Flavor-changing neutral-current processes in gauge models and strong interactions*

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The effect of strong interactions on flavor-changing neutral-current (FCNC) processes is discussed. In order that the FCNC semileptonic process $I \to F + \nu + \bar{\nu}$ does not take place to order G_F nor to order $G_F\alpha$, the hadronic matrix elements of weak-interaction currents must satisfy (A) $\langle F|J^0_\mu|I\rangle = 0$ to first order in the gauge coupling and (B) $\mathfrak{F}^{FI}_{\mu\nu}(k) = \int d^4x\, e^{-ikx}\langle F|T(J^+_\mu(x)J^-_\nu(0))\,|I\rangle = O(k^{-2})\mathcal{O}^{FI}_{\mu\nu}$ as $k = [\vec{k}^2 + (ik_0)^2]^{1/2} \to \infty$, where I and F are hadronic states of different flavors and J^0_μ and $J^\pm_\mu$ are neutral and charged weak-interaction currents, respectively, while $\mathcal{O}^{FI}_{\mu\nu}$ stands for the matrix elements of finite operators. By examining conditions (A) and (B), we show that if the FCNC quark process $q^I \to q^F + \nu + \bar{\nu}$ is suppressed to order G_F and to order $G_F\alpha$, the semileptonic process $I \to F + \nu + \bar{\nu}$ is also suppressed to the same order, where the $q^{I(F)}$ quark carries the same flavor as the hadronic state $I(F)$.

Experimentally it has been known that the flavor-changing neutral-current (FCNC) processes are suppressed[1] to order $G_F{}^2$ [or to order $G_F\alpha\epsilon$ (Ref. 2) with $\epsilon \sim 10^{-2}$]. Such conservation of flavor(s)[3,4] in neutral-current processes provides strong constraints on models of weak interactions. Recently several authors[5] have derived constraints for the representation content of SU(2) × U(1) gauge models[6] by requiring that neutral-current interactions of quarks in the model conserve *all* flavors.

In this paper we discuss the effects of strong interactions on the flavor (strangeness, charm, etc.,) changing neutral-current semileptonic process

$$I \to F + \nu + \bar{\nu}\,, \tag{1}$$

where I and F are hadronic states of different flavors. With the aid of current-algebra techniques,[7,8] we derive constraints for the hadronic matrix elements of weak-interaction currents so that the flavor is conserved in process (1) to order G_F and $G_F\alpha$. By examining constraints in terms of quark-bridge diagrams,[7,8] we show that if the FCNC quark process

$$q^I \to q^F + \nu + \bar{\nu} \tag{2}$$

is suppressed so as not to take place to order G_F nor to order $G_F\alpha$, there is no FCNC semileptonic process (1) to the same order in the gauge model with strong interactions.

It should be noted that a class of FCNC processes that may take place to order $G_F\alpha$ via induced effective photon vertices,[2,9] such as

$$I \to F + l + \bar{l} \quad (l = \mu \text{ or } e)\,, \tag{3}$$

are not regarded as FCNC semileptonic processes in this paper.[10]

THEOREMS ON THE FCNC SEMILEPTONIC PROCESSES

(A) Let $J^0_\mu(x)$ be the neutral current in a gauge model. Then the hadronic matrix element of $J^0_\mu(x)$ must vanish to first order in the gauge coupling in order to guarantee that process (1) is not observed to order G_F:

$$\langle F|J^0_\mu(x)|I\rangle = 0\,. \tag{4}$$

Namely, nondiagonal (flavor-changing: $I \to F$) terms are absent in $J^0_\mu(x)$. The theorem (A) is very trivial if we recall that the matrix element for the process (1) is described to order G_F by Fig. 1(a),

$$M_0 = -i\frac{G_F}{\sqrt{2}}\bar{\nu}\gamma_\mu\gamma_L\nu\langle F|J^0_\mu(0)|I\rangle\,. \tag{4$'$}$$

(B) Assume that condition (A), Eq. (4), is satisfied. In order that the process (1) does not take place to the induced order $G_F\alpha$, the hadronic matrix element for the induced neutral current must satisfy the constraint

$$\mathfrak{F}^{FI}_{\mu\nu}(k) = \int d^4x\, e^{-ikx}\langle F|T(J^+_\mu(x)J^-_\nu(0))|I\rangle$$
$$= O(k^{-2})\mathcal{O}^{FI}_{\mu\nu} \tag{5}$$

as $k \to \infty$, where $J^\pm_\mu(x)$ is the charged current in the model, and $k^2 = \vec{k}^2 + (ik_0)^2$. $\mathcal{O}^{FI}_{\mu\nu}$ stands for the matrix elements of finite tensor operators. It should be recalled that according to the power-counting theorem,[7,11] $\mathfrak{F}^{FI}_{\mu\nu}(k)$ in general behaves as $\mathfrak{F}^{FI}_{\mu\nu}(k) = O(k^{-1})$.[12]

Proof of (*B*). All the Feynman diagrams that contribute to the process (1) possibly to order $G_F\alpha$ are shown in Figs. 1(b) and 1(c). All other one-loop induced-neutral-current diagrams do not contribute to order $G_F\alpha$ since they are proportional to $\langle F|J^{0,\text{em}}_\mu(x)|I\rangle$ or $\langle F|T(J^0_{\mu_1}(x_1)\cdots J^0_{\mu_n}(x_n))|I\rangle$ and both J^0_μ and the electromagnetic current J^{em}_μ conserve flavors.

In order to make our discussion simpler, we consider the process (1) in an SU(2) × U(1) gauge model in the 't Hooft–Feynman gauge. The quantum chromodynamics (QCD) SU(3)-color gluon model is taken to account for strong interactions[7] and the matrix elements for the process (1) are evaluated to all orders in the gluon coupling. It should be stressed that in such a model with strong interactions the following CCC (continuity equations and commutation relations of currents)[7,8,13] hold:

$$i\partial^{\mu}J_{\mu}^{\pm}(x)=[Q^{\pm}(x_0),H(x)]=M_W S^{\pm}(x)\,,$$
$$[J_0^{+}(x),J_{\mu}^{-}(y)]\delta(x_0-y_0)=-i\delta^3(\vec{x}-\vec{y})[2J_{\mu}^{0}(x)+2(1-R)J_{\mu}^{\mathrm{em}}(x)]\,,\ \text{etc.}\,, \tag{6}$$

where $R=M_W^2/M_Z^2=\cos^2\theta_W$ and $S^{\pm}$ are the charged scalar currents that couple to Higgs scalars $\phi^{\mp}$.

With the aid of current-algebra technique and the CCC (6), it is easy to observe that the leading contributions of M_{I} and M_{II} [in Fig. 1(b)] to order $G_F\alpha$ offset each other:

$$M_{(\mathrm{I,II})}=\pm\frac{G^2g^2}{4M_Z^2}\bar{\nu}\gamma_{\mu}\gamma_L\nu\int\frac{d^4k}{(2\pi)^4}\frac{2k_{\mu}}{(k^2-M_W^2)^2}[\mathfrak{F}_{\rho\rho}^{FI}(k)+\mathfrak{F}_S^{FI}(k)]$$
$$+\text{terms proportional to }\langle F|J_{\mu}^{0}(x)|I\rangle\text{ or }\langle F|J_{\mu}^{\mathrm{em}}(x)|I\rangle+O(G_F^2)\,, \tag{7}$$

where $G^2/4M_Z^2=g^2/4M_W^2=G_F/\sqrt{2}$ and $g^2/4\pi=\alpha/\sin^2\theta_W$. $\mathfrak{F}_{\rho\sigma}^{FI}$ and $\mathfrak{F}_S^{FI}$ are defined, respectively, by

$$\mathfrak{F}_{\rho\sigma}^{FI}(k)=\int d^4x\,e^{-ikx}\langle F|T(J_{\rho}^{+}(x)J_{\sigma}^{-}(0))|I\rangle\,,$$
$$\mathfrak{F}_S^{FI}(k)=\int d^4x\,e^{-ikx}\langle F|T(S^{+}(x)S^{-}(0))|I\rangle\,. \tag{7$'$}$$

Thus, if condition (A) is satisfied, $M_{\mathrm{I}}+M_{\mathrm{II}}=O(G_F^2)$.

The box diagram in Fig. 1(c) is evaluated to be

$$M_{\mathrm{III}}=+\frac{g^4}{4}\int\frac{d^4k}{(2\pi)^4}\bar{\nu}(-g_{\rho\sigma}k\!\!\!/+i\epsilon_{\mu\nu\rho\sigma}k_{\mu}\gamma_{\nu}\gamma_5)\gamma_L\nu\frac{1}{(k^2-M_W^2)^2(k^2+2pk-m_{\mu}^2)}\mathfrak{F}_{\rho\sigma}^{FI}(k)$$
$$+\text{terms proportional to }\langle F|J_{\mu}^{0}(x)\text{ or }J_{\mu}^{\mathrm{em}}(x)|I\rangle+O(G_F^2)\,. \tag{8}$$

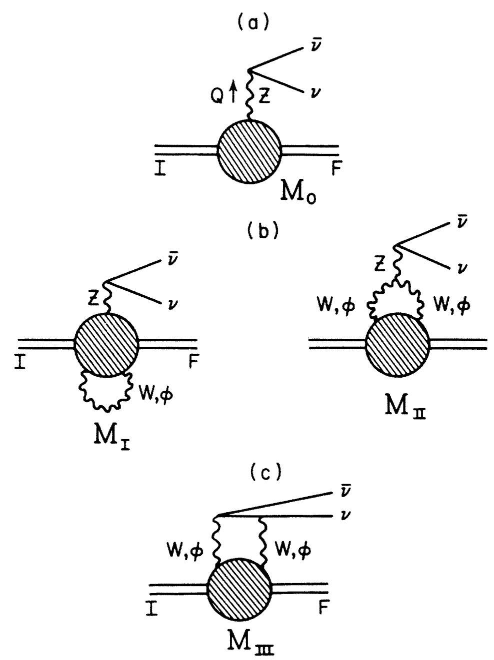

FIG. 1. Diagrams contributing to the FCNC semileptonic process $I\rightarrow F+\nu+\bar{\nu}$. (a) M_0 to order G_F. (b) Radiative corrections to the effective neutral-current vertex, M_I and M_{II}, to order $G_F\alpha$. (c) Box diagram to order $G_F\alpha$.

It immediately follows that

$$\mathfrak{F}_{\rho\sigma}^{FI}(k)=O(k^{-2})\mathcal{O}_{\rho\sigma}^{FI}\ \text{as}\ k\rightarrow\infty\,, \tag{5$'$}$$

where $k^2=\vec{k}^2+(ik_0)^2$, in order that the contribution from M_{III} is of order G_F^2. Q.E.D.

RELATION TO THE FCNC QUARK PROCESSES

Let $|(I,I_3)^j\rangle$ be an eigenstate of weak interactions, corresponding to an I_3^j component of a weak isospin multiplet (I^j,Y^j). The quark eigenstate of the strong interactions and the mass matrix is a linear superposition of $\{|(I,I_3)^j\rangle\}$ with $Q_q=I_3^j+\frac{1}{2}Y^j$:

$$|q\rangle=\sum_j a_{q,j}|(I,I_3)^j\rangle\,,$$
$$\sum_j a_{q,j}a_{q',j}=\delta_{q,q'}\,. \tag{9}$$

In order to relate the above theorems on semileptonic processes to quark processes (2) and constraints on the representation content of the SU(2) × U(1) gauge model, we analyze the hadronic matrix elements of currents $\langle F|J_{\mu}^{0}|I\rangle$ and $\mathfrak{F}_{\rho\sigma}^{FI}(k)$ in terms of the bridge diagrams consisting of two quark lines of different flavors,[12] q^I and q^F [Figs. 2(a) and 2(b)]. The conditions (A) and (B), given by Eqs. (4) and (5), respectively, then reduce to

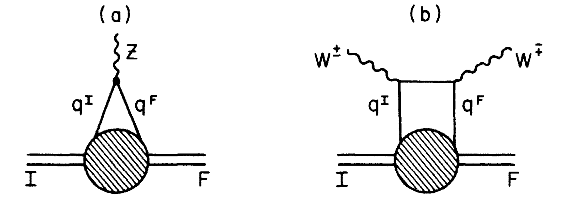

FIG. 2. Bridge diagrams for (a) $\langle F|J^0_\mu|I\rangle$ and (b) $\mathfrak{F}^{FI}_{\rho\sigma}$, consisting of two quark lines of different flavors.

$$(\mathrm{A}')\quad \langle F|J^0_\mu(0)|I\rangle \sim \sum_j I_3^j a_{q^F,j} a_{q^I,j} \mathcal{O}^{FI}_\mu = \text{const} \times \delta_{q^F,q^I} \mathcal{O}^{FI}_\mu \qquad (10)$$

and

$$(\mathrm{B}')\quad \mathfrak{F}^{FI}_{\rho\sigma}(k) = \sum_j a_{q^F,j} a_{q^I,j} \{\mathcal{O}^{FI}_{\rho\sigma;S}[I^j(I^j+1) - (I_3^j)^2] + \mathcal{O}^{FI}_{\rho\sigma;A}[I_3^j]\} O(k^{-1}) = \mathcal{O}^{FI}_{\rho\sigma;(S,A)} \delta_{q^F,q^I} O(k^{-1}) , \qquad (11)$$

where $\mathcal{O}^{FI}_\mu$ and $\mathcal{O}^{FI}_{\rho\sigma;S(A)}$ are the hadronic matrix elements of vector and (anti) symmetric tensor operators, and $q^{I(F)}$ carry the same flavors as the hadronic states $I(F)$. It is trivial to identify these two constraints (A′) and (B′) to be the conditions[5] that the FCNC quark process $q^I \to q^F + \nu + \bar{\nu}$ does not occur to order G_F nor to order $G_F\alpha$.[14] Therefore we are led to the following theorem (C) on the effects of strong interactions on the FCNC semileptonic process:

(C) If the FCNC quark process (2) is suppressed so as not to take place to order G_F nor to order $G_F\alpha$ in a gauge model, there is no corresponding FCNC semileptonic process (1) to the same order in the model with strong interactions.[15]

With this theorem (C), we may now relate the absence of FCNC processes to order G_F or to order $G_F\alpha$ in experiments to constraints on the representation content of a quark gauge model.

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[2]M. K. Gaillard and B. W. Lee, Phys. Rev. D 10, 897 (1974).

[3]Charm conservation in neutral currents has not been confirmed yet in experiments. If charm is not conserved in neutral currents to order G_F nor to order $G_F\alpha$, then D^0-$\bar{D}^0$ mixing will result in detection of $S=\pm 2$ final states as frequently as of $S=0$ final states in the e^+e^- colliding-beam experiment at SPEAR.

[4]As for the possible muon- (and electron-) number nonconservation in neutral currents, see the references in R. Marshak *et al.*, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969). See also S. Frankel, in *Muon Physics*, edited by C. S. Wu and V. W. Hughes (Academic, N.Y., 1975), Vol. II, p. 83.

[5]S. Glashow and S. Weinberg, Phys. Rev. D 15, 1958 (1977); F. Paige *et al.*, *ibid.* 15, 3416 (1977).

[6]Reviews of spontaneously broken gauge theory and gauge models are given by E. S. Abers and B. W. Lee, Phys. Rep. 9C, 1 (1974); S. Weinberg, Rev. Mod. Phys. 46, 255 (1974); M. A. B. Bég and A. Sirlin, Annu. Rev. Nucl. Sci. 24, 379 (1974).

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[9]The author is thankful to Professor A. Sirlin for discussions.

[10]Our discussion can be easily extended to the suppression of $\Delta S=2$ processes.

[11]S. Weinberg, Phys. Rev. 118, 838 (1960).

[12]$\mathfrak{F}^{FI}_{\rho\sigma}(k)$ behaves asymptotically $k^d \times$ power of ln$k$), where $d = 2 - \frac{3}{2}F - G$ for a given bridge consisting of F quark lines and G gluon lines (See Refs. 7 and 11.) If the largest d comes from diagrams with a bridge of two quarks of different flavors, then $d \le -1$. Recall that gluons that mediate strong interactions conserve flavors.

[13]In the QCD strong-interaction model, even the anomalous algebra of (axial-) vector currents becomes normal. M. A. B. Bég, Phys. Rev. D 11, 1165 (1975).

[14]If neutral currents conserve all the flavors carried by quarks of charge $Q_q = I_3^j + \frac{1}{2} Y^j$ in the "natural" way (See Ref. 5), i.e., I_3^j = common and I^j = common for all quarks of charge Q_q, the constraints (A') and (B') are satisfied for an arbitrary set of flavor-mixing parameters $\{a_{q,j}\}$.

[15]The converse is also true unless the hadronic matrix elements are suppressed dynamically in strong interactions. See, for example, G. Branco *et al.*, Phys. Rev. D 13, 104 (1977).