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Properties of axial-vector-current divergences in the constituent-quark basis*

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The properties of the axial-vector currents and their divergences are examined in the context of the quark model as quantized on the null plane. The SU(3) transformation properties of the divergences are not as they naively appear to be. It is found that correction terms to strong PCAC (partial conservation of axial-vector current) are expected, especially for kaons, and that these terms are likely to be proportional to one-body quark operators (and therefore octet operators), but that the proportionality constants may have a mixture of linear and quadratic dependence on the quark mass parameters. In view of this, some consequences of the one-body nature of the PCAC-correction terms are derived without making any assumptions of SU(3) algebraic structure. A universality relation which was recently obtained by Dominquez is a direct result of this line of reasoning; furthermore, assuming the pion PCAC correction Δ_π is small, a value for Δ_K of approximately 0.2 follows. This then is used to correct some old PCAC formulas, bringing them into agreement with experiment. Finally, the consequences of some assumptions concerning the SU(3) properties of the axial divergences are explored.

I. INTRODUCTION

In the usual approach to chiral symmetry,[1] one begins by considering the case in which the axial-vector current A_a^μ is exactly conserved; this is presumed to be related to the vanishing of the masses of the pseudoscalar-meson octet. In this case, a matrix element of A_a^μ can be written

$$\langle\beta|A_a^\mu|\alpha\rangle = if_\pi\langle\beta, P_a|\alpha\rangle(q^\mu/q^2) + \langle\beta|A_a^\mu|\alpha\rangle_N\,, \qquad (1)$$

where $q = p_\alpha - p_\beta$ and the meson-pole term is explicitly separated from the nonpole term, $\langle\beta|A_a^\mu|\alpha\rangle_N$. The conservation of A_a^μ then implies (using the standard null-plane notation[2])

$$if_\pi\langle\beta, P_a|\alpha\rangle = \lim_{q_\perp\to 0}\lim_{q^+\to 0}\langle\beta|q^- A_a^+|\alpha\rangle\,, \qquad (2)$$

where the order of limits is such as to eliminate the pole terms from A_a^+. The result is the same as that which one would obtain via the strong partially conserved axial-vector current (PCAC) hypothesis for nonzero meson mass. It can be written as

$$(m_\alpha{}^2 - m_\beta{}^2)\langle\beta|\hat{Q}_a^5|\alpha\rangle = if_a(2\pi)^3 2p_\alpha^+\delta(p_\alpha^+ - p_\beta^+) \times \delta^{(2)}(p_\alpha^\perp - p_\beta^\perp)\langle\beta, P_a|\alpha\rangle\,, \qquad (3)$$

where

$$\hat{Q}_a^5 = \int d^4x\, \delta(x^+) A_a^+(x)\,. \qquad (4)$$

This relates the matrix elements of the null-plane charges $\hat{Q}_a^5$ to pionic transitions between arbitrary hadronic states α and β.

When $m \neq 0$, there is no ambiguity in defining the limit $q_\perp, q^+ \to 0$, since A_a^+ then has no pion-pole contribution. What one shows in this way[3] is that strong PCAC reduces smoothly in the limit $m_a{}^2 \to 0$ to the result Eq. (3). This does not mean that there cannot be other terms in Eq. (3) that vanish in the zero-mass limit; such terms would enter for example if PCAC were modified by addition of terms proportional to m_a which were not singular in the chiral-symmetric limit. The question of the existence of such terms is unsettled, and we turn next to an examination of this problem

One begins[4] by defining the nucleon-nucleon matrix element of the axial-vector current as

$$\langle N(p')|A_a^\mu(x)|N(p)\rangle = i\bar{u}(p')\tfrac{1}{2}\lambda_a[g_A(q^2)\gamma^\mu + h_A(q^2)q^\mu] \times \gamma_5 u(p)e^{-iqx}\,, \qquad (5)$$

where $q = p' - p$.

Taking the divergence, one has

$$\langle N(p')|\partial_\mu A_a^\mu(0)|N(p)\rangle = D(q^2)\bar{u}(p)\lambda_a\gamma_5 u(p)\,, \qquad (6)$$

where

$$D(q^2) = Mg_A(q^2) + \tfrac{1}{2}q^2 h_A(q^2)\,, \qquad (7)$$

which can be separated into a contribution from the pion pole plus the background:

$$D(q^2) = D_{\text{pole}}(q^2) + \bar{D}(q^2)\,,$$
$$D_{\text{pole}} = f_\pi g_{\pi NN}(m_\pi{}^2 - q^2)^{-1}\,. \qquad (8)$$

Evaluating the pole term at $q^2 = 0$ leads to

$$D(0) = Mg_A = f_\pi g_{\pi NN} + \bar{D}(0)\,. \qquad (9)$$

One then assumes an unsubtracted dispersion relation for

$$\bar{D}(0) = \frac{1}{\pi}\int_{9m_\pi{}^2}^{\infty} \mathrm{Im}\bar{D}(t)\frac{dt}{t}\,. \qquad (10)$$

The conventional (strong) PCAC relation,[1]

$$\partial_\mu A_\pi^\mu = m_\pi^2 f_\pi \Phi_\pi \,, \tag{11}$$

where Φ_π is the pion field operator, connects $\bar{D}(q^2)$ to the pionic form factor $K(q^2)$ of the nucleon. With this assumption, one finds

$$\mathrm{Im}\bar{D}(t) = m_\pi^2 f_\pi g_{\pi NN} \frac{\mathrm{Im}K(t)}{m_\pi^2 - t} \,, \tag{12}$$

where $K(0) = 1$. Thus

$$\Delta_{\pi NN} \equiv 1 - \frac{M g_A}{f_\pi g_{\pi NN}} = \frac{m_\pi^2}{\pi} \int_{9m_\pi^2}^{\infty} \frac{dt\, \mathrm{Im}K(t)}{t(t - m_\pi^2)} \,. \tag{13}$$

If one neglects the continuum integral at $t=0$ one obtains the Goldberger-Treiman relation. Within the context of strong PCAC, however, the validity of the formula is not dependent on pole dominance. Although $\bar{D}$ is proportional to m_π^2, so is the residue of the pole term. From this point of view, the relation is simply a consequence of SU(2) $\times$ SU(2) symmetry.[5] However, this leads to difficulties since one must now explain why $\Delta_{\pi NN}$ is as large[4] as it is (namely, $\Delta_{\pi NN} = 0.058 \pm 0.013$) in spite of the fact that it is proportional to a small quantity, m_π^2.

Various possibilities have been considered: $\Delta_{\pi NN}$ may move into agreement with theory, $\bar{D}(q^2)$ may require a subtraction, strong PCAC may not be valid, and a heavy pion (which is not a Goldstone boson) may exist.[6] Drell[7] has emphasized that $\bar{D}(q^2)$ in general will receive contributions from other physical states in the $J^P = 0^-$ channel that cannot be described in terms of a resonance; for example, there may be a direct coupling of $\partial_\mu A_\pi^\mu$ from the vacuum to the 3π continuum with the quantum numbers of the pion. Finally, it is necessary to call attention to the recent work of Jones and Scadron,[4] where a dispersion-theoretic estimate of $\Delta_{\pi NN}$ is shown to give roughly half of the experimental value, which is far from the older estimates of an order of magnitude smaller than experiment. We will proceed with the attitude that there is likely to be a small unexplained discrepancy for the pion case, while (as we discuss below) there is more likely to be a large unexplained discrepancy for the kaon case.

The PCAC hypothesis may be tested in other ways than in Goldberger-Treiman relations. For example, there are the various sum rules of the Adler-Weisberger type.

Because of the difference between the K and the π masses one might expect that the continuations to zero mass would be different for pions and kaons; however, this is not necessarily true. By combining Adler-Weisberger–type relations for πK and $K\pi$ scattering, one obtains the following[8] sum rule:

$$f_K^2 \int \frac{d\nu}{\nu} [\sigma_{``K^-"\pi^+}(\nu) - \sigma_{``K^+"\pi^+}(\nu)] = f_\pi^2 \int \frac{d\nu}{\nu} [\sigma_{``\pi^-"K^+}(\nu) - \sigma_{``\pi^+"K^+}(\nu)] \,, \tag{14}$$

where $\sigma_{``a"b}(\nu)$ denotes the total cross section for particle a continued to zero-mass scattering on particle b on the mass shell, all at laboratory energy ν. If the equality Eq. (14) held for physical particles as well as for zero-mass incident particles, then the decay constants f_π and f_K would be equal. At the time of its derivation, this was believed to be true. However, it is now known that $f_K/f_\pi \simeq 1.25$, so one must look for a source of this 25% discrepancy. A possible solution will be proposed below.

Recently, Dominguez[9] has proposed an extension of the usual strong PCAC hypothesis by postulating an SU(3) family of heavy pseudoscalar mesons in an effort to account for corrections to Goldberger-Treiman relations and other soft-pion and soft-kaon theorems. He shows that there is a universality among the corrections to the various Goldberger-Treiman relations for $\Delta S \neq 0$ decays. The soft-kaon theorem for K_{l3} is modified and is in agreement with experiment. Other applications to soft pion theorems are made also.

We show how these and other related results may be obtained in the quark model. The consequences of quark additivity are deduced first, without any assumptions of SU(3) algebraic structure. Only then do we analyze the rather complex SU(3) structure of the divergence operators, abstract their properties, and obtain some phenomenological results.

II. QUARK STRUCTURE OF CURRENT DIVERGENCE

When studying the properties of nonconserved operators, such as the axial-vector currents, one should first express all quantities in terms of charges and moments integrated on the null plane. Such a formulation of the theory has many advantages over the older infinite-momentum approach.[10] The principle advantage is that nonconserved charges still annihilate the vacuum, so that Coleman's theorem is no longer valid. That is to say,

$$\langle 0 | \hat{Q}_a^5 | P_b \rangle = 0 = \langle 0 | A_a^+ | P_b \rangle \,, \tag{15}$$

but nevertheless

$$\begin{aligned} \int d^4x\, \delta(x^+) \langle 0 | \partial_\mu A_a^\mu(x) | P_b(q) \rangle &= i \langle 0 | q^- A_a^+ | P_b(q) \rangle \\ &= (2\pi)^3 \delta(q^+) \delta^{(2)}(q_\perp) \delta_{ab} i f_a m_a^2 \end{aligned} \tag{16}$$

is nonzero, and

$$(m_\beta{}^2 - m_\alpha{}^2)\langle\beta|\hat{Q}_a^5|\alpha\rangle = i\left\langle\beta\left|\int d^4x\,\delta(x^4)\partial_\mu A_a^\mu(x)\right|\alpha\right\rangle . \tag{17}$$

Recently, this last relation has been used to study pion-emission processes in the context of strong PCAC and the Melosh transformation.[11]

Even in the free-quark model, it is easy to see that the SU(3) transformation properties of the divergences $\partial_\mu A_a^\mu$ are not as they naively appear to be.[12] The symmetry-breaking part of the Hamiltonian is taken to be[13]

$$\mathcal{H}' = m_u(\bar{u}u + \bar{d}d) + m_s\bar{s}s \ , \tag{18}$$

so that the divergences may be written in the form

$$\partial_\mu A_a^\mu = 2m_u v_i \ , \quad a = 1,2,3 \tag{19}$$

$$\partial_\mu A_a^\mu = (m_u + m_s)v_i \ , \quad a = 4,5,6,7 \ , \tag{20}$$

where

$$v_i = i\bar{q}\gamma_5\lambda_a q \ , \tag{21}$$

and analogous expressions for A_a^μ. We will assume that insofar as algebraic properties of operators are concerned one may use the free-quark model as a guide.[14]

In order to study the SU(3) properties of the v_i we must first express them[15] in terms of "good" quark fields q_+:

$$v_a \sim q_+^\dagger(x)\lambda_a\sigma_3(\vec{\gamma}_\perp\cdot\vec{\partial}_\perp + m_u)\frac{1}{\eta}q_+(x) + \text{H.c.} + \cdots \ , \tag{22}$$

for $a = 1,2,3$, and analogous expressions for the other v_a. The centered dots stand for terms arising from interaction; these terms are independent of the quark mass parameters. Therefore

$$v_a = A_a + 2m_u B_a + \cdots \ , \quad a = 1,2,3 \tag{23}$$

$$v_a = A_a + (m_s + m_u)B_a + \cdots \ , \quad a = 4,5,6,7 \ , \tag{24}$$

where A_a and B_a are one-body quark operators lying in distinct octets. This conclusion is not modified by Melosh transformation.[16]

As we have pointed out,[12] since we know from experience that $\partial_\mu A_a^\mu$ $(a = 1,2,3)$ is a good interpolating field for the pion, it is reasonable to expect that the SU(3)-rotated quantity with the quantum number of the kaon should be a good interpolating field for the kaon. But this quantity is not $\partial_\mu A_a^\mu$ $(a = 4,5,6,7)$. In fact, it is clear that factoring out the mass parameter will not help, since v_a for $a = 1,2,3$ and v_a for $a = 4,5,6,7$ are not SU(3) partners.

Now $\partial_\mu A_\pi^\mu$ is a sum of a part proportional to m_u^2 and one proportional to m_u. Since m_u is presumably small, the former term will be negligible except for possible contributions to a pion pole, since $m_\pi{}^2/(m_\pi{}^2 - q^2)$ is large for q^2 small even if $m_\pi{}^2$ is small. Therefore, corrections to pion-pole dominance can arise from the second term, which is linear in m_u. This observation is independent of how much of the pion-pole term comes from the m_u^2 or from the m_u term.

It is consistent for the m_u^2 term to dominate the pole term, and this would give the results of Scadron and co-workers.[17] That is, it is possible that the pion and kaon field operators are, respectively,

$$\Phi_\pi = q_+^\dagger\lambda_\pi\sigma_3\frac{1}{\eta}q_+ \ , \tag{25}$$

$$\Phi_K = q_+^\dagger\lambda_K\sigma_3\frac{1}{\eta}q_+ \ , \tag{26}$$

which are obviously SU(3) partners in an octet. In this case one would have

$$\partial_\mu A_\pi^\mu = 4m_u{}^2\Phi_\pi + O(m_u) \ , \tag{27}$$

$$\partial_\mu A_K^\mu = (m_u + m_s)^2\Phi_K + O(m_u + m_s) \ , \tag{28}$$

and the correction terms would have vanishing matrix elements between the vacuum and one-meson states.

On the other hand, if the terms proportional to m_u^2, $(m_u + m_s)^2$ can be entirely ignored, then $\partial_\mu A_\pi^\mu/2m_u$ and $\partial_\mu A_K^\mu/(m_u + m_s)$ are SU(3) partners. Furthermore, if the pole parts of these operators are SU(3)-related, then the nonpole parts are so related as well. This is the line of reasoning used by Gell-Mann *et al.*[13] and others.

The lesson to be drawn from this analysis is that it is quite plausible for strong PCAC to require correction terms, especially for kaons, that these terms are likely to be proportional to one-body quark operators (and therefore octet members), but that the proportionality constants may be linear or quadratic (or a mixture of these) in the quark mass parameters.

III. QUARK-ADDITIVITY CONSEQUENCES

One of the fundamental assumptions underlying the phenomenology of the quark model is that contributions to a given process from individual quarks may be added independently.[18] For conserved quantities such as electric charge, the additivity assumption is trivially satisfied. On the other hand, for nonconserved quantities such as axial charges, additivity is nontrivial and is a statement of dynamics.[19]

In order to make definite predictions about algebraic properties of transition matrix elements one must specify the transformation properties of the transition operator as well as those of the states. It is generally true that a kind of octet dominance is automatically incorporated in the quark model when transitions are described by one-body quark operators. However, these operators connecting say one hadron multiplet H to another H' need not belong to the same octet. This can sometimes be rather subtle, as for example in the case of the transition operators u_i and v_i, the scalar and pseudoscalar densities.[15]

The additivity principle implies[18] that

$$\langle H'|T|H\rangle = \langle q'\|T\|q\rangle \,, \tag{29}$$

where q, q' are the "active" quarks participating in the transition, while the other, "spectator," quarks play no role in the process in this approximation. The overlap integral involving these spectator quarks has been set equal to unity; in taking ratios, the effect of the overlap mismatch should be negligible. One should note that in this way one relates matrix elements of baryon transitions to corresponding matrix elements of meson transitions, something that one cannot achieve via symmetry arguments alone. For example, in this way one may obtain all the SU(3) results for pionic decays of hadron resonances, as well as other SU(3) relations for decays involving a particular meson, simply by the assumption that the relevant matrix elements are determined by one-body quark operators with the proper quantum numbers. However, π decays and K decays are not related unless additional SU(3) or higher-symmetry assumptions are made.

In the case at hand, we are interested in the corrections to strong PCAC which arise from the part of $\partial_\mu A_a^\mu$ which is orthogonal to the state P_a. As discussed above, this may be thought of as a field operator for a heavy pion, but this picture is not necessary. The null-plane analysis of $\partial_\mu A_a^\mu$ suggests that it may be reasonable to assume that this nonpole term is a one-body operator; we will abstract this property from the free-field equations.

Let us write the axial-vector-current divergences as a sum of a pole term $(\partial_\mu A_a^\mu)_{\rm pole}$ and a background term δ_a:

$$\partial_\mu A_a^\mu \equiv (\partial_\mu A_a^\mu)_{\rm pole} + \delta_a \,. \tag{30}$$

Then, with the assumption that only one-body operators are important, quark additivity implies that the ratio

$$f_a \Delta_a \equiv -\frac{\langle\beta|\delta_a|\alpha\rangle}{\langle\beta|(\partial_\mu A_a^\mu)_{\rm pole}|\alpha\rangle} \tag{31}$$

is independent of β, α. Note that we have not yet made any assumptions as to the SU(3) transformation properties of δ_a, so the Δ_a are not related at this point. This universality relation has been obtained by Dominguez.[9]

The net effect of the PCAC correction, according to Eqs. (31), (32) is that f_a is to be replaced by $f_a(1-\Delta_a)$. This result may now be used to correct various relations that follow from the strong PCAC hypothesis. For example, the πK analog of the Adler-Weisberger relation, Eq. (14), now becomes

$$f_\pi^{\,2}(1-\Delta_\pi)^2 = f_K^{\,2}(1-\Delta_K)^2 \,. \tag{32}$$

Of course, the quantities Δ_a enter into corrections to PCAC relations involving baryons as well. In particular, the Goldberger-Treiman relation and its octet analogs may be written

$$(m_{B'} + m_B) g_A^{B'B} = f_a g_{B'Ba}(1-\Delta_a) \,, \tag{33}$$

when $g_A^{B'B}$ is the axial-vector coupling constant for the $B' \to B$ transition and $g_{B'Ba}$ is the strong-coupling constant for $B' \to B + P_a$.

Note that the same quantities Δ_a appear here as appeared before in purely mesonic processes.

If the correction to pion PCAC is negligible then by using the experimental value for f_K/f_π of 1.25 we find from Eq. (32)

$$\Delta_K = 0.2 \,, \tag{34}$$

which is consistent with the presently known experimental value of this quantity, as deduced[1] from $\Lambda \to Ne\nu$ decay,

$$\Delta_K(\text{expt}) = 0.30 \pm 0.15 \,. \tag{35}$$

Chen[20] has made a theoretical estimate of this using strong PCAC and has found a value too small to explain the observed correction.

A further application of these ideas may be made in modifying the classic results of PCAC and current algebra for K_{l3} decay. The soft-pion[21] and soft-kaon[22] results will now read

$$\frac{f_K}{f_\pi} = [f_+(m_K^{\,2}) + f_-(m_K^{\,2})](1-\Delta_\pi) \,, \tag{36}$$

$$\frac{f_\pi}{f_K} = [f_+(m_\pi^{\,2}) - f_-(m_\pi^{\,2})](1-\Delta_K) \,. \tag{37}$$

The most recent experimental data[23] are consistent with

$$f_-(m_\pi^{\,2}) \ll f_+(m_\pi^{\,2}) \approx 1 \,, \tag{38}$$

which then implies that the correction Δ_K cannot be negligible. In fact, with the previously deduced value of $\Delta_K \approx 0.2$ agreement with experiment is obtained for Eq. (37). Assuming a linear fit to the form factors $f_\pm(t)$, which is consistent with the data, it is easy to check that Eq. (36) is in agreement with experiment also.

Another way to state the above is in the following theoretically biased manner. Assuming Δ_π negligible, K^* dominance for $f_+(t)$, $f_+(0) = 0.97$ as estimated by Pagels then Eqs. (36), (37) imply

$$\frac{f_K}{f_\pi}=2f_+(0)\left(1+\frac{m_K{}^2+m_\pi{}^2}{2m_{K^*}{}^2}\right)-1=1.27\,, \tag{39}$$

providing a theoretical estimate of f_K/f_π which is remarkably good. (We have dropped a term proportional to $[f_-(m_K{}^2)-f_-(m_\pi{}^2)]$ which is presumably very small.)

As another example of a corrected PCAC result we briefly examine meson-baryon scattering σ terms. Significant effects are expected only for kaon scattering, as we have seen; in this case, one finds[24]

$$f_K{}^2F(0,0,0,0)=-\sigma_{KN}\,, \tag{40}$$

where $F(\nu,t,q^2,q'^2)$ is the isospin-symmetric KN scattering amplitude. Clearly, any analysis of scattering data will only determine $\sigma_{KN}/f_K{}^2$. The modifications of PCAC that we have inferred will effectively multiply f_K by $(1-\Delta_K)$ or approximately 0.8; thus σ_{KN} will be 0.64 times the value obtained without this correction. In spite of the fact that the correction here is quite large, little can be deduced with any certainty for two reasons: First, the data analysis is not unambiguous and estimates of σ_{KN} vary[24] from 180 MeV to 600 MeV; second, theoretical expectations for σ_{KN} depend on the ratio of σ_{KN} to $\sigma_{\pi N}$, which depends on the details of how SU(3) is broken.[17] In the $(\overline{3},3)$ model of chiral-symmetry breaking, for example, it can be shown that[25]

$$2\sigma_{KN}-\sigma_{\pi N}=242\text{ MeV} \tag{41}$$

if one ignores the (small) isospin-zero KN amplitude. We will discuss this problem below when we take up the whole question of the SU(3) transformation properties of the strong-PCAC modifications δ_a introduced earlier. In this section we have carefully avoided this question in an effort to determine those consequences of the present theoretical framework that are independent of SU(3).

In a similar manner, one may consider meson-meson scattering. Some time ago, Osborn[26] studied the set of all soft-meson theorems for all scattering amplitudes of the pseudoscalar octet, with two particles off shell, using PCAC for each pseudoscalar state and current algebra. He found that a linear expansion in the invariants is uniquely determined by the absence of exotic σ terms for $\pi\pi$, πK, and KK scattering so long as $f_\pi=f_K$ is required for consistency. In the present context, it is easy to see that the same result obtains if one uses the effective couplings $f_\pi(1-\Delta_\pi)$ and $f_K(1-\Delta_K)$, which are constrained to be equal by Eq. (32).

The modification of PCAC mentioned above can be described as simply the use of the same constant f for both pion and kaon PCAC, where $f\approx f_\pi$. This has the effect of increasing kaon decay amplitudes considerably; such a prescription has been used in phenomenological analyses.[27]

IV. ALGEBRAIC STRUCTURE OF CURRENT DIVERGENCES

In this section we shall return to the axial-vector-current divergences which were discussed briefly in Sec. II. Recall that we found there that these operators did not have simple SU(3) transformation properties even in the free-quark model. In view of this, we were reluctant to draw any conclusions about the structure of these operators except the presumably weak assumption of their dominant one-body nature (which in fact does hold true in the absence of interaction). Having done this, and having explored some of the consequences of this, we now proceed to the more speculative analysis of the SU(3) properties of the v_i.

The ratio of pion to kaon masses satisfies[12]

$$\frac{f_\pi m_\pi{}^2}{f_K m_K{}^2}=\frac{2m_u}{m_u+m_s}\frac{\langle 0|v_\pi|\pi\rangle}{\langle 0|v_K|K\rangle}\,, \tag{42}$$

which involves not only the mass parameters m_u, m_s but also the ratio of matrix elements of the v_i. This latter quantity clearly depends on the SU(3) properties of the v_i. In an expansion[19] in terms of quark transverse momentum $p_\perp$, the term with $L_z=0$ give[12]

$$\langle 0|v_\pi|\pi\rangle=2m_u(h_1+h_2/M^2)+h_3/M+\cdots\,, \tag{43}$$

$$\langle 0|v_K|K\rangle=(m_u+m_s)(h_1+h_2/M^2)+h_3/M+\cdots\,, \tag{44}$$

where h_i are reduced matrix elements such that h_2/h_1, h_3/h_1 are of order $\langle p_\perp{}^2\rangle$ and M is the quark mass appearing in the Melosh transformation. If one takes only the leading term, which corresponds to ignoring transverse motion of quarks, one obtains[14,17]

$$\frac{f_K m_K{}^2}{f_\pi m_\pi{}^2}=\left(\frac{m_s+m_u}{2m_u}\right)^2, \tag{45}$$

so that

$$\frac{m_s}{m_u}\cong 7\quad(\text{Gunion }et\ al.)\,. \tag{46}$$

On the other hand, if the term in h_3 is dominant, then

$$\frac{f_K m_K{}^2}{f_\pi m_\pi{}^2}=\frac{m_s+m_u}{2m_u}\,, \tag{47}$$

so that

$$\frac{m_s}{m_u}\cong 30\quad(\text{Gell-Mann }et\ al.)\,. \tag{48}$$

These two extremes correspond, respectively, to the divergences $\partial_\mu A_a^\mu$ being proportional to the quark mass parameters m_u, m_s quadratically or linearly. The ratio m_s/m_u is clearly very sensitive to this behavior of $\partial_\mu A_a^\mu$ and so it is of interest to determine other quantities which depend on this ratio.

Consider Eq. (31) for the special cases in which $\beta = K, \pi$; taking ratios, one finds

$$\frac{f_K \Delta_K}{f_\pi \Delta_\pi} = \frac{\langle \pi | \delta_K | \alpha \rangle}{\langle K | \delta_\pi | \alpha \rangle} \tag{49}$$

for arbitrary state α. We have argued above that (independent of the ratio m_s/m_u) the PCAC-correction terms are such that $\delta_\pi/2m_u$ and $\delta_K/(m_s + m_u)$ are SU(3) partners. Incorporating this into Eq. (49) implies

$$\frac{f_K \Delta_K}{f_\pi \Delta_\pi} = \frac{m_u + m_s}{2m_u}, \tag{50}$$

so that the two extreme cases described above correspond to

$$\frac{\Delta_K}{\Delta_\pi} = \left(\frac{f_\pi}{f_K}\right)^{1/2} \frac{m_K}{m_\pi} \approx 3.2, \tag{51}$$

$$\frac{\Delta_K}{\Delta_\pi} = \frac{m_K^{\,2}}{m_\pi^{\,2}} \approx 12.5. \tag{52}$$

Furthermore, using Eq. (32), one finds for these two cases

$$\Delta_K = 0.27, \quad \Delta_\pi = 0.08, \tag{53}$$

$$\Delta_K = 0.21, \quad \Delta_\pi = 0.017. \tag{54}$$

The second solution, which follows from the approach of Gell-Mann *et al.*,[13] has been used by Dashen and Weinstein[28] to derive various sum rules for the Goldberger-Treiman discrepancies. (A recent discussion of these sum rules is given in Ref. 4.)

V. SUMMARY

Within the context of the quark model, as quantized on the null plane, the SU(3) transformation properties of the axial-vector-current divergences are not as they naively appear to be. Correction terms to strong PCAC are expected, and we have argued that these are likely to be important for kaons especially. Such corrections may be dominated by one-body (and therefore octet) quark operators; however, since the dependence of these operators on the quark mass parameters even in the free-quark model is not simple, it is unlikely that it would be simple in the interacting case. Therefore we were led to examine the consequences of the assumption of quark additivity for the PCAC-correction terms without making any assumptions of SU(3) algebraic structure. A universality relation, Eq. (31), is a direct result of this line of reasoning. Furthermore, current algebra implies Eq. (32), which, together with the assumption that the pion PCAC discrepancy Δ_π is small, leads to an estimate for Δ_K of approximately 0.2. This then was used to correct some old PCAC formulas, bringing them into agreement with experiment. Finally, we returned to the problem of the SU(3) properties of the axial-vector-current divergences and explored the consequences of some assumptions of simple behavior.

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