Inelastic screening and total nuclear cross sections*

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We calculate high-energy neutron-lead total cross sections in the range of laboratory momenta from 5 to 400 GeV/c including inelastic-screening effects to all orders. Lowest-order screening corrections are seen to be dominant. The results are compared with experiment.

I. INTRODUCTION

High-energy total nucleon-nucleus cross sections have been of considerable interest for some time, and more so since the advent of accelerators with energies up to a few hundred GeV. The straight forward Glauber[1] picture has provided a framework for calculation of small-angle scattering cross sections and through the optical theorem the total cross section.

It was recognized some years ago[2-5] that the Glauber picture could be generalized to allow for a regenerated elastic amplitude, resulting from intermediate diffractively excited resonances. The incident nucleon in this picture diffractively excites on one nucleon of the nucleus and eventually, perhaps after interactions with several nucleons, de-excites, returning to its ground state, contributing then to the elastic scattering amplitude. We call this contribution to elastic scattering inelastic screening.

Until now, reliable calculations have been done including only the lowest-order screening (regeneration) effects.[6-8] A nucleon scatters elastically, off as many nucleons as Glauber theory dictates including possibly diffractive excitation on one nucleon followed eventually by de-excitation on another. The diffractively excited nucleon may elastically scatter as well of course. The Glauber picture further includes damping due to all types of production processes.

Several authors have studied the question of the validity of the Glauber picture in high-energy nucleon-nucleus elastic scattering mainly from the point of view of Regge theory.[9] This question deserves further study. The question we address ourselves to in this paper is the following. Glauber theory augmented by diffractive inelastic-screening effects calculated in lowest order provides a good description of the total cross section.[8,9] What then is the contribution of higher-order inelastic-screening effects? We will include to all orders resonance-resonance and nucleon-resonance couplings. Provided one still has good agreement with experiment the job of the theorist then is clear. One must find a fundamental basis for the picture adopted. Further the experimentalist will be challenged to produce as accurate measurements as possible to test fully the picture presented.

II. THE FORMALISM

We will consider complex nuclei as targets having atomic mass number $A \gg 1$. For this case the problem of elastic scattering can be treated through a coupled-channel optical model[3,5] which automatically includes inelastic screening effects. The coupled-channel equations are[3]

$$[\nabla^2 + k_\alpha{}^2 - U_{\alpha\alpha}(\vec{r})]\psi_\alpha(\vec{r}) = \sum_{\beta\neq\alpha} U_{\alpha\beta}(\vec{r})\psi_\beta(\vec{r}), \quad (1)$$

$$U_{\alpha\beta}(\vec{r}) = -4\pi f_{\alpha\beta}(0) A\rho(\vec{r}). \quad (2)$$

Equations (1) describe the coherent production of particles α given an incident particle 1 in one of the channels α. The sum on the right-hand side of (1) is over all channels β which couple diffractively to channel α. The wave function $\psi_\alpha(\vec{r})$ describes the coherently produced amplitude in the channel α which is elastic scattering for $\alpha = 1$. The $f_{\alpha\beta}(t)$ are two-body diffractive amplitudes at four-momentum t corresponding to producing particle β on a nucleon with incident particle α. These amplitudes have the dimension of length so that the elastic scattering amplitude relates to the differential scattering cross section through $d\sigma^{el}/dt = |f_{11}(t)|^2$. The quantity $\rho(\vec{r})$ is the average nucleon single-particle density in the nucleus convoluted with the nucleon interaction range. The wave number k_α is the three-momentum of particle α in the laboratory frame. We are clearly free to ignore nuclear target motion since the dominant production and scattering is at very small angles. We are thus characteristically in the region where the eikonal model is applicable.

We consider a wave with impact vector $\vec{b}$ traveling in the z direction, and assume that the produced waves are also in the same direction with unaltered impact parameter. Then one obtains using the usual eikonal methods and writing

$$\psi_\alpha(\vec{b},z)=e^{ik_\alpha z}\varphi_\alpha(\vec{b},z)\,, \tag{3}$$

$$\frac{d}{dz}\varphi_\alpha(\vec{b},z)=\frac{1}{2ik_\alpha}\sum_\beta U_{\alpha\beta}(\vec{b},z)e^{i(k_\beta-k_\alpha)z}\varphi_\beta(\vec{b},z)\,. \tag{4}$$

The amplitude for the outgoing channel α, with $\alpha=1$ the incident channel, is then

$$\begin{aligned}F_{\alpha 1}&=-\left(\frac{k_\alpha}{k_1}\right)^{1/2}\frac{1}{4\pi}\int e^{-i\vec{k}_\alpha\cdot\vec{r}}\sum_\beta U_{\alpha\beta}(\vec{r})\psi_\beta(\vec{r})d^3r\\&=\left(\frac{k_\alpha}{k_1}\right)^{1/2}\frac{k_\alpha}{2\pi i}\int e^{i\vec{q}_\alpha\cdot\vec{b}}d^2b\,[\varphi_\alpha(\vec{b},\infty)-\delta_{\alpha 1}]\,,\end{aligned} \tag{5}$$

using Eq. (4) with $\vec{q}_\alpha=\vec{k}_\alpha-\vec{k}_1$. The coherent production cross section is

$$\frac{d\sigma_\alpha^{(c)}}{d\Omega}=|F_{\alpha 1}|^2=\frac{k_\alpha{}^2}{\pi}\frac{d\sigma_\alpha^{(c)}}{dt}\,. \tag{6}$$

For $\alpha=1$ Eq. (6) corresponds to elastic scattering.

III. CALCULATION OF THE TOTAL CROSS SECTION

The total cross section is calculated from (5) taking $\alpha=1$ and using the optical theorem. We must know the two-body diffractive amplitudes $f_{\alpha\beta}(0)$ to carry out this program. The magnitudes of these amplitudes will determine how many states α,β contribute to the coupled equations (4). We can use experimental data to determine $|f_{1\alpha}(0)|$ as follows. Murthy *et al.*[7] have deduced from proton-proton interaction data the following empirical form for the diffractive part of the reaction

$$p+p\to p+M: \tag{7}$$

$$\begin{aligned}\left.\frac{d^2\sigma}{dt\,dM^2}\right|_{t=0}&=26.47(M^2-1.17)-35.969(M^2-1.17)^2\\&\quad+18.47(M^2-1.17)^3-4.13(M^2-1.17)^4\\&\quad+0.341(M^2-1.17)^5,\quad 1.17<M^2<5\ \text{GeV}^2,\\&=4.4/M^2,\quad M^2>5\ \text{GeV}^2\end{aligned} \tag{8}$$

We take the phase of the amplitudes $f_{1\alpha}$ $(\alpha\neq 1)$ to be as for elastic scattering. For resonance-resonance amplitudes following triple-Pomeron considerations of Henyey[10] we write

$$f_{\alpha\beta}(0)=ik_\alpha\left(\frac{0.24}{M_\alpha M_\beta}\right)|M_\beta^2-M_\alpha^2|^{1/2}\quad \alpha,\beta\neq 1\,. \tag{9}$$

The strength of the amplitude (9) is determined from p-p reactions through extrapolation. We take all elastic amplitudes $f_{\alpha\alpha}$ as equal with the phase given by proton-proton elastic scattering at the appropriate energy.[11] Since we are mainly interested in an energy region where the elastic amplitudes are predominantly imaginary, this should be good enough. We will comment further on inelastic phases below.

We have divided the mass continuum into a grid with a finite number of masses. The resulting calculated total cross sections are we reckon accurate to about 0.3%. We have made calculations for the total cross section for the heaviest nucleus for which data are available, namely lead, for laboratory momenta from 5 to 400 GeV/c. The amount of computer time required has limited us at the high-energy end of the range. We have used four intermediate states in our grid at 5 GeV/c up to twenty states at 400 GeV/c.

Figure 1 shows our calculated total nucleon-lead cross sections along with neutron-lead measured cross sections.[7,12-17] Curve (a) is the result with no inelastic screening. Curve (b) is the result including only direct coupling to channel 1, i.e., we take $f_{\alpha\beta}=0$, $\alpha,\beta\neq 1$. This is essentially the result of Ayre and Longo.[8] We have, in these calculations, chosen the effective nuclear density function as follows. We have taken the $\rho(r)$ determined in the photoproduction of ρ^0 mesons[18] and have increased the radius parameter by 0.13 F corresponding to the fact that the ratio

$$\frac{d\sigma(p,p)}{dt}\Big/\frac{d\sigma(\gamma,\rho)}{dt}\approx e^{2t}\,.$$

Our radius parameter, using a Wood-Saxon form is then $R=6.95$ F for Pb. We use the surface thickness parameter $c=0.545$ F as in photoproduction.[18] The function $\rho(r)$ has the form

$$\rho(r)=\frac{\rho_0}{1+\exp[(r-R)/c]}\,. \tag{10}$$

We see very good agreement between calculation (b) and the experimental data. At the upper end of the energy scale the effect of inelastic screening is about 7%. It is of utmost importance then to see the effect of higher-order screening corrections. To this end we now include terms corresponding to Eq. (9). The resulting curve is labeled (c) in Fig. 1. We include one more curve corresponding to changing the sign of all the nonelastic diffractive amplitudes $f_{\alpha\beta}$, $\alpha\neq\beta$. This ambiguity in sign is present for all nonelastic amplitudes. The corresponding curve is labeled (d). It is of course possible that some of the $f_{\alpha\beta}$ have positive imaginary phases and some have negative ones. We note in this connection that alternating the signs of the amplitudes $f_{\alpha\beta}$ so that

$$f_{\alpha\beta}\to(-1)^{\alpha-\beta}f_{\alpha\beta} \tag{11}$$

is equivalent to alternating the sign of the wave function in each channel so that $\varphi_\alpha\to(-1)^\alpha\varphi_\alpha$. We find then that since the cross section is independent of the sign of the wave function we get the same answer as with all $f_{\alpha\beta}$ having positive imaginary phases

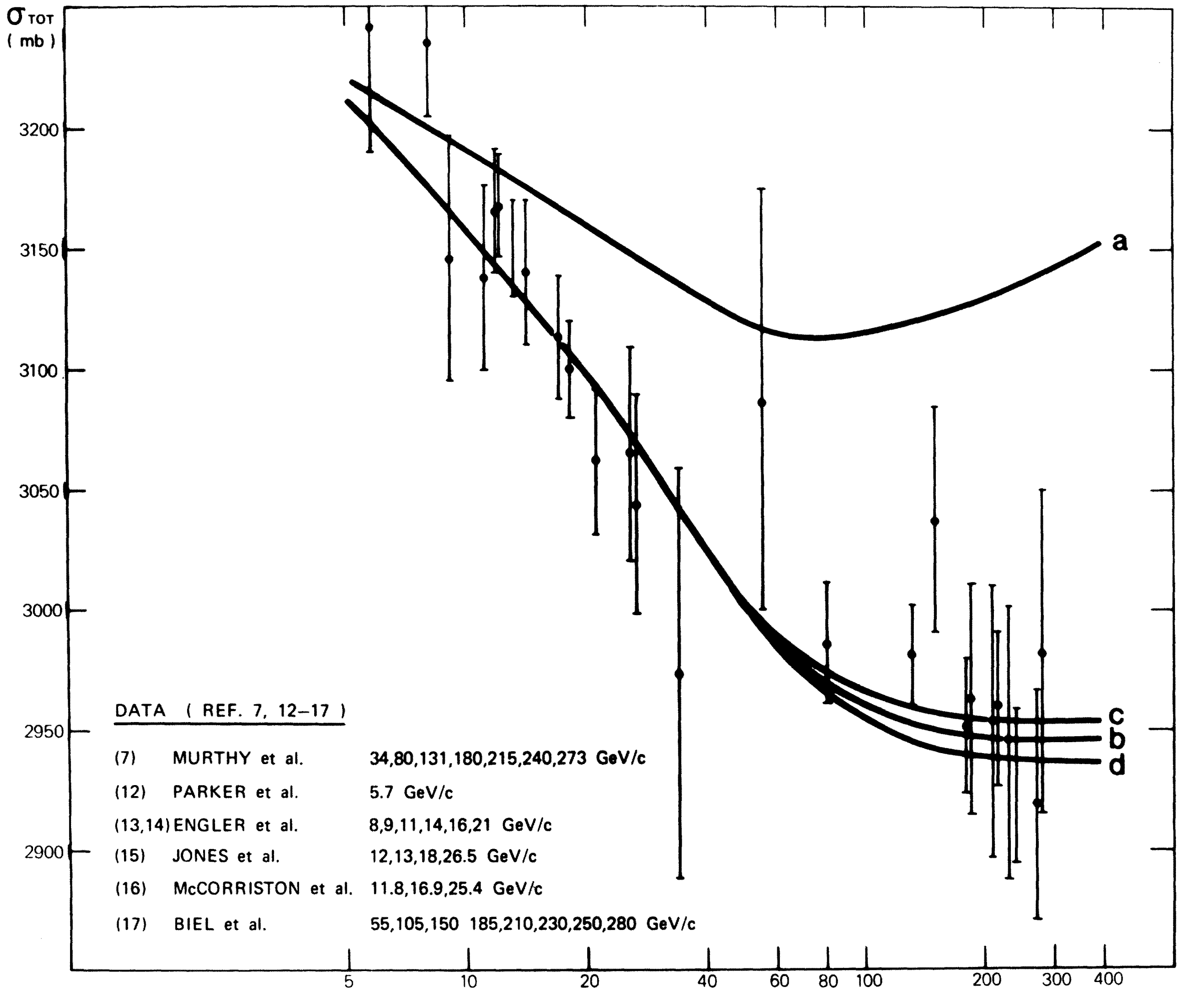

FIG. 1. Calculated neutron-lead total cross section as a function of laboratory momentum. Curve *a* no inelastic screening, *b* inelastic screening included in the approximation of zero resonance-resonance coupling, c inelastic screening with nonzero resonance-resonance coupling for diffractive production amplitudes, *d* as *c* but with sign changed for diffractive production amplitudes. The experimental data is from Refs. 7 and 12–17.

IV. CONCLUSIONS

We see from the results of our calculations that we get very good agreement with measured total neutron-nucleus total cross sections including inelastic screening. The coupled-channel procedure is a highly convergent one. Inelastic screening corrections calculated in lowest order, that is coupling only the incident channel to inelastic diffractive channels, is the dominant effect assuming the validity of Eq. (9). Higher-order couplings between diffractively excited states lead to only small corrections. Whereas the dominant screening correction is of the order of 7%, the higher-order corrections, regardless of phase, are only a fraction of a percent. These results on the one hand are a challenge to the experimentalist to reduce the errors in the measurements as far as possible. On the theoretical side, it is clear that a justification of the methods adopted here for calculation, from the point of view of as fundamental an approach as possible, is of great interest.

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