Quark additivity and magnetic moments of charmed hadrons

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Magnetic moments and transition moments of charmed hadrons have been calculated using the state vectors and the additivity principle of the quark model.

I. INTRODUCTION

Quarks were first introduced by Gell-Mann[1] as fundamental constituents of hadrons. This not only gave physical content to SU(3) and SU(6) but also proved extremely useful in predicting relations among magnetic moments, strong and weak decay widths,[2] etc. of hadrons. It was surprising that some of the predictions such as $\mu_p/\mu_n=-\frac{3}{2}$ of SU(6) were in excellent agreement with experiment. The basic assumptions underlying these calculations are to (i) set the space part of the overlapping integrals of hadron wave functions equal to unity and (ii) use the principle of additivity according to which some properties of a hadron are described as a sum of the contributions from the constituent quarks or antiquarks.

Recently, however, a great deal of attention has been given to the notion of understanding hadrons as consisting of four quarks,[3] u, d, s, and c, where u, d, s are the conventional Gell-Mann triplet and c is called a charmed quark. This scheme seems to provide the most probable answers to the questions raised in the context of the discovery of narrow resonances at 3.1 and 3.7 GeV.[4] These resonances have been interpreted as $c\bar{c}$ bound states.[5] Acceptance of this hypothesis naturally leads to possible existence of hadrons with nonzero charm quantum numbers. The explicit quark structures of these hadrons are given in Ref. 5. The recent discovery of a state at 1.865 GeV (see Ref. 6) seems to provide evidence for existence of charmed hadrons. Thus by a straightforward extension of SU(3) and SU(6) to SU(4) and SU(8) one can calculate the observable properties of the charmed hadrons. Working along this line Okubo, Mathur, and Borchardt[7] have predicted the masses of charmed hadrons and Choudhury and Joshi[8] have calculated their magnetic moments. However, quark structures and calculation of hadronic properties become physically more transparent when represented in terms of state vectors.[9] Thirring[2] has calculated magnetic moments and transition moments of uncharmed hadrons using their state vectors and the quark additivity principle. In this paper we wish to extend the formalism to charmed hadrons.

II. MAGNETIC MOMENTS

We write the magnetic moment of hadron A between the maximum z component of the spin as

$$\mu(A)=\langle\phi(A,J_z=J)|M|\phi(A,J_z=J)\rangle\,. \tag{1}$$

Assuming that the magnetic moment operator M is the sum of the magnetic moment operators M_{q_i} of the constituent quarks and the M_{q_i}'s in turn are proportional to the charges of the quarks, we write

$$M=\mu_p\sum Q_i\sigma_{i_z}\,, \tag{2}$$

where μ_p is the proton magnetic moment.[10]

The calculated magnetic moments of the charmed particles obtained by using Eqs. (2), (1), and the corresponding wave functions presented in the Appendix are listed below:

$$\mu(D^+)=\mu(D^0)=\mu(F^+)=\mu(\eta_c')$$
$$=\mu(\bar{D}^0)=\mu(D^-)=\mu(F^-)=0\,, \tag{3}$$

$$\mu(D^{*0})=\mu(\bar{D}^{*0})=\mu(\phi_c)=0\,, \tag{4}$$

$$\mu(D^{*+})=\mu(F^{*+})=-\mu(D^{*-})=-\mu(F^{*-})=\mu_p\,, \tag{5}$$

$$\mu(C_1^{++})=\mu(C_0^+)=\mu(A^+)=\mu(A^0)=-\mu(C_1^0)=-\mu(S^0)$$
$$=-\mu(T^0)=\mu(X_u^{++})=\tfrac{2}{3}\mu_p\,, \tag{6}$$

$$\mu(X_d^+)=\mu(X_s^+)=\mu_p\,, \tag{7}$$

$$\mu(C_1^+)=\mu(S^+)=0\,, \tag{8}$$

$$\mu(C_1^{*++})=\mu(X_u^{*++})=\mu(R^{*++})=2\mu_p\,, \tag{9}$$

$$\mu(C_1^{*+})=\mu(S^{*+})=\mu(X_d^{*+})=\mu(X_s^{*+})=\mu_p\,, \tag{10}$$

$$\mu(C_1^{*0})=\mu(S^{*0})=\mu(T^{*0})=0\,. \tag{11}$$

These values, as expected, have exact agreement with the SU(8)-invariant result of Ref. 8.

III. TRANSITION MOMENTS

One finds that though the charmed mesons have zero magnetic moments, these, like the uncharmed mesons, also seem to possess a sort of inner magnetic moment which manifests itself in transitions to spin-1 states. Thus for the transition moments $\langle\phi(s=1,s_z=0)|M|\phi(s=0,s_z=0)\rangle$ we get

$$\langle D^{*+}|M|D^+\rangle = \langle D^{*-}|M|D^-\rangle = \langle F^{*+}|M|F^+\rangle = \langle F^{*-}|M|F^-\rangle = -\tfrac{1}{3}\mu_p \qquad (12)$$

and

$$\langle D^{*0}|M|D^0\rangle = \langle \bar{D}^{*0}|M|\bar{D}^0\rangle = -\tfrac{4}{3}\mu_p \ . \qquad (13)$$

For the charmed baryons analogous to the $\Sigma\Lambda$ transition in the uncharmed sector we get

$$\langle C_1^+|M|C_0^+\rangle = -\langle S^+|M|A^+\rangle = \frac{1}{\sqrt{3}}\mu_p \ . \qquad (14)$$

Finally for the transition moments $\langle \phi(s=\tfrac{3}{2}, s_z=\tfrac{1}{2})|M|\phi(s=\tfrac{1}{2}, s_z=\tfrac{1}{2})\rangle$ our results are

$$\langle C_1^{*+}|M|C_0^+\rangle = -\langle S^{*+}|M|A^+\rangle = (\tfrac{2}{3})^{1/2}\mu_p \ , \qquad (15)$$

$$\langle C_1^{*+}|M|C_1^+\rangle = \langle S^{*+}|M|S^+\rangle = (\sqrt{2}/3)\mu_p \ , \qquad (16)$$

and

$$\langle C_1^{*0}|M|C_1^0\rangle = \langle S^{*0}|M|S^0\rangle = \langle T^{*0}|M|T^0\rangle = \langle X_d^{*+}|M|X_d^+\rangle = \langle X_s^{*+}|M|X_s^+\rangle = 2\sqrt{2}/3 \ . \qquad (17)$$

In conclusion it may be noted that our results depend crucially on the c quark having the same magnetic moment as the u quark. If quark magnetic moments are inversely proportional to their masses,[11] quite different results will be obtained.

APPENDIX

The state vectors for charmed hadrons are

$$\phi(D^+) = (1/\sqrt{2})(\bar{d}\uparrow c\downarrow - \bar{d}\downarrow c\uparrow), \quad \phi(D^0) = (1/\sqrt{2})(\bar{u}\uparrow c\downarrow - \bar{u}\downarrow c\uparrow), \quad \phi(F^+) = (1/\sqrt{2})(\bar{s}\uparrow c\downarrow - c\uparrow \bar{s}\downarrow),$$

$$\phi(\eta_c') = (1/\sqrt{24})(\bar{u}\uparrow u\downarrow - \bar{u}\downarrow u\uparrow + \bar{d}\uparrow d\downarrow - \bar{d}\downarrow d\uparrow + \bar{s}\uparrow s\downarrow - \bar{s}\downarrow s\uparrow - 3\bar{c}\uparrow c\downarrow + 3\bar{c}\downarrow c\uparrow),$$

$$\phi(D^-) = (1/\sqrt{2})(\bar{c}\uparrow d\downarrow - \bar{c}\downarrow d\uparrow), \quad \phi(\bar{D}^0) = (1/\sqrt{2})(\bar{c}\uparrow u\downarrow - \bar{c}\downarrow u\uparrow), \quad \phi(F^-) = (1/\sqrt{2})(\bar{c}\uparrow s\downarrow - \bar{c}\downarrow s\uparrow),$$

$$\phi(D^{*+}, J_z=1) = c\uparrow\bar{d}\uparrow, \quad \phi(D^{*0}, J_z=1) = c\uparrow\bar{u}\uparrow, \quad \phi(F^{*+}, J_z=1) = c\uparrow\bar{s}\uparrow,$$

$$\phi(\phi_c, J_z=1) = c\uparrow\bar{c}\uparrow, \quad \phi(D^{*-}, J_z=1) = d\uparrow\bar{c}\uparrow, \quad \phi(\bar{D}^{*0}, J_z=1) = \bar{c}\uparrow u\uparrow, \quad \phi(F^{*-}, J_z=1) = \bar{c}\uparrow s\uparrow,$$

$$\phi(C_1^{++}, J_z=\tfrac{1}{2}) = (1/\sqrt{18})(c\uparrow u\downarrow u\uparrow + c\uparrow u\uparrow u\downarrow + u\downarrow u\uparrow c\uparrow + u\downarrow c\uparrow u\uparrow + u\uparrow c\uparrow u\downarrow + u\uparrow u\downarrow c\uparrow - 2c\downarrow u\uparrow u\uparrow - 2u\uparrow u\uparrow c\downarrow - 2u\uparrow c\downarrow u\uparrow),$$

$$\phi(C_1^+, J_z=\tfrac{1}{2}) = \tfrac{1}{6}(c\uparrow u\uparrow d\downarrow + c\uparrow d\downarrow u\uparrow + u\uparrow d\downarrow c\uparrow + u\uparrow c\uparrow d\downarrow + d\downarrow c\uparrow u\uparrow + d\downarrow u\uparrow c\uparrow + c\uparrow d\uparrow u\downarrow + c\uparrow u\downarrow d\uparrow + d\uparrow u\downarrow c\uparrow + d\uparrow c\uparrow u\downarrow$$
$$+ u\downarrow c\uparrow d\uparrow + u\downarrow d\uparrow c\uparrow - 2c\downarrow u\uparrow d\uparrow - 2c\downarrow d\uparrow u\uparrow - 2u\uparrow d\uparrow c\downarrow - 2u\uparrow c\downarrow d\uparrow - 2d\uparrow c\downarrow u\uparrow - 2d\uparrow u\uparrow c\downarrow),$$

$$\phi(C_1^0, J_z=\tfrac{1}{2}) = (1/\sqrt{18})(c\uparrow d\downarrow d\uparrow + c\uparrow d\uparrow d\downarrow + d\downarrow d\uparrow c\uparrow + d\downarrow c\uparrow d\uparrow + d\uparrow c\uparrow d\downarrow + d\uparrow d\downarrow c\uparrow - 2c\downarrow d\uparrow d\uparrow - 2d\uparrow d\uparrow c\downarrow - 2d\uparrow c\downarrow d\uparrow),$$

$$\phi(C_0^+, J_z=\tfrac{1}{2}) = (1/\sqrt{12})(u\uparrow d\downarrow c\uparrow + u\uparrow c\uparrow d\downarrow + d\downarrow c\uparrow u\uparrow + d\downarrow u\uparrow c\uparrow + c\uparrow u\uparrow d\downarrow$$
$$+ c\uparrow d\downarrow u\uparrow - u\downarrow d\uparrow c\uparrow - u\downarrow c\uparrow d\uparrow - d\uparrow c\uparrow u\downarrow - d\uparrow u\downarrow c\uparrow - c\uparrow u\downarrow d\uparrow - c\uparrow d\uparrow u\downarrow),$$

$$\phi(S^+, J_z=\tfrac{1}{2}) = \tfrac{1}{6}(c\uparrow s\uparrow u\downarrow + c\uparrow u\downarrow s\uparrow + s\uparrow u\downarrow c\uparrow + s\uparrow c\uparrow u\downarrow + u\downarrow c\uparrow s\uparrow + u\downarrow s\uparrow c\uparrow + c\uparrow u\uparrow s\downarrow + c\uparrow s\downarrow u\uparrow + u\uparrow s\downarrow c\uparrow + u\uparrow c\uparrow s\downarrow$$
$$+ s\downarrow c\uparrow u\uparrow + s\downarrow u\uparrow c\uparrow - 2c\downarrow s\uparrow u\uparrow - 2c\downarrow u\uparrow s\uparrow - 2s\uparrow u\uparrow c\downarrow - 2s\uparrow c\downarrow u\uparrow - 2u\uparrow c\downarrow s\uparrow - 2u\uparrow s\uparrow c\downarrow),$$

$$\phi(S^0, J_z=\tfrac{1}{2}) = \tfrac{1}{6}(c\uparrow s\uparrow d\downarrow + c\uparrow d\downarrow s\uparrow + s\uparrow d\downarrow c\uparrow + s\uparrow c\uparrow d\downarrow + d\downarrow c\uparrow s\uparrow + d\downarrow s\uparrow c\uparrow + c\uparrow d\uparrow s\downarrow + c\uparrow s\downarrow d\uparrow + d\uparrow s\downarrow c\uparrow + d\uparrow c\uparrow s\downarrow$$
$$+ s\downarrow c\uparrow d\uparrow + s\downarrow d\uparrow c\uparrow - 2c\downarrow s\uparrow d\uparrow - 2c\downarrow d\uparrow s\uparrow - 2s\uparrow d\uparrow c\downarrow - 2s\uparrow c\downarrow d\uparrow - 2d\uparrow c\downarrow s\uparrow - 2d\uparrow s\uparrow c\downarrow),$$

$$\phi(A^+, J_z=\tfrac{1}{2}) = (1/\sqrt{12})(s\uparrow u\downarrow c\uparrow + s\uparrow c\uparrow u\downarrow + u\downarrow c\uparrow s\uparrow + u\downarrow s\uparrow c\uparrow + c\uparrow s\uparrow u\downarrow$$
$$+ c\uparrow u\downarrow s\uparrow - u\uparrow s\downarrow c\uparrow - u\uparrow c\uparrow s\downarrow - s\downarrow c\uparrow u\uparrow - s\downarrow u\uparrow c\uparrow - c\uparrow u\uparrow s\downarrow - c\uparrow s\downarrow u\uparrow),$$

$$\phi(A^0, J_z=\tfrac{1}{2}) = (1/\sqrt{12})(s\uparrow d\downarrow c\uparrow + s\uparrow c\uparrow d\downarrow + d\downarrow c\uparrow s\uparrow + d\downarrow s\uparrow c\uparrow$$
$$+ c\uparrow s\uparrow d\downarrow + c\uparrow d\downarrow s\uparrow - d\uparrow s\downarrow c\uparrow - d\uparrow c\uparrow s\downarrow - c\uparrow s\downarrow d\uparrow - c\uparrow d\uparrow s\downarrow - s\downarrow d\uparrow c\uparrow - s\downarrow c\uparrow d\uparrow),$$

$$\phi(T^0, J_z=\tfrac{1}{2}) = (1/\sqrt{18})(-2s\uparrow c\downarrow s\uparrow - 2s\uparrow s\uparrow c\downarrow - 2c\downarrow s\uparrow s\uparrow + c\uparrow s\downarrow s\uparrow$$
$$+ s\uparrow c\uparrow s\downarrow + s\downarrow c\uparrow s\uparrow + s\uparrow s\downarrow c\uparrow + c\uparrow s\uparrow s\downarrow + s\downarrow s\uparrow c\uparrow),$$

$$\phi(X_u^{++}, J_z=\tfrac{1}{2}) = (1/\sqrt{18})(2c\uparrow u\downarrow c\uparrow + 2c\uparrow c\uparrow u\downarrow + 2u\downarrow c\uparrow c\uparrow - c\uparrow c\downarrow u\uparrow - c\uparrow u\uparrow c\downarrow$$
$$- c\downarrow u\uparrow c\uparrow - u\uparrow c\downarrow c\uparrow - u\uparrow c\uparrow c\downarrow - c\downarrow c\uparrow u\uparrow),$$

$$\phi(X_d^+), J_z=\tfrac{1}{2}) = (1/\sqrt{18})(2c\uparrow d\downarrow c\uparrow + 2c\uparrow c\uparrow d\downarrow + 2d\downarrow c\uparrow c\uparrow - c\uparrow c\downarrow d\uparrow$$
$$- c\uparrow d\uparrow c\downarrow - c\downarrow d\uparrow c\uparrow - d\uparrow c\downarrow c\uparrow - d\uparrow c\uparrow c\downarrow - c\uparrow c\downarrow d\uparrow),$$

$$\phi(X_s^+, J_z=\tfrac{1}{2}) = (1/\sqrt{18})(2c{\uparrow}s{\downarrow}c{\uparrow} + 2c{\uparrow}c{\uparrow}s{\downarrow} + 2s{\downarrow}c{\uparrow}c{\uparrow} - c{\uparrow}c{\downarrow}s{\uparrow} - c{\uparrow}s{\uparrow}c{\downarrow}$$
$$- c{\downarrow}s{\uparrow}c{\uparrow} - s{\uparrow}c{\downarrow}c{\uparrow} - s{\uparrow}c{\uparrow}c{\downarrow} - c{\downarrow}c{\uparrow}s{\uparrow}),$$

$$\phi(C_1^{*++}, J_z=\tfrac{3}{2}) = (1/\sqrt{3})(c{\uparrow}u{\uparrow}u{\uparrow} + u{\uparrow}u{\uparrow}c{\uparrow} + u{\uparrow}c{\uparrow}u{\uparrow}),$$

$$\phi(C_1^{*+}, J_z=\tfrac{3}{2}) = (1/\sqrt{6})(c{\uparrow}u{\uparrow}d{\uparrow} + u{\uparrow}d{\uparrow}c{\uparrow} + d{\uparrow}c{\uparrow}u{\uparrow} + c{\uparrow}d{\uparrow}u{\uparrow} + d{\uparrow}u{\uparrow}c{\uparrow} + u{\uparrow}c{\uparrow}d{\uparrow}),$$

$$\phi(C_1^{*0}, J_z=\tfrac{3}{2}) = (1/\sqrt{3})(c{\uparrow}d{\uparrow}d{\uparrow} + d{\uparrow}d{\uparrow}c{\uparrow} + d{\uparrow}c{\uparrow}d{\uparrow}),$$

$$\phi(S^{*+}, J_z=\tfrac{3}{2}) = (1/\sqrt{6})(c{\uparrow}s{\uparrow}u{\uparrow} + c{\uparrow}u{\uparrow}s{\uparrow} + s{\uparrow}u{\uparrow}c{\uparrow} + u{\uparrow}s{\uparrow}c{\uparrow} + u{\uparrow}c{\uparrow}s{\uparrow} + s{\uparrow}c{\uparrow}u{\uparrow}),$$

$$\phi(S^{*0}, J_z=\tfrac{3}{2}) = (1/\sqrt{6})(c{\uparrow}s{\uparrow}d{\uparrow} + c{\uparrow}d{\uparrow}s{\uparrow} + s{\uparrow}d{\uparrow}c{\uparrow} + d{\uparrow}s{\uparrow}c{\uparrow} + d{\uparrow}c{\uparrow}s{\uparrow} + s{\uparrow}c{\uparrow}d{\uparrow}),$$

$$\phi(T^{*0}, J_z=\tfrac{3}{2}) = (1/\sqrt{3})(c{\uparrow}s{\uparrow}s{\uparrow} + s{\uparrow}s{\uparrow}c{\uparrow} + s{\uparrow}c{\uparrow}s{\uparrow}),$$

$$\phi(R^{*++}, J_z=\tfrac{3}{2}) = c{\uparrow}c{\uparrow}c{\uparrow},$$

$$\phi(X_u^{*++}, J_z=\tfrac{3}{2}) = (1/\sqrt{3})(c{\uparrow}c{\uparrow}u{\uparrow} + c{\uparrow}u{\uparrow}c{\uparrow} + u{\uparrow}c{\uparrow}c{\uparrow}),$$

$$\phi(X_d^{*+}, J_z=\tfrac{3}{2}) = (1/\sqrt{3})(c{\uparrow}c{\uparrow}d{\uparrow} + c{\uparrow}d{\uparrow}c{\uparrow} + d{\uparrow}c{\uparrow}c{\uparrow}),$$

$$\phi(X_s^{*+}, J_z=\tfrac{3}{2}) = (1/\sqrt{3})(c{\uparrow}c{\uparrow}s{\uparrow} + c{\uparrow}s{\uparrow}c{\uparrow} + s{\uparrow}c{\uparrow}c{\uparrow}),$$

where the arrow on the right-hand side of each quark gives its spin direction.

For vector mesons with $J_z=0$, ϕ's can be constructed by noting that the corresponding spin wave function is $(1/\sqrt{2})(\uparrow\downarrow + \downarrow\uparrow)$. Similarly for the $\frac{3}{2}^+$ baryons with $J_z=\frac{1}{2}$ the corresponding spin function to be used is $(1/\sqrt{3})(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$. For states with $J_z=-\frac{1}{2}$, -1, and $-\frac{3}{2}$ the spin wave functions are obtained by just flipping $\uparrow$ to $\downarrow$ and vice versa in spin states of $J_z=\frac{1}{2}$, 1, and $\frac{3}{2}$, respectively. ϕ's, then, can be constructed by reading off the corresponding unitary spin part.

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