PHYSICAL REVIEW D VOLUME 16, NUMBER 5 1 SEPTEMBER 1977

Effective-gluon model description of inclusive meson production in $\pi^{\pm}p$ and pp collisions*

George W. Look and Ephraim Fischbach
Physics Department, Purdue University, West Lafayette, Indiana 47907
(Received 17 January 1977; revised manuscript received 18 March 1977)

The recently introduced effective-gluon (EG) model is applied to inclusive meson production at large transverse momentum ($p_\perp$) from $\pi^{\pm}p$ and pp interactions. In the EG model the dependence of the inclusive differential cross section on $p_\perp$ and center-of-momentum energy $\sqrt{s}$ is characterized by *a single universal constant* $B = 18\ \mathrm{GeV}^2$; this constant is obtained by fitting to the inclusive data for $pp \to cX$ with $c = \pi^{\pm}$, π^0, $K^{\pm}$, p, and $\bar{p}$. Since the EG model requires that the inclusive differential cross sections for $\pi^{\pm}p \to cX$ should be characterized by the *same* constant B, the dependence of these cross sections on $p_\perp$ and $\sqrt{s}$ should follow from the previous analysis of $pp \to cX$ without the introduction of any additional parameters. The only new information required is the form of the momentum distribution function for partons in the incident pion. Several models for this function are considered and it is shown that all of the alternatives lead to approximately the same predictions. Comparison of theory and experiment indicates that the EG model description of the recent Fermilab data of Donaldson *et al.* is good to approximately 15–20% per data point over a range of more than 10^5 in the magnitude of the inclusive cross section.

An effective-gluon (EG) model of the strong parton-parton interaction has been developed[1,2] to describe the inclusive production of particles with large transverse momentum ($p_\perp$) in high-energy p-p collisions. In the EG model, the parton-parton amplitude which results from the assumption of single massless-vector-gluon exchange[3] is multiplied by the phenomenological function

$$F^2(\hat{t}) = (1 - \hat{t}/B)^{-2}\,, \qquad B = 18\ \mathrm{GeV}^2\,, \tag{1}$$

where $\hat{t}$ is the parton-parton four-momentum transfer, and the value of B is obtained by fitting the available experimental data for the inclusive production $pp \to \pi^0, \pi^{\pm}, K^{\pm}, p, \bar{p}$ at large $p_\perp$.[4-6] The dependence of these invariant cross sections on the center-of-momentum (c.m.) energy of the p-p system, $\sqrt{s}$, and on $x_\perp$ ($=2p_\perp/\sqrt{s}$) are both described by the *single universal constant* B. An additional normalization constant is required for each observed final-state particle to fix the overall scale of the cross sections.

The basic structure of the model is that developed by Berman, Bjorken, and Kogut (BBK),[3] in which the inclusive production takes place via the following steps: The initial hadrons fragment into quarks which then scatter from each other with large momentum transfer. The scattered quarks then "decay" into the final hadrons, one of which is the observed particle. The quark scattering and decay processes are common to *all* inclusive scattering processes, regardless of the type of initial particles, and consequently the constant B and the normalization constants are uniquely determined from the p-p scattering data. In order to calculate the large-$p_\perp$ cross sections for processes involving incident particles other than the proton, the only new information needed is a knowledge of the momentum distributions of the partons within the colliding particles.

In the case of pions, a reasonable form of the parton distribution functions can be postulated, and inclusive cross sections for $\pi^{\pm}p \to cX$ ($c = \pi^0$, $\pi^{\pm}$, $K^{\pm}$, p and $\bar{p}$) can be calculated using the EG model. Following the arguments used by Kuti and Weisskopf (KW)[7] and McElhaney and Tuan[8] in deriving distribution functions for the proton, we assume that the pion is composed of valence and sea, or core, quarks. Thus the π^+ contains one u and one $\bar{d}$ valence quark, while the π^- contains one $\bar{u}$ and one d valence quark, and each contains an SU(3)-symmetric sea of quark-antiquark pairs. For the π^+ distribution functions we write

$$\begin{aligned} u^{\pi^+}(x) &= u_v^{\pi^+}(x) + c(x),\\ \bar{d}^{\pi^+}(x) &= \bar{d}_v^{\pi^+}(x) + c(x),\\ \bar{u}^{\pi^+}(x) &= d^{\pi^+}(x) = s^{\pi^+}(x) = \bar{s}^{\pi^+}(x) = c(x), \end{aligned} \tag{2a}$$

and for the π^-,

$$\begin{aligned} \bar{u}^{\pi^-}(x) &= \bar{u}_v^{\pi^-}(x) + c(x)\\ d^{\pi^-}(x) &= d_v^{\pi^-}(x) + c(x),\\ u^{\pi^-}(x) &= \bar{d}^{\pi^-}(x) = s^{\pi^-}(x) = \bar{s}^{\pi^-}(x) = c(x). \end{aligned} \tag{2b}$$

In Eq. (2) the subscript v denotes the valence contribution, $c(x)$ is the contribution from the parton sea, and x is the fraction of the pion's momentum carried by the parton. The normalization condition specifying the number of valence quarks is

$$\int_0^1 u_v^{\pi^+}(x)dx = \int_0^1 \bar{d}_v^{\pi^+}(x)dx = \int_0^1 \bar{u}_v^{\pi^-}(x)dx = \int_0^1 d_v^{\pi^-}(x)dx = 1\,. \tag{3}$$

Using the isospin and charge-conjugation invariance of the strong interactions, we can argue that

$$u_v^{\pi^+}(x) = d_v^{\pi^-}(x) = \bar{d}_v^{\pi^+}(x) = \bar{u}_v^{\pi^-}(x). \quad (4)$$

Thus, to calculate the πp inclusive cross sections, we need to specify only two distribution functions for the pions, one describing the valence-quark distribution, and one describing the $q\bar{q}$ sea.

It has been argued by a number of authors[3,9,10] that the behavior of the elastic electromagnetic form factor at large momentum transfer q^2 can be related to the scaling behavior of the inelastic electromagnetic form factor, or, equivalently in the parton model, to the parton distributions near $x=1$ (where x is the fraction of the particle's momentum carried by the leading parton). If the elastic electromagnetic form factor of a hadron h is $F_h(q^2) \propto (q^2)^{-r}$, then, as discussed by Drell and Yan[9] and Bloom and Gilman,[10] the large-x behavior of the valence partons should be of the form $(1-x)^{2r-1}$. If, for example, we consider the proton where $F_p \propto (q^2)^{-2}$, the distribution of the valence partons should be $(1-x)^3$ near $x=1$. This has been utilized in the development of detailed distribution functions for the proton.[7,8,11]

The preceding arguments suggest that near $x=1$ the valence parton distribution functions for pions are proportional to $(1-x)$, which corresponds to the experimentally observed behavior $F_\pi(q^2) \propto (q^2)^{-1}$.[12] We assume that the small-x behavior of the valence distribution of the pion is the same as that assumed for the proton distribution functions in Refs. 7 and 8. Combining these assumptions yields

$$d_v^{\pi^-}(x) = \beta(1-x)/\sqrt{x}\,. \quad (5)$$

The constant β can be determined from the normalization condition of Eq. (3) and gives

$$d_v^{\pi^-}(x) = 0.75(1-x)/\sqrt{x}\,. \quad (6)$$

We expect that the arguments used above to motivate our choice of $d_v^{\pi^-}(x)$ are best for values of $x \cong 1$, and thus regions of intermediate and high $x_\perp$ should provide the best experimental tests of the EG-model predictions of the s and $x_\perp$ dependence of $Ed\sigma/d^3p$. Regions of relatively small $x_\perp$ are naturally more sensitive to the functional form of the valence distributions near $x=0$ (and to the form of the sea distribution discussed below) which is more speculative than the large-x behavior.

To specify the distribution of core quarks and antiquarks in the pion we assume that the parton sea is universal in the sense that it is the same for all hadrons. Hence for the sea we use the expression for $c(x)$ determined for the proton by McElhaney and Tuan[8] and used in the EG calculations of Ref. 1:

$$c(x) = 0.1(1-x)^{3.5}/x\,. \quad (7)$$

As will be shown below, the EG-model predictions for $x_\perp \gtrsim 0.1$ are not particularly sensitive to the form of the pion sea distribution.

The $(1-x)$ form for the distribution of valence quarks in a pion used here is consistent with what was assumed for the behavior of the quark decay function in Ref. 1, and hence we shall focus our attention on results obtained using this function. Other forms of this distribution have been suggested and we have also explored the results of using quark distribution (and decay) functions of the form $xu^\pi(x) = \text{constant}$ and $xu^\pi(x) = (1-x)^2$. The first form is of the type suggested by Field and Feynman,[13] and the latter form results from the structure-function analysis of Farrar and Jackson.[14] The effect of using these distributions (in conjunction with the EG-model quark-quark cross section) will be discussed below.

The invariant cross sections predicted by the EG model for the processes $\pi^\pm p \to \pi^0, \pi^\pm, K^\pm$ are shown in Figs. 1–5 for several Fermilab energies and using the distribution function of Eqs. (6) and

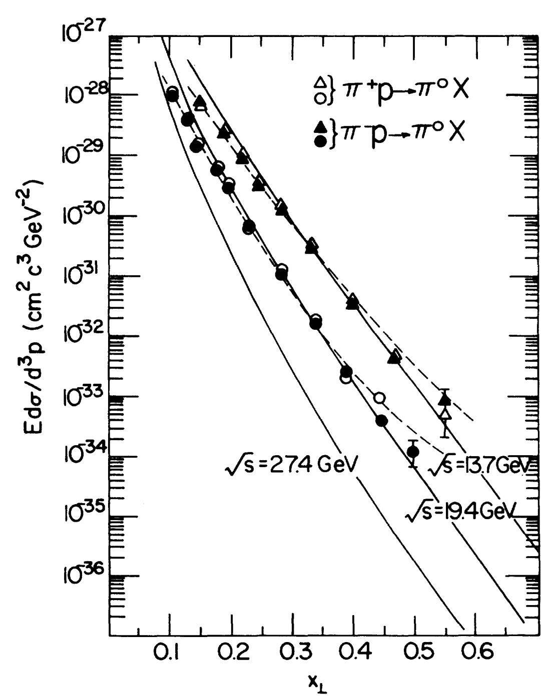

FIG. 1. Invariant cross section versus $x_\perp$ for $\pi^\pm p \to \pi^0 X$ at $\theta_{\mathrm{c.m.}} = 90°$. The three solid curves are the EG-model predictions using the quark distributions of Eqs. (6) and (7). The dashed curves use distribution functions of the form $xu^\pi(x) = \text{constant}$. The EG model predicts the near equality of $Ed\sigma/d^3p\,(\pi^\pm p \to \pi^0 X)$. The experimental data are from Ref. 15.

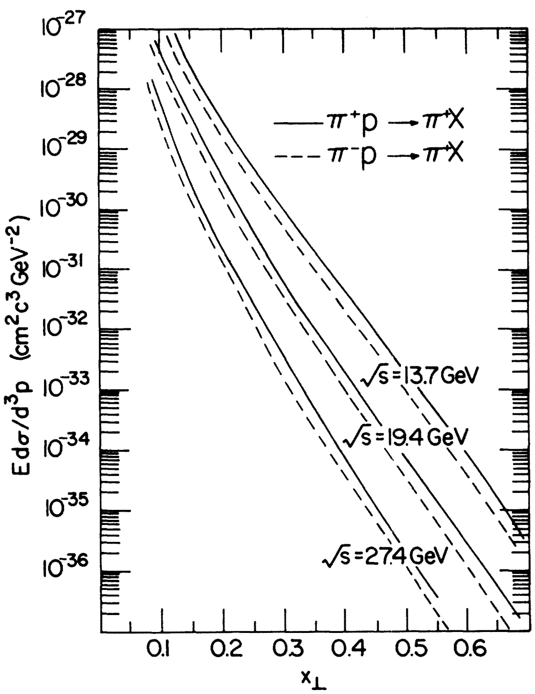

FIG. 2. EG-model predictions for the invariant cross sections for the processes $\pi^{\pm}p \to \pi^+X$ at $\theta_{c.m.} = 90°$. The curves correspond to incident laboratory momenta p_L $= 100$, 200, and 400 GeV/c.

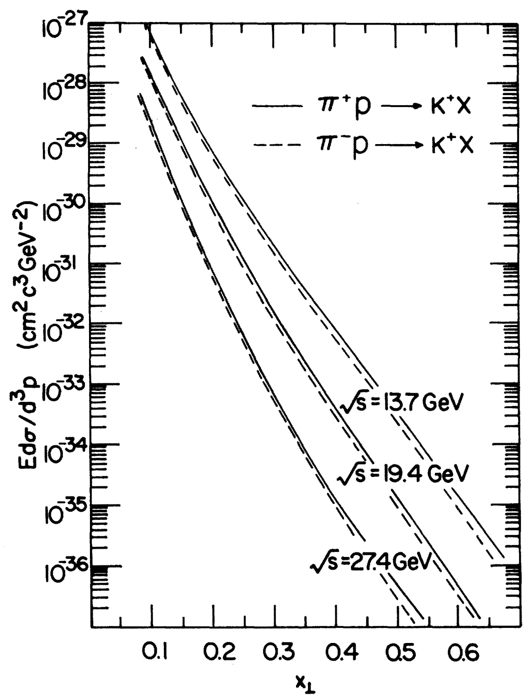

FIG. 4. EG-model predictions for the invariant cross sections for the processes $\pi^{\pm}p \to K^+X$ at $\theta_{c.m.} = 90°$. The curves correspond to incident laboratory momenta p_L $= 100$, 200, and 400 GeV/c.

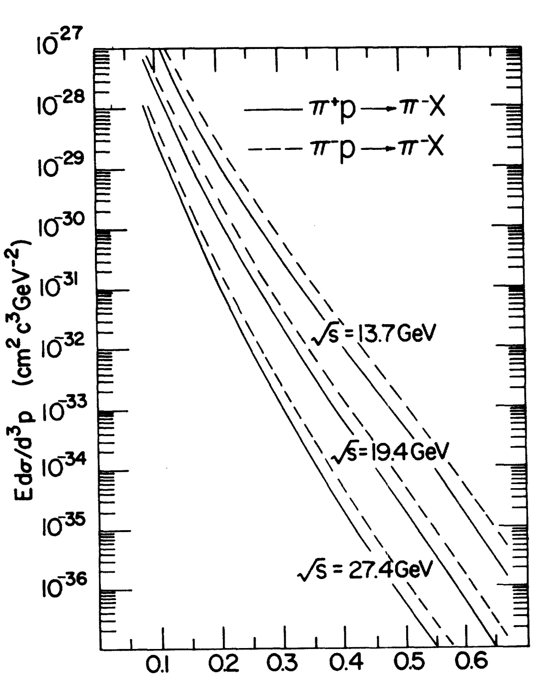

FIG. 3. EG-model predictions for the invariant cross sections for the processes $\pi^{\pm}p \to \pi^-X$ at $\theta_{c.m.} = 90°$. The curves correspond to incident laboratory momenta p_L $= 100$, 200, and 400 GeV/c.

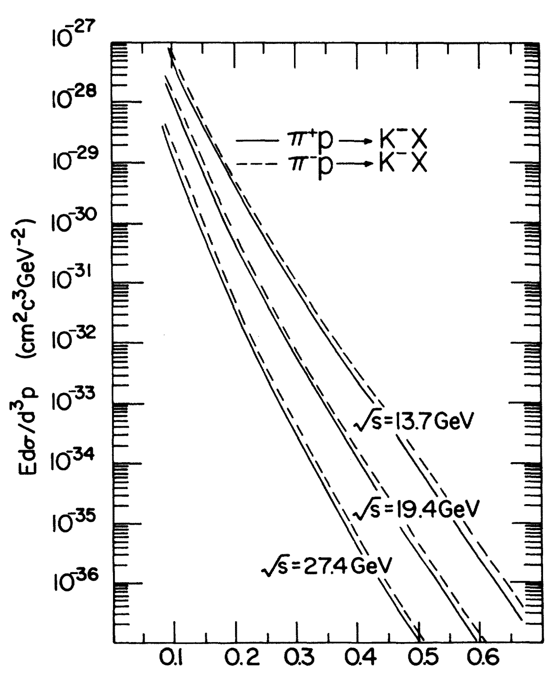

FIG. 5. EG-model predictions for the invariant cross sections for the processes $\pi^{\pm}p \to K^-X$ at $\theta_{c.m.} = 90°$. The curves correspond to incident laboratory momenta p_L $= 100$, 200, and 400 GeV/c.

(7). We have not included predictions for p and $\bar{p}$ production since the present model is expected to be less reliable for baryon production in the region of $p_\perp$ typically measured at Fermilab energies. The details of the method of calculation are given in Ref. 1, and the only change needed to calculate the πp cross sections from the equations given there is to replace the distribution functions of one of the incident protons in the $pp \to cX$ ($c=\pi^0, \pi^\pm, K^\pm$) calculations by the appropriate pion distribution functions. The differences among the predicted cross sections arise primarily from the EG-model assumption that the scattered quark appears as a valence quark of the observed particle, and also reflect differences in the normalizations found in Ref. 1. We emphasize that, once the pion distribution functions are given, there are *no free parameters* in the πp calculations, since all normalizations are determined by the pp data. The normalizations determined there are, however, generally uncertain by (10–15)%, and this uncertainty is also present in the cross sections presented here.

The experimental points shown in Fig. 1 are taken from the recent Fermilab results of Donaldson *et al.*[15] The EG model predicts the near-equality[16] of the $\pi^+ p \to \pi^0 X$ and $\pi^- p \to \pi^0 X$ inclusive cross sections, and thus data for both are presented. This equality is a general property of hard-scattering models[17,18] and should hold for all values of $x_\perp$. This equality does not necessarily hold in other models of inclusive scattering.[19,20] In view of the crude nature of our assumptions leading to the pion distribution functions, the agreement between the EG-model predictions and experiment is extremely good, particularly in the region $x_\perp > 0.2$. That the agreement is best for larger values of $x_\perp$ is precisely what we expect, as discussed above. We feel that this agreement lends support to the idea that a single quark-quark scattering mechanism may be responsible for all of the observed high-$p_\perp$ inclusive processes.

In Fig. 1 we also show (with dashed lines) the $\pi^\pm p \to \pi^0 X$ cross sections obtained using $u^\pi(x) = \text{constant}/x$ for the distribution of quarks in the incident pion and using the same form to describe the quark decay into a pion. In this case the normalization of the cross sections has been determined directly from the $\pi^+ p \to \pi^0 X$ data of Ref. 15, and it is not the same as the normalization found in Ref. 1 using a different quark decay function. These curves demonstrate that a variety of different distribution functions can account for the limited πp data. They also suggest that a substantially larger body of data is needed before a detailed form of these distributions can be determined directly from strong-interaction inclusive measurements. Calculations based on $xu^\pi(x) = (1-x)^2$ lead to cross sections which fall much more rapidly with $x_\perp$ than do the experimental values of $E_{\pi^0} d\sigma/d^3p_{\pi^0}$, and hence we have not included these results in Fig. 1. In Figs. 2–5 we present only the curves obtained using Eqs. (6) and (7). These curves demonstrate both the shape and the magnitude of the cross sections expected from the EG model.

In Figs. 6, 7 we show the effect of changing the sea contribution to the parton distribution functions of the pion for the processes $\pi^- p \to \pi^\pm X$. The lower curve at each energy corresponds to $c^{\pi^-}(x) = 0$, and the upper curve to $c^{\pi^-}(x) = 0.056(1-x)^{3/2}/x$. The latter function is obtained by applying the derivation leading to the KW[7] distributions to the pion. In this model

$$d_v^{\pi^-}(x) \propto (1-x)^{\gamma-1/2}/\sqrt{x}$$

and (8)

$$c(x) \propto (1-x)^\gamma / x,$$

where γ is a common parameter. For $d_v^{\pi^-}(x) \propto (1-x)/\sqrt{x}$ we must have $\gamma = \frac{3}{2}$ and $c^{\pi^-}(x) \propto (1-x)^{3/2}/x$. The constant 0.056 is then determined by assuming that the ratio of the momentum carried by the valence quarks to the momentum carried by the sea is the same for pions and protons.[21] This should provide a maximal estimate

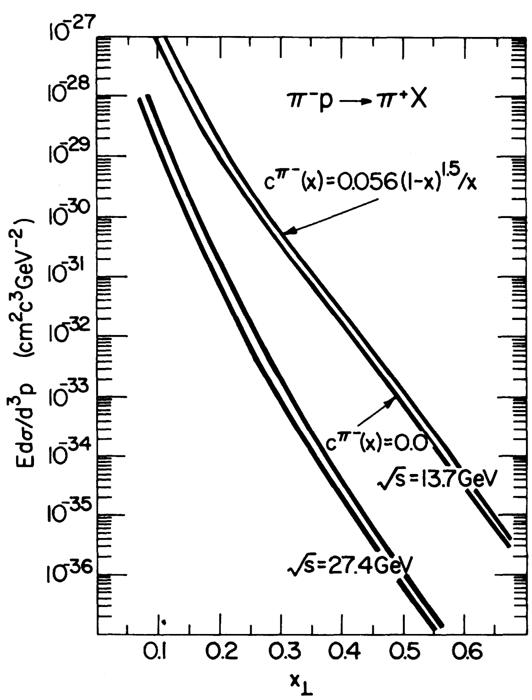

FIG. 6. Sensitivity of the EG-model cross section for $\pi^- p \to \pi^+ X$ to the assumed form of $c^{\pi^-}(x)$.

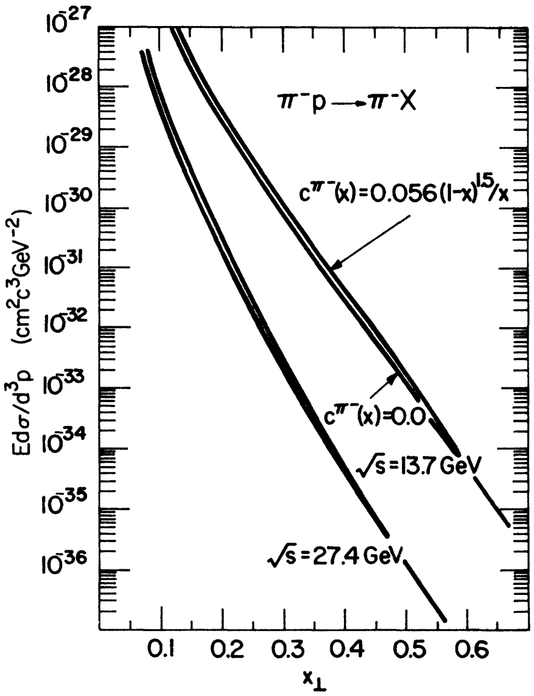

FIG. 7. Sensitivity of the EG-model cross sections for $\pi^- p \to \pi^- X$ to the assumed form of $c^{\pi^-}(x)$.

for the sea contribution at large $x_\perp$. We see from Fig. 7 that the use of these appreciably different estimates for the parton sea does not significantly affect the predicted cross sections, and hence the major contribution to the cross section must arise from the scattering of the valence partons in the pion.

Finally, we present in Fig. 8 the EG-model predictions for the ratios

$$R(\pi^{\pm 0}) = \frac{Ed\sigma/d^3p(pp\to\pi^0,\pi^\pm X)}{Ed\sigma/d^3p(\pi^- p\to\pi^0,\pi^\pm X)}\,. \qquad (9)$$

As is seen in this figure, the pp cross sections are expected to fall more rapidly with $x_\perp$ than the πp cross sections, a result which can be traced to the difference in the parton distributions of the proton and pion. The general shape of the $(pp \to \pi^0 X)/(\pi^- p \to \pi^0 X)$ ratio predicted here is in agreement with the Fermilab data,[15] although the $x_\perp \lesssim 0.3$ values predicted here are lower than those measured. This again reflects the fact, commented upon earlier, that the valence distribution functions for the partons within the pion should be more reliable at larger values of x. Hence we do not consider the numerical disagreements at lower $x_\perp$ significant and we emphasize that the major features of the ratio are correctly predicted.

The numerical values of the ratios are, of course, dependent upon the exact form of the pion distributions, but there are a number of general features of the ratios which will hold for the EG model regardless of the precise form of these distributions, and which will be true, to some extent, in all direct quark-quark scattering models.[17] General features of quark-quark scattering models which are reflected in Fig. 8 include: (1) The decrease of the ratios $R(\pi^{\pm 0})$ with increasing $x_\perp$, which results from the relative behavior of the valence parton distributions of the pion and proton, reflects the more rapid decrease of the proton distributions at large $x_\perp$. (2) $R(\pi^+) > R(\pi^0) > R(\pi^-)$ reflects the quark content of the incident π^- and p. The actual difference in the ratios can be used to test different models since they will depend upon the detailed assumptions concerning which quarks can scatter to form the observed particle.

One feature of Fig. 8 which is true for the EG model but is not, in general, true for other models of quark-quark scattering which have been recently proposed,[13] is the dependence of ratios $R(\pi^{\pm 0})$ on

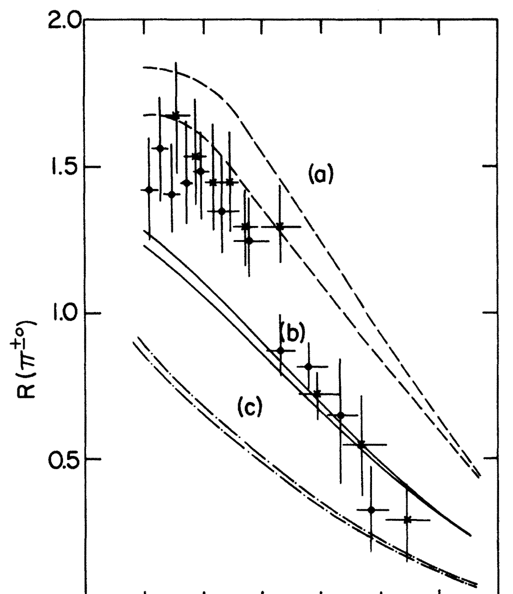

FIG. 8. Inclusive-cross-section ratios (a) $(pp\to\pi^+X)/(\pi^- p\to\pi^+X)$, (b) $(pp\to\pi^0X)/(\pi^- p\to\pi^0X)$, and (c) $(pp\to\pi^-X)/(\pi^- p\to\pi^-X)$. For (a) and (b) the upper curve gives the EG-model predictions at $p_L = 100$ GeV/c, $\theta_{c.m.} = 90°$ and the lower curve the predictions for $p_L = 400$ GeV/c, $\theta_{c.m.} = 90°$. For the π^- curves (c), the lower curve is at $p_L = 100$ GeV/c and the upper curve at $p_L = 400$ GeV/c. The experimental data are from Ref. 15 for $R(\pi^0)$ at $p_L = 100$ GeV/c (shown with × 's) and $p_L = 200$ GeV/c (shown with closed circles).

the center-of-momentum energy, $\sqrt{s}$. Since, in the EG model, the energy dependence of $Ed\sigma/d^3p$ is a function of both s and $x_\perp$ and can be different for different particles (it depends upon the relative behavior of the quark distribution and decay functions which contribute to each scattering), the ratios will depend upon the energy of the incident particles. As is shown in Fig. 8, this effect is expected to be small over the range of Fermilab energies. The details of this energy behavior will depend upon the exact functions used to describe the quark distributions and decays, and the results presented here are intended to be representative of the predictions of the EG model. In order to make more exact predictions, it is necessary to have a better knowledge of the correct quark distributions. Conversely, if the detailed quark distribution and decay functions needed in describing pp scattering are assumed to be known, then the EG model can be used to determine the details of the distribution of quarks within the pion.

In the above calculations (and in those of Ref. 1) we have presented the addition of the scale-breaking term of Eq. (1) as a modification of the basic quark-quark scattering amplitude. It is possible, however, to interpret this term as an effect of scale breaking in the parton distribution and/or decay functions.[2] The success of Eq. (1), which was obtained from the pp scattering data, in describing the πp scattering results seems to indicate that the interpretation of $F^2(\hat{t})$ as a q-q scattering modification may be correct. Since different distribution functions are needed to describe pp and πp scattering, the interpretation that the scale breaking occurs in the structure functions would require that the same $F(\hat{t})$ modifies both the pion and proton distributions. Because of the approximations used in determining the pion distributions, the interpretation of our results is not, however, without ambiguities, and the possibility that other interpretations are correct cannot be positively ruled out.

---

*Work supported in part by the Energy Research and Development Administration.

[1]E. Fischbach and G. W. Look, Phys. Rev. D 15, 2576 (1977), and in *High Energy Physics with Polarized Beams and Targets*, proceedings of the Argonne Conference, 1976, edited by M. L. Marshak (AIP, New York, 1976), p. 258.

[2]R. C. Hwa, A. J. Speissbach, and M. J. Teper, Phys. Rev. Lett. 36, 1418 (1976).

[3]S. M. Berman, J. D. Bjorken, and J. B. Kogut, Phys. Rev. D 4, 3388 (1971); S. D. Ellis and M. B. Kislinger, *ibid.* 9, 2027 (1974).

[4]F. W. Büsser *et al.*, Phys. Lett. 46B, 471 (1973); 55B, 232 (1975).

[5]B. Alper *et al.*, Nucl. Phys. B87, 19 (1975); B100, 237 (1975).

[6]J. W. Cronin *et al.*, Phys. Rev. Lett. 31, 1426 (1973); Phys. Rev. D 11, 3105 (1975); D. Antreasyan *et al.*, Phys. Rev. Lett. 38, 112 (1977); 38, 115 (1977).

[7]J. Kuti and V. F. Weisskopf, Phys. Rev. D 4, 3418 (1971).

[8]R. McElhaney and S. F. Tuan, Phys. Rev. D 8, 2267 (1973); Nucl. Phys. B72, 487 (1974).

[9]S. D. Drell and T.-M. Yan, Phys. Rev. Lett. 24, 181 (1970).

[10]E. D. Bloom and F. J. Gilman, Phys. Rev. Lett. 25, 1140 (1970).

[11]G. Altarelli, N. Cabibbo, L. Maiani, and R. Petronzio, Nucl. Phys. B69, 531 (1974).

[12]M. L. Perl, *High Energy Hadron Physics* (Wiley, New York, 1974), p. 453; M. L. Goldberger, D. E. Soper, and A. Guth, Phys. Rev. D 14, 1117 (1976).

[13]R. D. Field and R. P. Feynman, Phys. Rev. D 15, 2590 (1977).

[14]G. Farrar and D. Jackson, Phys. Rev. Lett. 35, 1416 (1975).

[15]G. Donaldson *et al.*, Phys. Rev. Lett. 36, 1110 (1976).

[16]The inclusive cross sections for $\pi^+ p \to \pi^0 X$ and $\pi^- p \to \pi^0 X$ are exactly equal if the scattering of identical quarks is not allowed, as would be the case if colored gluons were exchanged. Including such a contribution (as is done in the EG model), gives rise to a small difference between $\pi^\pm p \to \pi^0 X$, which difference can then be used to explore the color and flavor properties of the gluon.

[17]A discussion of the general features of hard-scattering models which can be tested in πp scattering is given by S. Fredriksson, Phys. Rev. Lett. 37, 1373 (1976).

[18]B. L. Combridge, Phys. Rev. D 10, 3849 (1974); Phys. Lett. 62B, 222 (1976).

[19]K. Kinoshita and Y. Kinoshita, Phys. Lett. 66B, 471 (1977).

[20]M. Teper, Phys. Lett. 59B, 455 (1975).

[21]We use the ratio obtained from the MKW functions of Ref. 8. This same assumption used in conjunction with the functional form of Eq. (7) gives the same normalization $[c^{\pi^+}(x) = 0.1(1-x)^{3.5}/x]$ as the assumption of the exact quality of $c^p(x)$ and $c^{\pi^-}(x)$.