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Theoretical models for the angular distribution of massive muon pairs produced in hadronic collisions

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We point out that accurate measurements of the angular distributions of direct muons produced in hadronic collisions will have important consequences for various theoretical models for such processes. A number of models based on concepts of quark-partons and gluons are discussed within the context of the preliminary data available at present.

I. INTRODUCTION

Production of massive μ pairs in hadronic collisions has been studied extensively recently both experimentally and theoretically. It has already provided insight into the dynamics of various theoretical models based on the concepts of quark-partons and gluons. Ideally one would like to have data for Q^2 [(mass of dimuon)^2], $Q_\parallel$ (longitudinal momentum of the dimuon), and Q_T (transverse momentum) distributions along with the angular distribution of the muons in a range of values of each of these variables. It will be some time in the future before such complete high-statistics data become available. But already some preliminary data for angular distributions averaged over the other variables have been obtained.[1] In the past, several authors have considered Q^2, $Q_\parallel$, and Q_T distributions of the dimuon in various models, but the angular distributions of muons have not been considered. In the present work we discuss various theoretical models for dimuon production, with special emphasis on the angular distributions. We show that even from such averaged data some interesting conclusions follow. Q^2, $Q_\parallel$, and Q_T distributions, for one of the models, are considered separately.[2] We hope to present a more detailed analysis of angular distributions when complete data become available. One of the purposes of this work is to point out that accurate measurements of such distributions will have very important consequences for various theoretical models, since predictions differ considerably from one model to another.

The preliminary results which follow from a recent experiment[1] give the muon angular distribution in the dimuon rest frame as $1+\alpha\cos^2\theta$, where the angle θ is measured with respect to the incident beam axis. There are considerable variations in the measured α values. But they are found to be consistent with 1 (although they could be different from 1) for continuum μ pair in the $Q\approx(1.9\text{–}2.3)$-GeV region. In the ψ resonance region ($Q=3.1$ GeV), however, α is found to be consistent with zero ($\alpha=-0.26\pm0.19$ for incident protons, $\alpha=0.26\pm0.25$ for incident pions). If future analyses uphold this conclusion, it will already be an indication that the production mechanism for continuum μ pairs production is entirely different from that of ψ production.

The complete angular distribution of the muon is given by the expression[3]

$$W(\theta,\phi)=1-\rho_{11}\sin^2\theta-\rho_{00}\cos^2\theta+\rho_{1,-1}\sin^2\theta\cos 2\phi + \sqrt{2}\,\mathrm{Re}\rho_{10}\sin^2\theta\cos\phi, \quad (1)$$

where the ρ's are the density matrix elements. The angles (θ,ϕ) are measured with respect to some convenient axes. In the Gottfried-Jackson (GJ) frame the z axis is along the direction of the incident beam in the dimuon rest frame. In the helicity frame it is opposite to the direction of the momentum of the recoiling missing mass in the dimuon rest frame. Averaging over the azimuthal angle ϕ, $W(\theta,\phi)$ can be written as

$$W(\theta)=1+\frac{3\rho_{11}-1}{1-\rho_{11}}\cos^2\theta, \quad (2)$$

where we have used the normalization condition $\rho_{00}+2\rho_{11}=1$. Hence

$$\alpha=\frac{3\rho_{11}-1}{1-\rho_{11}}. \quad (3)$$

In general ρ_{11} (and hence α) will be a function of other kinematic variables such as $Q_\parallel$, Q_T, etc. At present, however, data only give the average value of α. If m_1, m_2, and Q are the masses of the incident particle, target particle, and dimuon, and m_X is the missing mass, the crossing angle χ which relates the GJ frame (t-channel) to the s-channel helicity frame is given by

$$\cos\chi=[(s+Q^2-m_x^2)(t+Q^2-m_1^2)+2Q^2\Delta]/D, \quad (4)$$

where

$$\Delta = m_X^2 - m_2^2 + m_1^2 - Q^2 \tag{5}$$

and

$$D = [(Q+m_1)^2 - t]^{1/2}[(Q-m_1)^2 - t]^{1/2} \times [s-(Q+m_X)^2]^{1/2}[s-(Q-m_X^2)^2]^{1/2} \tag{6}$$

Here s and t are the usual Mandelstam variables for the process

$$m_1 + m_2 \to Q + m_X \, .$$

Then $\rho_{11}^{t(\mathrm{GJ})}$ is related to ρ_{11}^s by

$$\rho_{11}^t = \frac{(1-2\rho_{11}^s)}{2}\sin^2\chi + \frac{\rho_{11}^s(1+\cos^2\chi)}{2}\,, \tag{7}$$

assuming that $\rho_{1,-1}$ and $\rho_{1,0}$ are zero.

Next we consider various quark-parton and gluon models. Throughout this work we assume, for simplicity, that the quark-partons are on the mass shell. Some remarks about this assumption are made in Sec. IX.

In the following, we will write equations explicitly for the case where the μ pair is produced through an intermediate heavy photon. Equations for the case of ψ can be readily obtained by using a Breit-Wigner form instead of $1/Q^2$.

II. DRELL-YAN (DY) MECHANISM WITH LIGHT (NONCHARMED) QUARKS AND POINT COUPLINGS

The Drell-Yan mechanism[4] is the most popular model for this process. Here, as shown in Fig. 1, a quark (parton) from one hadron annihilates an antiquark (antiparton) from the other hadron. If neither beam nor target contains a valence antiquark, the antiquark is supposed to come from the sea of $q\bar{q}$ pairs. It is often stated that the distribution is of the form $1+\cos^2\theta$. Here we consider some more exact expressions. Let k_1, k_2, p_1, p_2, and Q be the four-momenta of the q-$\bar{q}$ and μ^+-μ^- pairs and the intermediate heavy photon or ψ particle. Then the matrix element for pointlike (γ_μ) couplings is given by

$$M = \bar{v}(k_2)\gamma^\mu u(k_1)\frac{1}{Q^2}\,\bar{u}(p_1)\gamma_\mu v(p_2)\,. \tag{8}$$

Squaring and summing over the spins, one finds

$$|M|^2 \propto 1 + \frac{4m^2}{Q^2} + \beta^2\cos^2\theta\,, \tag{9}$$

where

$$\beta^2 = 1 - 4m^2/Q^2 \tag{10}$$

is the square of the velocity of the quarks in the c.m. system. θ is the angle between $\vec{k}_1$ and $\vec{p}_1$ in the same system. We have neglected the lepton masses but keep the quark mass (m) nonzero. Setting $m=0$, one gets the well-known Drell-Yan result $1+\cos^2\theta$. So for very light quarks one should have this distribution for relatively large values of Q^2. This should be true for both the heavy photon and the ψ meson if the production mechanism is the Drell-Yan process. For very low values of Q^2, the value of α will be sensitive to the quark mass m. If the magnetic moment of the quark is assumed to be the same as that of the proton, a typical value of $m=0.336$ GeV is obtained. In the $\rho(\omega)$ mass region this gives

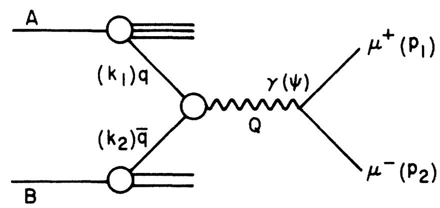

FIG. 1. Drell-Yan model.

$$\alpha = \frac{1-4m^2/Q^2}{1+4m^2/Q^2} = 0.14.$$

On the other hand, if $m=m_{\rho/2}$, $\alpha=0$. So for light quarks the distribution should be essentially isotropic. In fact, experimentally it has been found that, at Fermilab energies, for $\rho(\omega)$ production the distribution is close to being isotropic.[5] We note in passing that, for pion exchange (for ρ production) and ρ exchange (for ω production), the distribution should be $\sin^2\theta$ and $1+\cos^2\theta$, respectively. At low energies (11.2-GeV/c π beam) results consistent with one-pion exchange + absorption were found.[3] But at higher energies the quark models presumably should work better.

III. DY MODEL WITH HEAVY QUARKS

As we noted above, for $m \approx \frac{1}{2}Q$, $\alpha=\beta^2\approx 0$. Now the mass of a charmed quark is believed to be about half the mass of the ψ meson. So, in such a case, one would automatically get an isotropic distribution in ψ production. Some authors have already considered the Q^2, $Q_\parallel$, and Q_T distribution in such models.[6] The number of charmed-quark $c\bar{c}$ pairs in the sea associated with nucleons and pions is presumably very small. But this is compensated by a large coupling of $c\bar{c}$ to ψ. In the $q\bar{q}$ model, that coupling is forbidden by the Zweig rule and hence it is small. In the $c\bar{c}$ model one has to arrange carefully so that excessive numbers of charmed particles are not produced (in agreement with experiments) and there are some difficulties. But, on the whole, at present the model is not ruled out. The isotropic angular distribution will give strong support to such a model if other predicted distributions agree with experiment.

IV. DY MODEL WITH STRUCTURE IN THE $q\bar{q}$ COUPLINGS TO γ AND ψ

The scaling observed in electroproduction puts strong restrictions on the structure one could allow in the $q\bar{q}\,\gamma$ vertex. However, as suggested by Drell and Chanowitz[7] and West,[8] one can introduce a $\sigma_{\mu\nu}$ term with a form factor to describe the anomalous magnetic moment of the quark if it exists. It can be argued that even if the fundamental quark vertex is pointlike (γ_μ type), renormalization effects due to exchanges of gluons could produce a $\sigma_{\mu\nu}$ term analogous to the anomalous magnetic moment term for the elementary electron.[8] Now with two form factors $F_1(Q^2)$ and $F_2(Q^2)$ Eq. (8) becomes

$$M=\bar{v}(k_2)[\gamma^\mu F_1(Q^2)+i\sigma^{\mu\nu}Q_\nu\mu_Q F_2(Q^2)]u(k_1)\times(1/Q^2)\bar{u}(p_1)\gamma_\mu v(p_2)\,. \tag{11}$$

The leptonic vertex has been kept γ_μ type. For the $q\bar{q}\,\gamma$ case μ_Q is the anomalous magnetic moment of the quark. Squaring and spin-averaging lead to (for small m and large Q^2)

$$|M|^2\propto F_1{}^2(Q^2)(1+\beta^2\cos^2\theta)+F_2{}^2(Q^2)\mu_Q{}^2(1-\beta^2\cos^2\theta)\,. \tag{12}$$

This gives the distribution function

$$W(\theta)\propto 1+\frac{\beta^2(1-\mu_Q{}^2Q^2)}{(1+\mu_Q{}^2Q^2)}\cos^2\theta\,, \tag{13}$$

where we have set $F_2(Q^2)/F_1(Q^2)=1$. West[8] found that values of μ_Q of 0.1 to 0.2 GeV^{-1} are consistent with electroproduction, e^+e^- annihilation data, and also with the quark-model assumption that the quark magnetic moment be the same as that of the proton (with quark mass $\approx\frac{1}{3}$ proton mass). Taking $\beta=1$, $Q=3.1$ GeV, this gives $\alpha=0.83$ and 0.44 for $\mu_Q=0.1$ and 0.2 GeV^{-1}, respectively. $\alpha=0$ would require $\mu_Q=0.32$ GeV^{-1}. Also, the value of α will decrease as Q^2 increases. As Q^2 increases the distribution changes from $1+\cos^2\theta$ to 1 and then to $1-\cos^2\theta$. This is for the case of heavy photons. In principle the hadronic $q\bar{q}\psi$ vertex could be completely different from the $q\bar{q}\gamma$ vertex, and it is possible that the $q\bar{q}\gamma$ vertex may be pointlike and that the $q\bar{q}\psi$ vertex has a structure. In the latter case there is essentially no restriction on the value of μ_Q. It seems that such models may be arranged so that they are not in contradiction to existing experiments and could even be forced on us by future experiments, although the elegance and simplicity of the parton picture will be somewhat lost.

V. DY MODEL WITH SPIN-0 PARTONS (OR QUARKS?)

This is not a very attractive hypothesis, but we consider it for the sake of completeness. The relevant matrix element is given by

$$M=\bar{u}(p_1)(\not{k}_1-\not{k}_2)v(p_2)\,. \tag{14}$$

Then spin-averaging leads to

$$|M|^2\propto 1-\frac{4m^2}{Q^2}-\beta^2\cos^2\theta\,. \tag{15}$$

For light quarks, this gives the $1-\cos^2\theta$ distribution, as is well known. It should be noted that electroproduction data do favor spin-$\frac{1}{2}$ partons.

In all four DY models, the distribution has been calculated with respect to the direction of the quark-antiquark three-momenta in the c.m. system of the dimuon. The relationship of this axis to the beam axis will depend on the dynamical details of the model (i.e., probability functions for the quarks to have different momenta). We consider two simple cases in view of the crudeness of the present data which are given with respect to the incident beam axis and are averaged over all $Q_\parallel$ and Q_T values of the dimuon. In the first case we assume that the quark-antiquark have essentially only longitudinal momenta; the transverse momenta are extremely small. Then the values of α are approximately the same as the ones given above, even with respect to the beam axis. In the second case, we assume that the q-$\bar{q}$ are essentially moving in the same direction as the heavy photon (or ψ or dimuon) they produce. Then we use Eqs. (3), (4), (5), (6), and (7) to obtain α relative to the beam axis. The results are averaged over $Q_\parallel$ and Q_T to obtain $\bar{\alpha}$. As suggested by the experimental fits,[1] the weight factor $(1-X_F)^4e^{-2Q_T}$ ($X_F=2Q_\parallel/\sqrt{s}$) is used. It turns out that in this case the value of α does change substantially by such averaging. For example, $\alpha=1$ leads to $\bar{\alpha}=0.36$. On the other hand, however, it seems unlikely that any amount of averaging can lead from $\alpha=1$ to $\bar{\alpha}\approx 0$.

So far we have considered quark-antiquark annihilation models. Next in order of complexity is the constituent-interchange model of Blankenbecler, Brodsky, and Gunion,[9] in which a quark from one hadron scatters on a hadron (meson) emitted by the other hadron and produces a photon or a vector meson and anything else.

VI. CONSTITUENT INTERCHANGE MODEL (CIM)

This model has been quite successful in the large-Q_T region, but application to the entire Q_T range is only currently being made.[2] For the sake of definiteness we assume that, as in Fig. 2, the dominating hard-scattering subprocess is the one in which a quark is scattered by a π meson to produce a quark and a dimuon. The final quark then combines with the rest of the hadronic states which

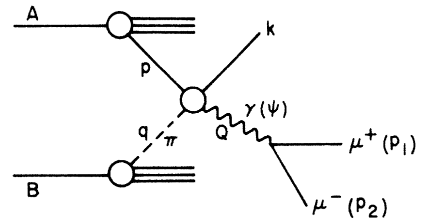

FIG. 2. Constituent-interchange model (CIM).

are not detected. This process is analogous to $\pi N \to \gamma N$ or ρN. We consider the dominant contribution to be given by an electric Born model. Our treatment will be somewhat similar to the one given by Sachrajda and Blankenbecler, who consider an s-channel pole term for this process.[10] Let p, q, k, and Q be the four-momenta of the initial quark, initial meson, final quark, and dimuon, respectively. Then $s'=(p+q)^2$, $t'=(p-k)^2$, $u'=(p-Q)^2$. In the gauge-invariant electric Born model there are two quark (s' and u') pole terms and one pion (t') pole. (See Fig. 3.) The matrix element is given by

$$T(s',t',u')=eg\bar{u}(k)\gamma_5\mathcal{T}u(p)\,, \tag{16}$$

where

$$\mathcal{T}=A\frac{2\epsilon\cdot k+(\gamma\cdot\epsilon)(\gamma\cdot Q)}{s'-m_q^2}+B\frac{2\epsilon\cdot p-(\gamma\cdot Q)(\gamma\cdot\epsilon)}{u'-m_q^2}+C\frac{2\epsilon\cdot q-\epsilon\cdot k}{t'-m_\pi^2}\,;$$

m_q and m_π are the masses of the quark and the meson, respectively. The values of the constants A,B,C depend on the charge states under consideration. e is the proton charge and g is the $qq\pi$ coupling constant. The values of (A,B,C) are easily found to be (in this order)

$$\pi^0 u\to\gamma u:\ (\tfrac{2}{3},\tfrac{2}{3},0)\,,$$
$$\pi^0 d\to\gamma d:\ (-\tfrac{1}{3},-\tfrac{1}{3},0)\,,$$
$$\pi^+ d\to\gamma u:\ (\tfrac{2}{3},-\tfrac{1}{3},1)\sqrt{2}\,,$$
$$\pi^- u\to\gamma d:\ (\tfrac{1}{3},-\tfrac{2}{3},1)\sqrt{2}\,.$$

u and d refer to the up and down quarks with charges $\frac{2}{3}$ and $-\frac{1}{3}$, respectively. From Eq. (16) one can read off the Ball amplitudes for the process and obtain the s-channel helicity amplitudes $H^{\lambda}_{\lambda_f,\lambda_i}(s',t')$. λ, λ_f, and λ_i are the helicities of the off-shell photon (or ψ meson) and final and initial quarks, respectively.[11] The nonzero $H^{\lambda}_{\lambda_f,\lambda_i}$ are as follows:

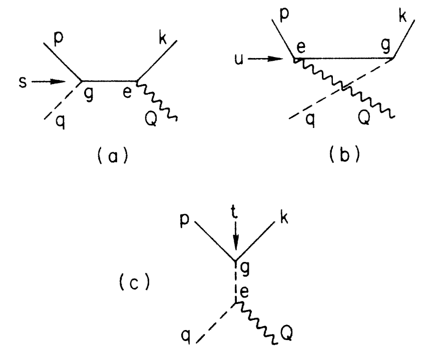

FIG. 3. (a) s-channel quark pole for CIM. (b) u-channel quark pole for CIM. (c) t-channel meson pole for CIM.

$$H^0_{+-}(s',t')=-\frac{Q(-t')^{1/2}}{m_q}\left[\frac{A}{2(s'-m_q^2)}+\frac{C}{2(t'-m_\pi^2)}\right], \tag{17}$$

$$H^1_{+-}(s',t')=-\frac{1}{\sqrt{2}\,m_q}\left[\frac{As'}{2(s'-m_q^2)}-\frac{Bs'}{2(u'-m_q^2)}-\frac{At'}{4(s'-m_q^2)}-\frac{Bt'}{4(u'-m_q^2)}-\frac{Ct'}{t'-m_\pi^2}\right], \tag{18}$$

$$H^{-1}_{+-}(s',t')=-\frac{t'}{\sqrt{2}\,m_q}\left[\frac{A}{4(s'-m_q^2)}+\frac{B}{4(u'-m_q^2)}+\frac{C}{t'-m_\pi^2}\right]. \tag{19}$$

By squaring these, the s-channel density matrix elements are obtained. These will be useful in the future. However, present data give a distribution with respect to the beam axis (Gottfried-Jackson frame). Hence we obtain the amplitudes G by crossing,

$$G^m_{\lambda_f,\lambda_i}(s',t')=\sum_\lambda d_{m\lambda}{}^1(\chi)H^{\lambda}_{\lambda_f,\lambda_i}(s',t')\,. \tag{20}$$

$d_{m\lambda}{}^1(\chi)$ is the usual d function of the γ (ψ) crossing angle χ given by Eq. (4) with the replacement

$$m_1\to m_\pi\,,\quad m_2\to m_q,\ \text{and}\ m_X\to m_q\,. \tag{21}$$

In terms of G's the GJ (t-channel) density matrix elements are given by

$$\rho_{00}(s',t')=|G_0|^2/(|G_0|^2+|G_1|^2+|G_{-1}|^2)\,,$$
$$\rho_{11}(s',t')=(|G_1|^2+|G_{-1}|^2)/2(|G_0|^2+|G_1|^2+|G_{-1}|^2)\,, \tag{22}$$

etc. Then the angular distribution for γ or $\psi\to\mu^+\mu^-$ is obtained from Eq. (2). Now ρ_{11} is a function of s' and t'. To compare with experiment we have to average over these variables. In the CIM the basic equation for the total inclusive cross section of γ (or ψ) is given by

$$Q_0 \frac{d^3\sigma}{d^3Q} = \int_0^1 dx_1 \int_0^1 dx_2\, G_{q/H}(x_1) G_{\pi/H}(x_2)\theta(s' - s'_{\text{th}}) \times Q_0 \frac{d^3\sigma}{d^3Q}(p + q \to k + Q). \quad (23)$$

We have neglected the transverse-momentum distributions of the incoming quark and meson. $G_{q/H}$ and $G_{\pi/H}$ are the probabilities of finding the quark and the π meson in the two hadrons with fractions x_1 and x_2 of the longitudinal momenta. Then $s' = x_1 x_2 s$, where s is the square of the total c.m. energy of the two incoming hadrons. In the quark-meson c.m. system we have

$$t' = t_{\min} - 2|\vec{q}|\,|\vec{Q}|(1 - \cos\theta_c), \quad (24)$$

$$u' = 2m_q^2 + m_\pi^2 + Q^2 - s' - t', \quad (25)$$

where $|\vec{q}|$, $|\vec{Q}|$, and θ_c are, respectively, the incoming-quark three-momentum, the $\gamma(\psi)$ three-momentum, and the scattering angle in this system. To produce experimentally observed rapid falloff in Q_T, we multiply the integrand by a phenomenological factor e^{-2Q_T} as before.[1] The G functions are determined from electroproduction and neutrino data. For example, for the up quark in the proton, we have

$$G_{q/p}(x) = \frac{0.2(1-x)^7}{x} + \frac{1.89(1-x)^7}{\sqrt{x}} + \begin{cases} 90.2x^{3/2}e^{-7.5x}, & x \le 0.35 \\ 5(1-x)^3, & x \ge 0.35. \end{cases} \quad (26)$$

For $G_{\pi/p}$ one can consider various cases. At one extreme we can take it to be the same as the sea-quark distribution in proton, i.e.,

$$G_{\pi/p}(x) = 0.2(1-x)^5/x. \quad (27)$$

At the other extreme it can be taken to be the same as the one for the up quark in the proton. It should be emphasized that since we are interested only in the averaged angular distribution, and not the actual magnitude of the cross section, such differences are not crucial here. Actually we have verified numerically that this is so. The average values of α obtained using Eq. (23) are given below with corresponding values of (A, B, C) for different charged states

$$\begin{aligned} &\alpha = 0.55, \quad (\tfrac{2}{3}, \tfrac{2}{3}, 0), \\ &\alpha = 0.36, \quad (\tfrac{2}{3}, -\tfrac{1}{3}, 1)\sqrt{2}, \\ &\alpha = 0.48, \quad (\tfrac{1}{3}, -\tfrac{2}{3}, 1)\sqrt{2}. \end{aligned} \quad (28)$$

It is also interesting to note that part of the u-channel diagram is the Drell-Yan $q\bar{q}$ annihilation process. In fact, it is amusing to note that if we ignore gauge invariance and set s and t pole terms equal to zero the value of α obtained is 0.35, which is extremely close to the Drell-Yan averaged value 0.36.

The above results show the range of variation with different charged states. With a full isospin treatment of the CIM some average results in this range can be expected. So CIM does lead to values of α smaller than 1.

VII. CIM WITH MESON-MESON SCATTERING SUBPROCESS

In this model, which was considered by Chu and Koplik,[12] a meson is emitted by each hadron and they produce a heavy photon in analogy with the DY process. We do not consider this mechanism in detail but merely point out that if the mesons are spinless, the angular distribution will be similar to the case (Sec. IV) considered above. Next we consider gluon models.

VIII. GLUON MODELS

Such a model for ψ production has been considered by Ellis, Einhorn, and Quigg[13] and Carlson and Suaya.[14] In this model, as shown in Fig. 4, one gluon is emitted by each of the hadrons to produce an even charge conjugation state χ. This in turn decays into ψ and γ. ψ can then decay into muons. These authors did not consider implications for the angular distributions of the muons. It is clear that, if the intermediate state χ has a spin 0 (as one of the χ states should have), the decay products of ψ cannot have any correlation with the gluon axis or incident hadron axis. Thus an isotropic distribution will be obtained. If the intermediate state χ has a spin different from 0 (e.g., 2), the muons will have some angular correlation with the gluon axis and hence the beam axis. On the basis of the charmonium model it can be argued that the spin-2 intermediate state will be relatively suppressed as compared to the spin-0 intermediate state. Thus, if the value of $\alpha \approx 0$ is confirmed, it will lend support to the model with the spin-0 χ intermediate state. Of course the

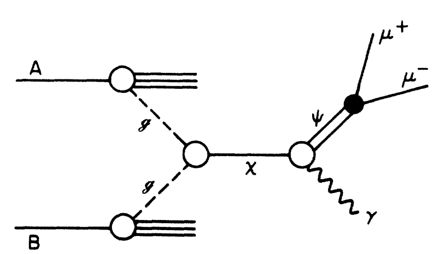

FIG. 4. Gluon model with intermediate state χ.

overall predictive power of the gluon models seems to be somewhat less than the quark-parton models.

IX. CONCLUDING REMARKS

In the above sections we have discussed various theoretical models for the angular distribution of muons produced in hadronic collisions. An important assumption was that the constituents act like on-mass-shell particles when they scatter off each other. This is the basis of the present quark-parton phenomenology. The structure functions (e.g., G functions used in the text) are assumed to damp the off-shell behavior very strongly (see, for example, Refs. 9 and 10). For off-shell constituents, the problem of violation of gauge invariance becomes a very serious one. In addition, the off-shell extrapolation is very ambiguous. Thus, for phenomenological purposes it is convenient to regard the constituents as on-mass-shell particles with some effective masses. These may or may not be the same as the spectroscopic masses. In fact, in Ref. 2, in order to fit Q^2, $Q_{\parallel}$, and Q_T distributions we have to choose $m_{\text{quark}} \simeq 1$ GeV. In the absence of a relativistic theory for quark binding even the spectroscopic masses are uncertain, although m_{quark} is believed to be about $\frac{1}{3}$ (nucleon mass) for noncharmed and nonstrange quarks and about $\frac{1}{2}$ (ψ mass) for the charmed quarks. In any case there is an unavoidable ambiguity about the precise value of the quark masses. Of course, the hope is that quarks which can be regarded as light or heavy in spectroscopy remain effectively light or heavy respectively in scattering processes also. In various models (especially the Drell-Yan model) an important ratio determining the angular distribution is m_{quark}^2/Q^2. In the off-shell case this can be roughly replaced by k^2/Q^2 (k = four-momentum of the quark). If k^2 is not too different from $m_{\text{effective}}^2$ or if Q^2 is much larger than either of them, the results will not change very much. A careful angular distribution analysis for various Q^2 and incident beam energies will shed considerable light on such effective masses involved in the models.

Different models discussed here differ considerably in their predictions. Hence in the future, good angular distribution data in both helicity and Gottfried-Jackson frames in different Q^2, $Q_{\parallel}$, and Q_T bins will give critical tests for various models. It will be also interesting to see if the angular distributions of the muons depend on the nature of the incident beam. Simultaneous analyses of all these distributions (density matrices, Q^2, $Q_{\parallel}$, Q_T, etc.) will give constraints on various models and will have important consequences for understanding the role of the constituents in the scattering of hadrons.

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