Estimation of the annihilation component in $\bar{p}p$ interactions

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We derive formulas to estimate the inclusive distribution of charged pions due to annihilation which utilize $\bar{p}p$-pp differences in a way that does not violate charge symmetry.

I. INTRODUCTION

At low energies, explicit identification of the annihilation channels in $\bar{p}p$ interactions is possible though difficult.[1] Because of the large number of ambiguities, at energies over 20 GeV/c, direct identification of annihilation channels in a bare bubble chamber is no longer possible. Various efforts have been made to estimate the annihilation topological cross sections by relating them to the difference in $\bar{p}p$ and pp cross sections.[2] This approach is motivated by the belief that the excess in $\bar{p}p$ cross sections over the pp cross sections is due to the number of extra final states available in $\bar{p}p$ that are forbidden in pp, namely the annihilation channels. For total cross sections, at high energies, this approach works reasonably well. The difference is

$$\sigma_{\text{tot}}(\bar{p}p) - \sigma_{\text{tot}}(pp) \propto s^{-0.6} ,$$

as would be expected from unitarity considerations, if the difference was entirely due to annihilations.[3]

II. INCLUSIVE DISTRIBUTIONS

The subtraction method works reasonably well for neutral-particle inclusive distributions, e.g. π^0's.[4] The excess in π^0 rapidity distributions is concentrated in the central region, as would be expected from annihilations. We will now prove that the naive subtraction between $\bar{p}p$ and pp inclusive cross sections when applied to charged pions violates charge symmetry. Symmetry considerations, however, suggest alternate formulas, which show good agreement with annihilation data at 12 GeV/c.[1]

Define the inclusive invariant cross sections:

$$\begin{aligned} &E\frac{d^3\sigma}{d^3p}(pp\to\pi^+ + X) \equiv A_1 ,\\ &E\frac{d^3\sigma}{d^3p}(pp\to\pi^- + X) \equiv A_2 ,\\ &E\frac{d^3\sigma}{d^3p}(\bar{p}p\to\pi^+ + X) \equiv B_1 ,\\ &E\frac{d^3\sigma}{d^3p}(\bar{p}p\to\pi^- + X) \equiv B_2 ,\\ &E\frac{d^3\sigma}{d^3p}(\bar{p}p\to\pi^+ + X)_{\text{annihilations}} \equiv C_1 ,\\ &E\frac{d^3\sigma}{d^3p}(\bar{p}p\to\pi^- + X)_{\text{annihilations}} \equiv C_2 . \end{aligned} \tag{1}$$

A_1, A_2, B_1, B_2, C_1, C_2 are functions of $\vec{p}$ the center-of-mass momentum of the pion, and s, the overall center-of-mass energy squared. Let P denote the inversion operator defined by

$$PA_1(\vec{p}, s) \equiv A_1(-\vec{p}, s) = A_1(\vec{p}, s) ,$$

the last equation following from symmetry in pp interactions. We denote the last equation briefly as

$$PA_1 = A_1; \tag{2}$$

similarly,

$$PA_2 = A_2 . \tag{3}$$

For $\bar{p}p$, however,

$$PB_1 = B_2 \text{ and } PB_2 = B_1 ,$$

and

$$PC_1 = C_2 \text{ and } PC_2 = C_1 \tag{4}$$

for annihilations. These equations follow from C invariance in $\bar{p}p$ interactions.

The operators 1, P form an Abelian group[5] and therefore have one-dimensional irreducible representations only. The one-dimensional representations for P are clearly $+1$ and -1. The two A's belong to the representation $+1$, i.e., they are even under inversion. The B's and C's are a mixture of odd and even. However, one can form the linear combinations B_1+B_2 and B_1-B_2 which are even (belong to +1) and odd (belong to −1), respectively. Similarly C_1+C_2 is even under inversion and C_1-C_2 is odd.

To estimate C_1+C_2 by the subtraction method, one may write

$$C_1+C_2=(B_1+B_2)-(A_1+A_2)\ . \quad (5)$$

Equation (5) is valid from a charge-symmetry point of view since both sides of the equation are even. One can motivate (5) further by analyzing $\bar{p}p\to\pi^{\pm}$ in terms of a four-component model involving beam and target fragmentation, central production, and annihilation and relating all the components except annihilation to those in pp using charge symmetry.[6]

However, C_1-C_2 is odd and cannot be expressed as a difference involving the A's since the A's are pure even. Thus C_1-C_2 cannot be found by subtraction.[7] Hence C_1 and C_2 cannot be found individually. Q.E.D.

For π^0's, however, since the π^0 is its own antiparticle, $C_1=C_2$ and $C_1-C_2\equiv 0$. Equation (5) alone is sufficient to yield the π^0 information. If the π^0 information were available by subtraction, one would think that the $\pi^{\pm}$ information would also be forthcoming. If one hypothesizes this to be so, the only remedy to the situation would be to write an expression for C_1-C_2 that does not involve the A's. The substitution $C_1-C_2=B_1-B_2$ is patently incorrect, since this would imply that the nonannihilation components $N_1\equiv B_1-C_1$ and $N_2\equiv B_2-C_2$ into π^+ and π^-, respectively, were equal throughout phase space. This is not the case at 12 GeV/c.[1] The only symmetric relation between the C's, B's, and N's that one can write that is odd under inversion is

$$\frac{C_1-C_2}{C_1+C_2}=\frac{B_1-B_2}{B_1+B_2}=\frac{N_1-N_2}{N_1+N_2}\ . \quad (6)$$

Each component of Eq. (6) is odd under inversion, so (6) is a valid equation as far as charge symmetry is concerned. Equation (6) implies that the percentage excess of π^+ over π^- in any part of phase space is independent of whether it is annihilation or nonannihilation. This may be intuitively understood if one pictures the pion being emitted by a fireball that has an asymmetric charge distribution; the π^+'s are emitted preferentially from regions of excess positive charge with the additional proviso that the emission of any single pion is decoupled from the decay of the rest of the fireball. However, its ultimate justification must come from the data.

Combining (5) and (6) leads to

$$C_1=\left[\frac{(B_1+B_2)-(A_1+A_2)}{(B_1+B_2)}\right]B_1 \quad (7a)$$

and

$$C_2=\left[\frac{(B_1+B_2)-(A_1+A_2)}{(B_1+B_2)}\right]B_2\ . \quad (7b)$$

Note that under inversion, each side of (7a) goes into the corresponding side of (7b).

III. COMPARISON WITH DATA

The A's, B's, and C's can be expressed as a function of x and p_T^2:

$$\frac{Ed^3\sigma}{d^3p}=\frac{2Ed^2\sigma}{\pi\sqrt{s}dxdp_T^2}\ .$$

Statistics permit comparison only after p_T^2 integration. Since Eq. (5) is linear in the cross sections, it remains valid for the integrated cross sections. However, Eq. (6) is nonlinear and one requires empirical arguments to show that the integrated quantities may be used there as well.

One may write

$$B_1(x,p_T^2)=B_1'(x)e^{-\beta(x)p_T^2}\ ,$$
$$B_2(x,p_T^2)=B_2'(x)e^{-\beta'(x)p_T^2}\ . \quad (8)$$

It is found experimentally that the mean values of p_T^2 as a function of x for π^+ and π^- are identical in $\bar{p}p$ interactions at 100 GeV/c[8]; the χ^2/DF for the hypothesis that they are equal is 3.12/6. At 12 GeV/c, the p_T^2 dependences of π^+ and π^- cross sections, integrated over the backward center-of-mass hemisphere, are similar.[1] The assumption that $\langle p_T^2\rangle$ is the same for π^+ and π^- implies that $\beta(x)\equiv\beta'(x)$. Dividing (7a) by (7b) yields $C_1/C_2=B_1/B_2$, which is independent of p_T^2. Hence C_1 and C_2 should also have the same slope function, $\alpha(x)$ in p_T^2. This implies that

$$\frac{C_1-C_2}{C_1+C_2}=\frac{C_1'(x)-C_2'(x)}{C_1'(x)+C_2'(x)}=\frac{B_1-B_2}{B_1+B_2}=\frac{B_1'(x)-B_2'(x)}{B_1'(x)+B_2'(x)}\ , \quad (9)$$

which is independent of p_T^2 and equal to the value obtained if integrated cross sections are used.

Hence one may substitute the integrated cross sections in both (5) and (6) and therefore in (7a) and (7b). Figure 1 is a comparison of the predictions of (7a) and (7b) with the explicit annihilation data at 12 GeV/c.[9] The agreement between the prediction and the data is seen to be excellent. Also shown are curves predicted by the naive subtraction formulas $C_1=B_1-A_1$, $C_2=B_2-A_2$, the first of which disagrees drastically in shape with data in the forward hemisphere. Since the π^+ and π^- data are charge-symmetric, they cross the $x=0$ line

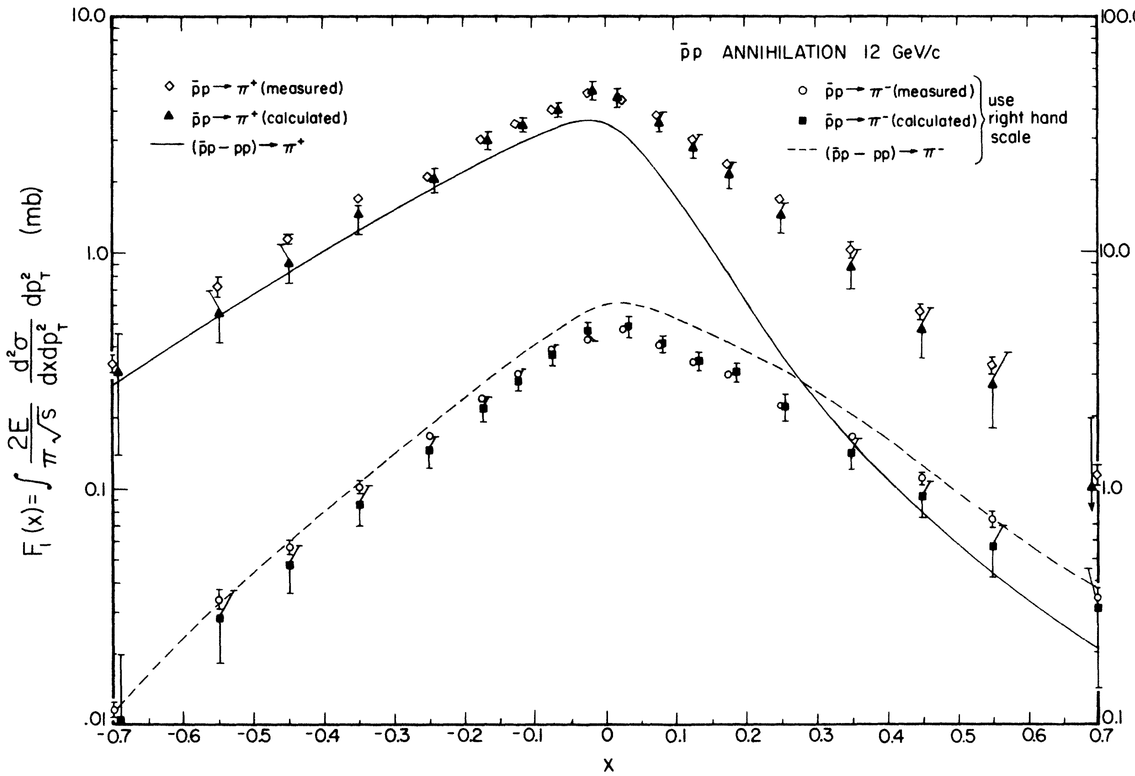

FIG. 1. Comparison of explicit annihilation data at 12 GeV/c with predictions of the derived formulas. Note the different scales for the two sets of data. The curves are the predictions of the charge-symmetry-violating subtraction formulas.

at 4.58 ± 0.11 mb. The dashed curve crosses $x = 0$ at 5.81 ± 0.13 mb and the full curve at 3.54 ± 0.09 mb. The discrepancy between the naive subtraction formulas and the data is thus greatest at $x = 0$. The predictions of (7a) and (7b), on the other hand, cross $x = 0$ at 4.68 ± 0.35 mb, well within errors of the data.

We have thus shown that it is possible to use the integrated quantities

$$F_1(x) = \int \frac{2Ed^2\sigma}{\pi\sqrt{s}\,dx\,dp_T^2}\, dp_T^2$$

in (7a) and (7b). Equally, one may use the integrated quantities $d\sigma/dx$, since for a given x, p_T^2, the energy of a π^- is the same as that of a π^+. It is not immediately obvious that the relations can be used with the quantities $d\sigma/dy$, where y is the center-of-mass rapidity defined as $\frac{1}{2}\ln[(E + p_L)/(E - p_L)]$. The following argument can be used to show that to a good approximation they can:

$$C_1(x, p_T^2) = C_1'(x)e^{-\alpha p_T^2} = \frac{1\, d^2\sigma}{\pi\, dy\, dp_T^2} ,$$

therefore

$$\frac{d\sigma}{dy} = \pi \int_{y\,=\,\mathrm{const}} C_1'(x)e^{-\alpha p_T^2}\, dp_T^2 .$$

For $y = \text{constant}$, $p_T^2 = f(x)$. Using the mean-value theorem, this leads to

$$\frac{d\sigma}{dy} = \pi C_1'(\bar{x}) \int_{y\,=\,\mathrm{const}} e^{-\alpha f(x)} f'(x)\, dx ,$$

where $\bar{x}$ is a value of x in the range of integration. A similar expression follows for π^-. Since the shapes of $C_1'(x)$ and $C_2'(x)$ are similar, the values of $\bar{x}$ for each case will be close to the other. Substitution in (6) shows that to a good approximation, $d\sigma/dy$ can be used in (7a) and (7b).

To conclude, we have derived expressions for the inclusive distributions for charged pions in $\bar{p}p$ annihilation. These formulas contain the π^0 distributions as a special case. It has been shown that naive subtraction results in the loss of charge symmetry. The derived formulas, though to some extent heuristic, show good agreement with experimental data at 12 GeV/c, and may thus be useful in predicting annihilation distributions at higher energies, where explicit annihilation information is unavailable.

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[1]D. Gall *et al.*, in Proceedings of the 1975 Symposium on $\bar{p}p$ interactions, Loma-Koli, p. 414 (unpublished).

[2]J. G. Rushbrooke *et al.*, Phys. Lett. 59B, 303 (1975).

[3]P. D. Ting, Phys. Rev. 181, 1942 (1969).

[4]R. Raja *et al.*, Phys. Rev. D 15, 627 (1977).

[5]The application of group theory, though in this instance trivial, would be essential when more than two symmetry operators are present, e.g. in the case of polarized beams.

[6]It must be pointed out, however, that at 100 GeV/c, subtraction between topological cross sections fails for two prongs; the difference between $\bar{p}p$ and pp two-prong inelastic cross sections is negative. For higher multiplicities, however, the subtraction procedure works well, the parameters estimated using subtraction extrapolating smoothly to lower-energy annihilation data (Ref. 2). However, since Eq. (5) deals with inclusive cross sections, it has very little contribution from low multiplicities and significant contribution from high multiplicities. At 100 GeV/c, the $\bar{p}p$ two-prong inclusive cross section is only 2% of the total inclusive cross section (see Ref. 8). The error introduced in (5) due to two prongs would thus be less than 2% at 100 GeV/c.

[7]The naive subtraction $C_1 = B_1 - A_1$, $C_2 = B_2 - A_2$ implies $C_1 - C_2 = (B_1 - B_2) - (A_1 - A_2)$ and thus involves A's, violating charge symmetry.

[8]The author is grateful to the Cambridge-Fermilab-MSU collaboration for making this information available.

[9]For 12-GeV/c annihilation data, we have averaged the backward π^+'s with the forward π^-'s and vice versa, a valid procedure due to C invariance.