Muon-number-nonconservation effects in a gauge theory with $V + A$ currents and heavy neutral leptons

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Since in gauge theories eigenstates of weak interactions are in general not mass eigenstates, we would not expect flavor conservation. In particular this should also hold for the leptonic flavors: muon numbers and electron numbers, etc. The apparent conservation of muon number in the standard $V - A$ theory should be interpreted as reflecting the fact that neutrino masses (if not identically zero) are almost degenerate when viewed on the normal mass scale. In theories containing $V + A$ currents, the right-handed muon and electrons are expected to couple to intermixing heavy leptons in the GeV range. In such theories muon-number-violation effects will be dramatically larger. However, when constructing new models of leptons one should be mindful that flavor-changing neutral-current processes such as $\mu \to e\gamma$ and $\mu \to ee\bar{e}$ are suppressed experimentally. This indicates the need for a "leptonic Glashow-Iliopoulos-Maiani cancellation mechanism." We have proposed a way to incorporate these features in an $SU_2 \times U_1$ gauge theory. Basically it involves the addition to the standard Weinberg-Salam theory of right-handed doublets with the electron and muon coupled to orthogonal "heavy neutrinos." This leads to an electronic neutral current which is purely vector and its attendant suppression of parity-violation effects in high-Z atoms. Muon-number-nonconservation effects involving only familiar particles are higher-order weak processes and are naturally of the order G_F^2. In this paper we give details of our calculations of $\mu \to e\gamma$, $\mu \to ee\bar{e}$, $K_L \to e\bar{\mu}$, $K \to \pi e\mu$, and μe conversion in a nucleus, etc. In order to have a "natural" theory, we have incorporated recent suggestions made by Bjorken, Lane, and Weinberg about the Higgs structure for such a model. This modification increases the rate for $\mu \to e\gamma$ by a factor of 25 but does not materially affect other processes. For a heavy-lepton mass-difference and mixing-angle combination of $\sin\phi\cos\phi[m(N_1)^2 - m(N_2)^2] \simeq 1$ GeV^2, the branching ratio for $\mu \to e\gamma$ is 4×10^{-10}; that for $\mu \to ee\bar{e}$ is around 10^{-11}; the μe conversion rate can be as large as 10^{-9} when compared to the ordinary muon capture in the nucleus. If there is a heavy quark b coupled to the u quark through the $V + A$ current, this conversion rate will be decreased by a factor of 30. The rates for the muon-number-nonconserving K decay are more sensitive to the relative lepton masses. For example the branching ratio of $K_L \to e\bar{\mu}$ is about 10^{-10} for $m(N_1)/m(N_2) \simeq 4$, and a 1.8-GeV charmed quark.

I. INTRODUCTION

It is becoming increasingly plausible that modern gauge theories provide the correct theoretical framework to understand particle interactions. One of the attractive features of such a possibility is that there is a natural link between dynamics and the various approximate symmetries observed in nature. Spontaneous symmetry breaking in gauge theories of weak and electromagnetic interactions generally brings about fermion masses that are not diagonal with respect to the weak eigenstates. This brings about mixing angles among gauge couplings of physical states. We have the Cabibbo angle, and strangeness is not conserved in weak interactions. Furthermore, the off-diagonal fermion masses specify the size of the induced higher-order neutral-current couplings: The Glashow-Iliopoulos-Maiani[1] (GIM) cancellation mechanism works in such a way that the strangeness-changing ($s \leftrightarrow d$) neutral-current effects are controlled by the quark masses: $\sin\theta_C \cos\theta_C(m_c^2 - m_u^2)$. In this paper we shall discuss an analogous symmetry for the leptons and the possibility of its being broken: the separate conservations of the muon and electron lepton numbers. We have already presented our principal results in earlier communications.[2,3] Here we provide details of our calculations on the various induced muon-number-changing ($\mu \leftrightarrow e$) electromagnetic and neutral-current effects.[4-6]

In the following section we offer a general theoretical discussion of muon-number-changing currents in gauge theories. Readers familiar with such broad questions may wish to skip over this section and proceed directly to Sec. III where our model is stated, and to specific calculations in Secs. IV and V. In Sec. IV leptonic processes $\mu \to e\gamma$, $\mu \to ee\bar{e}$ and muonium-antimuonium transitions are discussed. Section V contains calculations on semileptonic processes: μe conversion in a nucleus, $K_L \to e\bar{\mu}$, and $K \to \pi e\bar{\mu}$. In the concluding section, we summarize our results, together with existing experimental limits, in a table. In the Appendix various details of our computations are provided. Throughout this paper we shall restrict our considerations to muon-number-nonconservation effects mediated by gauge bosons. The alternative mechanism via Higgs exchanges has been proposed by Bjorken and Weinberg.[7]

II. GENERAL FEATURES OF MUON-NUMBER-CHANGING GAUGE COUPLINGS

We shall first briefly review some of the general features about flavor-number conservation in the standard $V-A$ theory. Then we will discuss what we can expect for weak-interaction theories with both $V-A$ and $V+A$ currents.

A. The standard model

Consider the standard $SU_2 \times U_1$ gauge theory of electromagnetic and weak interactions[8]

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \quad e_R , \; \mu_R \tag{2.1}$$

and

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L , \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L , \quad u_R , \; d_R , \; c_R , \; s_R \tag{2.2}$$

where the weak eigenstates d' and s' are linear combinations of the mass eigenstates d and s:

$$\begin{aligned} d' &= \cos\theta_C \, d + \sin\theta_C s , \\ s' &= -\sin\theta_C d + \cos\theta_C s . \end{aligned} \tag{2.3}$$

The Cabibbo angle θ_C represents a rotation between these two sets of orthogonal states. In gauge theory, there is, of course, a natural explanation for the necessity of such a rotation. The mass terms of the fermions in such theories arise from their couplings to the Higgs scalars which develop vacuum expectation values. If we allow for the most general, gauge-invariant, Yukawa couplings then we would expect both the diagonal $\bar{d}'d'$ and $\bar{s}'s'$ as well as the off-diagonal $\bar{d}'s'$ and $\bar{s}'d'$ terms. The relation in Eq. (2.3) reflects the need of a unitary transformation which diagonalizes the mass matrix. Thus, except for accidental symmetries, we would anticipate mixings among states of the same charge and helicity.

Another outstanding feature of the quark gauge couplings is the GIM mechanism[1]: Absence of the direct strangeness-changing neutral-current coupling and the induced couplings is suppressed by an order of $G_F \sin\theta_C \cos\theta_C (m_c^2 - m_u^2)$.[9] Thus the nonconservation of the strange-quark number in these processes is directly controlled by the quark mass matrix.

In the standard theory (2.1) with massless neutrinos, lepton-number flavors are conserved. This is brought about by the neutrino mass degeneracy. There the mass eigenstates can be chosen to be the eigenstates of weak interactions; so there does not exist a physically meaningful angle corresponding to θ_C. However, we may assume that neutrinos are not strictly massless and they possess different masses. From our previous discussion we would indeed expect a nontrivial mixing angle between the neutrino states. However, the physical consequence of such a mixing will in practice still be unobservable. Since we also have a GIM mechanism here (after all, the fourth quark was invented on the basis of lepton-quark analogy),[10] all the $\mu \leftrightarrow e$ transitions must be proportional to neutrino mass differences. (For example, with $\Delta m_\mu^2 \lesssim 25$ eV^2,[11] the branching ratio for $\mu \to e\gamma$ is less than 10^{-43}.[12]) In other words, on a typical mass scale neutrinos for all practical intents and purposes are massless, hence degenerate. In this limit we can always "rotate the mixings away," leading to a muon-number-conserving gauge interaction.

B. Gauge theories with both $V-A$ and $V+A$ currents

Our main concern in this paper is to discuss muon-number nonconservation in weak-interaction theories with heavy leptons and right-handed currents. These two new features are, of course, intimately connected. It has been suggested that new elementary fermions exist and that some of their weak couplings are $V+A$ in nature.[13] The observed $V-A$ character is supposed to break down when new degrees of freedom are excited at high energies. There are indications that right-handed currents are already playing an important role in present-day high-energy neutrino experiments.[14] They presumably involve quarks beyond charm. Thus it seems to us entirely plausible that the true weak-interaction theory involves leptonic $V+A$ currents as well. The right-handed muon and electron are expected to couple to intermixing heavy leptons in the GeV range.[15] In such theories muon-number-violation effects will be dramatically larger. As already reported in Ref. 2, we found that a reasonable lepton mass difference would lead us to predict branching ratios for $\mu \to e\gamma$ and $K_L \to e\bar{\mu}$ not extremely small when compared to their present experimental limits.

III. A MODEL OF LEPTONS

In constructing new models of leptons with right-handed muons and electrons belonging to nontrivial representations of the weak isospin group, one should be mindful of the need of a "natural" GIM mechansim[16] for the leptons as well. By this we mean specifically the following[17]:

(1) The absence of a direct $\bar{\mu}(a+b\gamma_5)\gamma_\lambda e$ neutral

current. Otherwise the model will run afoul of the stringent experimental limits on $\mu \to ee\bar{e}$ (Ref. 18) and μe conversion in a nucleus.[19] This requires that the third components of weak isospin for the right-handed electron and muon have the same value: $t_3(e_R) = t_3(\mu_R)$.[16]

(2) The cancellation of the leading one-loop contributions to μe transitions. This cancellation should take place for both the left-left (LL) and right-right (RR) type, as well as for the (LR) and (RL) type. The conditions imposed by this requirement on the theory may be stated most simply by an examination of diagrams where the intermediate fermions are labeled by the weak eigenstates. We require the leading terms of diagrams such as shown in Fig. 1 to vanish. These cancellations are required in order to maintain the induced muon-number-changing effects at the G_F^2 level. That the leading helicity-nonflip diagrams such as Fig. 1(a) should vanish can be trivially satisfied: All it requires is that electron and muon (for a given helicity) be coupled to independent orthogonal states [hence one of the gauge couplings in Fig. 1(a) is necessarily zero]. That diagrams of the type shown in Fig. 1(b) should vanish requires in this graph either

(2a) the absence of one of the gauge couplings or

(2b) the absence of the appropriate off-diagonal mass term [shown as a cross in Fig. 1(b)].

Of course, the original standard model with only $V-A$ currents, and extensions thereof, already satisfy these requirements. They become significant statements for models where e_R and μ_R belong to nontrivial representations of the gauge group.[20]

We have proposed a model of leptons[2,21]

$$\begin{pmatrix} n_e \\ e \end{pmatrix}_L , \quad \begin{pmatrix} n_e \\ e \end{pmatrix}_R , \quad \begin{pmatrix} n_\mu \\ \mu \end{pmatrix}_L , \quad \begin{pmatrix} n_\mu \\ \mu \end{pmatrix}_R \quad \ldots , \tag{3.1}$$

where we have already taken the mass matrix for charged leptons to be diagonal. The weak eigenstates $(n_e)_{L,R}$ and $(n_\mu)_{L,R}$ are then expected to be linear combinations of their respective mass eigenstates. In particular we have for the heavy neutral leptons

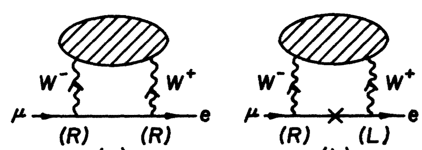

FIG. 1. Leading loop contributions to μe transition amplitudes. (The shaded blob stands for any relevant interactions between the two W lines.) Type (a) involves no helicity flip by mass-insertion interactions; type (b) involves one mass insertion to flip the helicity once. The intermediate fermions are supposed to be weak eigenstates.

$$\begin{aligned} (n_e)_R &= (\cos\phi N_1 + \sin\phi N_2)_R , \\ (n_\mu)_R &= (-\sin\phi N_1 + \cos\phi N_2)_R , \end{aligned} \tag{3.2}$$

where $N_{1,2}$ are the mass eigenstates with masses $m_{1,2}$ in the GeV range. Clearly this model satisfies the GIM requirement (1) since $t_3(e_R) = t_3(\mu_R) = -\frac{1}{2}$. We have placed the right-handed electrons and muon in weak isodoublets (instead of, say, in triplets.) This is mainly for aesthetic reasons, but it also has the implication that the electronic neutral current is purely vector:

$$J_\lambda^Z(e) = (-\tfrac{1}{2} + \sin^2\theta_W)\bar{e}\gamma_\lambda e \tag{3.3}$$

(θ_W is the Weinberg angle). This is compatible with the experimental report that parity violation is small in high-Z atoms.[22]

It is clear that in this model the requirement (2a) of a natural GIM mechanism for the (LR) transition cannot be satisfied. However, taking into account a crucial observation made by Bjorken, Lane, and Weinberg,[23] one notices that the alternative situation (2b) can be arranged in a rather simple manner. Condition (2b) states that the mass terms ($\bar{n}_e n_\mu$ + H.c.) should be absent. Since the product of two doublets can either be a singlet or a triplet, these off-diagonal terms automatically disappear if there is no Higgs triplet which develops vacuum expectation values. This comes about because there is now only one type of source (vacuum expectation values of singlet Higgs and/or gauge-invariant bare mass terms) which can give rise to terms bilinear in e and μ as well as in n_e and n_μ. When we diagonalize the ($e-\mu$) mass matrix (which we are free to do without loss of generality) we automatically diagonalize the ($n_e - n_\mu$) mass matrix as well. Furthermore, the diagonal elements for both matrices are simply the electron and muon masses. From this Bjorken, Lane, and Weinberg[23,24] further conclude that for our model (3.1) and (3.2) the contents of N_1 and N_2 in $(n_e)_L$ and $(n_\mu)_L$ are uniquely determined. For definiteness, we assume that there are two left-handed singlet neutral leptons n_σ, n_τ and no corresponding right-handed singlets. This way we are guaranteed to have two massless neutrinos after spontaneous symmetry breaking has taken place. The weak eigenstates $(n_e)_L$ and $(n_\mu)_L$ are related to the mass eigenstates $(N_1, N_2, \nu_3, \nu_4)_L$ [see Eqs. (A24) and (A27) in Appendix A]

$$\begin{aligned} (n_e)_L &= \left(\frac{m_e}{m_1}\cos\phi N_1 + \frac{m_e}{m_2}\sin\phi N_2 + U_{e3}\nu_3 + U_{e4}\nu_4\right)_L , \\ (n_\mu)_L &= \left(-\frac{m_\mu}{m_1}\sin\phi N_1 + \frac{m_\mu}{m_2}\cos\phi N_2 + U_{\mu3}\nu_3 + U_{\mu4}\nu_4\right)_L , \end{aligned} \tag{3.4}$$

where U_{ai} are elements of the unitary matrix that diagonalizes the mass matrix [see Eq. (A25)]. Since ν_3 and ν_4 are degenerate neutrino mass eigenstates, we are free to *define*

$$\begin{aligned} U_{e3}\nu_3 + U_{e4}\nu_4 &\equiv a\tilde{\nu}_e\,, \\ U_{\mu 3}\nu_3 + U_{\mu 4}\nu_4 &\equiv b\tilde{\nu}_e + c\tilde{\nu}_\mu\,. \end{aligned} \tag{3.5}$$

The constants a, b, and c can be determined by the two normalization conditions and one orthogonality condition for n_e and n_μ:

$$a = \{1 - m_e{}^2[(\cos\phi/m_1)^2 + (\sin\phi/m)^2]\}^{1/2}\,, \tag{3.6}$$

$$b = m_e m_\mu \sin\phi\cos\phi(m_1{}^2 - m_2{}^2)/(am_1{}^2 m_2{}^2)\,, \tag{3.7}$$

$$c = \{1 - m_\mu{}^2[(\sin\phi/m_1)^2 + (\cos\phi/m_2)^2] - b^2\}^{1/2}\,. \tag{3.8}$$

The presence of a nonvanishing b indicates that the neutrino states as defined in the reaction

$$\begin{aligned} \pi^+ &\to e^+ + \nu_e \\ &\to \mu^+ + \nu_\mu \end{aligned}$$

are not entirely orthogonal. For example, the "muon neutrino" ν_μ which accompanies the muon in the π decay has a small probability ($\sim b^2$) of producing an electron when it scatters off a nucleon. Treiman, Wilczek, and Zee[6] termed this phenomenon "neutrino sharing."

The mixing of the heavy leptons $N_{1,2}$ in n_{eL} and $n_{\mu L}$ is small because heavy leptons cannot be lighter than the kaons. For simplicity let us set $m_e = 0$, then $a = 1$, $b = 0$ in Eqs. (3.6) and (3.7). The violation of μ_e (and hadron-lepton) universality is basically measured by the amount c deviates from unity. From Eq. (3.8) it is clear that this deviation is

$$\Delta \simeq \tfrac{1}{2}\left[(\sin\phi m_\mu/m_1)^2 + (\cos\phi\, m_\mu/m_2)^2\right]. \tag{3.9}$$

The experimental bounds, after radiative corrections, on Δ are not much better than half a percent.[25] We note that this requirement can be easily satisifed for $m_{1,2} \gtrsim 1$ GeV.

An inevitable consequence of our model is the existence of at least two heavy neutral leptons. In this paper we shall not discuss the production and decay properties of these particles. Interesting observations on these questions have already been made by other authors.[26] In connection with our discussion of muon-number nonconservation, we make the obvious remark that once these heavy leptons N_i are produced, we can observe spectacular muon-number and electron-number violations in their decay modes: $N_i \to \mu\pi$, $e\pi$, $\mu\bar{\mu}\nu$, $e\bar{e}\nu$, $e\bar{\mu}\nu$, etc.

IV. LEPTONIC PROCESSES

In this section we discuss the muon-number-nonconserving leptonic processes: $\mu \to e\gamma$, $\mu \to ee\bar{e}$, and $(\mu\bar{e}) \to (e\bar{\mu})$, etc. Details of our calculation of the induced $\mu e\gamma$, μeZ vertices and W-exchange box diagrams are given in Secs. B, C, and D of the Appendix. Throughout our computations we shall set the electron mass equal to zero.

A. $\mu \to e\gamma$[27]

For the general case of the electromagnetic current operator between states of a muon and an electron, the matrix element is of the form

$$\begin{aligned} T_\lambda &= \langle e(p')\,|J_\lambda(0)\,|\,\mu(p)\rangle \\ &= \bar{u}_e(p')[(f_M + f_M{}^5\gamma_5)im_\mu\sigma_{\lambda\nu}q^\nu \\ &\qquad + (f_E + f_E{}^5\gamma_5)(\gamma_\lambda q^2 - q_\lambda\gamma\cdot q)]u_\mu(p)\,, \end{aligned} \tag{4.1}$$

where the transition form factors f_M, $f_M{}^5$, f_E, and $f_E{}^5$ are functions of q^2, with $q = p - p'$. This structure for T_λ is dictated by the requirement of current conservation, $T_\lambda q^\lambda = 0$. The most general form for the on-shell ($q^2 = 0$) $\mu \to e\gamma$ amplitude is then given by

$$\begin{aligned} \mathfrak{M}(\mu \to e\gamma) &= T_\lambda\epsilon^\lambda \\ &= \bar{u}_e(f_M + f_M{}^5\gamma_5)im_\mu\sigma_{\lambda\nu}q^\nu u_\mu\epsilon^\lambda\,, \end{aligned} \tag{4.2}$$

where ϵ^λ is the photon polarization vector, $\epsilon^\lambda q_\lambda = 0$. The decay rate is

$$\Gamma(\mu \to e\gamma) = \frac{m_\mu{}^5}{8\pi}\left(|f_M|^2 + |f_M{}^5|^2\right). \tag{4.3}$$

The angular distribution[28] of the outgoing electron with respect to the initial muon polarization vector ($\hat{\mu}$) is

$$I^-(\theta) = (1 + \alpha\cos\theta)\,; \tag{4.4}$$

for $\mu^+ \to e^+\gamma$ it is

$$I^+(\theta) = (1 - \alpha\cos\theta)\,, \tag{4.5}$$

where $\cos\theta = \hat{p}'\cdot\hat{\mu}$ and α is the asymmetry parameter

$$\alpha = \frac{2\,\mathrm{Re}(f_M^* f_M{}^5)}{|f_M|^2 + |f_M{}^5|^2}\,. \tag{4.6}$$

In the model of Eqs. (3.1), (3.2), and (3.4) the decay proceeds through the diagrams shown in Fig. 2 (they are unitary gauge diagrams, hence only physical particles appear). To ensure that our result is (non-Abelian) gauge invariant, we have carried out the calculation in the general ξ gauge and verified that our final result is independent of the parameter ξ.[29] (See Sec. B of the Appendix for details).

The contributions from diagrams (b) and (c) are proportional to γ_λ, hence they must be cancelled by the corresponding contribution from diagram (a). Let us now consider the $\sigma_{\lambda\nu}q^\nu$ contribution coming from diagram (a).

(i) The (LL) diagrams. With each gauge coupling being of the $(V-A)$ type, the leading contribution for each intermediate fermion $\sim eg^2/M_W^2$. This would yield a branching ratio $\Gamma(\mu\to e\gamma)/\Gamma(\mu\to e\nu\bar{\nu}) = O(\alpha/\pi)$. This is just the famous result known since the late 1950's that in a weak-interaction theory with intermediate vector bosons and one neutrino the $\mu\to e\gamma$ rate would be much too large.[30] This result played a pivotal role in the formulation of the two-neutrino hypothesis. Here, these leading (LL) amplitudes are independent of the masses of the intermediate leptons, and the coupling structures are such that they precisely cancel. This satisfies the requirement (2) discussed in Sec. III; that such a cancellation must take place is obvious when the intermediate fermions are labeled by the weak eigenstates: $(n_e)_L$ and $(n_\mu)_L$. The next leading term must be proportional to the square of the intermediate lepton masses. Hence only $(N_{1,2})_L$ contribute. But in this model their coupling to μ and e is such that they also cancel each other.

(ii) The (RR) diagrams. Again there is a GIM cancellation of the leading term. The surviving contribution is calculated in Appendix B [see Eq. (B13)]:[31]

$$f_M(RR) = -f_M{}^5(RR) = \tfrac{1}{2}\kappa \tag{4.7}$$

with

$$\kappa = \frac{e}{16\pi^2}\left(\frac{g^2}{8M_W{}^2}\right)\delta_N\,. \tag{4.8}$$

Ths suppression factor is

$$\delta_N = \sin\phi\cos\phi(m_1{}^2 - m_2{}^2)/M_W{}^2\,; \tag{4.9}$$

m_1 and m_2 are masses of lepton $N_{1,2}$.

(iii) The (RL) diagram. The leading contributions by each intermediate lepton $N_{1,2}$ are proportional to $m_{1,2}$. Still, the couplings of this model are such that they precisely cancel in the sum. One can easily understand the origin of this rather remarkable cancellation if the intermediate states are labeled by the weak eigenstates: the sum of $N_{1,2}$ contributions must be proportional to the $\bar{n}_e n_\mu$ mass term, which is absent in this model. This is of course our condition (2b) discussed in Sec. III. The next leading contribution is negligibly small because the coupling is proportional to the electron mass.

(iv) The (LR) diagrams. The surviving "three-mass-insertion" contribution is of the same order as the (RR) diagrams [see Eq. (B13)]:

$$f_M(LR) = -f_M{}^5(LR) = -3\kappa\,. \tag{4.10}$$

Independent calculations by Bjorken *et al.*[23] agree with this result. This, when combined with the amplitude in Eqs. (4.7) and (4.8), leads to a branching ratio for $\mu\to e\gamma$ as

$$B(\mu\to e\gamma) = \frac{\Gamma(\mu\to e\gamma)}{\Gamma(\mu\to e\bar{\nu}_e\nu_\mu)} = \frac{75}{32}\left(\frac{\alpha}{\pi}\right)\delta_N{}^2\,, \tag{4.11}$$

where we have used $g^2/8M_W{}^2 = G_F/\sqrt{2}$ and $\Gamma(\mu\to e\nu_\mu\bar{\nu}_e) = G_F{}^2 m_\mu{}^5/192\pi^3$. For an off-diagonal mass $\sin\phi\cos\phi(m_1{}^2 - m_2{}^2) = 1$ GeV^2 and an $M_W \simeq 60$ GeV (corresponding to a Weinberg angle $\sin^2\theta_W \simeq \frac{1}{3}$) we have $B(\mu\to e\gamma) \simeq 4\times 10^{-10}$.

If these decays are in fact observed,[32] then the next step should be the measurement of the asymmetry parameter α through Eqs. (4.4) or (4.5). Our model predicts it to be -1 [see Eqs. (4.6), (4.7), and (4.10)] corresponding to an outgoing right-handed electron.

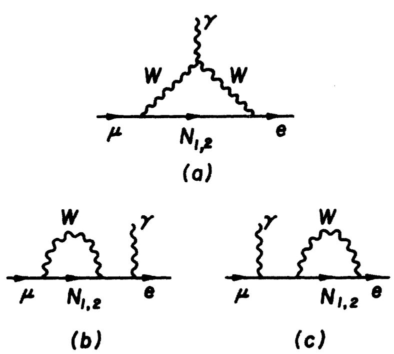

FIG. 2. One-loop contribution to $\mu\to e\gamma$. The intermediate fermions are neutral leptons. The dominant contributions to the final result come from $N_{1,2}$ in diagrams with (LR) and (RR) couplings.

B. $\mu\to ee\bar{e}$

There are basically three classes of one-loop diagrams contributing to the decay $\mu\to ee\bar{e}$. They are shown in Fig. 3.

Treiman, Wilczek, and Zee[6] were the first to emphasize the importance of the ratio $\Gamma(\mu\to 3e)/\Gamma(\mu\to e\gamma)$ for this model. They have already calculated the $\mu\to 3e$ rate by keeping only amplitudes proportional to $\ln(M_W{}^2/m^2)$. In this approximation

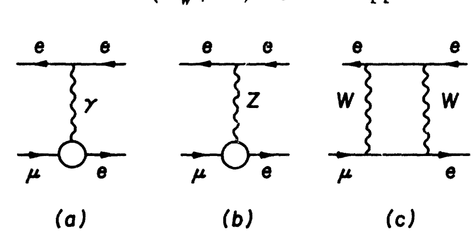

FIG. 3. Three classes of diagrams for $\mu\to 3e$ decay via photon and weak-intermediate-vector-boson exchanges.

only the "weak amplitudes" in Figs. 3(b) and 3(c) contribute, and the calculation is relatively straightforward.[33] Since *a priori* this "leading log approximation" may not be all that reliable ($\ln M_W^2/m^2$ is expected to be about 6 for a reasonable range of heavy-lepton masses), we have carried out a detailed calculation by keeping all the constant terms. In this case diagrams belonging to Fig. 3(a) must be included.

For the momenta assignment of

$$\mu(p) \to e(k_1) + e(k_2) + \bar{e}(k_3),$$

we have the amplitude

$$\mathfrak{M}(\mu \to 3e) = \mathfrak{M}(k_1, k_2) - \mathfrak{M}(k_2, k_1) \tag{4.12}$$

with

$$\mathfrak{M}(k_1, k_2) = \mathfrak{M}^\gamma(k_1, k_2) + \mathfrak{M}^W(k_1, k_2), \tag{4.13}$$

where $\mathfrak{M}^\gamma$ is the one-photon-exchange amplitude and $\mathfrak{M}^W$ is the "weak amplitude" corresponding to both the Z- exchange and two W-exchange contributions.

The one-loop-induced $\mu e \gamma$ vertex is calculated in Sec. B of the Appendix. From Eqs. (B1), (B13), and (B14) we can easily compute the amplitude corresponding to Fig. 3(a).

$$\mathfrak{M}^\gamma(k_1, k_2) = -\{\bar{u}_e(k_1)(1-\gamma_5)[i f_M m_\mu \sigma_{\lambda\nu} q^\nu + f_E(\gamma_\lambda q^2 - \not{q} q_\lambda)] u_\mu(p)\} \times [\bar{u}_e(k_2)\gamma^\lambda v_e(k_3)]/q^2, \tag{4.14}$$

where $q = p - k_1$, and

$$f_M = -\tfrac{5}{2} e\kappa, \quad f_E = 2e\kappa \tag{4.15}$$

with

$$\kappa = \frac{e}{16\pi^2}\left(\frac{g^2}{8M_W^2}\right)\delta_N. \tag{4.16}$$

As noted in Sec. B of the Appendix, f_M is computed in the general ξ gauge and is explicitly verified to be (weak-group) gauge-invariant. This must be so since it is the physical amplitude for $\mu \to e\gamma$. On the other hand, the results for f_E as well as the "weak amplitude" have been obtained only in the

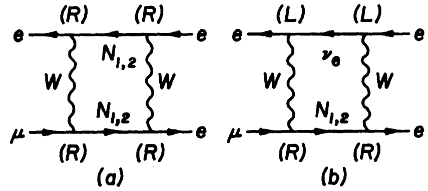

FIG. 4. Two types of box diagrams for $\mu \to e e \bar{e}$: (a) N graphs and (b) ν graphs.

't Hooft gauge. While each diagram is gauge-dependent, the sum of all the diagrams should be gauge-independent.

The weak amplitude can be parametrized as[34]

$$\mathfrak{M}^W(k_1, k_2) = [\bar{u}_e(k_1)(1-\gamma_5)\gamma_\lambda u_\mu(p)] \times \{\bar{u}_e(k_2)[g_L \tfrac{1}{2}(1+\gamma_5) + g_R \tfrac{1}{2}(1-\gamma_5)]\gamma^\lambda v_e(k_3)\}. \tag{4.17}$$

The diagram in Fig. 3(b) is

$$-iG^Z[\bar{u}_e(k_1)(1-\gamma_5)\gamma_\lambda u_\mu(p)]\frac{ig}{\cos\theta_W}(-\tfrac{1}{2} + \sin^2\theta_W) \times [\bar{u}_e(k_2)\gamma^\lambda v_e(k_3)]. \tag{4.18}$$

G^Z is the one-loop effective $\mu e Z$ coupling constant, and it has been calculated in Appendix [Eq. (C8)]:[35]

$$G^Z = 2\cot\theta_W \kappa \left[\ln\frac{m^2}{M_W^2} + \begin{pmatrix}4\\3\end{pmatrix}\right]. \tag{4.19}$$

Thus the contribution of Fig. 3(b) to g_L and g_R in Eq. (4.17) is

$$g_L(Z) = g_R(Z) = \frac{e\kappa}{\sin^2\theta_W}(-1 + 2\sin^2\theta_W)\left[\ln\frac{m^2}{M_W^2} + \begin{pmatrix}4\\3\end{pmatrix}\right]. \tag{4.20}$$

Figure 3(c) includes two types of box diagrams, as shown in Fig. 4. The N diagrams and ν diagrams belong to "type a" and "type b" box graphs as discussed in Appendix [see Eqs. (D11) and (D12)]. For Fig. 4(a) we also need to calculate the quantity[35]

$$\cos^2\phi[I(x_1, x_1) - I(x_2, x_1)] + \sin^2\phi[I(x_1, x_2) - I(x_2, x_2)] \simeq \left[\ln\frac{m^2}{M_W^2} + \begin{pmatrix}\frac{5}{2}\\ 1+\cos^2\phi\end{pmatrix}\right](x_1 - x_2), \tag{4.21}$$

where $x_1 = m_1^2/M_W^2$, $x_2 = m_2^2/M_W^2$. For Fig. 4(b) we note

$$I(x_1, 0) - I(x_2, 0) \simeq \left[\ln\frac{m^2}{M_W^2} + \begin{pmatrix}2\\1\end{pmatrix}\right](x_1 - x_2). \tag{4.22}$$

We then find their contribution to the weak amplitude of Eq. (4.17) as

$$g_L(2W, N_{1,2}) = 0, \tag{4.23}$$

$$g_R(2W, N_{1,2}) = \frac{e\kappa}{\sin^2\theta_W}\left[\ln\frac{m^2}{M_W^2} + \begin{pmatrix}\frac{5}{2}\\ 1+\cos^2\phi\end{pmatrix}\right], \tag{4.24}$$

and

$$g_L(2W, \nu) = \frac{e\kappa}{\sin^2\theta_W}\left[4\ln\frac{m^2}{M_W^2} + \begin{pmatrix}8\\ 4\end{pmatrix}\right], \tag{4.25}$$

$$g_R(2W, \nu) = 0\,. \tag{4.26}$$

Combining Eqs. (4.20) and the above results we have

$$g_L = \frac{e\kappa}{\sin^2\theta_W}\left[(3+2\sin^2\theta_W)\ln\frac{m^2}{M_W^2} + \begin{pmatrix}4+8\sin^2\theta_W\\ 1+6\sin^2\theta_W\end{pmatrix}\right], \tag{4.27}$$

$$g_R = \frac{e\kappa}{\sin^2\theta_W}\left[2\sin^2\theta_W\ln\frac{m^2}{M_W^2} + \begin{pmatrix}-\frac{3}{2}+8\sin^2\theta_W\\ -2+6\sin^2\theta_W+\cos^2\phi\end{pmatrix}\right]. \tag{4.28}$$

We have calculated the decay rate[36] for the amplitudes defined in Eqs. (4.12), (4.13), (4.14), and (4.17):

$$\Gamma(\mu\to 3e) = \frac{m_\mu^5 e^2\kappa^2}{384\pi^3}\left\{4\left(4\ln\frac{m_\mu}{2m_e} - \frac{13}{6}\right)|f_M|^2 - 12\,\mathrm{Re}(f_M^* f_E) + 3|f_E|^2 + |g_L|^2 + 2|g_R|^2 - 2\,\mathrm{Re}[(f_E - 2f_M)^*(g_L + 2g_R)]\right\}. \tag{4.29}$$

In writing this, we have rescaled all the amplitudes f_M, f_E, g_L, and g_R by taking out the common factor $e\kappa$. The branching ratio is given by

$$B(\mu\to 3e) = \frac{\Gamma(\mu\to ee\bar{e})}{\Gamma(\mu\to e\nu\bar{\nu})} = \frac{1}{64}\left(\frac{\alpha}{\pi}\right)^2 \delta_N^2(\Gamma^\gamma + \Gamma^W + \Gamma^{\gamma W}). \tag{4.30}$$

Taking the results from Eqs. (4.15), (4.27), and (4.28), with $(e\kappa)$ factored out,[35]

$$\Gamma^\gamma = 4\left(4\ln\frac{m_\mu}{2m_e} - \frac{13}{6}\right)|f_M|^2 - 12\,\mathrm{Re}(f_M^* f_E) + 3|f_E|^2 \simeq 478\,, \tag{4.31}$$

$$\Gamma^W = |g_L|^2 + 2|g_R|^2 \simeq \begin{pmatrix}2260\\ 3476\end{pmatrix}, \tag{4.32}$$

$$\Gamma^{\gamma W} = -2\,\mathrm{Re}[(f_E - 2f_M)^*(g_L + 2g_R)] = \begin{pmatrix}882\\ 1092\end{pmatrix}. \tag{4.33}$$

Here we have taken $\ln(m^2/M_W^2) \simeq -6$ and $\sin^2\theta_W \simeq \frac{1}{3}$. From Eq. (4.11) we then obtain the ratio, which depends on $\ln(m^2/M_W^2)$ only

$$R = \frac{B(\mu\to 3e)}{B(\mu\to e\gamma)} = \left(\frac{\alpha}{\pi}\right)\frac{1}{150}(\Gamma^\gamma + \Gamma^W + \Gamma^{\gamma W}) \simeq \begin{pmatrix}0.06\\ 0.08\end{pmatrix}. \tag{4.34}$$

These results are close to the value $R \simeq 0.07$ obtained by keeping only the $\ln(m^2/M_W^2)$ terms. This is due to some complicated cross-term cancellation and not to dominance of logarithmic terms.

C. Muonium-antimuonium transition

The process $e\bar{\mu} \to \mu\bar{e}$ has been studied in connection with the possibility that there is a "multiplicative scheme" for the μ- and e-lepton-number conservation.[37] Namely the μ-type and e-type leptons corresponds to opposite "parity" states. If this is the case, then $\mu\to e\gamma$ will be forbidden while $e\bar{\mu}\to\mu\bar{e}$ is allowed. A particularly interesting physical reaction of this type is that muonium can spontaneously turn into an antimuonium.[38] Feinberg and Weinberg have made a detailed analysis of this conversion in vacuum and in various environments.[39]

In our model this process can also occur, albeit

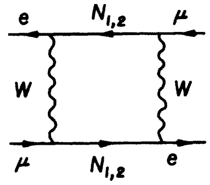

FIG. 5. The box diagram for $\mu\bar{e}\to e\bar{\mu}$.

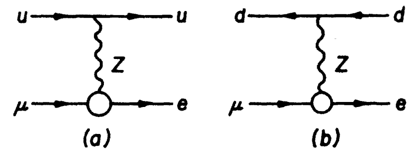

FIG. 6. Feynman graphs for (a) $\mu u \to eu$ and (b) $\mu d \to ed$ processes with an effective $\mu e Z$ vertex.

of order G^2, much like the situation for the $(K^0\text{-}\bar{K}^0)$ transitions. The effective Lagrangian is easy to calculate. It corresponds to the diagram in Fig. 5.

From the result in Sec. D of the Appendix (this is a "type a" box) we easily obtain[40]

$$\mathcal{L}_{\text{eff}} = g_{\text{eff}}[\bar{e}(1-\gamma_5)\gamma_\lambda \mu][\bar{\mu}(1-\gamma_5)\gamma^\lambda e] \tag{4.35}$$

with

$$g_{\text{eff}} = \frac{G_F^{\,2}}{16\pi^2}\sin^2\phi\cos^2\phi(m_1^{\,2}-m_2^{\,2})\begin{pmatrix}\ln\dfrac{m_1^{\,2}}{m_2^{\,2}}\\ 1\end{pmatrix}. \tag{4.36}$$

The present generation of experiments[41] can probably make a measurement at the level of $g_{\text{eff}} \simeq G_F$.

V. SEMILEPTONIC PROCESSES

In this section we calculate $\mu \to e$ conversion in a nucleus, $K_L \to e\mu$ and $K \to \pi e \mu$, etc. To study these semileptonic processes we adopt the following simple approach: We first calculate the elementary lepton-quark interactions; the amplitude so obtained is used as the effective Lagrangian for the appropriate semileptonic process. This involves evaluating the matrix elements of the two-body free quark operators between hadron states. For weak interactions of quarks we shall, for simplicity, use the standard model of Eq. (2.1). However, in Sec. V A, where we discuss μe conversion in a nucleus, we shall also consider the possibility that this model may have to be extended to include right-handed currents and new heavy quarks.

A. μe conversion in a nucleus

A muon trapped in the field of nucleus (A, Z), after it undergoes transition to the K shell, is normally "captured" in the reaction

$$\mu + (A,Z) \to \nu_\mu + (A, Z-1). \tag{5.1}$$

We shall study the possibility of it being, on a rare occasion, "converted" into an electron in the reaction

$$\mu + (A,Z) \to e + (A,Z). \tag{5.2}$$

This is an interesting and important process to study[42] since stringent bounds already exist[19] and significant improvements over these limits are feasible in the not-too-distant future.[43]

The basic theoretical work on this subject is a paper by Weinberg and Feinberg.[44] Once we have obtained the effective Lagrangian in this model we shall lean heavily upon that work to obtain our final estimates.

The relevant quark-lepton reaction for this process is

$$\mu + u \to e + u, \tag{5.3}$$

$$\mu + d \to e + d. \tag{5.4}$$

The calculation is similar to that for $\mu \to ee\bar{e}$. It involved the same three classes of diagrams shown in Fig. 3. However, for this process we shall use the leading log approximation. From our experience with the $\mu \to 3e$ calculation, it probably does not introduce a large error. Since the uncertainties due to strong interactions and nuclear physics will be considerably larger, this expediency is perhaps justified. Our task then is simply to calculate the $\ln(m^2/M_W^{\,2})$ contributions coming from the Z-exchange and the two W-exchange diagrams (see Fig. 6 and Fig. 7). With only these "weak amplitudes," the effective Lagrangian may be parameterized as

$$\begin{aligned}\mathcal{L}_{\text{eff}} = {} & (e\kappa/\sin^2\theta_W)\ln(m^2/M_W^{\,2})[\bar{e}(1-\gamma_5)\gamma_\lambda\mu] \\ & \times\{\bar{u}[\alpha_L(1+\gamma_5)/2+\alpha_R(1-\gamma_5)/2]\gamma^\lambda u + \bar{d}[\beta_L(1+\gamma_5)/2+\beta_R(1-\gamma_5)/2]\gamma^\lambda d\}.\end{aligned} \tag{5.5}$$

In calculating the weak amplitude we shall allow for the possibility that u_R and/or d_R may belong to some weak isodoublets. Their contributions will be specified by the parameters ρ_u and ρ_d:

$$\rho_u = 1 \ \text{ if } \begin{pmatrix} u \\ b \end{pmatrix}_R \text{ is present}, \quad \rho_u = 0 \text{ otherwise}, \tag{5.6}$$

$$\rho_d = 1 \ \text{ if } \begin{pmatrix} t \\ d \end{pmatrix}_R \text{ is present}, \quad \rho_d = 0 \text{ otherwise}, \tag{5.7}$$

where b and t are new heavy quarks.

Let us first calculate the Z-exchange contributions: The amplitude for the graph in Fig. 6(a) is

$$-iG^Z[\bar{e}(1-\gamma_5)\gamma_\lambda\mu](ig/\cos\theta_W)\{\bar{u}[(\tfrac{1}{2}-2\sin^2\theta_W/3)(1+\gamma_5)/2+(\tfrac{1}{2}\rho_u-2\sin^2\theta_W/3)(1-\gamma_5)/2]\gamma^\lambda u\}. \tag{5.8}$$

The amplitude for the graph in Fig. 6(b) is

$$-iG^Z[\bar{e}(1-\gamma_5)\gamma_\lambda\mu](ig/\cos\theta_W)\{\bar{d}[(-\tfrac{1}{2}+\sin^2\theta_W/3)(1+\gamma_5)/2+(-\tfrac{1}{2}\rho_d+\sin^2\theta_W/3)(1-\gamma_5)/2]\gamma^\lambda d\}, \tag{5.9}$$

where G^Z is given by Eq. (C8) in the Appendix. They contribute to Eq. (5.5) as

$$\alpha_L(Z)=1-4\sin^2\theta_W/3\,,\quad \alpha_R(Z)=\rho_u-4\sin^2\theta_W/3\,,$$
$$\beta_L(Z)=-1+2\sin^2\theta_W/3\,,\quad \beta_R(Z)=-\rho_d+2\sin^2\theta_W/3\,. \tag{5.10}$$

We next compute the contributions from the box graphs in Fig. 7. They correspond precisely to the four "types" as classified in Sec. D of the Appendix [see Fig. 16 and Eq. (D12)]. Two of them have also been evaluated in our calculation for the decay $\mu\to ee\bar{e}$ [see (4.24) and (4.25)]. Their contributions are simply

$$\alpha_L(2W)=-1\,,\quad \alpha_R(2W)=-4\rho_u\,,$$
$$\beta_L(2W)=4\,,\quad \beta_R(2W)=\rho_d\,. \tag{5.11}$$

From Eqs. (5.10) and (5.11), we obtain

$$\alpha_L=-4\sin^2\theta_W/3\,,$$
$$\alpha_R=-3\rho_u-4\sin^2\theta_W/3\,,$$
$$\beta_L=3+2\sin^2\theta_W/3\,,$$
$$\beta_R=2\sin^2\theta_W/3\,. \tag{5.12}$$

Now we can proceed to place the effective Lagrangian of Eqs. (5.5) and (5.12) between the initial and final nuclear states. For this purpose we rewrite it in terms of the isoscalar and isovector vector and axial-vector currents:

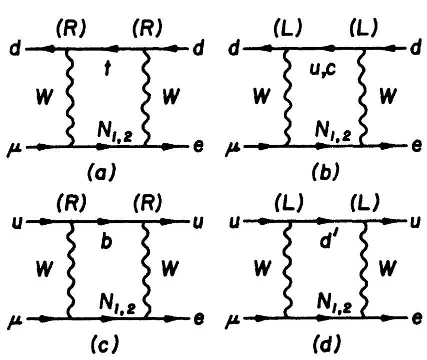

FIG. 7. Box diagrams for $\mu q\to eq$ with both $V-A$ and $V+A$ quark couplings.

$$\mathcal{L}_{\rm eff}=(e\kappa/\sin^2\theta_W)\ln(m^2/M_W^2)[\bar{e}(1-\gamma_5)\gamma^\lambda\mu]\times(G_V^{(0)}V_\lambda^{(0)}+G_V^{(1)}V_\lambda^{(0)}+G_A^{(0)}A_\lambda^{(0)}+G_A^{(1)}A_\lambda^{(1)})\,, \tag{5.13}$$

where

$$G_V^{(0)}=(\alpha_L+\alpha_R+\beta_L+\beta_R)/2\,, \tag{5.14}$$

$$V_\lambda^{(0)}=(\bar{u}\gamma^\lambda u+\bar{d}\gamma_\lambda d)/2\,,\quad\text{etc.} \tag{5.15}$$

For coherent processes the axial-vector current contributions are suppressed, and the vector current matrix elements are

$$\langle(A,Z)'|V_0^{(0)}|(A,Z)\rangle=\tfrac{3}{2}AF_V^{(0)}\,, \tag{5.16}$$

$$\langle(A,Z)'|V_0^{(1)}|(A,Z)\rangle=(Z-A/2)F_V^{(1)}\,, \tag{5.17}$$

where $F_V^{(0)}$ and $F_V^{(1)}$ are the isoscalar and isovector nuclear form factors with $F_V^{(0)}(0)=F_V^{(1)}(0)=1$. For our case they are, of course, evaluated at $q^2=m_\mu^2$.[44] The μe transition rate is then given by

$$\omega(\mu\to e)=(1/2\pi^2)(Z_{\rm eff}^4Z^{-1}\alpha^3m_\mu^5)\times[(e\kappa/\sin^2\theta_W)\ln(m^2/M_W^2)]^2\times|\tfrac{3}{2}AG_V^{(0)}F_V^{(0)}+(Z-A/2)G_V^{(1)}F_V^{(1)}|^2\,. \tag{5.18}$$

We have assumed that the density of the initial K-shell muons is $Z_{\rm eff}^4Z^{-1}\alpha^3m_\mu^3$.[44] Compared to the ordinary muon capture (here we ignore some nuclear physics factor; see comment e in Table I)

$$\omega(\mu\to\nu)=(1/4\pi^2)Z_{\rm eff}^4\alpha^3m_\mu^5G_F^2(C_V^2+3C_A^2) \tag{5.19}$$

($C_V\simeq1$ and $C_A\simeq1.25$), we have, for isoscalar nucleus,

$$\frac{\omega(\mu\to e)}{\omega(\mu\to\nu)}=6.2\times10^{-3}E(\rho_u)(A^2|F_V^{(0)}|^2/Z)\times[(\alpha/\pi)\delta_N\ln(m^2/M_W^2)]^2\,, \tag{5.20}$$

or, in terms of the $\mu\to e\gamma$ branching ratio,

$$\frac{\omega(\mu\to e)}{\omega(\mu\to\nu)}=2.6\times10^{-3}E(\rho_u)(A^2|F_V^{(0)}|^2/Z)(\alpha/\pi)\times[\ln(m^2/M_W^2)]^2B(\mu\to e\gamma)\,, \tag{5.21}$$

where

$$E(\rho_u)=\left(\frac{3-3\rho_u}{\sin^2\theta_W}-\frac{4}{3}\right)^2. \tag{5.22}$$

TABLE I. Summary of our principal results for the branching ratios of muon-number-nonconserving processes. (For the reaction $\mu Z\to eZ$ it is the μe conversion rate divided by the ordinary μ capture rate $\mu Z\to\gamma Z$.) The theoretical predictions of our model all contain a combination of the mixing angles and heavy-lepton mass differences as $[\sin\phi\cos\phi(m_1^2-m_2^2)/M_W^2]^2$. The numerical estimates shown in the second column are given for $\Delta m^2=\sin\phi\cos\phi(m_1^2-m_2^2)=1$ GeV^2 and $M_W=60$ GeV. [In our previous communications (Refs. 2 and 3) we took $M_W=50$ GeV in our numerical estimates.]

Process	Our result	Experimental bound	Comments
$\mu\to e\gamma$	4×10^{-10}	$<2.2\times10^{-8}$ (Ref. 51)	
$\mu\to ee\bar{e}$	3×10^{-11}	$<1.9\times10^{-9}$ (Ref. 18)	a,b
$\mu Z\to eZ$	$10^{-10}/C$	$<1.6\times10^{-8}$ (Ref. 19)	a,c,d,e
	$3\times10^{-12}/C$		a,c,e,f
$K_L\to e\bar{\mu}$	$\sim10^{-10}$	$<1.6\times10^{-9}$ (Ref. 52)	g
$K^+\to\pi^+e\bar{\mu}$	$\sim10^{-13}$	$<1.4\times10^{-8}$ (Ref. 53)	g
$K_S\to\pi^0e\bar{\mu}$	$\sim10^{-15}$	...	g

[a] We have taken $\ln(M_W^2/m^2)\simeq6$.
[b] We list here the average value for the two cases of $m_1^2\approx m_2^2$ and $m_1^2\gg m_2^2$.
[c] This is the "leading log approximation" result with maximal coherent effects for nuclei near Cu.
[d] For standard $V-A$ four-quark theory.
[e] C is a nuclear physics factor which can be as small as 0.1 for Cu. H. Primakoff, Revs. Mod. Phys. 31, 802 (1959) and S. Weinberg and G. Feinberg, Phys. Rev. Lett. 3, 244 (E) (1959). We would like to thank Dr. A. Sanda for calling our attention to these references.
[f] For theories include the $V+A$ current: $\bar{b}(1-\gamma_5)\gamma_\lambda u$.
[g] For asymmetric lepton masses. We took $m_1/m_2\simeq4$ and $m_c\simeq1.8$ GeV.

Weinberg and Feinberg[44] have estimated that the "coherent factor" $(A^2|F_V^{(0)}|^2/Z)$ reaches a maximum of about 30 for nuclei near copper. Taking this value, $\ln(m^2/M_W^2)\simeq-6$, and assuming the absence of right-handed couplings for the u quark $E(0)\simeq60$, we obtain from Eq. (5.21)

$$\frac{\omega(\mu\to e)}{\omega(\mu\to\nu)}\simeq0.4B(\mu\to e\gamma)\,, \tag{5.23}$$

or, for $\sin\phi\cos\phi(m_1^2-m_2^2)\simeq1$ GeV^2,

$$\frac{\omega(\mu\to e)}{\omega(\mu\to\nu)}\simeq1.6\times10^{-10}\,. \tag{5.24}$$

This is to be compared with the present experimental limit of 1.6×10^{-8}. (Here we ignore a nuclear physics factor, see comment e in Table I.)

It is interesting to note that the presence of a right-handed current, $\rho_u=1$, would significantly *suppress* this μe transition. From Eq. (5.22) we see that the suppression factor is

$$\epsilon=E(1)/E(0)\simeq\tfrac{1}{33}\,; \tag{5.25}$$

the $V+A$ current for the d quark makes no contribution to this process. We note in particular that a weak-interaction model which has had considerable phenomenological success[45]

$$\begin{pmatrix}u\\d'\end{pmatrix}_L,\ \begin{pmatrix}c\\s'\end{pmatrix}_L,\ \begin{pmatrix}u\\b\end{pmatrix}_R,\begin{pmatrix}c\\g\end{pmatrix}_R\ d_R,\ s_R,\ b_L,\ g_L$$

would have $\rho_u=1$ and thus a small rate for $\mu\to e$ conversion.

Lastly, for the incoherent processes the μe transition rate is given by

$$\begin{aligned}\omega'(\mu\to e)=&(1/2\pi^2)(Z_{\rm eff}{}^4Z^{-1}\alpha^3m_\mu{}^5)[(e\kappa/\sin^2\theta_W)\ln(m^2/M_W^2)]^2\\&\times\{Z[(3G_V^{(0)}F_V^{(0)}/2+G_V^{(1)}F_V^{(1)}/2)^2+3(3G_A^{(0)}F_A^{(0)}/2+G_A^{(1)}F_A^{(1)}/2)^2]\\&+(A-Z)[(3G_V^{(0)}F_V^{(0)}/2-G_V^{(1)}F_V^{(1)}/2)^2+3(3G_A^{(0)}F_A^{(0)}/2-G_A^{(1)}F_A^{(1)}/2)^2]\}\,.\end{aligned} \tag{5.26}$$

B. Muon-number-nonconserving K decays

The elementary interactions

$$\begin{pmatrix}s\\d\end{pmatrix}+\mu\to\begin{pmatrix}d\\s\end{pmatrix}+e \tag{5.27}$$

which change both muon number and strangeness are simply given by the box graphs shown in Fig. 8. From this we calculate the effective Lagrangian for muon-number-nonconserving K decays. The box graph shown in Fig. 8 is of "type b" as classified in Sec. D of the Appendix. From the calculations given there we have[46]

$$\begin{aligned}\mathcal{L}_{\rm eff}&=g^4(64\pi^2M_W^2)^{-1}\sin\phi\cos\phi\sin\theta_C\cos\theta_C K[\bar{e}(1-\gamma_5)\gamma^\lambda\mu][\bar{s}(1+\gamma_5)\gamma_\lambda d+\bar{d}(1+\gamma_5)\gamma_\lambda s]\\&=(\sin\theta_C G_F/\sqrt{2})\epsilon K[\bar{e}(1-\gamma_5)\gamma^\lambda\mu][\bar{s}(1+\gamma_5)\gamma_\lambda d+\bar{d}(1+\gamma_5)\gamma_\lambda s]\,,\end{aligned} \tag{5.28}$$

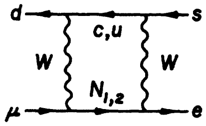

FIG. 8. The Feynman graph for $s\mu \to de$.

where

$$\epsilon = (\alpha/2\pi)\sin\phi\cos\phi\cos\theta_C/\sin^2\theta_W, \qquad (5.29)$$

$$K = [I(x_1, 0) - I(x_2, 0)] - [I(x_1, y) - I(x_2, y)], \qquad (5.30)$$

$$I(x, y) = [J(x) - J(y)]/(x - y), \qquad (5.31)$$

$$J(x) = (1 - x)^{-1} + x^2 \ln x/(1 - x)^2, \qquad (5.32)$$

with $x_i = m_i^2/M_W^2$, $y = m_c^2/M_W^2$, and $m_u^2/M_W^2 \simeq 0$. Fermion masses are expected to be small when compared to M_W; this rather complicated expression for K can be simplified. The resulting limits depend on the relative sizes of lepton and quark masses:

$$K = (m_c^2/M_W^2)\ln(m_1^2/m_2^2), \quad \text{for } m_1 \gg m_2 \text{ with } m_2 \gtrsim m_c \qquad (5.33)$$

$$K = (m_1^2 - m_2^2)/2M_W^2, \quad \text{for } m_1 \approx m_2 \approx m_c \qquad (5.34)$$

$$K = m_c^2(m_1^2 - m_2^2)/(m^2 M_W^2), \quad \text{for } m_1 \approx m_2 \gg m_c. \qquad (5.35)$$

$K_L \to e\bar{\mu}$. The amplitude for $K_L \to e\bar{\mu}$ is given by

$$\mathfrak{M}(K_L \to e\mu) = (\sin\theta_C G_F/\sqrt{2})\epsilon K[\bar{e}(1 - \gamma_5)\gamma^\lambda\mu] \times \langle 0|\bar{s}\gamma_5\gamma_\lambda d + \bar{d}\gamma_5\gamma_\lambda s|K_L\rangle. \qquad (5.36)$$

A straightforward SU_3 calculation[47] can relate this hadronic matrix element to that for the leptonic decays of K^+:

$$\langle 0|\bar{s}\gamma_5\gamma_\lambda d + \bar{d}\gamma_5\gamma_\lambda s|K_L\rangle \simeq \sqrt{2}\,\langle 0|\bar{s}\gamma_5\gamma_\lambda u|K^+\rangle. \qquad (5.37)$$

From this we can immediately conclude (recall we set $m_e = 0$) that

$$\frac{\Gamma(K_L \to e\bar{\mu})}{\Gamma(K^+ \to \nu\bar{\mu})} = 2\epsilon^2 K^2. \qquad (5.38)$$

Folding in the branching ratio of $K^+ \to \nu\mu^+$ and taking into account the different lifetimes for K^+ and K_L, we obtain

$$B(K_L \to e\bar{\mu}) = \frac{\Gamma(K_L \to e\bar{\mu})}{\Gamma(K_L \to \text{all})} \simeq 5.5\epsilon^2 K^2, \qquad (5.39)$$

where ϵ and K are given by Eqs. (5.29) and (5.30), respectively. The various limiting expressions of K are given by Eqs. (5.33)–(5.35). As an illustrative example for $(m_1/m_2) \simeq 4$ and $\sin\phi\cos\phi \simeq \frac{1}{2}$, we obtain $B(K_L \to e\bar{\mu}) \simeq 10^{-10}$, with $m_c \simeq 1.8$ GeV.[48]

$K \to \pi e\bar{\mu}$. The amplitude for $K \to \pi e\bar{\mu}$ is given by

$$\mathfrak{M}(K \to \pi e\bar{\mu}) = (\sin\theta_C G_F/\sqrt{2})\epsilon K[\bar{e}(1 - \gamma_5)\gamma^\lambda\mu] \times \langle\pi|\bar{s}\gamma_\lambda d + \bar{d}\gamma_\lambda s|K\rangle. \qquad (5.40)$$

This is to be compared to the amplitude for $K^+ \to \pi^0\nu\mu^+$. Again SU_3 calculations yield[49]

$$\langle\pi^0|\bar{s}\gamma_\lambda d + \bar{d}\gamma_\lambda s|K_S\rangle = \langle\pi^+|\bar{s}\gamma_\lambda d + \bar{d}\gamma_\lambda s|K^+\rangle = (-1/\sqrt{2})\langle\pi^0|\bar{s}\gamma_\lambda u|K^+\rangle. \qquad (5.41)$$

Folding in the branching ratio of $K^+ \to \pi^0\nu\mu^+$, etc. we obtain

$$\frac{\Gamma(K^+ \to \pi^+ e\mu^+)}{\Gamma(K^+ \to \text{all})} \simeq 1.5 \times 10^{-2}\epsilon^2 K^2 \qquad (5.42)$$

and

$$\frac{\Gamma(K_S \to \pi^0 e\mu^+)}{(K_S \to \text{all})} \simeq 10^{-4}\epsilon^2 K^2. \qquad (5.43)$$

For $m_1/m_2 \simeq 4$, $\sin\phi\cos\phi \simeq \frac{1}{2}$, these branching ratios are 3×10^{-13} and 2×10^{-15}, respectively.

VI. SUMMARY

In renormalizable gauge theories we must allow for all possible fermion and Higgs-scalar couplings that are compatible with the requirement of gauge invariance. Unless we impose some discrete symmetries beforehand, these would include scalar couplings to fermionic weak eigenstates belonging to different multiplets. When spontaneous symmetry breaking takes place and Higgs scalars develop vacuum expectation values, these Yukawa couplings generate mass terms which are off-diagonal with respect to weak eigenstates. It is this source of flavor-changing interactions in gauge theories[50] which we have discussed, in connection with the question of possible muon-number nonconservation. These off-diagonal masses are in effect the coupling constants that determine the magnitude of the corresponding flavor-changing interactions.

As was pointed out in our earlier communications,[2,3] in models where the muon and electron have both the $V-A$ and $V+A$ couplings, one would *anticipate* that these off-diagonal mass terms would be in the GeV range. Consequently muon-number-changing effects would be dramatically larger than the simplest $V-A$ theory where one would *a priori* think that the off-diagonal mass terms should be comparable to the neutrino masses.

We have proposed a simple model of leptons, Eq. (3.1) [or see Eq. (A1)], which incorporates a "leptonic GIM mechanism." (The requirements of

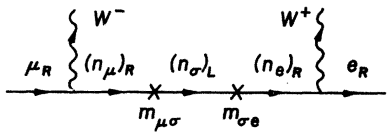

FIG. 9. The basic mechanism for μe transition in our model. There is another diagram where $(n_\sigma)_L$ is replaced by $(n_\tau)_L$.

a natural GIM mechanism in theories with $V+A$ currents are amplified in Sec. III.) In this model parity-violation effects in high-Z atoms are strongly suppressed. And the basic mechanism for μe transitions is the one shown in Fig. 9. The couplings of these two diagrams, by inspection, must be proportional to $\Delta m^2 = (m_{\mu\sigma} m_{\sigma e} + m_{\mu\tau} m_{\tau e})$. In terms of the physical masses and mixing angles this factor is precisely [see Eq. (A30)] the familiar combination of $\sin\phi \cos\phi (m_1{}^2 - m_2{}^2)$ which appears in every result of our calculations. If all Yukawa couplings [f_{ab} in Eq. (A3)] are comparable in magnitude, then we do not expect Δm^2 to be significantly smaller than the average heavy-lepton mass $\overline{m}^2 = \frac{1}{2}(m_1{}^2 + m_2{}^2)$ [see Eq. (A29)]. Thus a value of few GeV^2 seems reasonable to us. In presenting our results in numerical form we have set $\Delta m^2 = 1\ \mathrm{GeV}^2$. This should set the scale of our estimates. These estimates, together with the present experimental limits,[18,19,51-53] are listed in Table I.

Note added. While this manuscript was being typed, we received the following papers on muon-number nonconservation: M. A. B. Bég and A. Sirlin, Phys. Rev. Lett. 38, 1113 (1977); R. Decker and J. Pestieau, Univ. de Louvain report (unpublished); T. Kaneko, Meijo Univ. Report No. MJU-DP-703 (unpublished); S. M. Barr and S. Wandzura, Phys. Rev. D 15, 707 (1977); W. J. Marciano and A. Z. Sanda, Rockefeller Univ. report (unpublished). The last two papers also present calculations on $\mu \to e e \bar{e}$ and μe conversion in a nucleus. We would like to thank Dr. Marciano for pointing out a sign error in our original calculation for the $\mu \to e e \bar{e}$ amplitude.

APPENDIX

In this Appendix we shall provide some details of our calculations.[54] In Sec. A, we specify the Lagrangian of our model and present the relevant Feynman rules. We shall discuss important features of the lepton mass matrix. For the sake of completeness, some familiar relations for the $SU_2 \times U_1$ gauge models will also be included. The various physical processes discussed in this paper often involve similar Feynman diagrams: We need to compute the one-loop-induced $\mu e \gamma$ and $\mu e Z$ vertices, as well as the box diagrams corresponding to the exchange of two W intermediate vector bosons. These objects are computed in Sec. B, C, and D, respectively. The method for diagram calculations in gauge theories is by now well known. In particular we shall perform many of the calculations in the general ξ gauge.[29] Non-Abelian gauge invariance then demands that physically meaningful quantities must be independent of the ξ parameter. Some of the results we present here have been obtained before in connection with studies of the various strangeness-changing neutral-current effects in the standard model. A particularly useful reference is the paper by Gaillard and Lee.[55]

A. Lagrangian, mass matrix, and Feynman rules

The Lagrangian

In our $SU_2 \times U_1$ gauge model of weak and electromagnetic interactions, there are two doublets ($t = \frac{1}{2}$) of four-component fermions: one e type, one μ type, with weak hypercharge $y = -1$, and two left-handed two-component fermions with $t = y = 0$. (This is to ensure that after spontaneous symmetry breaking, there will still be two massless neutrinos):

$$\psi_a :\ \begin{pmatrix} n_e \\ e \end{pmatrix}, \ \begin{pmatrix} n_\mu \\ \mu \end{pmatrix} \text{ and } \psi_b :\ n_{\sigma L},\ n_{\tau L} \ . \qquad \text{(A1)}$$

There is one set of (complex) scalar fields ($t = \frac{1}{2}$, $y = 1$)

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad \text{(A2)}$$

The Lagrangian is given by (with $a = e$, μ and $b = \sigma, \tau$)

$$\mathcal{L} = -\tfrac{1}{4}\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} + i\bar{\psi}_a\gamma^\lambda D_\lambda\psi_a + i\bar{\psi}_b\gamma^\lambda D_\lambda\psi_b + (D_\lambda\Phi)^\dagger(D^\lambda\Phi) - V(\Phi) - m_a\bar{\psi}_a\psi_a - (f_{ab}\Phi\bar{\psi}_a\psi_b + \text{H.c.}), \qquad \text{(A3)}$$

where

$$\begin{aligned} \vec{W}_{\mu\nu} &= \partial_\mu\vec{W}_\nu - \partial_\nu\vec{W}_\mu + g\vec{W}_\mu\times\vec{W}_\nu, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\ D_\lambda\psi_a &= (\partial_\lambda - ig\vec{T}\cdot\vec{W}_\lambda - ig'\tfrac{1}{2}YB_\lambda)\psi_a, \\ D_\lambda\Phi &= (\partial_\lambda - ig\tfrac{1}{2}\vec{\tau}\cdot\vec{W}_\lambda - ig'\tfrac{1}{2}B_\lambda)\Phi \ , \end{aligned} \qquad \text{(A4)}$$

and

$$V(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \rho(\Phi^\dagger\Phi)^2 \ .$$

$V(\Phi)$ is such that spontaneous symmetry breaking takes place

$$\langle\Phi\rangle_0 = (1/\sqrt{2})\begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \text{(A5)}$$

with

$$v = (\mu^2/\rho)^{1/2} .$$

Shifting the scalar field

$$\Phi = \begin{pmatrix} \phi^\dagger \\ (\phi_1 + i\phi_2 + v)/\sqrt{2} \end{pmatrix} . \tag{A6}$$

The neutral ϕ_1 is the physical Higgs scalar meson. The remaining scalar fields $\phi^\pm$, ϕ_2 are the "would-be Goldstone bosons"; when combined with the original massless gauge fields they provide us with the three massive intermediate vector bosons $W_\lambda^\pm$, Z_λ. The Z_λ and the photon field A_λ are linear combinations of $W_\lambda{}^3$ and B_λ:

$$\begin{aligned} A_\lambda &= \cos\theta_W B_\lambda + \sin\theta_W W_\lambda{}^3 , \\ Z_\lambda &= -\sin\theta_W B_\lambda + \cos\theta_W W_\lambda{}^3 , \end{aligned} \tag{A7}$$

where θ_W is the Weinberg angle:

$$\tan\theta_W = g'/g . \tag{A8}$$

Substituting (A7) back into the original Lagrangian we have the couplings of A_λ and Z_λ to the fermions as

$$e(\bar\psi Q\gamma^\lambda\psi)A_\lambda + (g/\cos\theta_W)[\bar\psi(T_3 - \sin^2\theta_W Q)\gamma^\lambda\psi]\, Z_\lambda \tag{A9}$$

with

$$Q = T_3 + Y/2$$

$$e = g\sin\theta_W . \tag{A10}$$

The vacuum expectation value v gives rise to masses for the intermediate vector bosons:

$$M_W = \tfrac{1}{2}gv \tag{A11}$$

and

$$M_Z = M_W/\cos\theta_W . \tag{A12}$$

This last relation reflects the feature of this model that only a Higgs doublet develops vacuum expectation value. We also recall the famous relation

$$M_W = 2^{-5/4}G_F{}^{-1/2}e/\sin\theta_W \simeq (37/\sin\theta_W)\ \text{GeV}. \tag{A13}$$

Present experimental data are consistent with $\sin^2\theta_W \simeq \frac{1}{3}$. In numerical estimates we shall adopt this value, which also implies that $M_W \simeq 60$ GeV.

Lepton mass matrix

For the lepton masses, we have the bare mass term[56]

$$\begin{aligned} -\mathcal{L}_a &= m_a \bar\psi_a \psi_a \\ &= m_e(\bar e e + \bar n_e n_e) + m_\mu(\bar\mu\mu + \bar n_\mu n_\mu) \end{aligned} \tag{A14}$$

and the mass terms rising from the Yukawa couplings of the leptons to the scalar which develops vacuum expectation value

$$\begin{aligned} -\mathcal{L}_{ab} &= f_{ab}\frac{v}{\sqrt{2}}\bar\psi_a\psi_b + \text{h.c.} \\ &= m_{\sigma e}(\bar n_{\sigma L} n_{eR}) + m_{\tau e}(\bar n_{\tau L} n_{eR}) \\ &\quad + m_{\sigma\mu}(\bar n_{\sigma L} n_{\mu R}) + m_{\tau\mu}(\bar n_{\tau L} n_{\mu R}) + \text{H.c.} \end{aligned} \tag{A15}$$

with

$$m_{ab} = f_{ab}\, v/\sqrt{2} . \tag{A16}$$

We collect the mass terms of neutral leptons in Eqs. (A.14) and (A.15) (and also extend the "left-hand index" b to include all e, μ, σ, and τ types),

$$-\mathcal{L}_m = (\bar\Psi_a)_R \mathfrak{M}_{ab}(\Psi_b)_L + \text{H.c.} \tag{A17}$$

with

$$\mathfrak{M}_{ab} = \begin{pmatrix} m_e & 0 & m_{e\sigma} & m_{e\tau} \\ 0 & m_\mu & m_{\mu\sigma} & m_{\mu\tau} \end{pmatrix} ; \tag{A18}$$

Ψ_R and Ψ_L are weak eigenstates

$$(\Psi_a)_R = \begin{pmatrix} n_e \\ n_\mu \end{pmatrix}_R , \quad (\Psi_b)_L = \begin{pmatrix} n_e \\ n_\mu \\ n_\sigma \\ n_\tau \end{pmatrix}_L . \tag{A19}$$

The mass matrix is diagonalized with respect to the physical states

$$-\mathcal{L}_m = (\bar\Psi_i')_R \mathfrak{M}_{ij}'(\Psi_j')_L \tag{A20}$$

with

$$\mathfrak{M}_{ij}' = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \end{pmatrix} . \tag{A21}$$

The mass eigenstates

$$(\Psi_i')_R = \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}_R , \quad (\Psi_j')_L = \begin{pmatrix} N_1 \\ N_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}_L \tag{A22}$$

are related to the weak eigenstates in Eq. (A19) by unitary transformations

$$V\Psi_R' = \Psi_R , \tag{A23}$$

$$U\Psi_L' = \Psi_L , \tag{A24}$$

$$V^\dagger \mathfrak{M} U = \mathfrak{M}' \tag{A25}$$

with

$$V = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} . \tag{A26}$$

Equation (3.2) is just Eq. (A23). We can fix the elements of U in Eq. (A25). A straightforward calculation yields

$$U_{e1}=(m_e/m_1)\cos\phi,\quad U_{e2}=(m_e/m_2)\sin\phi,$$
$$U_{\mu 1}=-(m_\mu/m_1)\sin\phi,\quad U_{\mu 2}=(m_\mu/m_2)\cos\phi. \tag{A27}$$

Part of Eq. (A24) is shown in Eq. (3.4). The relationship between the initial six parameters in $\mathfrak{M}_{ab}$ and the four physical masses, two mixing angles can be worked out. We note that

$$\tan 2\phi=\frac{2\sum_b m_{\mu b}m_{be}}{\sum_b (m_{\mu b}^2-m_{eb}^2)}\ , \tag{A28}$$

$$m_1^2+m_2^2=\sum_{a,b} m_{ab}^2\ . \tag{A29}$$

For us the most interesting combination of mixing angle and mass factors is

$$\sin\phi\,\cos\phi(m_1^2-m_2^2)=-(m_{e\sigma}m_{\sigma\mu}+m_{e\tau}m_{\tau\mu})\ , \tag{A30}$$

which controls the size of μe transition processes. If Yukawa couplings f_{ab} are all comparable in magnitude, then the combination in (A30) should not be too different from the average of the heavy-lepton mass squared.

The Feynman rules

Here we shall provide all the relevant couplings used in our calculations. The gauge couplings of leptons are shown in Fig. 10. Some examples of these couplings in terms of the physical states are

$$G(\mu\mu\gamma)=-ie\gamma_\lambda\ , \tag{A31}$$
$$G(\mu\mu Z)=(ig/\cos\theta_W)(-\tfrac{1}{2}+\sin^2\theta_W)\gamma_\lambda\ , \tag{A32}$$
$$G_L(\mu N_1 W)=(-ig\sin\phi\, m_\mu/\sqrt{2}\,m_1)\gamma_\lambda(1-\gamma_5)/2\ , \tag{A33}$$
$$G_R(\mu N_1 W)=(-ig\sin\phi/\sqrt{2})\gamma_\lambda(1+\gamma_5)/2\ . \tag{A34}$$

Except for the case of unitary gauge, the contribution due to the "would-be Goldstone bosons" $(\phi^\pm,\phi_2)$ must be taken into account in order to maintain gauge invariance and to obtain unitary results. We give explicitly an example of their couplings to leptons: the case where ϕ^+ replaces the W^+ in Eqs. (A33) and (A34):

$$G(\mu\bar{N}_1\phi)=(-ig\sin\phi/\sqrt{2}M_W)(m_1-m_\mu^2/m_1)(1+\gamma_5)/2 \tag{A35}$$
$$=(-ig\sin\phi/\sqrt{2}M_W)\{(m_\mu/m_1)[m_1(1-\gamma_5)/2-m_\mu(1+\gamma_5)/2]+[m_1(1+\gamma_5)/2-m_\mu(1-\gamma_5)/2]\}. \tag{A36}$$

From the second expression, we can easily check that contributions due to these scalars precisely cancel the unphysical pole in the W propagator [the second term in Eq. (A37) below]. Comparing the expressions in Eqs. (A33), (A34), and (A36) one can easily read off the Yukawa couplings for other leptons.

The relevant trilinear gauge and gauge-scalar couplings are displayed in Fig. 11.

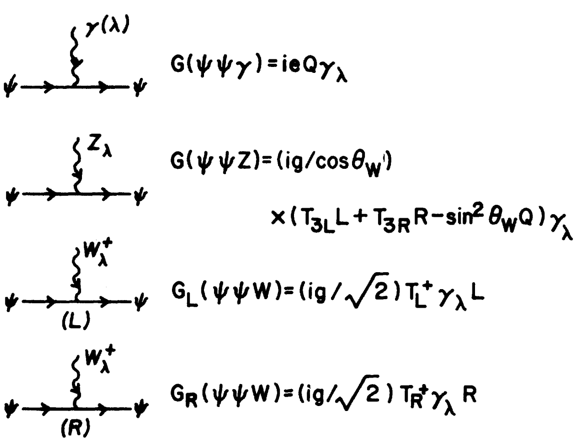

FIG. 10. Feynman rules for lepton gauge couplings where $T^+_{L,R}$ are the isospin-raising matrices for the left and right multiplets: $L=(1-\gamma_5)/2$ and $R=(1+\gamma_5)/2$.

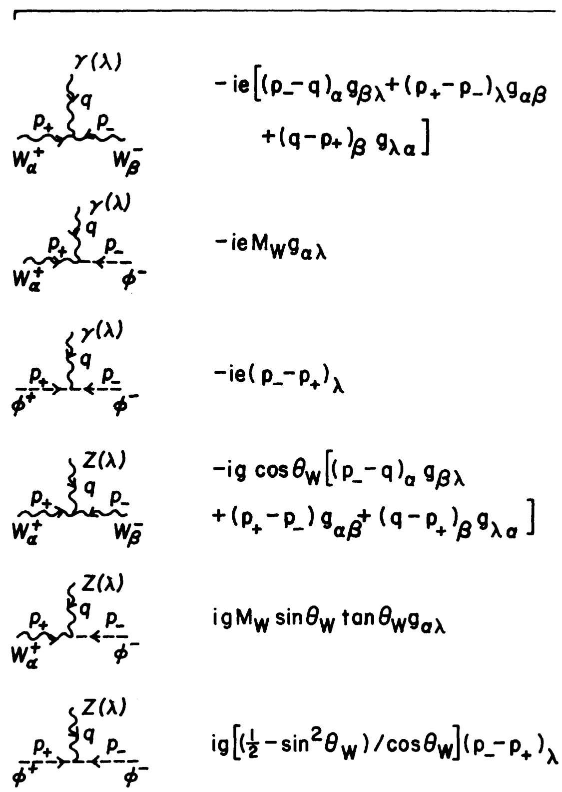

FIG. 11. Trilinear gauge couplings and the corresponding couplings with W bosons replaced by the unphysical Higgs scalars.

Many of our calculations are performed in the ξ gauge. In this general class of linear gauges, the propagators for the $W^{\pm}$ bosons and the unphysical Higgs scalars $\phi^{\pm}$ are given by

$$i\Delta_W^{\lambda\nu}(k,\xi) = -i[(g^{\lambda\nu} - k^\lambda k^\nu/M_W{}^2)(k^2 - M_W{}^2)^{-1} + (k^\lambda k^\nu/M_W{}^2)(k^2 - M_W{}^2\xi^{-1})^{-1}] \quad \text{(A37)}$$

and

$$i\Delta_\phi(k,\xi) = i(k^2 - M_W{}^2\xi^{-1})^{-1}. \quad \text{(A38)}$$

B. Induced $\mu e\gamma$ vertex

Because of the GIM cancellation mechanism discussed in the text, all the μe transitions proceed through the heavy leptons, which in the limit of $m_e = 0$ couple to the electron only through the right-handed currents. The most general form of the $\mu e\gamma$ transition vertex is given by [compare with Eq. (4.1)]

$$T_\lambda = \bar{u}_e(p')(1-\gamma_5)[im_\mu f_M \sigma_{\lambda\nu} q^\nu + f_E(\gamma_\lambda q^2 - q\cdot\gamma q_\lambda)]u_\mu(p), \quad \text{(B1)}$$

where $q = p - p'$. Current conservation requires that the "total charge" must be zero. In calculating the higher-order weak contributions to T_λ, we may ignore the diagrams [Figs. 2(b) and 2(c)] where the photon (real or virtual) is attached to the external fermion lines. They contribute to the μe charge, hence must be canceled by the corresponding contributions coming from other diagrams. Basically we must calculate the four diagrams in Fig. 12. We have calculated the f_M term in the general ξ gauges. The contributions due to each of the diagrams in Fig. 12 for the dominant right-right (RR) and left-right (LR) couplings are listed below: The (RR) amplitudes:

$$f_M(a) = 1 + 2u(\xi) - v(\xi)/3, \quad \text{(B2)}$$

$$f_M(b) = -5\xi/3, \quad \text{(B3)}$$

$$f_M(c) = 5\xi/6 - 7u(\xi)/3 + 4v(\xi)/3, \quad \text{(B4)}$$

$$f_M(d) = 5\xi/6 + u(\xi)/3 - v(\xi). \quad \text{(B5)}$$

The (LR) amplitudes:

$$f_M(a) = -6 - 4u(\xi) + 2v(\xi), \quad \text{(B6)}$$

$$f_M(b) = 2\xi, \quad \text{(B7)}$$

$$f_M(c) = -\xi + 2u(\xi) - v(\xi), \quad \text{(B8)}$$

$$f_M(d) = -\xi + 2u(\xi) - v(\xi). \quad \text{(B9)}$$

In each of the above expressions we have taken out a common factor of

$$(\kappa/2) = (e/32\pi^2)(g^2/8M_W{}^2) \times[\sin\phi\cos\phi(m_1{}^2 - m_2{}^2)/M_W{}^2]; \quad \text{(B10)}$$

u and v are the combinations of

$$u(\xi) = \xi/(\xi-1)[1 - \ln\xi/(\xi-1)], \quad \text{(B11)}$$

$$v(\xi) = \xi\ln\xi/(\xi-1). \quad \text{(B12)}$$

Thus we have

$$f_M = f_M(RR) + f_M(LR) = (1-6)\kappa/2 = -5\kappa/2. \quad \text{(B13)}$$

We have not completed our calculations of the μe "charge radius" f_E. However, Petcov[31] has calculated it in the standard V-A theory (in the 't Hooft gauge). Since the calculation with (LL) couplings is identical to the (RR) couplings and because for this amplitude the (LR) contribution is smaller by a factor of (m_μ/m_i), we can directly use his result to obtain

$$f_E = 2\kappa, \quad \text{(B14)}$$

where κ is given by Eq. (B10).

C. Induced μeZ vertex

Here we shall compute the one-loop induced μeZ vertex and keep only the leading γ_λ term.

Consider the Z-exchange contribution to the process $\mu e \to ee$ as shown in Fig. 13. We shall define the μeZ effective coupling G^Z as

$$\mathfrak{M}^Z(\mu e \to ee) = -iG^Z[\bar{e}(1-\gamma_5)\gamma_\lambda\mu](ig/\cos\theta_W) \times(-\tfrac{1}{2} + \sin^2\theta_W)(\bar{e}\gamma^\lambda e)], \quad \text{(C1)}$$

where we have taken the momentum transfer to be

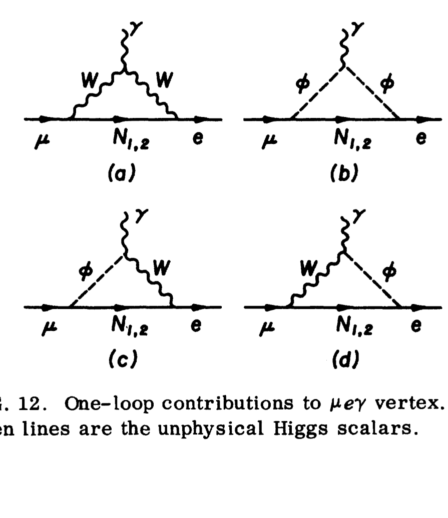

FIG. 12. One-loop contributions to $\mu e\gamma$ vertex. The broken lines are the unphysical Higgs scalars.

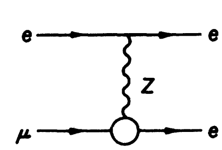

FIG. 13. The effective μeZ vertex in the process $\mu e \to ee$.

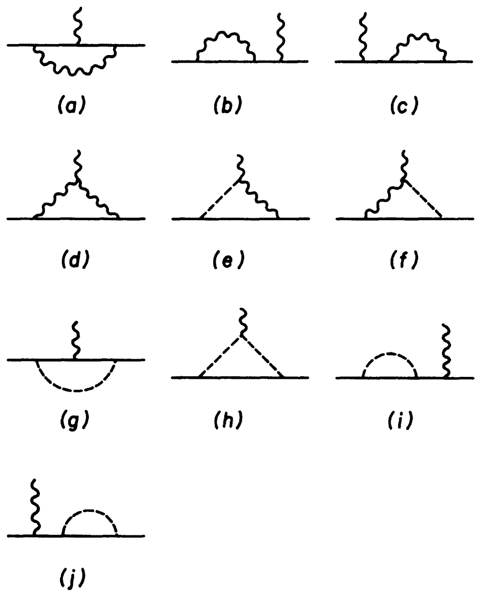

FIG. 14. One-loop contributions to the effective $\mu e Z$ vertex.

small compared to M_Z^2. The one-loop diagrams that contribute to G^Z are shown in Fig. 14. Since we are only interested in the leading contribution to the γ_λ term,[34] we are allowed to set the external momenta equal to zero in these loop integrands. This considerably simplified the computation. The results for each diagram are listed below. [In obtaining the final expression in Eq. (C2) we have distinguished two cases: (i) $m_1 \approx m_2$. In this case the m in the ln factor should be the average heavy-lepton mass and the constant factor is $\frac{3}{2}$; (ii) $m_1 \gg m_2$. They are $\ln(m_1^2/M_W^2)$ and $\frac{1}{2}$, respectively.] The results are

$$G^Z(a) = \ln(m^2/M_W^2) + \begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \end{pmatrix}, \tag{C2}$$

$$G^Z(b) + G^Z(c) = -\tfrac{1}{2} + \sin^2\theta_W, \tag{C3}$$

$$G^Z(d) = 3\cos^2\theta_W, \tag{C4}$$

$$G^Z(e) + G^Z(f) = 2\sin^2\theta_W, \tag{C5}$$

$$G^Z(g) = O[(m^2/M_W^2)\ln(m^2/M_W^2)], \tag{C6}$$

$$\begin{aligned} G^Z(h) = (-\tfrac{1}{2} + \sin^2\theta_W)\pi^{-(2-d/2)}\Gamma(2-d/2) \\ \times \int_0^1 d\alpha(1-\alpha)[(1-\alpha)M_W^2 \\ + \alpha m^2 - \alpha^2 m_\mu^2]^{-(2-d/2)}, \end{aligned} \tag{C7}$$

$$G^Z(i) + G^Z(j) = -G^Z(h).$$

Diagrams (h), (i), and (j) are individually divergent (we have used a dimensional regularization procedure with d = dimension). But both the divergent and constant terms cancel in the sum of these three graphs. In the above expressions we have factored out a constant $g^3 \sin\phi \cos\phi (m_1^2 - m_2^2) \times (64\pi^2 \cos\theta_W M_Z^2)^{-1}$.

The sum of Eqs. (C2)–(C5) is then[35]

$$G^Z = 2\cot\theta_W \kappa\left[\ln(m^2/M_W^2) + \begin{pmatrix} 4 \\ 3 \end{pmatrix}\right], \tag{C8}$$

where

$$\kappa = (e/16\pi^2)(g^2/8M_W^2)[\sin\phi\cos\phi(m_1^2 - m_2^2)/M_W^2]. \tag{C9}$$

D. Box diagrams

Consider the four-fermion process $1 + 3 \to 2 + 4$ by exchanging two W bosons, Fig. 15. We are interested in the limit where all the external momenta are small[34] (compared with M_W and masses of the heavy fermions that appear in the internal lines). With this approximation, this diagram can be easily calculated in the 't Hooft gauge[57]:

$$\begin{aligned} B(x,y) = -i(ig/\sqrt{2})^4 \int \frac{d^4k}{(2\pi)^4} [\bar{u}_R(4)\gamma^\nu(\gamma\cdot k + m_x)\gamma^\rho u_R(3)] \\ \times [\bar{u}_R(2)\gamma_\rho(\gamma\cdot k + m_y)\gamma_\nu u_R(1)]\left(\frac{-i}{k^2 - M_W^2}\right)^2\left(\frac{i}{k^2 - m_x^2}\right)\left(\frac{i}{k^2 - m_y^2}\right), \end{aligned} \tag{D1}$$

$$\begin{aligned} B(x,y) = (-ig^4/64\pi^4)\int d^4k(k^2/4)(k^2 - M_W^2)^{-2}(k^2 - m_x^2)^{-1}(k^2 - m_y^2)^{-1} \\ \times [\bar{u}(4)\gamma^\nu\gamma^\lambda\gamma^\rho\tfrac{1}{2}(1+\gamma_5)u(3)][\bar{u}(2)\gamma_\rho\gamma_\lambda\gamma_\nu\tfrac{1}{2}(1+\gamma_5)u(1)]. \end{aligned} \tag{D2}$$

After making the Wick rotation we can reduce the momentum integration to a simple form that can be integrated explicitly:

$$\begin{aligned} \int d^4k(k^2/4)(k^2 - M_W^2)^{-2}(k^2 - m_x^2)^{-1}(k^2 - m_y^2)^{-1} &= -(i\pi^2/4M_W^2)\int_0^\infty dt(t+1)^{-2}(t+x)^{-1}(t+y)^{-1} \\ &= -(i\pi^2/4M_W^2)I(x,y), \end{aligned} \tag{D3}$$

where $x = m_x^2/M_W^2$, $y = m_y^2/M_W^2$, and

$$I(x,y) = [J(x) - J(y)]/(x-y) \tag{D4}$$

with

$$J(x) = (1-x)^{-1} + (x^2 \ln x)/(1-x)^2 . \tag{D5}$$

The Dirac matrices can be simplified by the identity

$$\gamma^\nu\gamma^\lambda\gamma^\rho = g^{\nu\lambda}\gamma^\rho + g^{\lambda\rho}\gamma^\nu - g^{\nu\rho}\gamma^\lambda - i\epsilon^{\nu\lambda\rho\sigma}\gamma_5\gamma_\sigma . \tag{D6}$$

Thus we have

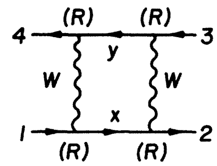

FIG. 15. The box diagram with intermediate fermions x and y.

$$[\gamma^\nu\gamma^\lambda\gamma^\rho(1+\gamma_5)/2][\gamma_\rho\gamma_\lambda\gamma_\nu(1+\gamma_5)/2] = 10[\gamma^\lambda(1+\gamma_5)/2][\gamma_\lambda(1+\gamma_5)/2] + \epsilon^{\nu\lambda\rho\sigma}\epsilon_{\rho\lambda\nu\tau}[\gamma_5\gamma_\sigma(1+\gamma_5)/2][\gamma_5\gamma^\tau(1+\gamma_5)/2] . \tag{D7}$$

Using

$$\epsilon^{\nu\lambda\rho\sigma}\epsilon_{\rho\lambda\nu\tau} = -6 g^\sigma_\tau , \tag{D8}$$

$$\gamma_5(1+\gamma_5) = (1+\gamma_5) , \tag{D9}$$

we have for the particular box diagram in Fig. 15

$$B(x,y) = -g^4(64\pi^2 M_W^2)^{-1} I(x,y)[\bar{u}(4)\gamma^\lambda \tfrac{1}{2}(1+\gamma_5)u(3)][\bar{u}(2)\gamma_\lambda \tfrac{1}{2}(1+\gamma_5)u(1)] . \tag{D10}$$

Next we shall classify four types of box diagrams as shown in Fig. 16 depending on their couplings and on the relative direction of momentum flows in the intermediate fermion lines.[58] The general results can then be written as

$$B(x,y) = -\zeta(g^2/16\pi^2)(g^2/8M_W^2)I(x,y) \times [\gamma^\lambda \tfrac{1}{2}(1+\eta\gamma_5)][(1-\gamma_5)\gamma_\lambda] . \tag{D11}$$

The two parameters ζ and η take on the following values for the four types of diagrams shown in Fig. 16:

$$\zeta(a) = -\zeta(d) = 1, \quad \zeta(b) = -\zeta(c) = 4,$$
$$\eta(a) = \eta(c) = 1, \quad \eta(b) = \eta(d) = -1. \tag{D12}$$

They are determined by the relations in Eqs. (D8) and (D9) and

$$\epsilon^{\nu\lambda\rho\sigma}\epsilon_{\nu\lambda\rho\tau} = 6 g^\sigma_\tau , \tag{D13}$$

$$\gamma_5(1-\gamma_5) = -(1-\gamma_5) . \tag{D14}$$

The overall minus sign for $\zeta(c)$ and $\zeta(d)$ reflects the direction of fermion flow in these diagrams.

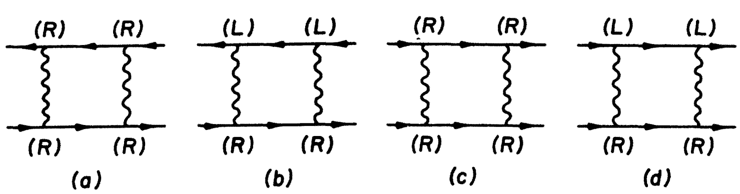

FIG. 16. Four types of box diagrams.

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†Work supported by the Energy Research and Development Administration.

[1]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[2]T. P. Cheng and L.-F. Li, Phys. Rev. Lett. 38, 381 (1977).

[3]T. P. Cheng and L.-F. Li, Univ. of Missouri—St. Louis and Carnegie-Mellon report in the Proceedings of the Coral Gables Conference, 1977, edited by A. Perlmutter (Plenum, New York, to be published).

[4]Many theorists have investigated these processes before the discovery of the two neutrinos. See in particular the work on $\mu\to e\gamma$ by G. Feinberg, Phys. Rev. 110, 1482 (1958) [Independent work on the same subject was reported by R. P. Feynman and M. Gell-Mann, at the meeting of the American Physical Society, 1957 (unpublished).] Subsequent papers on $\mu\to e\gamma$: P. Meyer and G. Salzman, Nuovo Cimento 14, 4214 (1959); G. Feinberg, P. Kabir, and S. Weinberg, Phys. Rev. Lett. 3, 527 (1959); S. P. Rosen, Nuovo Cimento 15, 7 (1960); M. E. Ebel and F. J. Ernst, *ibid.* 15, 173 (1960); B. L. Ioffe, Zh. Eksp. Teor. Fiz. 38, 1608 (1960) [Sov. Phys.—JETP 11, 1158 (1960)]. On μe conversion in a nucleus: S. Weinberg and G. Feinberg, Phys. Rev. Lett. 3, 111 (1959); N. Cabibbo and R. Gatto, Phys. Rev. 116, 1134 (1959); F. J. Ernst, Phys. Rev. Lett. 5, 478 (1960); J. Dreitlein and H. Primakoff, Phys. Rev. 126, 375 (1962). On $\mu\to 3e$: M. Bander and G. Feinberg, *ibid.* 119, 1427 (1960). For a review of these works and the experimental status up to the early 1970's see S. Frankel in *Muon Physics,* edited by V. W. Hughs and C. S. Wu (Academic Press, New York, 1974), Vol. II.

[5]Since the discovery of the two neutrinos, these processes have also been discussed in the literature; see for example N. Nakagawa, H. Okonogi, S. Sakata, and A. Royoda, Prog. Theor. Phys. 30, 727 (1963); B. Pontecorvo, Zh. Eksp. Teor. Fiz. 34, 247 (1958) [Sov. Phys.-JETP 7, 172 (1958)]; 53, 1717 (1967) [26, 984 (1968)]; P. Minkowski, SIN report, 1971 (unpublished); H. Primakoff and S. P. Rosen, Phys. Rev. D 5, 1784 (1972); S. Eliezer and D. Ross, *ibid.* 10, 3088 (1974); H. Fritzsch and P. Minkowski, Phys. Lett. 62B, 72 (1976); T. P. Cheng, Phys. Rev. D 14, 1367 (1976); A. Mann and H. Primakoff, *ibid.* 15, 655 (1977); S. M. Bilenky and B. Pontecorvo, Phys. Lett. 61B, 248 (1976); S. Barshay, *ibid.* 63B, 466 (1976); S. T. Petcov, Joint Inst. of Nuclear Research report, 1976 (unpublished).

[6]For very recent discussions of muon-number-nonconservation effects see F. Wilczek and A. Zee, Phys. Rev. Lett. 38, 531 (1977); J. D. Bjorken and S. Weinberg, *ibid.* 38, 622 (1977); W. Marciano and A. Sanda, *ibid.* 38, 1512 (1977); W. K. Tung, Illinois Inst. of Tech. report, 1977 (unpublished); H. T. Nieh, Phys. Rev. D 15, 3413 (1977); Report No. ITP-SB-77-12, 1977 (unpublished); S. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D 16, 152 (1977); T. Kaneko, Meijo Univ. report, 1977 (unpublished); V. Barger and D. V. Nanopoulos, Univ. of Wisconsin report, 1977 (unpublished); S. Barshay, Université Louis Pasteur report, 1977 (unpublished); J. E. Kim, Brown Univ. report, 1977 (unpublished); D. A. Dicus and V. L. Teplitz, Univ. of Texas and VPI report, 1977 (unpublished); H. Fritzsch, Calif. Inst. of Tech. report, 1977 (unpublished); B. W. Lee, S. Pakvasa, R. E. Shrock, and H. Sugawara, Phys. Rev. Lett. 38, 937 (1977); 38, 1230(E) (1977); E. Ma and S. Pakvasa, Univ. of Hawaii report, 1977 (unpublished); S. Glashow, Harvard Univ. report, 1977 (unpublished); J. D. Bjorken, K. Lane, and S. Weinberg, Phys. Rev. D 16, 1474 (1977).

[7]Bjorken and Weinberg, Ref. 6.

[8]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); Phys. Rev. D 5, 1412 (1972); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity* (Nobel Symposium No. 8), edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[9]There are situations where there is only a "logarithmic GIM cancellation." For example, the one-loop *sd* charge radius is suppressed by $G_F \sin\theta_c \cos\theta_c \ln(m_c^2/m_\mu^2)$. See, for example, Gaillard and Lee (Ref. 31).

[10]J. D. Bjorken and S. L. Glashow, Phys. Lett. 11, 255 (1964).

[11]Mann and Primakoff (Ref. 5) have estimated the upper bound for neutrino mass difference $(m_{\nu_1}{}^2 - m_{\nu_2}{}^2)^{1/2} \lesssim 5$ eV. This is based on the nonobservation of neutrino oscillation effects in HPWF neutrino experiments. The distance between the source of roughly 20-GeV neutrinos and the detector is ≈ 2 km.

[12]In Ref. 3 we have discussed this exercise in some detail. Many other authors have calculated the branching ratio of $\mu\to e\gamma$ based on the idea of intermixing neutrinos. The first attempt dates back to 1963: Nakagawa *et al.* (Ref. 5) obtained a BR $\simeq 10^{-17}$ for $\Delta m_\nu{}^2 \simeq 1$ MeV ^{2} and $M_W \simeq 1$ GeV. A recent calculation is by Mann and Primakoff (Ref. 5). The first such attempt in the context of a gauge theory is that of Eliezer and Ross (Ref. 5). The very recent calculation by Petcov (Ref. 5). agrees with our result. For some comments about the relevance of such a mechanism for $\mu\to e\gamma$ in a theory with $V+A$ currents, see T. P. Cheng (Ref. 5, footnote 35).

[13]F. Wilczek, A. Zee, R. Kingsley, and S. Treiman, Phys. Rev. D 12, 2765 (1975); H. Fritzsch, M. Gell-Mann, and P. Minkowski, Phys. Lett. 59B, 256 (1975); A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 3589 (1975); S. Pakvasa, W. A. Simmons, and S. F. Tuan, Phys. Rev. Lett. 35, 702 (1975).

[14]See, for examples, reviews by A. Mann, in *Proceedings of the XVIII International Conference on High Energy Physics. Tbilisi, U.S.S.R., 1976,* edited by N. N. Bogolubov *et al.* (JINR, Dubna, 1977); R. M. Barnett, in *Particles and Fields '76,* proceedings of the Annual Meetings of the Division of Particles and Fields of the American Physical Society, Brookhaven National Laboratory, edited by H. Gordon and R. F. Peierls (BNL, Upton, New York, 1977), p. D77.

[15]This is based on the presumption that the 1.8-GeV charmed quark and the 1.9-GeV heavy lepton U [M. Perl *et al.*, Phys. Rev. Lett. 35, 1489 (1975)] represent the mass scale for heavy elementary fermions.

[16]S. L. Glashow and S. Weinberg, Phys. Rev. D 15, 1958 (1977); see also E. Paschos, *ibid.* 15, 1966 (1977).

[17]We explicitly do not consider the question of flavor-changing effects mediated by physical Higgs. Restriction on such interactions are discussed in Ref. 16. See, however, Bjorken and Weinberg (Ref. 6). It should should be noted that the model discussed in this paper has only one scalar doublet [see Eq. (A6)] and the

physical Higgs couplings do not change the flavors of charged leptons.

[18]S. M. Korenchenko *et al.*, Zh. Eksp. Teor. Fiz. 70, 3 (19xx) [Sov. Phys.-JETP 43, 1 (1976)].

[19]D. A. Bryman *et al.*, Phys. Rev. Lett. 28, 1469 (1972).

[20]While in our model this requirement is satisfied through condition (2b), in one of the models proposed by Wilczek and Zee (Ref. 6) in which it is satisfied through (2a) by having e_R and μ_R couple to doubly charged leptons.

[21]The idea that there may be right-handed electron and muon doublets has of course occured to many authors. In fact our model is simply the (2-2) model as classified by J. D. Bjorken and C. H. Llewellyn Smith, Phys. Rev. D 7, 887 (1973). They have also been discussed by authors in connection with parity violation in atomic physics; see, for example, R. M. Barnett, SLAC Report No. SLAC-PUB-1821, 1976 (unpublished).

[22]P. E. G. Baird *et al.*, Nature (London) 284, 528 (1976).

[23]Bjorken, Lane, and Weinberg, Ref. 6.

[24]H. T. Nieh, Ref. 6.

[25]See, for example, D. Bailin and N. Dombey, Phys. Lett. 64B, 304 (1976); A. Sirlin, Nucl. Phys B71, 29 (1974).

[26]See, for example, J. D. Bjorken and C. H. Llewellyn Smith, Ref. 21; Treiman, Wilczek, and Zee, Ref. 6; R. M. Barnett, Talk at the Coral Gables Conference, 1977 (unpublished); J. Rosner, *ibid.* (unpublished).

[27]For similar calculations in modern gauge theories, see R. Schrock, Phys. Rev. D 9, 743 (1974); R. Bertlmann, H. Gross, and B. Lautrup, Nucl. Phys. 373, 523 (1974); S.-Y. Pi and J. Smith, Phys. Rev. D 9, 1498 (1974); M. Ahmed and G. Ross, Phys. Lett. 59B, 293 (1975); N. Vasanti, Phys. Rev. D 13, 1889 (1976); F. Wilczek and A. Zee, Nucl. Phys. B78, 461 (1976); H. Fritzsch and P. Minkowski, Cal Tech. Report No. CalT-68-538 (unpublished).

[28]A model-independent analysis of *CP*-violating decay correlations in $\mu \to e\gamma$ has been carried out by W. K. Tung (Ref. 6). Also, Treiman, Wilczek, and Zee (Ref. 6) have pointed out that in this and many other gauge theory models and *CP*-violating effects will be suppressed by the electron mass.

[29]K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D 6, 2923 (1972); Y.-P. Yao, *ibid.* 7, 1647 (1973).

[30]Feinberg, Feynman, and Gell-Mann (Ref. 4).

[31]This agrees with the independent calculation by Petcov (Ref. 5), who computed the (LL) amplitude of $\mu \to e\gamma$ via mixing neutrinos.

[32]Swiss Institute of Nuclear Research Physics Report No. 1, 1976 (unpublished).

[33]Similar types of diagrams have been calculated in the context of rare decays of the kaons. M. K. Gaillard and B. W. Lee, Phys. Rev. D 10, 897 (1974); M. K. Gaillard, B. W. Lee, and R. Shrock, *ibid.* 13, 2674 (1976).

[34]The contribution due to μe transition magnetic moment term will be down by powers of M_W^2.

[35]The upper constant corresponds to the case of $m_1 \approx m_2$ and the lower one corresponds to $m_1 \gg m_2$. This convention will be followed throughout this paper.

[36]The contribution coming from the one-photon exchange agrees, except for minor differences, to that as calculated by Bander and Feinberg (Ref. 4).

[37]G. Feinberg and S. Weinberg, Phys. Rev. Lett. 6, 381 (1961).

[38]B. Pontecorvo, Zh. Eksp. Teor. Fiz. 33, 549 (1957).

[39]G. Feinberg and S. Weinberg, Phys. Rev. 123, 1439 (1961).

[40]This result agrees with the corresponding K_1-K_2 mass-difference calculation by Gaillard and Lee (Ref. 33).

[41]See, for example, S. Frankel (Ref. 4).

[42]We would like to thank Professor S. Weinberg and Professor L. Wolfenstein for emphasizing to us the importance of this process, and for useful discussions.

[43]See, for example, experimental proposals at SIN (Ref. 2).

[44]Weinberg and Feinberg (Ref. 4).

[45]M. Barnett, Phys. Rev. Lett. 34, 41 (1975); Phys. Rev. D 11, 3426 (1975); 13, 671 (1976); F. Gürsey and P. Sikivie, Phys. Rev. Lett. 36, 775 (1976); P. Ramond, Nucl. Phys. B110, 214 (1976).

[46]Our results Eqs. (5.28) to (5.32) agree exactly with those obtained by E. C. Poggio and H. J. Schnitzer [Phys. Rev. D 15, 1973 (1977)] and, in appropriate limits, to those of Gaillard and Lee (Ref. 33). See also Barger and Nanopoulos (Ref. 6).

[47]The corresponding matrix element for K_S is zero.

[48]See, for example, E. Eichten *et al*. Phys. Rev. Lett. 34, 369 (1975); 36, 500 (1976).

[49]The corresponding matrix element for K_L and π° is zero.

[50]The alternative flavor-changing mechanism in gauge theory is via the coupling of the *physical* Higgs scalars. (This usually requires more complicated Higgs structure than is required for spontaneous symmetry breakings.) Thus both mechanisms actually spring from the same source: off-diagonal Yukawa couplings. This similarity is all the more apparent if one expresses our mass terms as "tadpole couplings"; see Fig. 9.

[51]S. Parker, H. L. Anderson, and C. Rey, Phys. Rev. 133, B768 (1964).

[52]A. R. Clark *et al.*, Phys. Rev. Lett. 26, 1667 (1971).

[53]E. W. Beier *et al.*, Phys. Rev. Lett. 29, 678 (1972).

[54]We shall follow all the conventions as used by J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[55]Gaillard and Lee (Ref. 33); also Gaillard, Lee, and Shrock (Ref. 33).

[56]Alternatively this can arise from the vacuum expectation values of singlet Higgs scalars.

[57]The contributions due to unphysical Higgs scalar exchanges are less important than two W exchanges.

[58]Namely, the relative signs of the momentum k in fermion propagators.