Pseudoscalar transitions between high-spin resonances

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Pseudoscalar transitions between high-spin resonances, which involve two or more independent coupling constants, are considered. The overall ratios among decay amplitudes are derived from the correlations between different types of decay amplitudes by assuming $SU(2)_W$ symmetry or ignoring the higher partial waves. The corresponding expressions for decay widths are then derived and used to test SU(3) symmetry for transitions between high-spin resonances. We find that SU(3) symmetry is severely broken for the decays of spin-7/2 baryons. When SU(3) symmetry is combined with time-reversal invariance, some interesting restrictions on the ratios between decay widths are obtained. The centrifugal-barrier factors are also discussed.

I. INTRODUCTION

This paper is concerned with the pseudoscalar transitions between high-spin resonances which involve two or more partial waves. One of the purposes is to derive the overall ratios among decay amplitudes from the correlations between different types of decay amplitudes by either neglecting the higher partial waves or assuming $SU(2)_W$ symmetry. The corresponding expressions for decay widths are then derived. The other purpose is to examine, by using the above derived expressions, the validity of SU(3) symmetry in the pseudoscalar transitions between high-spin resonances. The coupling constants are used to extract the SU(3) invariants rather than the partial-wave amplitudes as many authors do.[1] The usual wrong identification[2] of coupling constants with partial-wave amplitudes is also discarded.

The pseudoscalar transition between high-spin resonances involves two or more independent decay amplitudes. In the case of strong decays, the number of independent decay amplitudes is equal to the number of partial waves allowed. Therefore, when the resonances are baryons, the number of independent decay amplitudes is equal to $J+\frac{1}{2}$, where J is the lesser spin of the two baryons. When the resonances are bosons, it is equal to $J+1$ or J, depending upon whether the normality of the boson resonances changes or not. The normality of a resonance is defined to be positive if it belongs to the normal spin-parity series, and negative if it belongs to the abnormal series. In other words, the normality is defined as $P(-)^J$ for bosons and $P(-)^{J-1/2}$ for baryons, where P is the intrinsic parity of the resonance.

In the present work, we consider the following decay processes:

(1) A spin-J baryon decays into a pseudoscalar meson and a spin-$\frac{3}{2}$ baryon.

(2) A spin-J meson decays into a pseudoscalar meson and a spin-1 meson with normality opposite to that of the decaying meson.

(3) A spin-J meson decays into a pseudoscalar meson and a spin-2 meson with normality equal to that of the decaying meson.

(4) A spin-J meson decays into a pseudoscalar meson and a spin-2 meson with normality opposite to that of the decaying meson.

The former three decay processes involve two independent decay amplitudes, while the last one involves three. By means of the Carruthers[3] and Lee's[4,5] decompositions, we may express[6-8] the helicity and the partial-wave amplitudes in terms of the coupling constants or vice versa. The above correlations then enable us to express the decay widths in terms of any type of decay amplitudes as we wish.

Since the high partial waves have higher centrifugal barriers which lessen the decay probability, it is reasonable to neglect the high-partial-wave contributions and keep only the lowest-partial-wave contribution. Under the above approximation, the number of independent coupling constants reduces to one which can then be determined by one known decay width. The calculated coupling constant can then be used to predict other decay widths through SU(3) symmetry. The overall ratios among decay amplitudes under the above approximation can also be derived.

In the naive $SU(2)_W$ model, any collinear process possesses $SU(2)_W$ symmetry. We investigate the restrictions of $SU(2)_W$ on the coupling constants and derive the decay width expression under $SU(2)_W$. We obtain simple relations between coupling constants, and simple expressions for decay widths. These expressions are then used to calculate the SU(3)-invariant coupling constants and test SU(3). The overall ratios among decay amplitudes under $SU(2)_W$ are also presented.

Under time-reversal invariance, it can be shown that the coupling constants are real. Combining the reality condition for coupling constants with SU(3), we obtain some interesting restrictions on the ratios between decay widths which can also be used to examine the SU(3) symmetry. We find that for the decays of spin-$\frac{7}{2}$ baryons, SU(3) is not a good symmetry. We note that in the present analysis, the physical masses of resonances are used in the calculation. In this way, the symmetry breaking due to mass difference has already been taken care of. The centrifugal-barrier factors are also discussed at the end of this paper.

II. PSEUDOSCALAR TRANSITION BETWEEN SPIN-J AND SPIN-$\frac{3}{2}$ BARYONS

The real and dimensionless coupling constants for the process in which a spin-J baryon decays into a pseudoscalar meson and a spin-$\frac{3}{2}$ baryon are defined by the expression

$$\left(\frac{p_0 q_0}{Mm}\right)^{1/2}\langle \tfrac{3}{2}(q,\lambda')|j_\pi(0)|J(p,\lambda)\rangle = \bar{u}_\mu(q,\lambda')\Gamma' i\gamma_5\left(\delta_{\mu\nu_1}F + \frac{p_\mu q_{\nu_1}}{m^2}G\right) \times q_{\nu_2}\cdots q_{\nu_n}(p,\lambda)M_0^{1-n}u_{\nu_1\cdots\nu_n}(p,\lambda)\,, \tag{1}$$

where $n = J-\frac{1}{2}$, $M_0 = 1$ MeV, λ's are helicities, M and m are the masses of the spin-J and spin-$\frac{3}{2}$ baryons, respectively, and Γ's are 1 or $i\gamma_5$ according to whether the normality of the baryon is $+1$ or -1. By means of the Carruthers decomposition for the high-spin spinor, expression (1) can be expressed[7,9] in terms of the coupling constants and explicit kinematical factors. We can then define the helicity amplitudes F_λ, and the partial-wave amplitudes[4] $F^{(l)}$ as linear combinations of coupling constants. The expressions for helicity amplitudes are

$$F_{\pm 3/2} = (\pm 1)^{6}\eta_1\left(\frac{J+\frac{3}{2}}{2J-1}\right)^{1/2}F,$$
$$F_{\pm 1/2} = (\pm 1)^{6}\eta_1\frac{1}{\sqrt{6}}\left[\left(\frac{2q_0}{m}-\eta\right)F - \frac{2q^2M}{m^3}G\right], \tag{2}$$

where η is the product of the normalities of spin-J and spin-$\frac{3}{2}$ baryons, and q is the c.m. momentum. For partial-wave amplitudes, the expressions are

$$F^{(l)} = \left[\frac{q_0}{m}+1+\frac{2(2+\eta)}{2J-1}\right]F - \frac{q^2M}{m^3}G,$$
$$F^{(l+2)} = -\frac{m}{q_0+m}F + \frac{M}{m}G, \tag{3}$$

where $l = J-\frac{1}{2}$ if $\eta = 1$, and $l = J-\frac{3}{2}$ if $\eta = -1$. We note that the expression for the high-partial-wave amplitude $F^{(l+2)}$ is independent of the relative normality η. The derived expression for the decay width takes the form

$$\Gamma(J\to\tfrac{3}{2}+\pi) = \frac{q^{2J-2}}{24\pi}\,\frac{q_0-\eta m}{M}\,\frac{(J-\frac{3}{2})!}{(2J)!!}M_0^{3-2J}\left[3(J+\tfrac{3}{2})|F|^2 + (J-\tfrac{1}{2})\left|\left(\eta-\frac{2q_0}{m}\right)F + \frac{2Mq^2}{m^3}G\right|^2\right], \tag{4}$$

which can also be expressed in terms of helicity amplitudes or partial-wave amplitudes as we wish. We note that the above expressions are exact without involving any kinematical and dynamical assumptions except Lorentz invariance.

Since the high partial wave has the higher centrifugal barrier which lessens the decay probability, it is reasonable to assume that the decays are dominated by the low-partial-wave contribution. Therefore we set the high-partial-wave amplitude $F^{(l+2)}$ equal to zero. Under this approximation, all decay amplitudes become related to each other. The overall ratios can be easily derived:

$$F^{(l)}:F:G:F_{\pm 1/2}:F_{\pm 3/2} = \frac{2(2J+1+\eta)}{2J-1}:1:\frac{m^2}{M(q_0+m)}:(\pm 1)^{6}\eta_1\frac{2-\eta}{\sqrt{6}}:(\pm)^{6}\eta_1\left(\frac{J+\frac{3}{2}}{2J-1}\right)^{1/2}. \tag{5}$$

The decay-width expression (4) also takes a very simple form under the above approximation. The expression is

$$\Gamma(J\to\tfrac{3}{2}+\pi) = \frac{q^{2J-2}}{4\pi}\,\frac{(q_0-\eta m)M(q_0+m)^2}{m^4} \times\frac{(J-\frac{3}{2})!}{(2J)!!}\,\frac{(2J+1+\eta)}{2+\eta}|G|^2. \tag{6}$$

In the naive $SU(2)_W$ model, any collinear process possesses $SU(2)_W$ symmetry. Under this symmetry all decay amplitudes are again related to each other. The overall ratio among decay amplitudes can be shown to be

$$F^{(l)}:F^{(l+2)}:F:G:F_{\pm 1/2}:F_{\pm 3/2} = -\frac{q^2M}{m^3}:\frac{M}{m}:0:1:-\left(\frac{2}{3}\right)^{1/2}\frac{q^2M}{m^3}:0 \tag{7}$$

for the decay processes $(8+1, {}^2P, \frac{3}{2}^-) \to (10, {}^4S, \frac{3}{2}^+) + (8, {}^1S, 0^-)$ and $(8, {}^2D, \frac{5}{2}^+) \to (10, {}^4S, \frac{3}{2}^+) + (8, {}^1S, 0^-)$, where ${}^{2s+1}L$ is the quark configuration. In the above ratio (7), $F = F_{\pm 3/2} = 0$ means that F and $F_{\pm 3/2}$ are forbidden by $SU(2)_W$ symmetry. The corresponding expressions for decay widths are then

$$\Gamma((8+1, {}^2P, \tfrac{3}{2}^-) \to (10, {}^4S, \tfrac{3}{2}^+) + (8, {}^1S, 0^-)) = \frac{q^5(q_0+m)M}{18\pi m^6} |G|^2 \quad (8)$$

and

$$\Gamma((8, {}^2D, \tfrac{5}{2}^+) \to (10, {}^4S, \tfrac{3}{2}^+) + (8, {}^1S, 0^-)) = \frac{q^7(q_0+m)M}{45\pi m^6 M_0^2} |G|^2, \quad (9)$$

respectively. Similarly, for the decay processes $(8, {}^4P, \frac{5}{2}^-) \to (10, {}^4S, \frac{3}{2}^+) + (8, {}^1S, 0^-)$ and $(10, {}^4D, \frac{7}{2}^+) \to (10, {}^4S, \frac{3}{2}^+) + (8, {}^1S, 0^-)$, we have the following overall ratios under $SU(2)_W$:

$$F^{(l)} : F^{(l+2)} : F : G : F_{\pm 1/2} : F_{\pm 3/2} = \frac{4(J+1)}{2J-1} : 0 : 1 : \frac{m^2}{M(q_0+m)} : \pm \frac{1}{\sqrt{6}} : \pm \left(\frac{J+\frac{3}{2}}{2J-1}\right)^{1/2}$$
$$= \begin{Bmatrix} \frac{7}{2} \\ 3 \end{Bmatrix} : 0 : 1 : \frac{m^2}{M(q_0+m)} : \pm \frac{1}{\sqrt{6}} : \pm \begin{Bmatrix} 1 \\ (\frac{5}{6})^{1/2} \end{Bmatrix} \quad (10)$$

for

$$\begin{Bmatrix} J = \frac{5}{2} \\ J = \frac{7}{2} \end{Bmatrix}.$$

The corresponding expressions for decay widths are then

$$\Gamma((8, {}^4P, \tfrac{5}{2}^-) \to (10, {}^4S, \tfrac{3}{2}^+) + (8, {}^1S, 0^-)) = \frac{7q^5(q_0+m)M}{180\pi m^4 M_0^2} |G|^2 \quad (11)$$

and

$$\Gamma((10, {}^4D, \tfrac{7}{2}^+) \to (10, {}^4S, \tfrac{3}{2}^+) + (8, {}^1S, 0^-)) = \frac{q^7(q_0+m)M}{70\pi m^4 M_0^4} |G|^2, \quad (12)$$

respectively. As for the process $(8, {}^4P, \frac{5}{2}^-) \to (8+1, {}^2P, \frac{3}{2}^-) + (8, {}^1S, 0^-)$, the overall ratio among decay amplitudes under $SU(2)_W$ is

$$F^{(l)} : F^{(l+2)} : F : G : F_{\pm 1/2} : F_{\pm 3/2} = \tfrac{5}{2} : 0 : 1 : \frac{m^2}{M(q_0+m)} : (\tfrac{3}{2})^{1/2} : 1, \quad (13)$$

and the corresponding decay width is then

$$\Gamma((8, {}^4P, \tfrac{5}{2}^-) \to (8+1, {}^2P, \tfrac{3}{2}^-) + (8, {}^1S, 0^-)) = \frac{q^3(q_0+m)^3 M}{12\pi m^4 M_0^2} |G|^2. \quad (14)$$

In the above expressions, $F^{(l)}$ denotes the low-partial-wave amplitudes, and $F^{(l+2)}$ denotes the high-partial-wave amplitudes.

III. PSEUDOSCALAR TRANSITION BETWEEN SPIN-J AND SPIN-1 MESONS

There are two independent coupling constants for the decay process in which a spin-J meson decays into a pseudoscalar meson and a spin-1 meson with normality opposite to that of the decaying boson. The two independent coupling constants are defined by

$$(4p_0q_0)^{1/2}\langle 1(q,\lambda')|j_\pi(0)|J(p,\lambda)\rangle = \bar{\epsilon}_\mu(q,\lambda')\left(\delta_{\mu\nu_1}F + \frac{p_\mu q_{\nu_1}}{m^2}G\right)q_{\nu_2}\cdots q_{\nu_J} \times \epsilon_{\nu_1\cdots\nu_J}(p,\lambda)M_0^{2-J}, \quad (15)$$

where $M_0 = 1$ MeV. The above coupling constants are dimensionless and can be shown to be real owing to time-reversal invariance. By using Lee's decomposition[6] for the polarization tensor, we can express the invariant decay matrix element (15) in terms of coupling constants and an explicit kinematical factor, which then enable us to define the helicity amplitudes and the partial-wave amplitudes[7] in terms of coupling constants. The expressions for the helicity amplitudes are

$$F_{\pm 1} = \left(\frac{J+1}{2}\right)^{1/2} F, \qquad F_0 = \sqrt{J}\left(\frac{q_0}{m}F - \frac{q^2 M}{m^3}G\right). \quad (16)$$

For partial-wave amplitudes, we have the following expressions:

$$F^{(J-1)} = \left(1 + \frac{q_0}{m} + \frac{1}{J}\right)F - \frac{q^2 M}{m^3}G, \qquad F^{(J+1)} = -\frac{m}{q_0+m}F + \frac{M}{m}G. \quad (17)$$

The decay width formula can then be expressed in terms of any type of decay amplitudes. When expressed in terms of coupling constants, we have[4]

$$\Gamma(J \to 1 + \pi) = \frac{q^{2J-1}}{8\pi M^2} \frac{(J-1)!}{(2J+1)!!} \times \left[(J+1)|F|^2 + J\left|\frac{q_0}{m}F - \frac{q^2 M}{m^3}G\right|^2\right] M_0^{4-2J}. \quad (18)$$

Under the approximation of neglecting the high-partial-wave contribution, we obtain the following overall ratio among decay amplitudes:

$$F^{(J-1)}:F:G:F_0:F_{\pm1}=\left(2+\frac{1}{J}\right):1:\frac{m^2}{M(q_0+m)}:\sqrt{J}:\left(\frac{J+1}{2}\right)^{1/2}. \quad (19)$$

The decay width formula (18) becomes, under the above approximation,

$$\Gamma(J\to 1+\pi)=\frac{q^{2J-1}}{8\pi}\frac{(q_0+m)^2M_0^{4-2J}}{m^4}\times\frac{(J-1)!}{(2J-1)!!}|G|^2. \quad (20)$$

When $SU(2)_W$ symmetry is assumed, the overall ratio among decay amplitudes becomes

$$F^{(0)}:F^{(2)}:F:G:F_0:F_{\pm1}=-\frac{q^2M}{m^3}:\frac{M}{m}:0:1:-\frac{q^2M}{m^3}:0; \quad (21)$$

for the decay process $(8,{}^1P,1^+)\to(8+1,{}^3S,1^-)+(8,{}^1S,0^-)$. The corresponding decay width is then

$$\Gamma((8,{}^1P,1^+)\to(8+1,{}^3S,1^-)+(8,{}^1S,0^-))=\frac{q^5M_0^2}{24\pi m^6}|G|^2. \quad (22)$$

For the decay process $(8,{}^3P,1^+)\to(8+1,{}^3S,1^-)+(8,{}^1S,0^-)$. The overall ratio among decay amplitudes, under $SU(2)_W$ symmetry, is

$$F^{(0)}:F^{(2)}:F:G:F_0:F_{\pm1}=2:\frac{m^2}{q^2}:1:\frac{q_0m^2}{q^2M}:0:1. \quad (23)$$

The corresponding decay width becomes

$$\Gamma((8,{}^3P,1^+)\to(8+1,{}^3S,1^-)+(8,{}^1S,0^-))=\frac{q^5M_0^2}{12\pi q_0^2m^4}|G|^2. \quad (24)$$

IV. PSEUDOSCALAR TRANSITION BETWEEN SPIN-J AND SPIN-2 MESONS

The process in which a spin-J boson decays into a pseudoscalar meson and a spin-2 boson with the same normality as that of the decaying boson has two independent coupling constants which are defined by the expression

$$(4p_0q_0)^{1/2}\langle 2(q,\lambda')|j_\pi(0)|J(p,\lambda)\rangle=i\bar{\epsilon}_{\mu_1\mu_2}(q,\lambda')\epsilon_{\mu_1\nu_1\nu\rho}p_\nu q_\rho\left(\delta_{\mu_2\nu_2}F+\frac{p_{\mu_2}q_{\nu_2}}{m^2}G\right)q_{\nu_3}\cdots q_{\nu_J}\epsilon_{\nu_1\cdots\nu_J}(p,\lambda)M_0^{1-J}, \quad (25)$$

where $\epsilon_{\mu_1\nu_1\nu\rho}$ is the antisymmetric tensor. The coupling constants defined in the above expression are dimensionless, and can be shown to be real owing to time-reversal invariance. After using Lee's decomposition for the polarization tensor, the invariant decay matrix element (25) can be expressed in terms of coupling constants and explicit kinematical factors. We can thus define[8] the helicity and partial-wave amplitudes. The helicity-amplitude expressions are

$$F_{\pm2}=\pm(J+2)^{1/2}F,$$
$$F_{\pm1}=\pm(J-1)^{1/2}\left(\frac{q_0}{m}F-\frac{q^2M}{m^3}G\right), \quad (26)$$
$$F_0=0.$$

For partial-wave amplitudes, we have

$$F^{(J-1)}=\left(1+\frac{q_0}{m}+\frac{3}{J-1}\right)F-\frac{q^2M}{m^3}G,$$
$$F^{(J+1)}=-\frac{m}{q_0+m}F+\frac{M}{m}G. \quad (27)$$

The decay-width formula, when expressed in terms of coupling constants, is

$$\Gamma(J\to 2+\pi)=\frac{q^{2J-1}M_0^{2-2J}}{16\pi}\frac{(J-2)!(J+1)}{(2J+1)!!}\left[(J+2)|F|^2+(J-1)\left|\frac{q_0}{m}F-\frac{q^2M}{m^3}G\right|^2\right]. \quad (28)$$

Under the approximation of neglecting the high-partial-wave contribution, we obtain the following overall ratio among decay amplitudes:

$$F^{(J-1)}:F:G:F_0:F_{\pm1}:F_{\pm2}=2+\frac{3}{J-1}:1:\frac{m^2}{M(q_0+m)}:0:\pm(J-1)^{1/2}:\pm(J+2)^{1/2}. \quad (29)$$

The corresponding decay width becomes

$$\Gamma(J\to 2+\pi)=\frac{q^{2J-1}M_0^{2-2J}}{16\pi}\frac{M^2(q_0+m)^2}{m^4}\frac{(J-2)!(J+1)}{(2J-1)!!}|G|^2. \quad (30)$$

If $SU(2)_W$ symmetry is assumed, then the overall ratio among decay amplitudes becomes

$$F^{(2)}:F^{(4)}:F:G:F_0:F_{\pm1}:F_{\pm2}=(5+\sqrt{3}):(2-\sqrt{3})\frac{m^2}{q^2}:2:\frac{m^2(2q_0-\sqrt{3}m)}{q^2M}:0:\pm\sqrt{6}:\pm2\sqrt{5}\,; \tag{31}$$

for the decay process $(8,{}^3D,3^-)\to(8,{}^3P,2^+)+(8,{}^1S,0^-)$. The corresponding decay width becomes

$$\Gamma((8,{}^3D,3^-)\to(8,{}^3P,2^+)+(8,{}^1S,0^-))=\frac{13q^9M^2}{210\pi m^4(2q_0-\sqrt{3}m)^2M_0^4}\,|G|^2. \tag{32}$$

We also consider the decay process in which a spin-J boson decays into a pseudoscalar meson and a spin-2 boson with normality opposite to that of the decaying boson. There are three independent coupling constants in the above process, which are defined by the expression

$$(4p_0q_0)^{1/2}\langle 2(q,\lambda')|j_\pi(0)|J(p,\lambda)\rangle$$
$$=\bar{\epsilon}_{\mu_1\mu_2}(q,\lambda')\left(\delta_{\mu_1\nu_1}\delta_{\mu_2\nu_2}F+\delta_{\mu_1\nu_1}\frac{p_{\mu_2}q_{\nu_2}}{m^2}G+\frac{p_{\mu_1}p_{\mu_2}q_{\nu_1}q_{\nu_2}}{m^4}H\right)q_{\nu_3}\cdots q_{\nu_J}\epsilon_{\nu_1\cdots\nu_J}(p,\lambda)M_0^{3-J}\,. \tag{33}$$

The above-defined coupling constants are dimensionless and are real because of time-reversal invariance. After some calculations[4] by means of Lee's decomposition for the polarization tensor, we can define[8] the helicity and partial-wave amplitudes in terms of coupling constants. The expressions for helicity amplitudes are

$$\begin{aligned}
F_{\pm2}&=[(J+2)(J+1)]^{1/2}\tfrac{1}{2}F\,,\\
F_{\pm1}&=(J^2-1)^{1/2}\left(\frac{q_0}{m}F-\frac{q^2M}{2m^3}G\right)\,,\\
F_0&=\left[\frac{2J(J-1)}{3}\right]^{1/2}\left[\left(\frac{1}{2}+\frac{q_0^2}{m^2}\right)F-\frac{q_0q^2M}{m^4}G+\frac{q^4M^2}{m^6}H\right]\,,
\end{aligned} \tag{34}$$

and

$$\begin{aligned}
F^{(J-2)}&=\left[J^2+J+1+2(J^2-1)\frac{q_0}{m}+J(J-1)\frac{q_0^2}{m^2}\right]F-\left[J^2-1+J(J-1)\frac{q_0}{m}\right]\frac{q^2M}{m^3}G+J(J-1)\frac{q^4M^2}{m^6}H\,,\\
F^{(J)}&=-2\left(\frac{J}{3}+\frac{m}{q_0+m}\right)F+\left(1+\frac{2J}{3}\frac{q_0}{m}\right)\frac{M}{m}G-\frac{2J}{3}\frac{q^2M^2}{m^4}H\,,\\
F^{(J+2)}&=\frac{m^2}{(q_0+m)^2}F-\frac{M}{q_0+m}G+\frac{M^2}{m^2}H\,,
\end{aligned} \tag{35}$$

for the partial-wave amplitudes. The decay width formula when expressed in terms of coupling constants is

$$\begin{aligned}
\Gamma(J\to2+\pi)=\frac{q^{2J-3}M_0^{6-2J}}{8\pi M^2}\frac{(J-2)!}{(2J+1)!!}\Bigg[&\frac{(J+2)(J+1)}{2}|F|^2+2(J^2-1)\left|\frac{q_0}{m}F-\frac{q^2M}{2m^3}G\right|^2\\
&+\frac{2J(J-1)}{3}\left|\left(\frac{1}{2}+\frac{q_0^2}{m^2}\right)F-\frac{q_0q^2M}{m^4}G+\frac{q^4M^2}{m^6}H\right|^2\Bigg]\,.
\end{aligned} \tag{36}$$

Under the approximation of neglecting the higher partial waves, we obtain the overall ratio among decay amplitudes as follows:

$$F^{(J-2)}:\ F:G:H:F_0:F_{\pm1}:F_{\pm2}=4J^2-1:1:\frac{2m^2}{M(q_0+m)}:\frac{m^4}{M^2(q_0+m)^2}:\left[\frac{3J(J-1)}{2}\right]^{1/2}:(J^2-1)^{1/2}:\tfrac{1}{2}[(J+2)(J+1)]^{1/2}. \tag{37}$$

The decay width (36) becomes, under the above approximation,

$$\Gamma(J\to2+\pi)=\frac{q^{2J-3}M_0^{6-2J}}{32\pi}\frac{(q_0+m)^2}{m^4}\frac{(J-2)!}{(2J-3)!!}\,|G|^2\,. \tag{38}$$

If $SU(2)_W$ symmetry is assumed, the overall ratio among decay amplitudes becomes

$$F^{(0)}:F^{(2)}:F^{(4)}:F:G:H:F_0:F_{\pm 1}:F_{\pm 2}$$
$$=-(3+\sqrt{3})\frac{q^2M}{m^3}:\frac{2+\sqrt{3}}{\sqrt{3}}\frac{M}{m}:\frac{2-\sqrt{3}}{2}\frac{Mm}{q^2}:0:1:\frac{m^2(2q_0-\sqrt{3}m)}{2q^2M}:-\frac{q^2M}{m^3}:-\frac{\sqrt{3}}{2}\frac{q^2M}{m^3}:0; \quad (39)$$

for the decay $(8, {}^1D, 2^-) \to (8, {}^3P, 2^+) + (8, {}^1S, 0^-)$ and the decay width (36) becomes

$$\Gamma((8, {}^1D, 2^-) \to (8, {}^3P, 2^+) + (8, {}^1S, 0^-)) = (q^5M_0{}^2/48\pi m^6)\,|G|^2 . \quad (40)$$

For the decay process $(8, {}^3D, 2^-) \to (8, {}^3P, 2^+) + (8, {}^1S, 0^-)$, the overall ratio among decay amplitudes under $SU(2)_W$ symmetry is

$$F^{(0)}:F^{(2)}:F^{(4)}:F:G:H:F_0:F_{\pm 1}:F_{\pm 2}$$
$$=9+3\sqrt{3}:\frac{(2-\sqrt{3})m^2}{q^2}:\frac{(2-\sqrt{3})m^4}{q^4}:1:\frac{(2q_0-\sqrt{3}m)m^2}{q^2M}:\frac{m^4(m^2-\sqrt{3}mq_0+q_0{}^2)}{q^4M^2}:\sqrt{3}:\tfrac{3}{2}:\sqrt{3}. \quad (41)$$

The corresponding decay width is

$$\Gamma((8, {}^3D, 2^-) \to (8, {}^3P, 2^+) + (8, {}^1S, 0^-)) = \frac{9q^5M_0{}^2}{80\pi m^4}\frac{|G|^2}{(2q_0-\sqrt{3}m)^2} . \quad (42)$$

We note that the decay-width expressions under $SU(2)_W$ symmetry given in Secs. II, III, and IV, depend only on the quark spin, the quark orbital angular momentum, and the J^P of resonances, and are independent of the SU(3) assignments of multiplets.

V. SU(3) SYMMETRY AND TIME-REVERSAL INVARIANCE

The decay-width formulas derived in Secs. II, III, and IV can then be used to test SU(3) symmetry. The SU(3)-invariant coupling constants are obtained by dividing the coupling constants with a proper SU(3) isoscalar factor. We choose one or two inputs to determine the SU(3)-invariant coupling constants, then use these values to predict the values of decay widths of other decay processes and compare them with available data.

In the above calculation, we use the physical masses of particles. In this way, most of the symmetry breaking is expected to be automatically

TABLE I. Decay widths for pseudoscalar transitions between high-spin baryons. $\Gamma_{SU(2)_W}$ and $\Gamma_{G'=0}$ are the decay widths obtained by assuming $SU(2)_W$ symmetry and ignoring the higher partial waves, respectively. The mixing angle for $\Lambda(1690)$ and $\Lambda(1520)$ is $(-23\pm4)°$.

$\alpha, {}^{2s+1}L, J^P$	Decay modes	Γ_{exp} (MeV)	$\Gamma_{SU(2)_W}$ pred (MeV)	$\Gamma_{G'=0}$ pred (MeV)
$\underline{8}\oplus\underline{1}, {}^2P, \frac{3}{2}^-$	$N(1520)\to\Delta(1232)\pi$	48.4 ± 4.8	48.4 (input)	48.4 (input)
	$\Lambda(1690)\to\Sigma^*(1385)\pi$	23	28.13	32.83
	$\Lambda(1520)\to\Sigma^*(1385)\pi$	0.6 ± 0.1	~0	0.45
	$\Sigma(1670)\to\Sigma^*(1385)\pi$	8.6 ± 1.1	4.95	7.85
	$\Xi(1820)\to\Xi^*(1530)\pi$	10.5 ± 6.0	5.13	11.68
$\underline{8}, {}^2D, \frac{5}{2}^+$	$N(1688)\to\Delta(1232)\pi$	21 ±12	21 (input)	21 (input)
	$\Lambda(1815)\to\Sigma^*(1385)\pi$	16.0 ± 4.03	7.14	13.18
	$\Sigma(1915)\to\Sigma^*(1385)\pi$	...	8.05	6.22
	$\to\Delta(1232)\bar{K}$	...	23.16	18.86
	$\Xi(2030)\to\Xi^*(1530)\pi$	...	5.04	7.31
	$\to\Sigma^*(1385)\bar{K}$	...	2.29	4.61
$\underline{8}, {}^4P, \frac{5}{2}^-$	$N(1670)\to\Delta(1232)\pi$	90 ±22	90 (input)	90 (input)
	$\Lambda(1830)\to\Sigma^*(1385)\pi$	27 ±26	56.36	56.36
	$\Sigma(1765)\to\Sigma^*(1385)\pi$	11.6 ± 4.7	5.56	5.56
	$\to\Delta(1232)\bar{K}$	...	1.38	1.38
	$\Xi(1940)\to\Xi^*(1530)\pi$	...	10.5	10.5
	$\to\Sigma^*(1385)\bar{K}$	...	1.51	1.51
$\underline{10}, {}^4D, \frac{7}{2}^+$	$\Delta(1950)\to\Delta(1232)\pi$	26 ± 6.5	26 (input)	26 (input)
	$\to\Delta(1232)\eta$	3.1 ± 1.4	0.18	0.18
	$\to\Sigma^*(1385)K$	3.08 ± 1.46	4.12×10^{-3}	4.12×10^{-3}
	$\Sigma(2030)\to\Sigma^*(1385)\pi$	...	6.92	6.92
	$\to\Delta(1232)\bar{K}$	...	6.36	6.36

TABLE II. Decay widths for pesudoscalar transitions between high-spin bosons. The ϕ-ω mixing angle is $39\pm1°$, and $\Gamma_K(1760)=\Gamma(K(1760)\rightarrow K^*(1420)+\pi)$ and $\Gamma_g=\Gamma(g(1680)\rightarrow A_2(1310)+\pi)$.

$\alpha, {}^{2s+1}L, J^{PC}$	Decay modes	Exp Γ (MeV)	$\Gamma_{SU(2)_W}$ pred (MeV)	$\Gamma_{G'=0}$ pred (MeV)
$\underline{8}, {}^1P, 1^{+-}$	$B(1235)\rightarrow\phi(1019)\pi$	4.8 ± 0.8	4.8 (input)	4.8 (input)
	$\rightarrow\omega(783)\pi$	120 ± 20	120 (input)	120 (input)
	$Q_L(1240)\rightarrow K^*(892)\pi$	47 ± 19.52	143	14.23
	$Q_h(1350)\rightarrow K^*(892)\pi$	60.25 ± 25.24	769	20.70
	$\rightarrow\rho(770)K$	180.75 ± 32.97	299	18.82
	$\rightarrow\omega(783)K$	2.41 ± 1.24	158	38.9
$\underline{8}, {}^3P, 1^{++}$	$A_1(1100)\rightarrow\rho(770)\pi$	~300	300 (input)	300 (input)
	$E(1420)\rightarrow K^*(892)\bar{K}$	6 ± 2	5.25	89
	$Q_L(1240)\rightarrow K^*(892)\pi$	47 ± 19.52	85	94
	$Q_h(1350)\rightarrow K^*(892)\pi$	60.25 ± 25.24	424	137
	$\rightarrow\rho(770)K$	180.75 ± 32.97	174	124
	$\rightarrow\omega(783)K$	2.41 ± 1.24	2.41 (input)	2.41 (input)
$\underline{8}, {}^3D, 3^{--}$	$g(1680)\rightarrow A_2(1310)\pi$	seen	$\frac{\Gamma_K(1760)}{\Gamma_g(1680)}=0.18$	$\frac{\Gamma_K(1760)}{\Gamma_g(1680)}=0.18$
	$K(1760)\rightarrow K^*(1420)\pi$	seen		
$\underline{8}, {}^1D, 2^{-+}$	$A_3(1640)\rightarrow f(1270)\pi$	~300	300 (input)	300 (input)
	$L(1770)\rightarrow K^*(1420)\pi$	seen	165	509

taken into account. The predicted decay widths are given in Tables I and II for baryon and boson decays, respectively. In the above Tables, $\Gamma_{SU(2)_W}$ denotes the values of decay widths calculated based upon the decay width expressions which assume $SU(2)_W$ symmetry, and $\Gamma_{G'=0}$ denotes those values of decay widths calculated by ignoring the higher partial waves. The data used are taken from the Particle Data Group[10] and Samios *et al.*[11] The width of $\Delta(1950)\rightarrow\Sigma^*K$ is estimated from Chinowsky *et al.*[12] and Samios *et al.*[11]

It can be shown[7] that all the decay amplitudes defined in the preceding sections are real because of time-reversal invariance. The reality of coupling constants, when combined with SU(3) symmetry, may give significant restrictions on the ratios between decay widths, which will be discussed in the next section.

VI. DISCUSSIONS AND CONCLUSIONS

The formalism presented in this paper gives explicit kinematical factors to the decay widths. When the decay widths are expressed in terms of the partial-wave amplitudes (which can be done easily), these kinematical factors represent the product of the phase space and the centrifugal-barrier factor. In this way, the centrifugal-barrier factor can be extracted. We find that the centrifugal-barrier factor is proportional to $q^{2l}(q_0+m)^{-\eta}$ for the decay in which a spin-J baryon decays into a pseudoscalar meson and a spin-$\frac{3}{2}$ baryon, where l is the relative orbital angular momentum of the final particles. For the process in which a spin-J boson decays into a pion and a spin-1 or spin-2 boson, the centrifugal-barrier factor has the well-known form q^{2l}. Under the approximation of ignoring higher partial waves, the barrier factor is simply that for the lowest partial wave. If $SU(2)_W$ symmetry is assumed, then the q^2 dependence of centrifugal-barrier factors is the same for all partial waves allowed in a given decay process. If we require that all coupling constants remain finite when analytically continued to the threshold, i.e., $q^2\rightarrow0$, then, under $SU(2)_W$ symmetry, all centrifugal-barrier factors assume the same q^2 dependence as that for the highest partial wave as long as the highest partial wave is allowed under $SU(2)_W$, i.e., $q^{2l_{max}}(q_0+m)^{-\eta}$ for baryon decays and $q^{2l_{max}}$ for boson decays. The above conclusions are drawn directly from the partial-wave expressions for decay widths and from the overall ratios among decay amplitudes. We note that the above rules are valid even if the final particles have nonvanishing quark orbital angular momentum.

From Table I, we see that SU(3) symmetry is consistent with the baryon-decay data up to spin $\frac{5}{2}$, and the approximation in which the higher partial waves are ignored gives better results than $SU(2)_W$ symmetry does in general. For spin $\frac{7}{2}$, the predicted result for Σ^*K is three orders of magnitude smaller than the experimental value. This definitely implies a severe breaking of SU(3) symmetry for spin $\frac{7}{2}$. Owing to very limited data for bosons, we present only some SU(3) predictions in Table II. The Q-meson classification is still an open question. We put both Q_L and Q_h in 1^{+-} and 1^{++}

octets, and try to see which one fits better. From the calculated widths, it seems that Q_L tends to belong to 1^{++}, since both $\Gamma_{SU(2)_W}$ and $\Gamma_{G'=0}$ are close to the experimental data. But this is not conclusive. Some mixing effect may be needed to clarify this problem.

Owing to time-reversal invariance, all decay amplitudes defined in this paper can be shown to be real. When the reality condition for coupling constants is combined with SU(3) symmetry, some restrictions on the ratios between decay widths can be obtained; some interesting results are

$$0.05 \leq \frac{\Gamma(\Sigma(1765) \to \Sigma^*\pi)}{\Gamma(\Lambda(1830) \to \Sigma^*\pi)} \leq 0.11$$

and

$$0.42 \leq \frac{\Gamma(\Lambda(1830) \to \Sigma^*\pi)}{\Gamma(N(1670) \to \Delta\pi)} \leq 0.68 \ .$$

In the above calculations, we use the exact decay-width expression (4) without neglecting high partial waves or assuming $SU(2)_W$ symmetry. Experimentally, the above two ratios are 0.43 ± 0.45 and 0.3 ± 0.3, respectively. Therefore if SU(3) symmetry is good, then the experimental values should be improved. Otherwise, this implies a deviation from SU(3) symmetry for spin $\frac{5}{2}$. When the branching ratios for decays of spin-$\frac{7}{2}$ baryons are calculated, we obtain

$$1.25 \times 10^2 \leq \frac{\Gamma(\Delta(1950) \to \Delta\pi)}{\Gamma(\Delta(1950) \to \Delta\eta)} \leq 9.91 \times 10^2$$

and

$$3.92 \times 10^3 \leq \frac{\Gamma(\Delta(1950) \to \Delta\pi)}{\Gamma(\Delta(1950) \to \Sigma^* K)} \leq 7.60 \times 10^5 \ .$$

The above restrictions on branching ratios imply very narrow widths for the decays of $\Delta(1950) \to \Delta\eta$ and $\Delta(1950) \to \Sigma^* K$ of order of magnitude in the keV region, which are clearly in contradiction with the experimental values. This implies that SU(3) is not a good symmetry for the decays of spin-$\frac{7}{2}$ baryons into a pseudoscalar meson and a spin-$\frac{3}{2}$ baryon.

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