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# Rare muon decays, heavy leptons, and *CP* violation*

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We generalize and elaborate a model recently proposed by Cheng and Li. It is characterized by the breakdown of separate conservation laws for $e$ and $\mu$ lepton types and allows for such processes as $\mu \to e + \gamma$, $\mu \to 3e$. Our generalization incorporates the singly charged heavy lepton discovered in $e^+e^-$ colliding-beam experiments, and it opens up the possibility of introducing $CP$-violating effects in lepton reactions. We analyze the $\mu \to 3e$ decay process, both with respect to rate (relative to $\mu \to e + \gamma$) and spectrum, including $CP$-violating correlations. Other physically interesting features are also discussed for the generalized scheme: neutrino "sharing" and off-diagonal neutral-current effects.

## I. INTRODUCTION

A revival of theoretical interest has developed recently concerning a possible breakdown of the usually accepted lepton conservation laws—separate conservation of $e$-type and $\mu$-type lepton quantum numbers. Such a breakdown would reveal itself, for example, by observation of the processes $\mu \to e + \gamma$ or $\mu \to e + e + \bar{e}$. It is easy enough to construct models in which the separate conservation laws are overturned, the hitherto forbidden processes being nevertheless quantitatively suppressed. Indeed, several such schemes have already been elaborated.[1,2,3] In the present note we wish to pursue several further issues, in particular for the model proposed by Cheng and Li, but also more generally.

The new schemes inevitably introduce new parameters, so that the absolute rates for $\mu \to e\gamma$ and $\mu \to 3e$ decays are still theoretically adjustable. One may expect that the *ratio* of the two rates is somewhat less sensitive to the parameters. This question has already been addressed for the model of Wilczek and Zee.[2,4] We shall take it up here for the Cheng-Li scheme where, in the simplest version of the scheme, the ratio turns out to be rather insensitive to the unknown parameters.

A second issue that we address has to do with a generalization of the models of Refs. 1 and 2, designed to incorporate the singly charged heavy lepton (call it $L$) discovered by Perl and collaborators.[5] We adopt the "sequential" view, associating to $L$ a massless, left-handed neutrino $\nu_L$ and treating the pairs $(\nu_e, e)$, $(\nu_\mu, \mu)$, and $(\nu_L, L)$ on a similar footing. This opens up the possibility of $CP$-violating phases in the lepton interactions. We discuss the implications for the spectrum in $\mu \to 3e$ decay. For the Cheng-Li model, this incorporation of $(\nu_L, L)$ also introduces other new features of considerable physical interest, quite apart from the $CP$ question.

## II. GENERALIZED CHENG-LI SCHEME AND *CP* VIOLATION

The SU(2)×U(1) model of Cheng and Li, which we will shortly generalize, is based on the lepton pairs $(\nu_e, e)$ and $(\nu_\mu, \mu)$, plus two additional heavy, neutral leptons. The neutral heavy leptons enter in left-handed SU(2) singlets and in right-handed doublets. The doublets may be written initially in the form

$$\begin{pmatrix} \nu' \\ e \end{pmatrix}_{\text{left}}, \begin{pmatrix} \nu'' \\ \mu \end{pmatrix}_{\text{left}}, \begin{pmatrix} N' \\ e \end{pmatrix}_{\text{right}}, \begin{pmatrix} N'' \\ \mu \end{pmatrix}_{\text{right}}. \tag{1}$$

Right-handed $\nu'$ and $\nu''$ fields make no appearance in the theory. With no loss of generality the states $e$ and $\mu$ may be taken to be mass eigenstates (with masses $m_e, m_\mu$). In general, however, $\nu'$, $\nu''$, $N'$, $N''$ are not eigenstates of the mass matrix. In this neutral sector the physical particles (linear combinations of $\nu'$, $\nu''$, $N'$, $N''$) will consist of two massless neutrinos, $\nu_e$ and $\nu_\mu$, and two heavy leptons, $N_1$ and $N_2$ (masses $m_1$ and $m_2$). The simplest possibility for the mass matrix involves the introduction of a single Higgs doublet. For this situation, as we have learned from Bjorken, Lane, and Weinberg,[6] the doublets can be expressed in terms of the mass eigenstates in the following form:

$$\begin{gathered} \begin{pmatrix} \beta_e \nu_e + \dfrac{m_e}{m_1}\cos\theta N_1 + \dfrac{m_e}{m_2}\sin\theta N_2 \\ e \end{pmatrix}_{\text{left}}, \\ \begin{pmatrix} \cos\theta N_1 + \sin\theta N_2 \\ e \end{pmatrix}_{\text{right}}; \\ \begin{pmatrix} \beta_\mu \nu_\mu + b_{\mu e}\nu_e - \dfrac{m_\mu}{m_1}\sin\theta N_1 + \dfrac{m_\mu}{m_2}\cos\theta N_2 \\ \mu \end{pmatrix}_{\text{left}}, \\ \begin{pmatrix} -\sin\theta N_1 + \cos\theta N_2 \\ \mu \end{pmatrix}_{\text{right}}. \end{gathered} \tag{2}$$

In addition there are two left-handed singlets (combinations of $\nu_e$, $\nu_\mu$, $N_1, N_2$) orthogonal to the neutral combinations appearing in the left-handed doublets. The important point about the structures displayed in Eq. (2) is that the mixing effects on the left and right are correlated in a definite way (a single parameter $\theta$). This emerges naturally when one diagonalizes the mass matrix. It is crucial to the viability of the model, for, otherwise, interference between the left and right couplings might give an unacceptably large rate for $\mu \to e+\gamma$. Although a single Higgs doublet (in addition to direct mass couplings) suffices for the present scheme, we observe that one could add any number of Higgs singlets and doublets without altering the structures of Eq. (2). For simplicity we speak of this as the single-Higgs-doublet scheme. The essential assumption is that there are no Higgs triplets. These would alter the structure of Eq. (2) and endanger the suppression of $\mu \to e+\gamma$ decay.

The coefficients $\beta_e$, $\beta_\mu$, and $b_{\mu e}$ in Eq. (2) are determined by considerations of orthonormality. Since $m_e/m_i$ and $m_\mu/m_i$ are surely small, we can take

$$\beta_e \approx \beta_\mu \approx 1\,, \tag{3}$$

and for $b_{\mu e}$ we find

$$b_{\mu e} \simeq \frac{m_e m_\mu}{m_1^2 m_2^2}\,(m_2^2 - m_1^2)\sin\theta\cos\theta \ll 1\,. \tag{4}$$

One is clearly free to redefine the two physical neutrinos to be any two orthogonal combinations of $\nu_e$ and $\nu_\mu$. We have made a particular choice in the writing of Eq. (2), so that the left-handed electron couples to $\nu_e$, but the left-handed muon couples not only to an orthogonal neutrino $\nu_\mu$, but also, with tiny coefficient, to $\nu_e$ (neutrino "sharing"). However, this latter coupling is negligible for practical purposes ($b_{\mu e} \ll 1$).

Let us now turn to a generalization of the above scheme. We introduce the pair $(\nu_L, L)$, with $\nu_L$ taken to be a massless left-handed neutrino, and we generalize from two to three heavy neutral leptons, $N_1, N_2, N_3$ (masses $m_1, m_2, m_3$). The mixing parameters $\cos\theta$, $\sin\theta$, are similarly generalized. The SU(2) doublets now become

$$\begin{pmatrix} \beta_e \nu_e + m_e \sum_i \frac{a_i^e}{m_i} N_i \\ e \end{pmatrix}_{\text{left}}, \quad \begin{pmatrix} \sum_i a_i^e N_i \\ e \end{pmatrix}_{\text{right}}; \quad \begin{pmatrix} \beta_\mu \nu_\mu + b_{\mu e}\nu_e + m_\mu \sum_i \frac{a_i^\mu}{m_i} N_i \\ \mu \end{pmatrix}_{\text{left}}, \quad \begin{pmatrix} \sum_i a_i^\mu N_i \\ \mu \end{pmatrix}_{\text{right}};$$

$$\begin{pmatrix} \beta_L \nu_L + b_{L\mu}\nu_\mu + b_{Le}\nu_e + m_L \sum_i \frac{a_i^L}{m_i} N_i \\ L \end{pmatrix}_{\text{left}}, \quad \begin{pmatrix} \sum_i a_i^L N_i \\ L \end{pmatrix}_{\text{right}}; \tag{4}$$

and

$$a_i^{e*} a_i^e = a_i^{\mu*} a_i^\mu = a_i^{L*} a_i^L = 1\,,$$

$$a_i^{e*} a_i^\mu = a_i^{e*} a_i^L = a_i^{\mu*} a_i^L = 0\,. \tag{5}$$

Since $m_e/m_i$ and $m_\mu/m_i$ are presumably very small, we can take

$$\beta_e \approx \beta_\mu \approx 1 \tag{6}$$

and

$$b_{\mu e} \approx 0\,. \tag{7}$$

However, if some of the heavy neutral leptons are not too much more massive than the $L$ particle, the coefficient $\beta_L$ could depart appreciably from unity ($|\beta_L| \leq 1$) and $b_{L\mu}$ could be non-negligible. Adopting the approximations of Eqs. (6) and (7) we have

$$|\beta_L|^2 + m_L^2 \sum_i \frac{|a_i^L|^2}{m_i^2} = 1 \tag{8}$$

and

$$b_{L\mu} = -m_\mu m_L \sum_i \frac{a_i^{\mu*} a_i^L}{m_i^2}\,. \tag{9}$$

In all of the above parameterization we have allowed for the $CP$-violating possibility that the coefficients $a_i, \beta_i, b_{ij}$ are complex. In the original Cheng-Li scheme, with only two neutral heavy leptons $N_1$ and $N_2$, the phases can all be reabsorbed into redefinitions of the fields so that in fact, as in Eq. (2), the coefficients can be taken real. With $(\nu_L, L)$ added to the scheme along with the third heavy neutral lepton $N_3$, this reabsorption can no longer be carried out all the way. After allowable redefinitions one finds that the nine complex $a_i$ can all be expressed in terms of three real parameters plus one phase factor $e^{i\delta}$.[7] The possibility thus arises of $CP$-violating lepton interactions. Of course even if they are there in the couplings, they can show up observationally only for processes where phase interference comes into play. This could happen for the process $\mu \to 3e$, where $CP$-violating effects would reveal themselves via the existence of a spectral correlation term of the form $f(p_1, p_2)\, \vec{\sigma}\cdot\vec{P}_1\times\vec{P}_2$, where $\vec{\sigma}$ is the muon polarization, $\vec{P}_1$ and $\vec{P}_2$ are two final-state momenta. Even if the $\mu\to 3e$ reaction occurs at all, which for the present is the main issue, and even if the $CP$-violating phase differences were large, there are several reasons why this correlation term would be suppressed. Nevertheless, we keep this issue in mind in the following discussion and in the Appendix we discuss the decay spectrum in general terms, allowing for the possibility of $CP$ violation.

Here let us consider the amplitude for $\mu\to 3e$ decay in the framework of the Cheng-Li model. The diagrams are shown in Fig. 1. With the electron pair removed from the photon line, the diagram in Fig. 1(a) describes the process $\mu\to e+\gamma$, a magnetic-dipole transition. For the $\mu\to 3e$ process, if we were to retain *only* this magnetic-dipole amplitude, we would find that[8]

$$\left.\frac{\Gamma(\mu\to 3e)}{\Gamma(\mu\to e\gamma)}\right|_{\text{mag dipole}} \simeq \frac{1}{150}\,. \tag{10}$$

However, there are additional contributions to $\mu\to 3e$. One comes from the charge-radius amplitude incorporated in the diagram of Fig. 1(a). The remaining contributions are summarized in Figs. 1(b) and 1(c). The shaded box in Fig. 1(c) itself summarizes several diagrams. An analogous problem has already been analyzed by Gaillard and Lee,[9] in the framework of the Weinberg-Salam model, for the process $s+\bar{d}\to\mu^+ +\mu^-$, where $s$ denotes the strange quark, $d$ the down quark. Indeed, for the diagrams of both Figs. 1(b) and 1(c) we can take over bodily their results, with only the following modifications: In Fig. 1(c) the $eeZ$ vertex is purely vectorial for the Cheng-Li model under present discussion (in the Gaillard-Lee calculation there is also an axial-vector term); and in Fig. 1(b), whereas the diagrams involving $N_i$ exchange along the upper line are exactly as in the Gaillard and Lee calculation, the $\nu_e$ exchange line has no counterpart in their work. We assume throughout that the neutral-lepton masses $m_i$ are small compared to the $W$-boson mass $M$; moreover, we neglect terms formally of order unity relative to terms which are formally of order $\ln M^2/m_i^2$. With these approximations the contributions from Figs. 1(b) and 1(c) dominate over that coming from Fig. 1(a), and we find

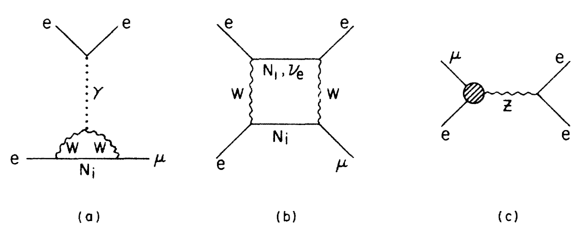


FIG. 1. Diagrams for $\mu\to e+e+\bar{e}$.

$$\text{amp}(\mu\to e_1+e_2+\bar{e}_3) = i\epsilon\frac{G_F}{\sqrt{2}}[(\bar{e}_{1R}\gamma_\lambda\mu)(A\bar{e}_{2R}\gamma_\lambda e_{3R}+B\bar{e}_{2L}\gamma_\lambda e_{3L}) - (e_1\leftrightarrow e_2)], \tag{11}$$

where the electron field is decomposed into right and left helicity parts according to $e=e_R+e_L$. The parameter $\epsilon$ is given by

$$\epsilon = \frac{\alpha}{2\pi\sin^2\theta_W}\sum_i \frac{a_i^{\mu *}a_i^e}{M^2}m_i^2\ln\frac{M^2}{m_i^2}\,, \tag{12}$$

with $\theta_W$ the Weinberg angle, and

$$A=2\sin^2\theta_W,\quad B=3+2\sin^2\theta_W\,. \tag{13}$$

For the branching ratio we then find

$$\frac{\Gamma(\mu\to 3e)}{\Gamma(\mu\to e\nu\bar{\nu})} = \frac{|\epsilon|^2}{16}(2A^2+B^2)\,. \tag{14}$$

We want to compare this with the $\mu\to e+\gamma$ branching ratio computed in the same model. The computation has been carried out by several authors.[10] Expressing their findings in the parameters of the generalized version of the model, one has

$$\frac{\Gamma(\mu\to e\gamma)}{\Gamma(\mu\to e\nu\bar{\nu})} = \frac{75}{32}\frac{\alpha}{\pi}\left|\sum_i a_i^{\mu *}a_i^e\frac{m_i^2}{M^2}\right|^2. \tag{15}$$

For the present rough purposes let us suppose that the masses $m_i$ are not too different among themselves. Then in the logarithmic terms of Eq. (12) we can replace each $m_i$ by the average heavy-neutral-lepton mass, $\bar{m}$, and we find

$$\frac{\Gamma(\mu\to 3e)}{\Gamma(\mu\to e\gamma)} = \frac{2}{25}\frac{\alpha}{\pi}\left(\ln\frac{M}{\bar{m}}\right)^2 \times\left(\frac{3+4\sin^2\theta_W+4\sin^4\theta_W}{\sin^4\theta_W}\right). \tag{16}$$

This is relatively insensitive to the unknown mass ratio $M/\bar{m}$; with $\ln(M/\bar{m})\approx 3$, $\sin^2\theta_W\approx\frac{1}{3}$, we find

$$\frac{\Gamma(\mu\to 3e)}{\Gamma(\mu\to e\gamma)} \simeq 0.07\,. \tag{17}$$

This is about one order of magnitude larger than the ratio we would have found keeping only the magnetic-dipole term corresponding to the diagram of Fig. 1(a) [see Eq. (10)] and indicates that our neglect of terms of order unity relative to terms of order $\ln(M^2/m_i^2)$ is not too misleading. But we *do* see that the magnetic contribution to the exact answer is not altogether negligible. This is important for the $CP$ tests in $\mu \to 3e$ decay, since it is the interference of the magnetic with the leading amplitudes that can give rise to $CP$-violating spectral correlations. However, an appreciable effect requires not only that the two amplitudes be at least roughly comparable in magnitude, it also requires that they have a noticeable phase difference. For the leading amplitude of Eq. (11) the phase is reflected in the factor

$$\sum_i a_i^{\mu *} a_i^e \frac{m_i^2}{M^2} \ln \frac{M^2}{m_i^2} .$$

In the magnetic amplitude the phase is that of the factor

$$\sum_i a_i^{\mu *} a_i^e \frac{m_i^2}{M^2} .$$

Insofar as the differences among the masses $m_i$ are small, so that we can set $\ln(M^2/m_i^2) \approx \ln(M^2/\overline{m}^2)$, the two factors, though perhaps complex, are seen to have the same phase. One has to go beyond this approximation, therefore, to reach the possibility of $CP$-violating phase interference; and the observational effects may well be small, even if $\mu \to 3e$ decay does occur (the main issue) and even if the *intrinsic* $CP$ violation were substantial. In the Appendix we parameterize the $\mu \to 3e$ amplitude in rather general terms, with a leading effective interaction of the form given in Eq. (11), to which we add an interaction of the sort that arises from the magnetic contribution of Fig. 1(a). The coefficients are allowed to be complex. The corresponding spectrum is worked out as a matter of possible future interest, quite apart from the $CP$ question, but we also display the spectral correlation term that would serve to test for $CP$ breakdown.

We conclude this section by describing briefly a natural generalization of the model of Ref. 2. In the original version of the model one has two doubly negatively charged heavy leptons $E_1$ and $E_2$ in addition to $(\nu_e, e)$, $(\nu_\mu, \mu)$. The SU(2) multiplets consist of two left-handed triplets: $(\nu_e, e, E_1 \cos\phi + E_2 \sin\phi)$, $(\nu_\mu, \mu, -E_1 \sin\phi + E_2 \cos\phi)$. To incorporate $(\nu_L, L)$ it is natural to add a third doubly charged lepton $E_3$, so that there are now three triplets:

$$\left(\nu_e, e, \sum a_i^e E_i\right), \quad \left(\nu_\mu, \mu, \sum a_i^\mu E_i\right), \quad \left(\nu_L, L, \sum a_i^L E_i\right) .$$

Again, the possibility of $CP$ violation arises because the $a_i$ can be complex (if so, in general, the phases cannot all be absorbed in redefinitions of the fields).

For the remaining discussion we return to the generalized scheme of Cheng and Li.

## III. NEUTRINO SHARING AND OFF-DIAGONAL NEUTRAL CURRENTS

The multiplet structure of the Cheng-Li model contains a number of physically interesting features, which become especially significant when the model is generalized to include the sequential leptons $(\nu_L, L)$. We have already emphasized the possibility of $CP$ violation. Let us now discuss several additional features.

### A. Constraints on heavy-lepton masses and breakdown of universality

As we see from Eq. (4), the parameter $\beta_L$ measures the strength of the $W$-boson coupling to the left-handed $(\nu_L, L)$ current. In the simplest single-Higgs-doublet) version of the model the analogous coefficients $\beta_\mu, \beta_e$ are essentially equal to unity $(m_e, m_\mu \ll m_i)$, as is required by the well-established universality of light-lepton and hadron charged-current couplings. However, as Eq. (8) shows, $|\beta_L|$ could conceivably be much smaller than unity, even in the simplest (single-Higgs-doublet) model, if some of the neutral leptons $N_i$ are not much heavier than the $L$ particle. Since $|\beta_L|$ cannot *exceed* unity, on the other hand, we have the constraint that not *all* of the neutral-lepton masses $m_i$ can be less than $m_L$. The parameter $\beta_L$ determines the rate for decay processes of the sort $L \to \nu_L + \cdots$, and the possibility arises that these rates are smaller than would be expected by universality considerations. Unfortunately, there seems to be no early prospect for experimental determination of absolute decay rates of such processes. In principle, however, the size of $\beta_L$ could be gotten at in another way, from decay of the charmed $F$ particle into lepton and neutrino, $(\nu_L, L)$ vs $(\nu_\mu, \mu)$. Here one has

$$\frac{\Gamma(F \to L + \nu_L)}{\Gamma(F \to \mu + \nu_\mu)} = |\beta_L|^2 \left(\frac{m_L}{m_\mu}\right)^2 \left(\frac{m_F^2 - m_L^2}{m_F^2 - m_\mu^2}\right)^2 .$$

### B. Neutrino sharing

The multiplets contain mixtures of the three neutrino types. As a matter of convention we have taken combinations such that: $e$ couples exclusively to $\nu_e$; $\mu$ couples primarily to $\nu_\mu$, but also with very small strength (in the single-Higgs-doublet model) to $\nu_e$; $L$ couples primarily to $\nu_L$, but also with small strength to $\nu_\mu$ and even smaller strength to $\nu_e$. These neutrino-sharing effects allow, for example, for $\pi^+ \to \mu^+ + \nu_e$ decay, and hence for the chain $\pi^+ \to \mu^+ + \nu_e$, $\nu_e + n \to p + e$; i.e., the classic "two-neutrino" experiment, if carried out with sufficient precision and careful tagging, would yield a non-null result. Of course, with $m_e \ll m_i$ the effect is very tiny. A small, but nevertheless more substantial rate is expected for

$\nu_\mu$ + nucleon $\rightarrow L$ + hadrons.

The amplitude is set by the parameter $b_{L\mu}$ [see Eq. (9)]. If the lightest of the $N_i$, say $N_1$, has mass $m_1$ comparable to $m_L$, and if the mass differences among the $m_i$ are substantial, the ratio of the cross section $\sigma_\nu(L)$ for $L$ production and $\sigma_\nu$(ord) for ordinary reactions $\nu$ + nucleon $\rightarrow \mu^-$ + hadrons could be as big as $\sigma_\nu(L)/\sigma_\nu(\text{ord}) \approx (m_\mu m_L/m_1^2)^2$.

### C. Off-diagonal neutral currents

In the model under discussion the neutral-current interactions contain off-diagonal couplings among the different heavy leptons $N_i$ and between the $N_i$ and neutrinos. This latter effect is especially interesting, since it leads to the process $\nu_\mu$ + nucleon $\rightarrow N_i$ + hadrons. The coupling strength is set by

$$\frac{m_\mu}{m_i}\left(a_i^\mu - a_i^L m_L^2 \sum_j \frac{a_j^{\mu *} a_j^L}{m_j^2}\right).$$

Thus, above threshold for $N_i$ production in a $\nu_\mu$ beam, one could have

$$\frac{\sum_i \sigma_\nu(N_i)}{\sigma_\nu(\text{ord})} \approx \left(\frac{m_\mu}{\overline{m}}\right)^2 .$$

If the heavy leptons are of order a few GeV this ratio might lie in the range $10^{-2} - 10^{-4}$.

Of course, through charged-current interactions of normal weak-interaction strength one also has the production reactions

$$e + \text{nucleon} \rightarrow N_i + \text{hadrons},$$

$$e^+ + e^- \rightarrow N_i + \overline{\nu}_e,$$

and through interactions of normal, weak strength

$$e^+ + e^- \rightarrow N_i + \overline{N}_j .$$

In these cases, however, one is competing against stronger (electromagnetic) reactions to normal channels.

As for modes of decay of the heavy leptons, the possibilities are fairly obvious: Through charged-current reactions of normal strength

$$N_i \rightarrow (e, \mu) + \text{hadrons},$$

$$N_i \rightarrow (e\overline{e}\nu_e, \mu\overline{\mu}\nu_\mu, e\overline{\mu}\nu_\mu, \mu\overline{e}\nu_e);$$

for $m_2 > m_1$,

$$N_2 \rightarrow N_1 + (\overline{e}e, \overline{\mu}\mu, \overline{e}\mu, \overline{\mu}e);$$

for $m_i > m_L$,

$$N_i \rightarrow L + (\overline{e}\nu_e, \overline{\mu}\nu_\mu),$$

$$N_i \rightarrow L + \text{hadrons};$$

and the converse, $N_i \leftrightarrow L$, for $m_L > m_i$; via off-diagonal neutral currents also $(m_2 > m_1)$

$$N_2 \rightarrow N_1 + \text{hadrons}.$$

One can imagine spectacular signals, e.g.,

$$\nu_\mu + \text{nucleon} \rightarrow N_2 + \cdots,$$

$N_2 \rightarrow N_1 + \mu + \overline{\mu}$, $N_1 \rightarrow \mu + \overline{\mu} + \nu_\mu$. For $e^+e^-$ collisions even more spectacular,

$$e^+ + e^- \rightarrow N_2 + \overline{N}_2,$$

$$N_2(\overline{N}_2) \rightarrow N_1(\overline{N}_1) + \mu\overline{\mu}, \quad N_1(\overline{N}_1) \rightarrow \mu\overline{\mu}\nu .$$

## APPENDIX

On any scheme where the process $\mu \rightarrow e + \gamma$ occurs, the process $\mu \rightarrow 3e$ must also inevitably take place, one term in the amplitude arising from magnetic-dipole coupling of $(e\mu)$ to a virtual photon which converts to an electron pair. For the $\mu \rightarrow 3e$ process this term has the momentum dependence characteristic of a magnetic-dipole transition. The remaining terms, arising from the charge-radius transition $\mu \rightarrow e$ + virtual photon and from nonphotonic diagrams [as in Figs. 1(b) and 1(c)] will be independent of external lepton momenta and will have the form of a direct current-current interaction. Provided that $\Gamma(\mu \rightarrow 3e)/\Gamma(\mu \rightarrow e\gamma)$ is much larger than $\frac{1}{150}$ [see Eq. (10)], these direct terms must dominate. We assume that this is the case. In parameterizing the transition amplitude we include both kinds of terms, but in displaying the decay spectrum we ignore the magnetic-dipole contributions except where they interfere with the leading amplitudes to produce $CP$-violating correlations. In the Cheng-Li model the muon always couples to a right-handed electron. This specialization is adopted in the following expressions. In the Wilczek-Zee scheme the muon, conversely, couples to a left-handed electron. To deal with this difference it is enough to change the sign of all spin-dependent terms in the final spectrum. For the most general case, where both kinds of couplings are envisaged, one can take advantage of the fact that the two kinds of terms contribute incoherently in the spectrum (provided that one sums over electron spins and ignores the electron mass, as we do), so the generalization is trivial. With these things understood we write for the $\mu \rightarrow 3e$ amplitude

$$\begin{aligned}\text{amp}(\mu \rightarrow e_1 + e_2 + \overline{e}_3) &= 4\sqrt{2}\,\frac{G_F}{\sqrt{2}}\Bigg[\frac{\alpha}{2}\overline{e}_{1R}\gamma_\lambda\mu\overline{e}_{2R}\gamma_\lambda e_{3R} + \beta\overline{e}_{1R}\gamma_\lambda\mu\overline{e}_{2L}\gamma_\lambda e_{3L} \\ &\quad - \frac{m_\mu\gamma}{q_1^2}\overline{e}_{1R}\sigma_{\lambda\nu}q_{1\nu}\mu(\overline{e}_{2R}\gamma_\lambda e_{3R} + \overline{e}_{2L}\gamma_\lambda e_{3L}) \\ &\quad - (1 \leftrightarrow 2)\Bigg], \end{aligned} \tag{A1}$$

where $q_i \equiv p_\mu - p_i$ is the four-vector momentum transfer between the muon and $i$th electron. The various numerical factors introduced into the parameterization are for later convenience. The coefficients $\alpha, \beta, \gamma$ are allowed to be complex (phase differences among them would reflect $CP$ violation); the term with coefficient $\gamma$ represents the magnetic-dipole contribution. Ignoring the magnetic contribution to the net decay rate, we have

$$\frac{\Gamma(\mu \to 3e)}{\Gamma(\mu \to e\gamma)} \simeq |\alpha|^2 + 2|\beta|^2 . \tag{A2}$$

For the decay spectrum we take as independent variables the energies $E_1$ and $E_2$ of the two identical electrons and their polar angles $\theta_1$, $\theta_2$, measured with respect to the muon-spin polarization. The muon spin is denoted by $\vec{\sigma}$ (its magnitude represents the degree of polarization). The decay spectrum is

$$d\omega = \frac{3}{\pi E_0^{\,4}}(1 - z_1^{\,2} - z_2^{\,2} - \lambda^2 + 2\lambda z_1 z_2)^{-1/2} \times f dE_1 dE_2 dz_1 dz_2 , \tag{A3}$$

where the normalization corresponds to

$$\int d\omega = |\alpha|^2 + 2|\beta|^2$$

and

$$z_i = \cos\theta_i , \quad E_0 = \tfrac{1}{2} m_\mu ,$$

$$\lambda = 1 + 2\frac{E_0(E_0 - E_1 - E_2)}{E_1 E_2} .$$

The spectral function $f$ is given by

$$\begin{aligned} f \simeq\ & |\alpha|^2 (E_1 + E_2 - E_0)(2E_0 - E_1 - E_2) + |\beta|^2 [(E_0 - E_1)E_1 + (E_0 - E_2)E_2] \\ & + |\alpha|^2 (E_1 + E_2 - E_0)\vec{\sigma}\cdot(\vec{P}_1 + \vec{P}_2) - |\beta|^2 [(E_0 - E_1)\vec{\sigma}\cdot\vec{P}_1 + (E_0 - E_2)\sigma\cdot\vec{P}_2] \\ & + \frac{E_1 - E_2}{(E_0 - E_1)(E_0 - E_2)}[(2E_0 - E_1 - E_2)\mathrm{Im}\beta\gamma^* - (E_1 + E_2 - E_0)\mathrm{Im}\alpha\gamma^*]\vec{\sigma}\cdot(p_1 \times p_2) . \end{aligned} \tag{A4}$$

Recall that we are dropping all terms containing the magnetic coefficient $\gamma$, except for the $CP$-violating correlation term $\vec{\sigma}\cdot(\vec{P}_1 \times \vec{P}_2)$.

Concerning $CP$-violating effects for the process $\mu \to e + \gamma$, one can easily see that these cannot arise for the models under discussion (the electron is produced in a definite helicity state, so the amplitude contains only one term, whose phase is unmeasurable). More general possibilities have however been discussed by Tung.[11]

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