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Quark elastic scattering in gauge theories and large-transverse-momentum hadron production

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Quark elastic scattering in a color gauge theory of quarks and gluons is investigated as a source of large-transverse-momentum hadrons. Nonscaling effects due to asymptotic freedom alone are not helpful in describing the data. Elastic quark-quark form factors derived in color gauge theories do describe the data if supplemented by a phenomenological assumption which probably amounts to antiscreening of the color of quarks in a proton. Transverse-momentum fluctuations of quarks in protons are quite important for understanding the normalization and shape of the theory. Patterns of scaling violations are predicted which may be compared with good future data to test the models.

I. INTRODUCTION

The production of particles at large values of transverse momentum p_T in hadronic reactions is commonly believed to occur through the hard scattering of constituents within the incident hadrons. If these constituents are pointlike quarks, then simple arguments suggest that the invariant single-particle cross section $Ed^3\sigma/d^3p$ should scale as p_T^{-4}.[1] However, at currently attainable energies the experimental data seem to scale roughly as p_T^{-8}.[2] This p_T scaling anomaly can be understood in the constituent-interchange model[3] (CIM) as the scattering of nonelementary constituents, such as mesons in a proton. The meson form factors then change the expected p_T^{-4} to the observed p_T^{-8}.

Recently Field and Feynman and others have formulated a model for large-p_T production that is based on quark-quark scattering and yet avoids the p_T^{-8} anomaly by the introduction of an empirical representation for the quark elastic-scattering cross section.[4] With the use of this one empirical function and with quark and hadron structure functions determined essentially from lepton scattering data, the model is found to successfully describe existing data on the large-p_T production of single particles, jets, and two-particle correlations on the same and opposite sides of the trigger hadron.[5] Indeed, since the Field-Feynman model is so remarkably efficient and accurate in describing the data, that model will be used as a benchmark to be compared with the models in this paper.

If the source of large-p_T particles is quark elastic scattering, then the theoretical origin of the empirical cross section used by Field and Feynman is a very interesting question. Two main approaches toward this question have been taken in the past:

(1) The CIM or some variant model is correctly describing large-p_T production, and the quark elastic-scattering contribution is either temporarily or permanently suppressed for reasons yet to be determined. Possible reasons for this have been investigated by Cahalan, Geer, Kogut, and Susskind[6] and more recently by Cutler and Sivers[7] within the framework of asymptotically free field theories. These investigations differ substantially in their conclusions and in fact both groups give inadequate treatments of several important points. Therefore, in Sec. III I have discussed this approach in rather careful detail, based on considerations from quantum chromodynamics (QCD) for colored quarks and gluons. Unfortunately, this whole approach ignores the question of how to interpret the success of the Field-Feynman model, and the fact that the CIM has yet to be formulated in a manner that accounts for the double-jet structure believed to have been observed in recent experiments.[2,5]

(2) Alternatively, one may insist that quark elastic scattering is the correct approach, but that various scale-breaking effects as seen, for example, in deep-inelastic electroproduction are changing p_T^{-4} to p_T^{-8}. If all the scale-breaking effects are coming from the decay of the final-state quarks to hadrons,[8] for example, the single-jet cross section will reveal the canonical p_T^{-4} (indeed, this possibility is still open experimentally). Other analyses have concluded that large-p_T production can be understood if the scale-breaking in the proton structure functions is parametrized with power-law quark form factors[9] or with logarithmic modifications[10] different from those predicted by asymptotic freedom. Unfortunately, the large-p_T kinematic regime is largely outside the regions where the form factors in both cases were fitted to electroproduction data, and therefore both analyses lack theoretical motivation. Attempts to use the logarithmic scale-breaking corrections predicted by asymptotic freedom[11] do not permit an

understanding of the data (see Sec. III).

In Sec. IV, results for the quark elastic-scattering amplitude calculated within the framework of QCD by Cornwall and Tiktopoulos[12] are used to calculate large-p_T particle production. Within the freedom of a certain phenomenological assumption which probably amounts to antiscreening of the color of quarks in a proton, this model is able to mimic the Field-Feynman results rather closely. It also turns out that the effect of transverse momentum of the quarks in a proton is very significant.

The results of Sec. IV are not theoretically firm and certainly fall short of any hoped-for "justification" of the Field-Feynman approach. It is hoped, however, that these results have enough phenomenological significance to excite better theoretical consideration of the questions raised. In fact, as discussed in Sec. V, this model does predict a certain pattern of scale-breaking effects which could possibly be detected in a careful data analysis.

II. KINEMATICS OF HIGH-p_T SCATTERING

Treatments of the kinematics relevant to large-p_T particle production are widely available,[13] so only a summary of the required formulas and notation will be given here. The invariant cross section for producing a hadron C at large p_T with momentum $\vec{p}$ in the reaction $A+B\to C+X$ is (neglecting transverse momenta)

$$Ed^3\sigma/d^3p=\int^1 dx_a \int^1 dx_b P_{a/A}(x_a)P_{b/B}(x_b)D_{C/c}(z_C) \times\frac{1}{\pi z_C}\,d\sigma/d\hat{t}(\hat{s},\hat{t},\hat{u};ab\to cd)\,. \quad (1)$$

The process is illustrated in Fig. 1. In Eq. (1) the

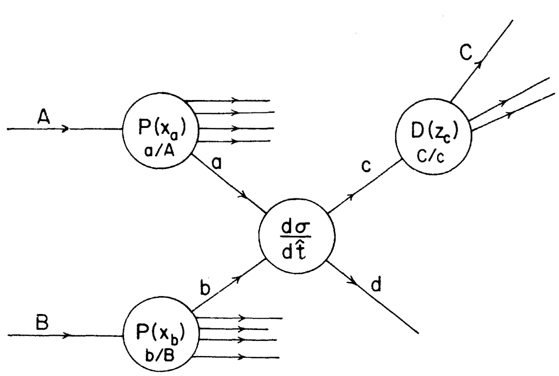

FIG. 1. Illustration of the constituent hard-subscattering process. Constituents a and b of hadrons A and B scatter through a large angle, followed by the fragmentation of the constituent c into hadrons, one of which (C) is detected.

probability functions $P(x)$ for quarks in hadrons and $D(z)$ for hadrons in quarks[14] are taken from Field and Feynman, where these functions were deduced from lepton scattering data. The longitudinal fractions $x_a=p_a/p_A$, $x_b=p_b/p_B$, and $z_C=p_C/p_c$ determine the $ab\to cd$ subreaction kinematics through the relations $\hat{s}=x_ax_bs$, $\hat{t}=x_at/z_C$, and $\hat{u}=x_bu/z_C$, where $s=(p_A+p_B)^2$, $t=(p_A-p_C)^2$, and $u=(p_B-p_C)^2$. The conditions $z_C\le 1$ and $\hat{s}+\hat{t}+\hat{u}=0$ fix the lower limits of integration at $x_a^{\min}=x_1/(1-x_2)$ and $x_b^{\min}=x_ax_2/(x_a-x_1)$, where $x_1=-u/s$ and $x_2=-t/s$. The possibility that the hadron C results from the fragmentation of the quark d is taken into account by carefully pairing the functions $P(x)$ and $D(z)$ with $d\sigma/d\hat{t}$ and $d\sigma/d\hat{u}$.[15]

Dimensional analysis and the absence of mass parameters to set a scale in Eq. (1) yields that if

$$\frac{d\sigma}{d\hat{t}}\sim \hat{s}^{-N/2}g\left(\frac{\hat{t}}{\hat{s}}\right),$$

then

$$E\frac{d^3\sigma}{d^3p}\sim p_T^{-N}f(x_T,\theta_{\text{c.m.}})\,,$$

where $x_T=2p_T/\sqrt{s}$, $\theta_{\text{c.m.}}$ is the scattering angle of hadron C in the $A+B$ center-of-momentum frame, and typically

$$f(x_T,\theta_{\text{c.m.}})\sim(1-x_T)^F$$

for $x_T\sim 1$ and $\theta_{\text{c.m.}}\sim 90°$. Usually the numbers N and F are used to characterize the cross section even for $x_T<1$.[16]

The inclusion of the effects of the transverse momentum k_T of quarks in hadrons (and to less extent, hadrons in quarks) in Eq. (1) is not trivial, but it is necessary to include these effects if any pretense of absolute normalization of the cross section is to be maintained. For calculational purposes, two approaches are used: (a) The integrals in Eq. (1) are evaluated independent of k_T, which is fixed at some average value $\langle k_T\rangle$ with the kinematical variables appropriately modified; (b) A Monte Carlo program[17] is used to perform additional k_T integrations given some model for the k_T dependence in $P(x,k_T)$ and $D(z,k_T)$. Generally, the Monte Carlo integration will give the correct answer if properly used. However, the $\langle k_T\rangle$ method is relatively easy to use and also gives reasonable results, and in any event, the theoretical uncertainty in the k_T effects (see Sec. IV) far exceeds the calculational uncertainty of either method. For these reasons, the $\langle k_T\rangle$ method is used in this paper. The two methods have been compared and generally agree to within about 20–50%.

In the $\langle k_T\rangle$ method, Eq. (1) is calculated using

$$\hat{s} = x_a x_b s - 2\langle k_T^2\rangle,$$

$$\hat{t} = \frac{x_a t}{z_C} + \sqrt{2}\,\langle k_T\rangle p_T,$$

$$\hat{u} = \frac{x_b u}{z_C} + \sqrt{2}\,\langle k_T\rangle p_T,$$

and

$$x_a^{\min} = \frac{x_1 + T}{1 - x_2},$$

$$x_b^{\min} = \frac{x_a x_2 + T}{x_a - x_1},$$

where $\langle k_T\rangle$ is the average transverse momentum of quarks in hadrons, and

$$T = (2\langle k_T^2\rangle - 2\sqrt{2}\,\langle k_T\rangle p_T)/s.$$

If the single particle C produced at large p_T in $A + B \to C + X$ is the product of a jet, one calculates the total jet cross section from Eq. (1) by setting $z = 1$ and $D(z) = 1$, obtaining

$$Ed^3\sigma/d^3p(\text{jet}) = \int^1 dx_a \int^1 dx_b P(x_a)P(x_b)\frac{1}{\pi z_C}\frac{d\sigma}{d\hat{t}}\,\delta(1 - z_C)$$
$$= \int^1 dx_a x_a P(x_a) x_b P(x_b)\frac{1}{\pi(x_a - x_1)}\frac{d\sigma}{d\hat{t}}, \quad (2)$$

where $x_b = x_a x_2/(x_a - x_1)$ and $x_a^{\min}$ is unchanged. Transverse-momentum effects are added by including in the integrand a factor

$$1 + \frac{x_1}{x_a^2 x_2}\frac{T}{s},$$

with $x_a^{\min}$, $\hat{s}, \hat{t}$, and $\hat{u}$ modified as in the single-particle case.

III. THE QCD BORN TERM AND ASYMPTOTIC FREEDOM

A. The QCD Born term

In the dominant hard-scattering subprocess in large-p_T single-particle production is quark-quark scattering, then one should expect[18] that the QCD Born term due to color-octet–gluon exchange (OGX),

$$\frac{d\sigma}{dt} = 2\pi\alpha^2\frac{2}{9}\left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{s}^2\hat{t}^2}\right), \quad (3)$$

will dominate Eq. (3). If the final quarks c and d have the same flavor, the interference term

$$\frac{d\sigma}{d\hat{t}}(\text{int}) = \frac{4\pi\alpha^2}{9}\left(-\frac{2}{3}\frac{1}{\hat{t}\hat{u}}\right)$$

will be included in the following calculations. Simple quark-quark scattering via single gluon exchange cannot be directly relevant to the currently observed large-p_T single-particle production, since Eq. (3) predicts that $Ed^3\sigma/d^3p$ scales as p_T^{-4}, and experimentally the scaling behavior p_T^{-8} is observed. In fact, Eq. (3) with $\alpha = 2$ (typical of a strong coupling constant) yields a cross section that exceeds the Field-Feynman benchmark by an order of magnitude at $p_T = 4$ GeV/c and $\sqrt{s} = 52.8$ (see Fig. 2).

There is an obvious resolution of this dilemma[6]: Asymptotic-freedom (AF) corrections might be suppressing the OGX contribution below the level of the observed cross section, thus unmasking the contributions of other hard-scattering subprocesses (e.g., the CIM-like $qM \to qM$) which naturally scale as p_T^{-8}. If this point of view is correct, then it is of some interest to determine when OGX with AF corrections will exceed extrapolations of the current data. Therefore, the effects of asymptotic freedom on the three components of Eq. (1), $P(x)$, $D(z)$, and $d\sigma/d\hat{t}$, must be considered in turn.

B. Asymptotic-freedom effects on $P(x)$

The structure functions $P(x)$ of quarks in hadrons used to calculate the OGX cross section are those determined by Field and Feynman by analyzing deep-inelastic electron and neutrino scattering. It is now widely believed that the Bjorken-scaling violations observed in deep-inelastic electropro-

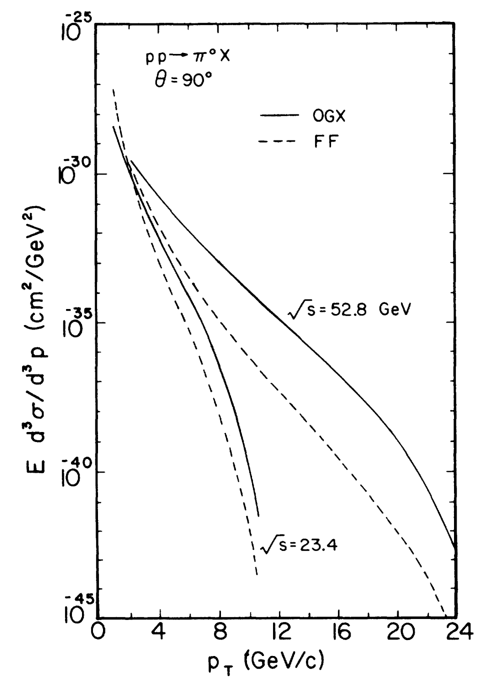

FIG. 2. Invariant cross section for octet-gluon exchange (OGX) with coupling constant $\alpha = 2$ compared with the Field-Feynman model (FF), which accurately represents the data.

duction are accounted for by the predictions of non-Abelian gauge theories of colored quarks and gluons.[11] A detailed analysis of the electroproduction and neutrinoproduction data permits evaluation of $F_2(x,Q^2)=xP(x,Q^2)$ given $F_2(x,Q_0^2)$, where Q_0^2 is some small value (e.g., $Q_0^2 \sim 4$ GeV^2), via the relation

$$\frac{\partial \ln F_2(x,Q^2)}{\partial \ln Q^2} \simeq d(x)$$

and the approximation that $\ln F_2(x,Q^2)$ varies linearly with $\ln Q^2$ at fixed x.[19] Thus

$$\frac{\ln F_2(x,Q^2) - \ln F_2(x,Q_0^2)}{\ln Q^2 - \ln Q_0^2} \simeq d(x). \quad (4)$$

The function $d(x)$ evaluated by Fox with $Q_0^2=4$ GeV^2 and $Q^2=2500$ GeV^2 is shown in Fig. 3. Note that $F_2(x,Q^2)$ decreases for large x and increases for small x as Q^2 increases. Qualitatively, this behavior is explained in very elegant terms by the physical picture of Kogut and Susskind[20]: At $Q^2 \sim Q_0^2$, the photon probes structures in the proton of size $R_0^2 \sim 1/Q_0^2$. As Q^2 increases to $Q_1^2 > Q_0^2$, the photon tends to break up the structures of size R_0^2 and momentum fraction x_0 into substructures of size $R_1^2 \sim 1/Q_1^2$ each having some momentum fraction $x_1 < x_0$. Thus as Q^2 increases, the quark population decreases at large x and increases at small x.

To the extent that such a picture describes (even heuristically) the physical processes behind the asymptotic-freedom corrections to $P(x,Q^2)$, one expects that a similar picture will hold if the photon probe is replaced by a gluon probe, as appropriate in large-p_T particle production which results from quark-quark scattering via gluon exchange ($Q^2=-\hat{t}$ is typically 10–300 GeV^2 in the region of currently measured data). However, it is not at all obvious that the function $d(x)$ obtained from the electroproduction data is the appropriate function to use. This is because a photon probes the charge density of a proton, while the gluon probes the color density, and the two densities may not be identical. More technically, the AF corrections to electroproduction are obtained by evaluating the anomalous dimensions of the operator-product expansion of two electomagnetic (color-singlet) currents $J_\mu^{em}(x)J_\nu^{em}(0)$, under the specific assumption that the symmetry group of the currents commutes with the color gauge group.[11] A similar calculation using color-octet currents has not yet appeared (to my knowledge). Therefore, the function $d(x)$ in Eq. (4) will be used henceforth, with the understanding that there may be large theoretical uncertainty in the application. In the actual calculations I have used $d(x)$ for valence quarks and $|d(x)|$ for sea quarks. Since the sea quarks do not contribute significantly for $x \gtrsim 0.2$, the prescription for that region is actually irrelevant.

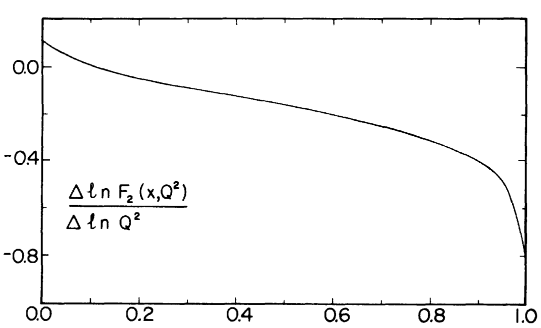

FIG. 3. The function $d(x)=\Delta \ln F_x(x,Q^2)/\Delta \ln Q^2$ provided by Fox (Ref. 19).

C. Asymptotic-freedom effects on $D(z)$

The structure functions $D(z)$ of hadrons in quarks do not share the same firm theoretical underpinning as the functions $P(x)$. However, both heuristic arguments[21] and explicit model-field-theory calculations[22] indicate that it may not be unreasonable to assume that the asymptotic-freedom corrections to $D(z,Q^2)$ are the same as those applied to $P(x,Q^2)$, especially for $z \sim 1$, which is the only region of importance for large-p_T single-particle production. The arguments are based on the Gribov-Lipatov reciprocity relation[23]

$$D(z) \simeq xP(x),$$

which should be approximately true in lowest-order perturbation theory and for $z\sim x \sim 1$. In particular, the model calculations support the view that the anomalous dimensions γ_N in electroproduction and $\tilde{\gamma}_N$ in e^+e^- annihilation should be about equal for $\bar{\alpha}$, the (running) coupling constant, small and N large. Since the large-N anomalous dimensions control the large-N moments of the structure functions and hence the behavior near $z \sim x \sim 1$, we conclude that a good guess is to use the function $d(x)$ in Eq. (4) to compute the asymptotic-freedom corrections to the structure functions $D(z)$. Of course, the same color-octet/singlet ambiguities that afflict $P(x)$ also apply to $D(z)$.

D. Asymptotic-freedom effects on $d\sigma/d\hat{t}$

If the quark-quark scattering were truly a short-distance-dominated process, then it might be appropriate to use the running coupling constant[24] $\bar{\alpha}(Q^2)$ in $d\sigma/d\hat{t}$. This could be the case if, for example, the quarks were far off the mass shell. In Sec. IV an alternate and more plausible point of view involving theoretically derived fixed-angle form factors[12] is presented. However, for the sake of argument and completeness, the effects of the running coupling constant will be pursued here. The reader should keep in mind that there are serious theoretical reservations concerning the use of $\overline{\alpha}(Q^2)$ in the present context.

The running coupling constant for four flavors and three colors is

$$\overline{\alpha}(Q^2)=\frac{\alpha_0}{1+(25/12\pi)\alpha_0\ln(Q^2/\mu^2)}=\frac{12\pi}{25\ln(Q^2/\Lambda^2)},$$

where the second relation defines Λ and eliminates the redundant parameter α_0. Neglect of the charm flavor changes $12\pi/25\to 4\pi/9$ and is insignificant here. Estimates of the renormalization point Λ vary; I have used the value $\Lambda^2=(500\ \mathrm{MeV}/c)^2$ favored by Fox[19] and others as a reasonable value.

E. Gluon contributions

The gluon momentum distribution in the proton cannot be measured directly and not much is known about it. It is possible that the gluons carry as much as 50% of the proton momentum. In order to set an upper limit on the gluon-scattering contribution to large-p_T production, I have fixed the gluon distribution in the most optimistic yet reasonable way:

$$P(x)=2.5x^{-1}(1-x)^4,$$

with the normalization chosen to give the gluons about 50% of the proton momentum. Including the contributions from (gluon) + $q\to$ (gluon) + q and the accompanying crossed-channel reactions,[25] the total contribution from gluons still amounts to only a few percent of the quark elastic scattering contribution and is therefore ignored in the following analysis.

F. Comparison with data

I have calculated the cross section for $pp\to\pi^0X$ at $\sqrt{s}=52.8$ GeV and $\theta_{\mathrm{c.m.}}=90°$ due to color-octet-gluon exchange (OGX), applying successively the asymptotic-freedom corrections to $P(x,Q^2)$, $D(z,Q^2)$, and $\overline{\alpha}(Q^2)$, with $\alpha=2$ until the last step. The results are displayed in Fig. 4, and compared to the Field-Feynman benchmark. Several points of interest may be noted:

(1) In the region of existing experimental measurements ($p_T\lesssim 6$–8 GeV/c, $x_T\lesssim 0.3$–0.5), the average values $\langle x_a\rangle$ and $\langle x_b\rangle$ in Eq. (1) are about 0.1 to 0.4, and therefore the asymptotic-freedom corrections to $P(x_a,Q^2)$ and $P(x_b,Q^2)$ are not very large. On the other hand, $\langle z_C\rangle\sim 0.7$–$0.9$, and the AF corrections to $D(z,Q^2)$ are relatively large. The corrections to the effective exponents N and F defined by

$$E\frac{d^3\sigma}{d^3p}\propto p_T^{-N}(1-x_T)^F$$

can be understood semiquantitatively:

$$\langle Q^2\rangle\simeq-\frac{\langle x_a\rangle t}{\langle z_C\rangle}\simeq\frac{2\langle x_a\rangle}{x_T\langle z_C\rangle}p_T{}^2\sim(7-5x_T)p_T{}^2,$$

for $\theta_{\mathrm{C.M.}}=90°$. Thus

$$E\frac{d^3\sigma}{d^3p}\simeq\left(E\frac{d^3\sigma}{d^3p}\right)_{\mathrm{OGX}}[(7-5x_T)p_T{}^2]^\delta,$$

where

$$\delta=d(\langle x_a\rangle)+d(\langle x_b\rangle)+d(\langle z_C\rangle)$$

and typically

$$-0.75\lesssim\delta\lesssim-0.25.$$

Then $N\to 4-2\delta$ and F also increases. Precise results are shown in Fig. 4(b). The p_T, x_T factorization is now only approximate and N and F depend weakly on both p_T and x_T: At fixed x_T F increases and N decreases as p_T increases. The cross section for OGX with AF corrections to $P(x,Q^2)$ and $D(z,Q^2)$ still exceeds the Field-Feynman benchmark.

(2) Adding the running coupling constant $\overline{\alpha}(Q^2)$ into the calculation depresses the OGX cross section below the Field-Feynman calculation and increases N by about one unit while decreasing F by a similar amount. Unlike other recent estimates,[7] we cannot conclude that observations at larger p_T at the CERN ISR will reveal the OGX contribution, since we see that N increases with x_T and keeps the OGX cross section below the Field-Feynman calculation. However, for $\sqrt{s}=52.8$ GeV^2 and $p_T\sim 4$–8 GeV/c, the OGX cross section is only a factor 2–3 below the Field-Feynman model, and such a factor can easily be made up by considering $\langle k_T\rangle$ smearing effects (see Section IV).

(3) Figure 5 shows the angular distributions for OGX and the Field-Feynman calculation for $p_T=4.0$ GeV/c. We see that a search for OGX contributions in data will benefit somewhat by looking away from $\theta_{\mathrm{C.M.}}=90°$. This peripherality in $\theta_{\mathrm{C.M.}}$ is only qualitatively sensitive to the AF corrections and should be quite reliable.

Any conclusions based on this model must be weighted by the large theoretical uncertainties involved: (a) Are the AF corrections to $P(x,Q^2)$ determined with a photon probe applicable when a

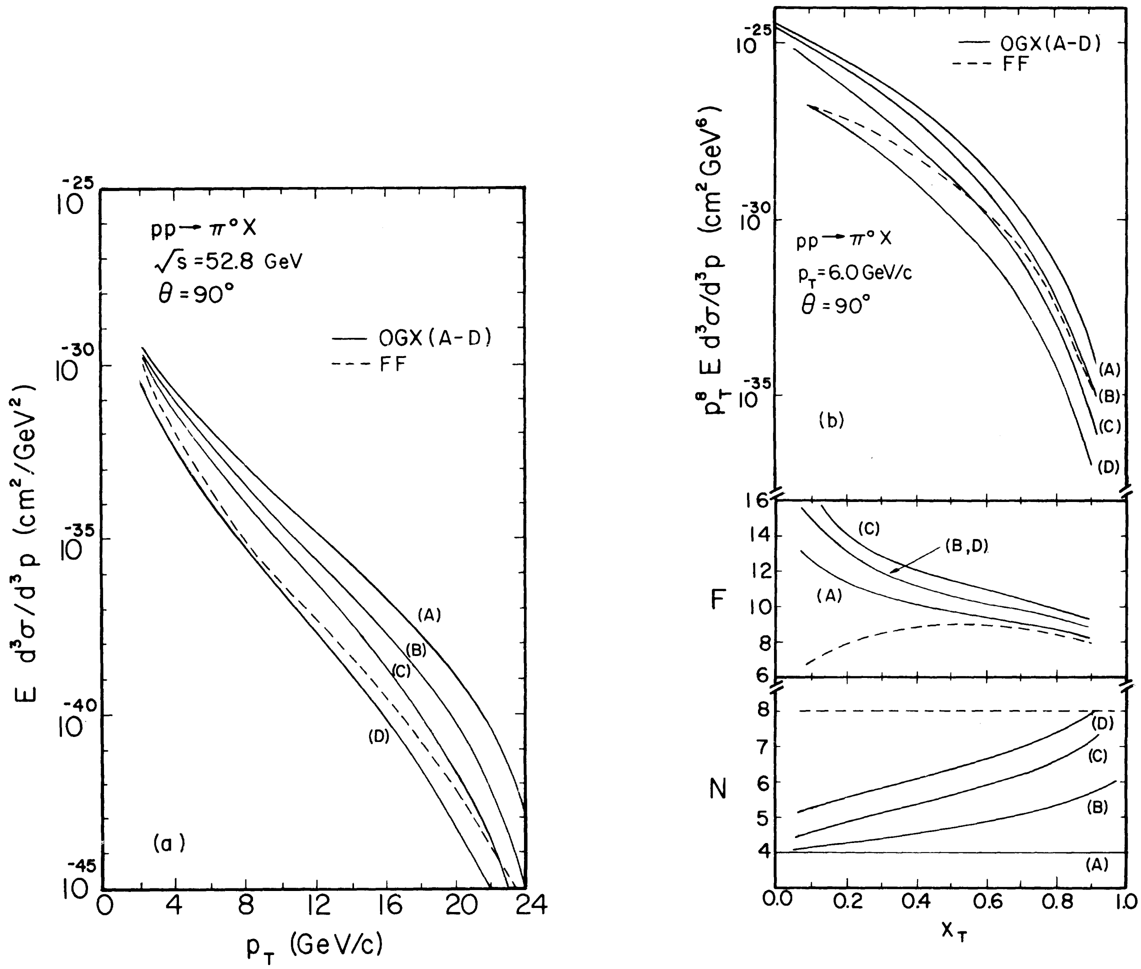

FIG. 4. Comparison of the Field-Feynman model and OGX with (A) $\alpha = 2$, (B) $\alpha = 2$ and $F(x, Q^2)$ included, (C) $\alpha = 2, F(x,Q^2), D(z,Q^2)$ included, (D) $\bar{\alpha}(Q^2), F(x,Q^2), D(z,Q^2)$ included. (a) Invariant cross section versus p_T. (b) Invariant cross section times $p_T{}^8$ versus x_T and the effective exponents F and N.

gluon probe is used? (b) Are the same corrections applicable to the quark decay functions $D(z)$? (c) Is the running coupling constant $\bar{\alpha}(Q^2)$ applicable when short-distance effects do not obviously dominate? (d) How important are $\langle k_T \rangle$ smearing effects? Within the context of the model defined above, we have seen that OGX will not dominate the Field-Feynman benchmark calculation until the next generation of accelerators is constructed. With colliding beams of (250×250)-GeV/c momentum, the OGX cross section [with $P(x,Q^2), D(z,Q^2), \bar{\alpha}(Q^2)$ included] for $pp \to \pi^0 X$ at 90° is 7.5×10^{-37} cm^2/GeV^2 at $p_T = 25$ GeV/c compared to 1.1×10^{-38} cm^2/GeV^2 for the Field-Feynman calculation. The single-jet cross sections are about 100 times larger. OGX cross sections this large are likely to seriously impair searches for $W^{\pm}$ and Z^0 production at large p_T.[26]

The Field-Feynman model assumes that the dominant hard subprocess in large-p_T production is quark-quark scattering, and agreement with experiment is obtained by lumping a good deal of ignorance into an empirical $d\sigma/d\hat{t}$ in Eq. (1). One might have hoped that by accounting for asymptotic freedom corrections as completely as possible, agreement between OGX and the data might be obtained. Apparently this is not the case. Therefore, we will turn to another approach still within the framework of QCD to account for the data.

IV. QUARK-QUARK SCATTERING IN QCD

A QCD form factors for quark-quark scattering

Recently Cornwall and Tiktopoulos[12] (CT) and others[27] have calculated the amplitude for quark-quark scattering at large fixed angle and $\hat{s}, \hat{t}, \hat{u} \gg m^2$, where m^2 is some mass scale, say $m^2 \lesssim 1$ GeV^2. The CT result is that in the leading-log approximation the perturbation-theory results ex-

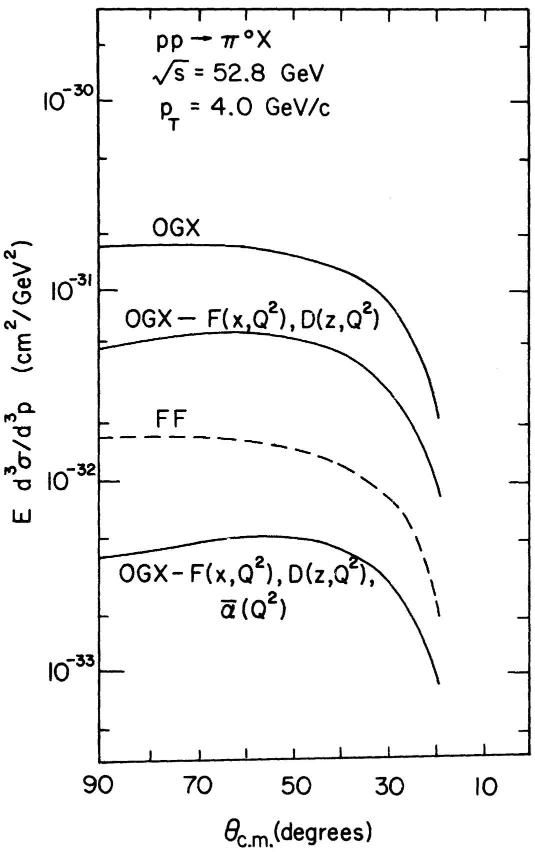

FIG. 5. The angular dependence of OGX compared with the Field-Feynman model. The value $\alpha = 2$ is used unless $\alpha = \bar{\alpha}(Q^2)$.

ponentiate, giving

$$\frac{d\sigma}{d\hat{t}} = \left(\frac{d\sigma}{d\hat{t}}\right)_B F^4(\hat{t}),$$

where $(d\sigma/dt)_B$ is the Born approximation [Eq. (3)], and $F(\hat{t})$ is the quark-quark color-singlet form factor

$$F(\hat{t}) = \exp\left[-\frac{\alpha}{4\pi}c_F \ln^2\left(-\frac{\hat{t}}{\mu^2}\right)\right], \tag{5}$$

or

$$F(\hat{t}) = \exp\left[-\frac{\alpha}{2\pi}c_F \ln^2\left(-\frac{\hat{t}}{m^2}\right)\right], \tag{6}$$

with $c_F = \frac{4}{3}$ for SU(3) color. Equation (5) results from calculating with the quarks on the mass shell and giving the gluons a mass μ, while Eq. (6) results from allowing the quarks to be off the mass shell ($p_i^2 = m^2 \neq m_{\text{quark}}^2$) while keeping the gluons massless. Presumably Eq. (6) is relevant for the present problem, and it will be used here. Whether the arguments in the logarithms are $\hat{s}$, $\hat{t}$, or $\hat{u}$ is irrelevant in the fixed-angle limit with all variables asymptotically large. I have used $\ln(-\hat{t}/m^2)$ in all calculations at $\theta_{\text{c.m.}} = 90°$.

Cornwall and Tiktopoulos have also suggested, on the basis of renormalization-group arguments, that Eqs. (5) and (6) could be modified to

$$F(t) = \exp\left[-\frac{c_F}{8\pi^2 b}\ln\left(-\frac{t}{\mu^2}\right)\ln\ln\left(-\frac{t}{\mu'^2}\right)\right] \tag{5'}$$

or

$$F(t) = \exp\left[-\frac{c_F}{4\pi^2 b}\ln\left(-\frac{t}{m^2}\right)\ln\ln\left(-\frac{t}{m'^2}\right)\right], \tag{6'}$$

where b is calculated from the asymptotic-freedom β function[24] $\beta(g) = -bg^3 + O(g^5)$. For four flavors and three colors $b = 25/48\pi^2$. Clearly, Eq. (6′) is much more speculative than Eq. (6).

We will henceforth assume that Eq. (6) [or Eq. (6′)] correctly describes large-p_T quark-quark scattering and proceed to build a model based on these form factors. In Figs. 6 and 7 the curves labeled CT I and CT II are based on Eq. (6) and Eq.

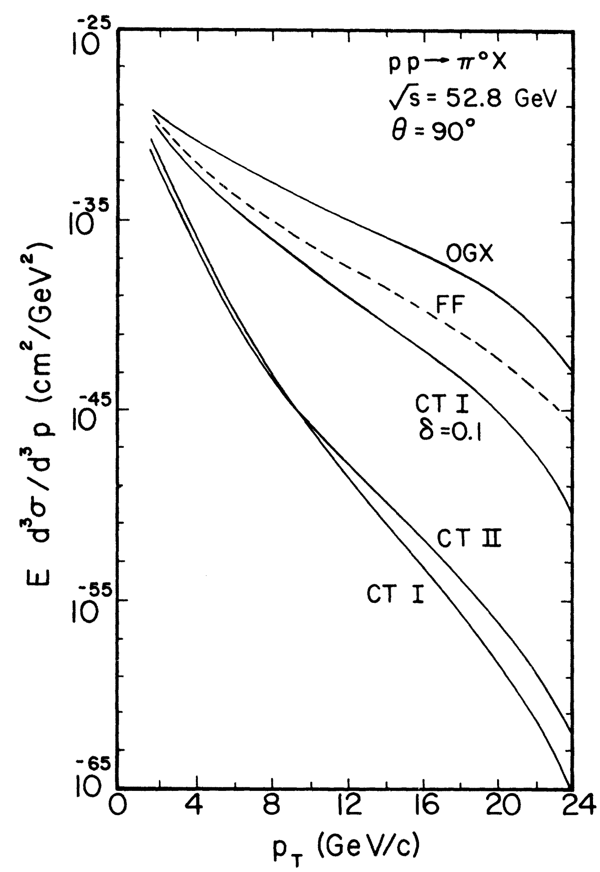

FIG. 6. OGX (with $\alpha = 2$) and the Field-Feynman model compared to models CT I [Eq. (6)], CT II [Eq. (6′)], and CT I($\delta = 0.1$) as described in the text.

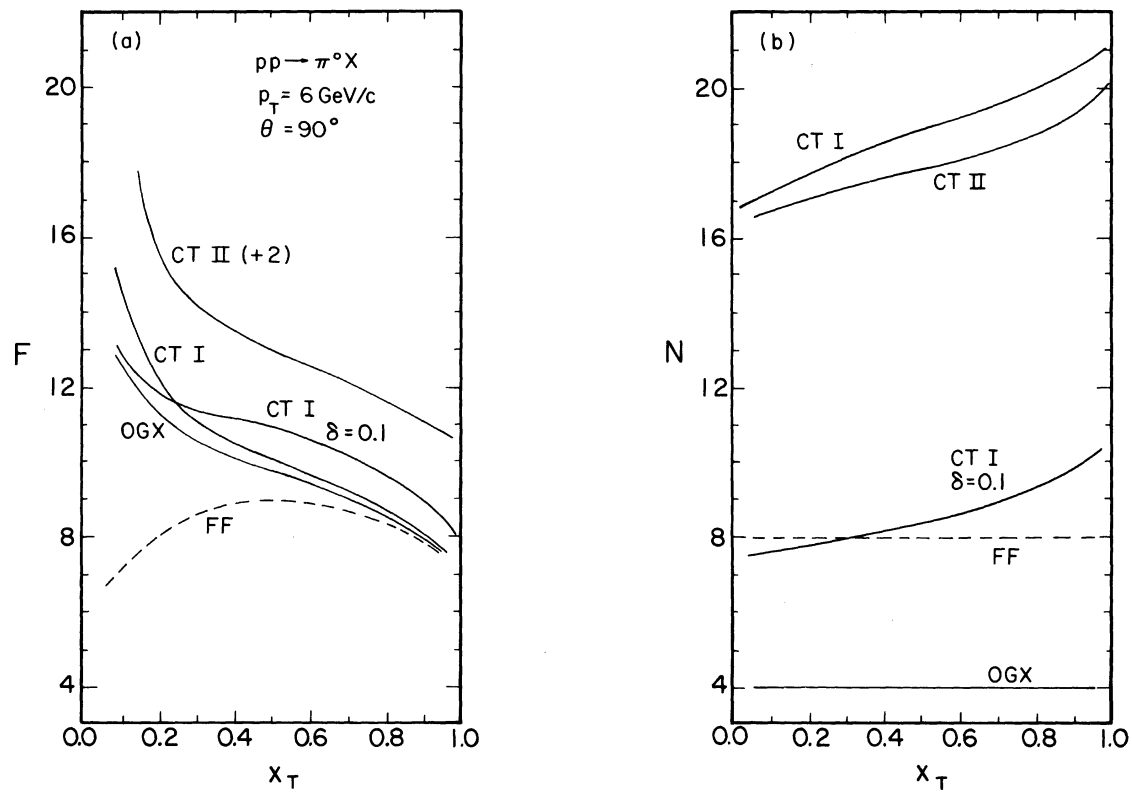

FIG. 7. (a) The effective exponents F and (b) N for models OGX ($\alpha = 2$), Field-Feynman, CT I, CT II, and CT I($\delta = 0.1$). For clarity, the value of F for model CT II has been displaced upward two units.

(6′), respectively. Asymptotic-freedom corrections to $P(x, Q^2)$ and $D(z, Q^2)$ are included as described in Sec. III, and the bare coupling constant $\alpha = 2$. Note that the running coupling constant $\overline{\alpha}(Q^2)$ is not used. The parameter m^2 is hard to determine theoretically and is taken as $m^2 = 1$ GeV^2 in the calculations. It will be shown in Sec. IV B below that consideration of $\langle k_T \rangle$ of quarks in hadrons indicates that the quarks are off the mass shell an amount of order 1 GeV^2. The value $m'^2 = 1$ GeV^2 is taken in Eq. (6′).

Clearly both models are a disaster as they stand. In particular, the effective exponents N and F are much larger than the Field-Feynman benchmark values. The remaining part of this section consists of a phenomenological demonstration that the model CT I can be modified in a reasonable way and agreement with data obtained. The same successful demonstration is also possible with model CT II, but since Eq. (6′) is much less firmly established theoretically, I have decided to neglect the model CT II in the following.

As the first step toward rescuing model CT I, we must modify the form factor $F(\hat{t})$ with a simple parameter δ in the exponential,

$$F(t) = \exp\left[-\frac{\alpha}{2\pi}\delta c_F \ln^2\left(-\frac{t}{m^2}\right)\right].$$

Figures 6 and 7 show that the value $\delta = 0.1$ gives much-improved values for F and especially N, the exponent controlling the p_T dependence. Theoretical motivation for the parameter δ will be discussed in Sec. IV C below; we turn now to a discussion of $\langle k_T \rangle$ smearing effects, as these effects must surely be present and are quite important.

B. $\langle k_T \rangle$ smearing effects

Since quarks are confined to a region of radius $R \sim 1$ F, they must have a minimum $\langle k_T^2 \rangle \sim (400$ MeV$/c)^2$ just from the uncertainty principle. However, when the proton is probed with a large-Q^2 beam, the quark transverse position is determined with a resolution of order $1/Q^2$, and $\langle k_T^2 \rangle$ for quarks must increase proportional to Q^2. This is the "Heisenberg microscope" effect.[28] In asymptotically free theories, it is probably true[29] that

$$\langle k_T^2 \rangle \sim \overline{\alpha}(Q^2) Q^2. \qquad (7)$$

It is not easy to determine $\langle k_T \rangle_{q/h}$ for quarks in hadrons directly from experiment. The most direct method is by measuring $\langle k_T\rangle_{\mu\mu}$ of high-mass μ pairs. If the μ pairs are produced by the Drell-Yan mechanism, then

$$\langle k_T\rangle_{q/h} \sim \frac{1}{\sqrt{2}}\langle k_T\rangle_{\mu\mu}.$$

Currently available μ-pair data[30] can be parametrized[31] as

$$\langle k_T^2\rangle_{\mu\mu} \simeq 0.6 + 0.09Q^2,$$

yielding

$$\langle k_T^2\rangle_{q/h} \simeq 0.20 + 0.14\bar{\alpha}(Q^2)Q^2 \tag{8}$$

if the form in Eq. (7) is used [where k_T^2 are Q^2 are in $(\mathrm{GeV}/c)^2$]. Equation (7) should be useful for $Q^2 > 1-2\ (\mathrm{GeV}/c)^2$. It is interesting that this value of $\langle k_T\rangle_{q/h}$ predicts the observed value of $R=\sigma_L/\sigma_T \simeq 0.18$ measured in electroproduction.[31] The value of $\langle k_T\rangle_{q/h}$ as given by Eq. (8) is shown in Fig. 8. Note that the rate of increase of $\langle k_T^2\rangle$ decreases as Q^2 increases.[29]

There is also considerable indirect evidence from ordinary hadron interactions that $\langle k_T\rangle_{q/h}$ is larger than one might naively expect based on pion production, where $\langle k_T\rangle_\pi \sim 330$ MeV/c. These ideas are discussed in detail by Levin and Ryskin.[32]

Naturally the whole basis of the parton model is destroyed if $\langle k_T^2\rangle/Q^2$ is not a small number. Fortunately, the values implied by Eq. (7) for that ratio do not contradict the impulse approximation. Furthermore, we see that it is not unreasonable to use $m^2 \sim 1\ \mathrm{GeV}^2$ as the amount that the quarks are off the mass shell for the large-Q^2 values 10–300 $(\mathrm{GeV}/c)^2$ of interest.

The relevance of the $\langle k_T\rangle$ considerations results from the so-called "trigger bias" effect in large-p_T production.[33] This effect means that it is easier for a quark to gain transverse momentum by a $\langle k_T\rangle$ fluctuation than from a hard scattering off another quark, since the latter process is damped by p_T^{-8}. Therefore, as discussed in Sec. II, I have included $\langle k_T\rangle$ effects by calculating the integrals in Eq. (1) assuming the quarks have transverse momenta in the trigger direction of exactly $\langle k_T\rangle$ as given by Eq. (7).[34] The transverse momenta of hadrons in quarks contributes a relatively small effect ($\langle k_T\rangle \sim 330$ MeV/c) and is neglected here.

Recently, Landshoff has suggested on the basis of the covariant parton model that $\langle k_T\rangle$ might increase as x increases.[35] Unfortunately, the suggestion has not been formulated in a manner that does not introduce further parameters undetermined by data, so I have not pursued the matter further. It is interesting to note, however, that physically the $\langle k_T(x)\rangle$ phenomenon is caused by the quark becoming more off the mass shell as x increases,[35,36] and we will see below that the parameter $\delta = 0.2$ can be accounted for in this way (among others).

The result of calculating the CT I model with $\delta = 0.1$ and $\langle k_T\rangle$ given by Eq. (7) is shown in Figs.

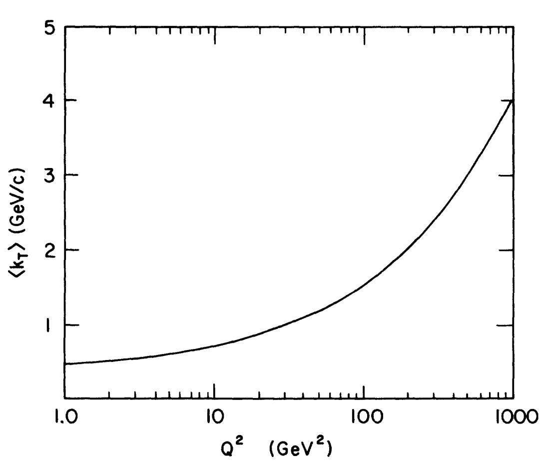

FIG. 8. The average value of transverse momentum of quarks in a proton as a function of the Q^2 used to observe the quarks [Eq. (8)].

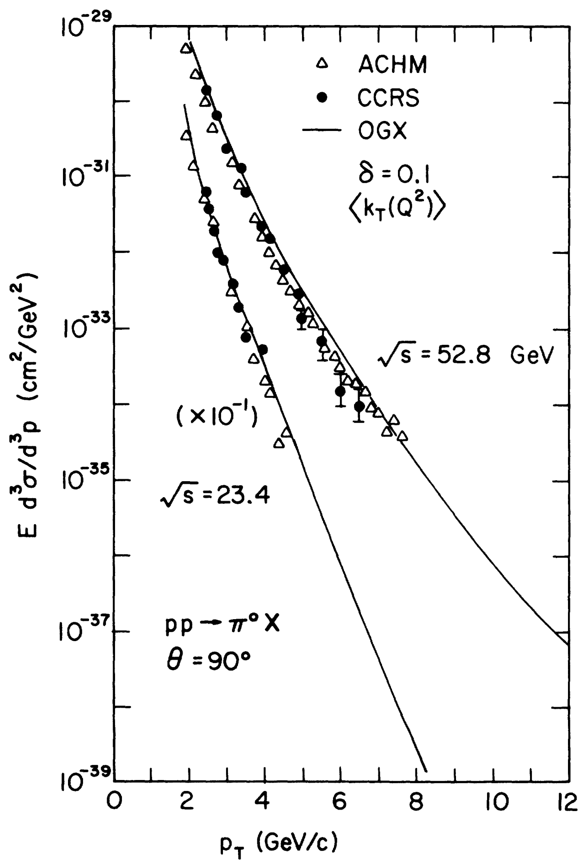

FIG. 9. Data from Ref. 2 compared to the model CT I with $\delta = 0.1$ and $\langle k_T(Q^2)\rangle$ determined by Eq. (8). The curve and data for $\sqrt{s} = 23.4$ GeV have been displaced down one decade for clarity.

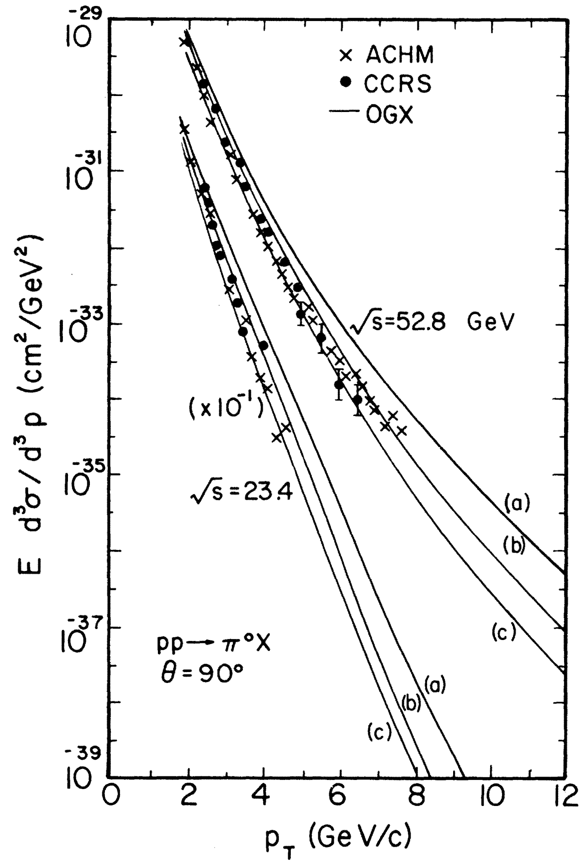

FIG. 10. Data from Ref. 2 compared to (a) the $m^2(x)$ model [Sec. IV C (3)], (b) the $\delta = 0.1$ model, and (c) the $\bar{\alpha}(Q^2)$ model [Sec. IV C(2)]. $\langle k_T \rangle$ effects are included in each curve. For clarity, the $\sqrt{s} = 23.4$ GeV set has been displaced down one decade.

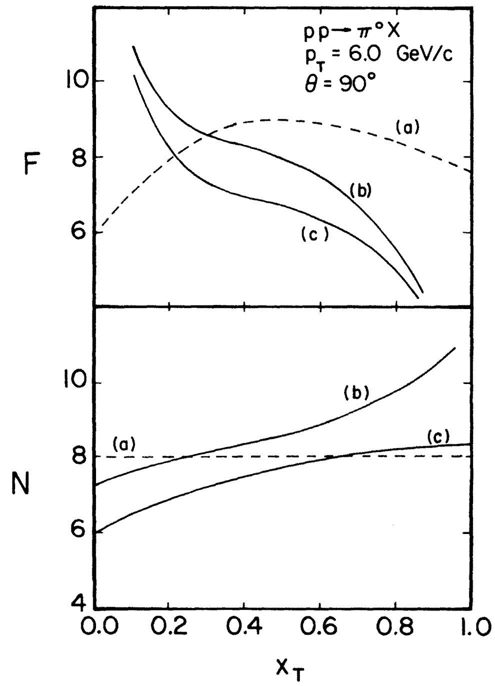

FIG. 11. The effective exponents F and N for (a) the Field-Feynman model, (b) the $\delta = 0.1$ and $\bar{\alpha}(Q^2)$ models, which give indistinguishable results, and (c) the $m^2(x)$ model.

9-11. The agreement with experiment is remarkable.

C. Motivation for the parameter δ

If the model CT I $(\delta, \langle k_T \rangle)$ is to be believed, then we must account physically for the value $\delta \simeq 0.1$. I can suggest at least three reasons to account for this:

(1) The form factors $F(\hat{t})$ in Eqs. (5) and (6) are supposed to be correct for the scattering of free colored quarks by colored gluons. This accounts for the group-theory factor c_F in the exponential. As stressed by Cornwall and Tiktopoulos,[12] the scattering of color-singlet objects is not suppressed: $c_F = 0$ in that case. It is possible that gluon bremsstrahlung is partially screening the color of the quarks which are bound in the color-singlet proton, thus creating an effective $c_F' \sim \delta c_F$. Whether or not this is reasonable probably depends on the mechanism for color neutralization and confinement (in particular, the time scale involved). A consequence of this line of argument is that the effective number of colors N_C seen by the gluons in this kinematic regime is $N_C \simeq 1.14$. In some sense the nearness of this number to unity indicates the consistency of using the color-singlet anomalous dimensions to calculate the AF corrections to $P(x, Q^2)$ and $D(x, Q^2)$.

Unfortunately I have no argument to estimate δ on the basis of color antiscreening considerations. However, the empirical value $\delta = 0.1$ suggests an intriguing conjecture that δ be identified with the running coupling constant $\bar{\alpha}(Q^2)$, as discussed next.

(2) The same leading-log analysis that yields Eqs. (5) and (6) has an interesting infrared (as opposed to fixed-angle) limit that has been suggested as a signal for quark confinement.[12] Since it is known that there is no signal for confinement order by order in perturbation theory,[37] it has been suggested that the leading-log analysis may be a method of transcending perturbation theory.[12,38] Recent attempts to adapt renormalization-group arguments to the infrared region have resulted in the suggestion that the coupling constant α in the exponentiated one-loop form-factor integral might in fact be the running coupling constant that couples the internal loop gluons to the quarks.[39] Thus we might conjecture that a similar effect is occurring in the

fixed-angle regime, yielding

$$F(Q^2) = \exp\left[-\frac{\overline{\alpha}(Q^2)}{2\pi} c_F \ln^2\left(\frac{Q^2}{m^2}\right)\right], \qquad (9)$$

with the difference that $\overline{\alpha}$ is evaluated at Q^2. This conjecture resembles the renormalization-group argument of CT that resulted in Eqs. (5′) and (6′), except for (phenomenologically crucial) numerical differences. In any event the theoretical motivation of Eq. (9) is at present far from clear. It is possibly related to the color antiscreening argument in the preceding paragraph. Calculations using this conjecture and $m^2 = 1$ GeV^2 are displayed in Figs. 10 and 11. Better agreement with the data is easily obtained by small adjustments of m^2 and the bare coupling constant α.

(3) The value for m^2 in Eq. (5) is particularly hard to guess in a reasonable way. The value $m^2 = 1$ GeV^2 has been used above since $\langle k_T \rangle$ considerations indicate that the quark is off the mass shell an amount of order 1 GeV^2. Landshoff and Polkinghorne[36] have suggested that m^2 can be determined by simple kinematics in the covariant parton model. They find

$$-m^2 = \frac{xs' + k_T^{\,2}}{(1-x)} - xM^2, \qquad (10)$$

where M is the proton mass and s' is the squared invariant mass of the debris left behind when the proton emits the quark with momentum fraction x (and before confining forces begin to act). As before, k_T is the transverse momentum of the single quark. The value of s' is somewhat arbitrary, but if I take the value $s' = 4$ GeV^2 used by Landshoff[35] and the values given in Eq. (7) for $\langle k_T^2 \rangle$, then the results shown in Fig. 12 for $-m^2$ are obtained. Using Eq. (10) for m^2 in Eq. (6) yields the results shown in Figs. 10 and 11. Once again it is clear that better agreement can be obtained by small adjustments of the parameters.

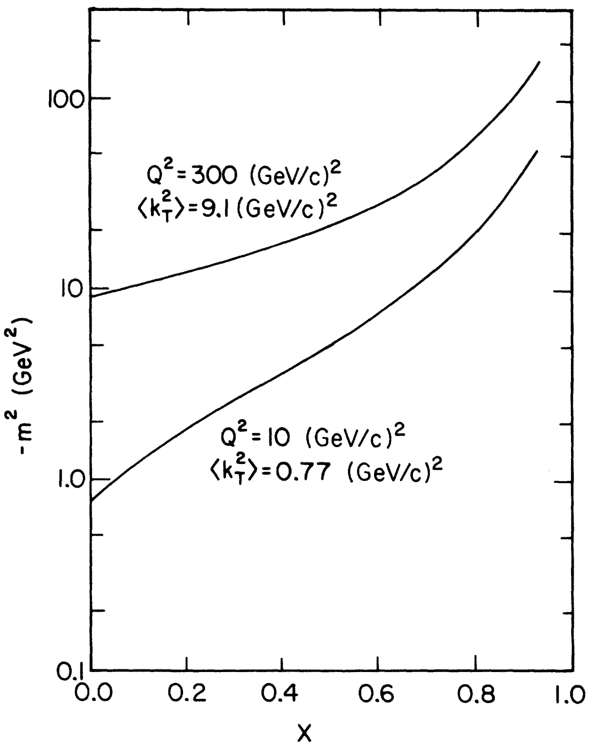

FIG. 12. The quantity $-m^2$ of Landshoff and Polkinghorne (Ref. 36) evaluated for different values of Q^2 and $\langle k_T^2 \rangle$.

D. Angular distributions and Regge behavior in QCD

Past attempts to identify the anomalous p_T behavior in large-p_T production with simple form-factor modifications to quark-quark scattering were plagued with incorrect peripheral angular dependences.[4] Indeed, the present analysis suffers the same problem if naively extended away from 90°.

The resolution of the problem is to note that the Cornwall-Tiktopoulos results apply when the invariants $\hat{s}, \hat{t}, \hat{u}$ grow large together. Calculation of the angular dependence at fixed p_T is correctly done using the formula for $F(\hat{t})$ given by Tyburski,[40]

$$F(t) = \exp\left[-\frac{\alpha}{2\pi} c_F' \frac{(-\hat{t}+\mu^2)}{\sqrt{\Delta}} \ln\left(\frac{-\hat{t}+\sqrt{\Delta}}{\hat{t}+\sqrt{\Delta}}\right) \ln\left(\frac{\hat{s}}{s_0}\right)\right], \qquad (11)$$

where $\Delta = \hat{t}^2 - 4\hat{t}m^2$, and a slight modification in the group theory factor calculated by Tyburski has been implemented so that Eq. (11) agrees with Eq. (6) in the large fixed-angle regime for $m^2 = \mu^2 = s_0/2$ and $\hat{s}, \hat{t}, \hat{u}$ asymptotically large. Angular distributions using Eqs. (6) and (11) with $\delta = 0.1$ in both are compared with the Field-Feynman result in Fig. 13. Once again the agreement is remarkable.

V. PREDICTED PATTERN OF SCALING VIOLATION

The variations of the effective exponents N and F shown in Fig. 11 indicate a definite pattern of scaling violation for the CT($\delta, \langle k_T \rangle$) model: N is a function of x_T, F is a function of p_T, and both N and F increase as p_T increases at fixed x_T. Examples of the predicted scaling violations are presented in Fig. 14, where the lines for fixed p_T are solid for $13.4 \le \sqrt{s} \le 63$ GeV and dotted otherwise, and the lines for fixed $\sqrt{s}$ are solid for $2 \le p_T \le 8$ (GeV/c)^2 and dotted otherwise. Clearly, the particular quantitative values shown in the figures are not reliable, but the qualitative pattern is reliable. It is doubtful whether any meaningful comparison can be made with present data.

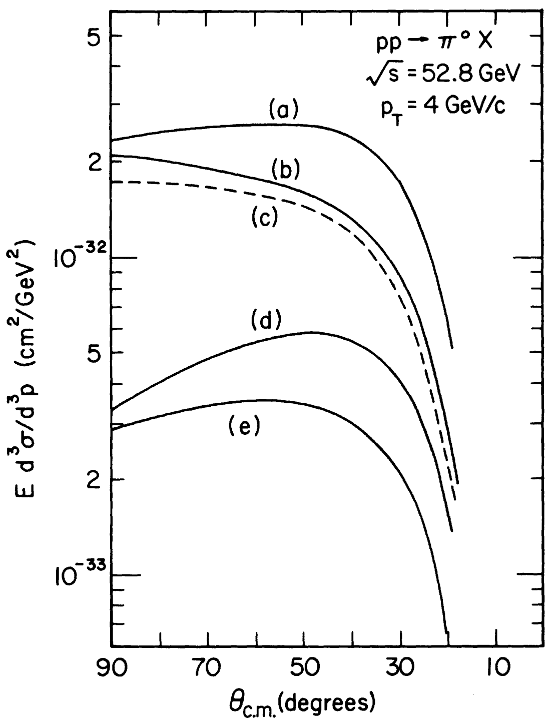

FIG. 13. Angular distributions of invariant cross sections for (a) model CT I with $\delta = 0.1$ and $\langle k_T \rangle$ effects included, (b) the Tyburski model with $\delta = 0.1$ and $\langle k_T \rangle$ effects included, (c) the Field-Feynman model, and (d), (e) same as (a), (b) but without $\langle k_T \rangle$ effects.

VI. CONCLUSIONS AND SUMMARY

The effects of asymptotic freedom and color gauge symmetry on the production of large-p_T particles by quark-quark scattering have been systematically investigated in an attempt to understand the striking phenomenological success of the Field-Feynman model. Apparently, asymptotic-freedom effects on the proton structure functions $P(x, Q^2)$ are not particularly important in regions of current experimental accessibiltiy ($0.1 \lesssim x_T \lesssim 0.4$). Asymptotic-freedom effects on the quark structure functions $D(z, Q^2)$ are potentially very significant; unfortunately, equally significant theoretical uncertainties are also present. Similarly, the use of the running coupling constant $\overline{\alpha}(Q^2)$ is also discussed; however, in the context of wide-angle quark elastic scattering, the use of $\overline{\alpha}(Q^2)$ is almost certainly incorrect: The quarks are simply not far off the mass shell. Using the most optimistic estimates that are still reasonable, gluon contributions to large-p_T production are found to be negligible.

Altogether, these results based only on asymptotic-freedom considerations lead to rather unsatisfying conclusions: With a bare coupling con-

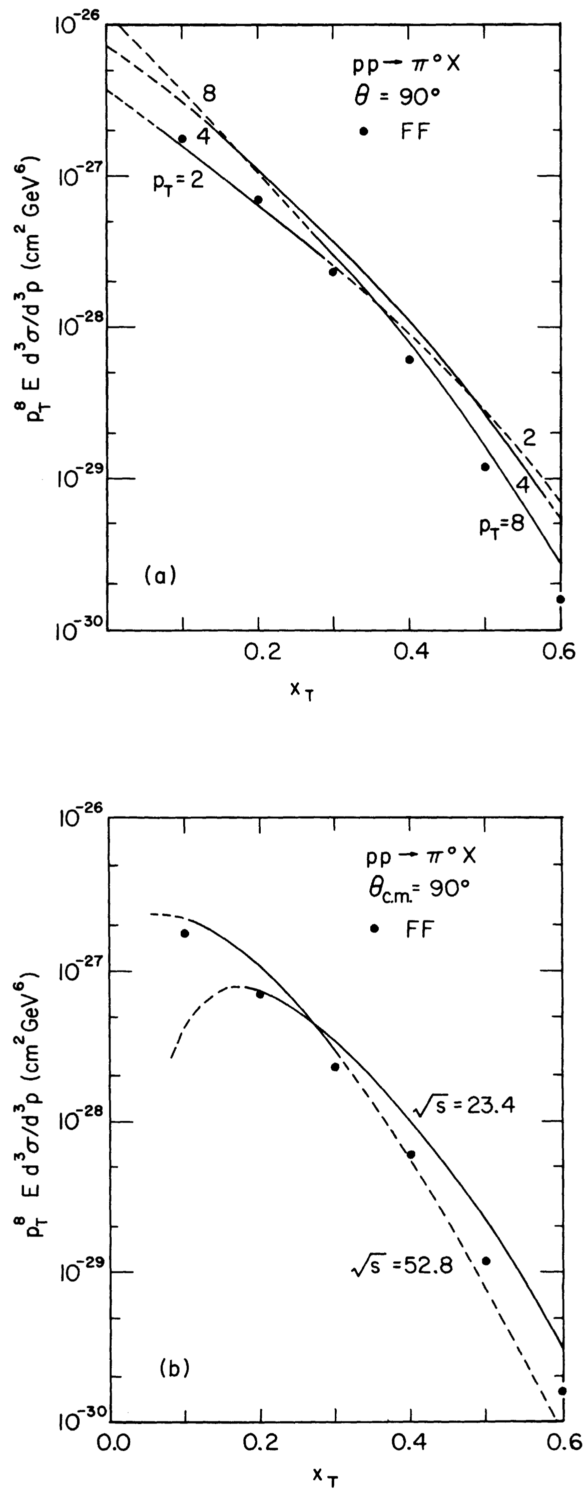

FIG. 14. Pattern of scaling violations for the model CT I with $\delta = 0.1$ and $\langle k_T \rangle$ effects included for (a) fixed values of p_T (——) $13.4 \leq \sqrt{s} \leq 63$ GeV, (---) otherwise, and (b) fixed values of $\sqrt{s}$; (——) $2 \leq p_T \leq 8$ GeV/c, (---) otherwise. The "data" points are from the Field-Feynman model.

stant of order unity, it is hard to understand why OGX is not experimentally visible; using $\bar{\alpha}(Q^2)$, which is theoretically dubious, OGX will not be seen until $\sqrt{s}$ exceeds several hundred GeV, and the success of the Field-Feynman calculation is still not understood.

We have seen that the use of form factors derived from color-gauge theories by Cornwall and Tiktopoulos does open the possibility of understanding existing data in terms of quark-quark scattering. The viability of this approach rests on a plausible but unproven phenomenological assumption that probably amounts to partial antiscreening of the color of quarks in protons when probed by color gluons. It is also possible that a simple kinematic effect proposed by Landshoff and Polkinghorne is responsible. In either case, it is quite important to consider the transverse-momentum fluctuations of quarks in protons if any pretense of absolute normalization is to be maintained. It is clearly of great interest to search for the pattern of scaling violations predicted by these models.

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[1]S. M. Berman and M. Jacob, Phys. Rev. Lett. 25, 1632 (1970).

[2]K. Eggert *et al.* (ACHM collaboration) Nucl. Phys. B98, 49 (1975); F. W. Büsser *et al.* (CCRS collaboration) *ibid.* B106, 1 (1976); for a full list of data references, see e.g. the review of M. Della Negra, in Proceedings of the VII International Colloquium on Multiparticle Production, Tutzing, 1976 (unpublished).

[3]J. F. Gunion, S. J. Brodsky, and R. Blankenbecler, Phys. Lett. 39B, 649 (1972); R. Blankenbecler, S. J. Brodsky, and J. F. Gunion, *ibid.* 42B, 461 (1972); most recently, S. J. Brodsky and J. F. Gunion, SLAC Report No. SLAC-PUB-1806, 1976 (unpublished).

[4]R. D. Field and R. P. Feynman, Phys. Rev. D 15, 2590 (1977); R. Baier, J. Cleymans, K. Kinoshita, and B. Peterson, Nucl. Phys. B118, 139 (1977).

[5]G. C. Fox, in *Particles and Fields '76*, proceedings of the Annual Meeting of the Division of Particles and Fields of the American Physical Society, Brookhaven National Lahoratory, edited by H. Gordon and R. F. Peierls (BNL, Upton, New York, 1977), p. G1.

[6]R. F. Cahalan, K. A. Geer, J. Kogut, and L. Susskind, Phys. Rev. D 11, 1199 (1975).

[7]R. Cutler and D. Sivers, Phys. Rev. D 16, 679 (1977).

[8]S. D. Ellis, Phys. Lett. 49B, 189 (1974).

[9]R. C. Hwa, A. J. Spiessbach, and M. J. Teper, Phys. Rev. Lett. 36, 1418 (1976); University of Oregon Report No. OITS-76, 1977 (unpublished).

[10]A. P. Contogouris, R. Gaskell, and A. Nicolaidis, Phys. Rev. D (to be published).

[11]D. J. Gross and F. Wilczek, Phys. Rev. D 9, 980 (1974); H. Georgi and H. D. Politzer, *ibid.* 9, 416 (1974); H. D. Politzer, Phys. Rep. 14C, 129 (1974).

[12]J. M. Cornwall and G. Tiktopoulos, Phys. Rev. Lett. 35, 338 (1975); Phys. Rev. D 13, 3370 (1976).

[13]S. D. Ellis and M. B. Kislinger, Phys. Rev. D 9, 2027 (1974); D. Sivers, S. Brodsky, and R. Blankenbecler, Phys. Rep. 23C, 1 (1976).

[14]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972).

[15]See Ellis and Kislinger (Ref. 13), Eq. (5.1).

[16]R. Blankenbecler, S. J. Brodsky, and J. Gunion, Phys. Rev. D 12, 3469 (1975).

[17]I have used a Monte Carlo program provided by R. D. Field and G. C. Fox.

[18]S. M. Berman, J. D. Bjorken, and J. B. Kogut, Phys. Rev. D 4, 3388 (1971); J. D. Bjorken, *ibid.* 8, 4098 (1973).

[19]G. C. Fox (unpublished).

[20]J. Kogut and L. Susskind, Phys. Rev. D 9, 697 (1974); 9, 3391 (1974).

[21]A. M. Polyakov, Zh. Eksp. Teor. Fiz. 60, 1572 (1971) [Sov. Phys.—JETP, 33, 850 (1971)]; in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California*, edited by W. T. Kirk (SLAC, Stanford, 1976), p. 855.

[22]A. Mueller, Phys. Rev. D 9, 963 (1974); C. G. Callan, Jr. and M. L. Goldberger, *ibid.* 11, 1542 (1974); 11, 1553 (1975); N. Coote, *ibid.* 11, 1611 (1975).

[23]V. N. Gribov and L. N. Lipatov, Yad. Fiz. 15, 1218 (1972) [Sov. J. Nucl. Phys. 15, 675 (1972)].

[24]D. Gross and F. Wilczek, Phys. Rev. Lett. 26, 1343 (1973); H. D. Politzer, *ibid.* 26, 1346 (1973).

[25]Ellis and Kislinger, Ref. 13.

[26]C. Quigg, Rev. Mod. Phys. 49, 297 (1977).

[27]E. C. Poggio and H. R. Quinn, Phys. Rev. D 12, 3279 (1975); P. Carruthers and F. Zachariasen, Phys. Lett. 62B, 338 (1976).

[28]W. Heisenberg, *The Physical Principles of the Quantum Theory* (Dover, New York, 1949).

[29]J. B. Kogut, Phys. Lett. 65B, 377 (1976).

[30]J. K. Anderson *et al.*, paper submitted to the XVIII Conference on High Energy Physics, Tbilisi, U.S.S.R., 1976 (unpublished).

[31]I. Hinchcliffe and C. H. Llewellyn Smith, Phys. Lett. 66B, 281 (1977).

[32]E. M. Levin and M. G. Ryskin, Zh. Eksp. Teor. Fiz. 69, 1537 (1975) [Sov. Phys.—JETP 42, 783 (1975)]; Invited Talk at the XVIII International Conference on High Energy Physics, Tbilisi, U.S.S.R. 1976 (unpublished).

[33]S. D. Ellis, M. Jacob, and P. V. Landshoff, Nucl. Phys. B108, 93 (1976); M. Jacob and P. B. Landshoff, *ibid.* B113, 395 (1976).
[34]The lower limits of integration are calculated using Eq. (7) evaluated at $\langle Q^2\rangle \simeq (7-5x_T)p_T^2$.
[35]P. V. Landshoff, Phys. Lett. 66B, 452 (1977).
[36]P. V. Landshoff and J. C. Polkinghorne, Phys. Rep. 5C, 1 (1972).
[37]T. Appelquist, J. Carazzone, H. Kluberg-Stern, and M. Roth, Phys. Rev. Lett. 36, 768 (1976).
[38]J. M. Cornwall, UCLA Report No. UCLA/76/TEP/10 (unpublished); E. C. Poggio, Harvard Report No. HUTP-76/A163 (unpublished).
[39]J. Frenkel, R. Meuldermans, I. Mohammad, and J. C. Taylor, Phys. Lett. 64B, 211 (1976); E. C. Poggio, Brandeis report, 1977 (unpublished); P. Carruthers, P. Fishbane, and F. Zachariasen, Phys. Rev. D 15, 3675 (1977).
[40]L. Tyburski, Phys. Rev. D 13, 1107 (1976).