{10}$$

The other form derived in Refs. 9 and 10 is[11] (for equal masses)

$$\left(-\vec{\nabla}^2 + 2m_w V + \frac{m_w}{w} V^2\right) = b^2\psi . \tag{11}$$

As pointed out there, the form (11) allowed very tight binding for scalar Coulomb potentials with large coupling, whereas (10) did not. This is no reason for preferring (11) over (10) in the present context.

The $L=0$ spectrum predicted by (10) and (11) can be found numerically, and the results together with the nonrelativistic results and the two relativistic results referred to above are given in Table I. The spectrum from (10) differs only slightly from the nonrelativistic results. In particular, the third excitation is off considerably. However, the spectrum from (11) predicts the ψ''' state at 4.415 to be 4.408, only 7 MeV off. The parameters used are $m_c = 0.942$ GeV and $k = 0.716$ GeV^2. These differ substantially from the more or less common results of the other methods in Table I.

Arik, Coon, and Yu[12] have proposed a three-parameter fit to the psion spectrum based on a Regge-like theory. It is of interest to note that their values of the fourth and fifth radial excitations (4.65 and 4.84 GeV) are very close to the values of 4.68 and 4.91 listed in Table I that results from the two-parameter approach based on Eq. (11).

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[11]The derivation of (10) and (11) are similar in that both involve (1) incorporating the scalar potential by generalizing the mass terms, and (2) imposing the constraint $p_1^2 - p_2^2 = m_2^2 - m_1^2$ (Ref. 8), i.e., requiring that it be a constant of the motion. However, the order in which these two steps are taken differs in the two derivations. Equation (10) can be obtained by imposing this constraint on $(w/M)[(p_1^2 + m_1^2)/2m_1 + (p_2^2 + m_2^2)/2m_2]$ to give $\tilde{p}_\mu \tilde{p}^\mu + m_w^2$ and then generalizing this by replacing m_w by $m_w + V$ (Refs. 9 and 10). Equation (11) can be obtained by generalizing $m_i \to \beta_i = m_i + m_j V/w$ $(i \neq j)$ and then imposing the constraint on $(w/B[(p_1^2 + \beta_1^2)/2\beta_1 + (p_2^2 + \beta_2^2)/2\beta_2]$ $(B = \beta_1 + \beta_2)$ (Refs. 9 and 10). Both (10) and (11) are c.m. forms. Now in each case, V must commute with the operator $p_1^2 - p_2^2$. This occurs if $V = V(|\tilde{x}|)$, where $|\tilde{x}|^2 \equiv (x_1 - x_2)^2 - (P \cdot (x_1 - x_2))^2/P^2$, $P = p_1 + p_2$. In the c.m. frame this is the square of the relative coordinate $\vec{r}$.

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