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Short-range part of the nuclear force

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A simple form of the quark theory accounts for the strong short-range repulsion between nucleons and makes it possible to estimate the magnitude of this part of the nuclear force.

Meson theory was originally developed to explain nuclear forces. It has, however, been successful only in describing the long-range part of the force which is determined mainly by the exchange of a single π meson. Ordinarily, it is believed that the short-range part of the nuclear force is dominated by the exchange of heavy mesons and more-complicated processes than single-meson exchange. Another point of view is that the short-range part of the nuclear force arises directly from the interaction of quarks and the forces which confine them in the nucleons. This idea has been advocated by Johnson[1] and others who have developed the "bag model." But the bag model does not lend itself to a simple treatment of the two-center problem[2] which the interaction of two nucleons involves. We adopt here a simple phenomenological description of the forces between quarks which is similar, for example, to that proposed by Schnitzer[3] as a model for the recently discovered ψ particles, and thereby make the calculation simpler.

The choice of the force assumed to act between quarks is essentially dictated by the requirement of explaining quark confinement and saturation in both baryons and mesons. We have taken our ideas mainly from a published lecture by Feynman.[4] The force between quarks is supposed to have a long range, as, for example, in Hooke's law. This is to explain the permanent binding of quarks in hadrons. But, to explain the absence of similar long-range forces between the observed mesons and baryons (saturation), an appropriate combination of attractive and repulsive forces is needed. This is given by the color hypothesis and an interaction between quarks by means of a color octet of vector bosons. The vector character of the interaction means that the forces between quarks will be spin-dependent, and the spin-dependent forces are related in a straightforward way to the central force[3,5]—this does not entail any additional assumption or additional parameters. There are spin-spin, spin-orbit, and tensor terms, but only the spin-spin interaction contributes in the cases considered here. These considerations lead to the following interaction potential acting between two quarks:

$$V_{ij} = -\lambda_i \cdot \lambda_j \left[v(r_{ij}) + \frac{1}{6}\left(\frac{\hbar}{M_Q c}\right)^2 \sigma_i \cdot \sigma_j \nabla^2 v(r_{ij}) \right], \tag{1}$$

where

$$v(r) = Kr^2/2 .$$

In this formula λ_i and λ_j are Gell-Mann's eight SU(3) matrices which operate on the color variables of particles i and j, M_Q is the quark mass and comes out of the nonrelativistic reduction of the Dirac velocity operator, and K is the Hooke's-law constant. A minus sign is incorporated in the formula because $\lambda_i \cdot \lambda_j$ has the character of being negative for particles which attract.[4]

There are two constants at our disposal: K and M_Q. They are chosen to give the observed masses for the nucleon (938 MeV) and the Δ particle (1236 MeV). These masses are the expectation values of the nonrelativistic three-quark Hamiltonian with the interaction potentials given by Eq. (1). The wave functions used in these expectation values are constructed according to the prescriptions given by Feynman, Kislinger, and Ravndal.[6] The color wave function is antisymmetric, the spin-unitary-spin wave functions are the appropriate symmetric functions, and the space wave function is the Gaussian

$$g(123) = \exp\{-\tfrac{1}{2}\gamma^2[(r_1 - R)^2 + (r_2 - R)^2 + (r_3 - R)^2]\}, \tag{2}$$

where $R = \frac{1}{3}(r_1 + r_2 + r_3)$. γ is adjusted according to the variational principle. The two parameters, given in terms of energies, are found to be $M_Q c^2 = 151$ MeV and $\hbar(K/M_Q)^{1/2} = 75$ MeV. This simple description of the nucleon's structure can be tested by computing the root-mean-square radius. We get

$$\langle (r-R)^2 \rangle^{1/2} = \frac{1}{\gamma} = 1.1 \text{ fm}, \tag{3}$$

which may be compared to a recent experimental estimate of 0.9 fm.[7] With the same two constants some meson masses can be computed. For the $\omega(783)$ and the $\rho(770)$ we get 767 MeV; for the $\pi(140)$ we get 370 MeV. The last is the only serious disagreement with observation. It may be noted that the color hypothesis enters the meson-mass calculation in an important way: it gives an effective interquark interaction twice as large in this case as in the baryons.[4]

Having given a plausible description of a single nucleon's structure in terms of quarks interacting with the potential of formula (1), we now wish to describe the interaction of two nucleons which are close together. We run into an unfortunate technical difficulty at this point. The proper procedure for calculating the interaction of two small groups of particles[8] is quite complicated. If we use space wave functions for the quarks which are localized about two fixed centers, we solve our mathematical problems but, of course, introduce substantial errors. Fortunately, the worst of the errors will subtract out of our final results, and others can be minimized by the use of a nonvariational wave function. In any event, the qualitative features in the final results should not be altered.

Antisymmetric six-quark wave functions are constructed from the three-quark wave functions. The space part is

$$G(1\cdots 6)=g(123;\tfrac{1}{2}X)g(456;-\tfrac{1}{2}X)\pm g(123;-\tfrac{1}{2}X)g(456;\tfrac{1}{2}X), \qquad (4)$$

where

$$g(123;Y)=\exp\{-\tfrac{1}{2}\beta^2[(r_1-Y)^2+(r_2-Y)^2+(r_3-Y)^2]\}. \qquad (5)$$

Parity determines the sign used in Eq. (4). $\frac{1}{2}X$ is the position of one nucleon and $-\frac{1}{2}X$ of the other. The color wave function is simply the product of two color-singlet wave functions for the two nucleons:

$$C(1\cdots 6)=C(123)C(456). \qquad (6)$$

The spin-unitary-spin wave function in the case where spin and isotopic spin are both 1 is

$$\phi(1\cdots 6)=N^{+}_{1/2}(123)N^{+}_{1/2}(456). \qquad (7)$$

The other cases require a sum of such terms multiplied by appropriate Clebsch-Gordan coefficients. Since the wave function, ψ, is already partially antisymmetric, the antisymmetrizing operator needs to contain only a few permutations:

$$A=1-\sum_{i=1}^{3}\sum_{j=4}^{6}P_{ij}. \qquad (8)$$

The effective interaction potential between two nucleons is the expectation value of the energy of the six quarks,

$$(A\psi|H|A\psi)/(A\psi|A\psi)=(\psi|H_{123}+H_{456}+V'|A\psi)/(\psi|A\psi), \qquad (9)$$

less the energy at infinite separation. (H_{123} is the Hamiltonian of one group of three particles, H_{456} is the Hamiltonian of the other group, and V' is the interaction between the two groups.) As might be expected, this effective potential is proportional to the square of an overlap integral. Because of this the range of the effective potential will depend very much on the nucleon's size as given by the space part of the wave function. If the parameter β in Eq. (5) is adjusted variationally, as γ in Eq. (2) was, the root-mean-square radius is very much too large, and the resulting effective potential will have too large a range. We will choose β to give the observed radius of the nucleon. As a result, there are large errors in $(\psi|H_{123}+H_{456}|A\psi)/(\psi|A\psi)$, but most of these are eliminated when the energy at infinite separation is subtracted.

The evaluation of the effective potential between nucleons is straightforward but too lengthy to include here. We will just give the results and make a few comments. It is convenient to divide the effective potential into two parts. The most important is

$$U_1(X)=(\psi|V'|A\psi)/(\psi|A\psi)=\left(\psi\left|\sum_{1}^{3}\sum_{4}^{6}V_{ij}\right|A\psi\right)\Big/(\psi|A\psi). \qquad (10)$$

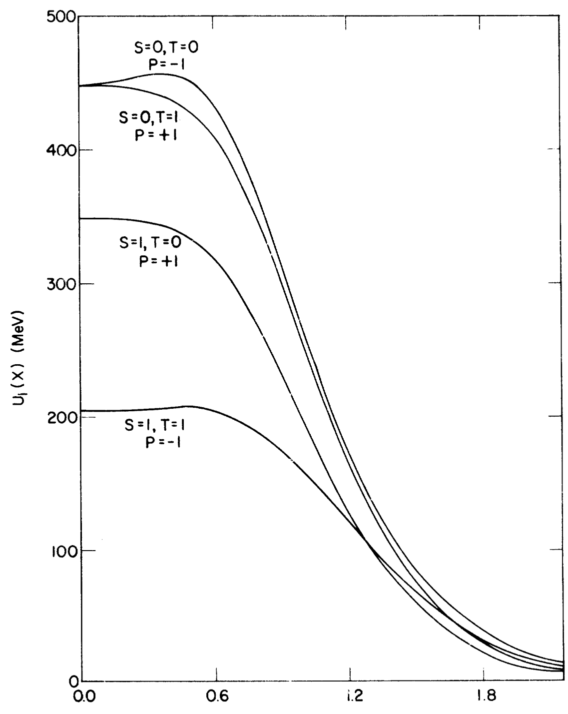

FIG. 1. Contribution from Eq. (10) to the approximate internucleon potential. S, T, and P are spin, isotopic spin, and parity.

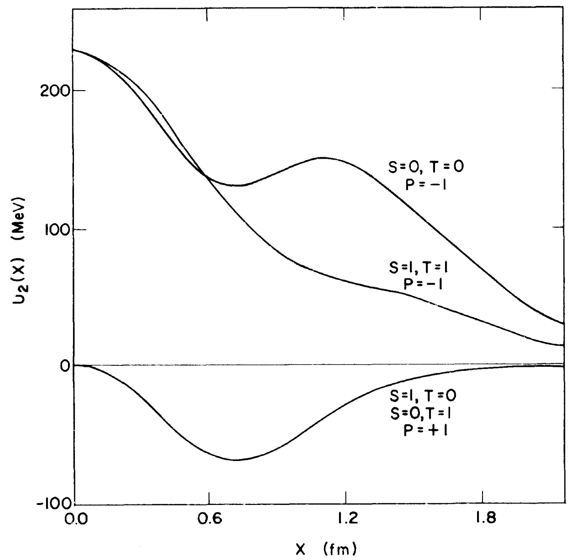

FIG. 2. Contribution from Eq. (11) to the approximate internucleon potential. S, T, and P are spin, isotopic spin, and parity.

There is an exact cancellation of the nonexchange terms in the numerator of Eq. (10). This is to be expected; otherwise there would be a long-range interaction between nucleons. The central-force part of V_{ij} and the spin-spin part contribute differently to the remaining exchange integrals. There is a partial cancellation between the central-force terms which becomes complete at zero separation while the spin-spin force terms add together and constitute the bulk of the interaction. The contribution from Eq. (10) is shown in Fig. 1.

The role of the spin-spin part of the interquark potential in giving a short-range repulsion between nucleons was pointed out in Ref. 1. Without the direct interaction between quarks, the bag model gives an attractive short-range force as in Ref. 2.

The other part of the effective potential is

$$U_2(X) = (\psi\,|H_{123}+H_{456}\,|A\psi)/(\psi\,|A\psi) - E_{123} - E_{456}\,,$$

where E_{123} and E_{456} are the isolated nucleon energies computed with the space wave functions of Eq. (5). There is a sizable contribution in the odd-parity (antibonding) states, which is to be expected. The contribution from Eq. (11) is shown in Fig. 2.[9]

We have tried to show here (i) that very simple considerations lead to a phenomenological interaction between quarks, (ii) that the spin-dependent part of this interaction accounts for the mass difference between nucleons and Δ particles, (iii) and that this interaction potential also explains the short-range repulsion of nucleons with the spin-dependent part being particularly important.

Note added in proof. C. DeTar has made a study of the internucleon potential for the $S=1$, $T=0$ case using the bag model [MIT Report No. CTP 631, 1977 (unpublished)].

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[9]If the reader wishes to compare these calculations as shown in Fig. 1 and Fig. 2 with a phenomenological potential, we recommend that developed by C. N. Bressel, A. K. Kerman, and B. Rouben [Nucl. Phys. A124, 624 (1969)]. They use repulsive square wells which have magnitudes of several hundred MeV and a radius of 0.7 fm.