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Nonscaling behavior—The proton, neutron, and deuteron structure functions*

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We present a simple parton-model interpretation of the approach to scaling observed in lepton scattering off protons and deuterons. Different final-state configurations are classified and their behavior predicted using quark-counting rules. Good fits to the proton data are obtained. Using a relativistic description of the deuteron, its elastic form factor and inelastic structure function are analyzed. An extraction of the neutron structure function is performed by fitting the deuteron data. Several characteristics of the resulting parametrizations are shown to support our general model. Further experimental consequences are described.

I. INTRODUCTION

The approximate validity of Bjorken scaling in deep-inelastic electroproduction[1] has had a considerable influence on the theory of hadrons. The currently most popular view of hadrons is that they are composite states of (almost) pointlike objects. The success of such of a picture and the models it leads to in interpreting the major features of both weak interactions[2] and certain limiting behaviors of electromagnetic and strong interactions (the mass spectra and the large mass and large transverse momentum behavior, for example) is striking and perhaps even better than one should expect. The next problem is to find the set of fundamental theories that leads to models in the above successful class.

The observation that asymptotically free gauge theories of strong interactions are capable of exhibiting scaling to within logarithmic factors whose powers are controlled by the anomalous dimensions in the theory[4] was an important step in this direction. The next question is whether or not these theories can quantitatively fit the various features of the data. This task has been undertaken by several groups who have stressed the importance of studying the nonscaling, or rather the approach to (approximate) scaling, behavior of the inelastic structure functions and of comparing features of the observed behavior with the predictions of a basic theory. In particular, Tung[5] has compared the predictions of asymptotically free theories to those of conventional theories and De Rújula, Georgi, and Politzer[6] have examined and defended a study using asymptotically free quantum chromodynamics (QCD) theory in a series of papers and talks. The practical problems of carrying out such a program have been discussed by Gross, Treiman, and Wilczek,[7] who have examined uncertainties in making mass-dependent corrections. Other authors[8] have discussed possible difficulties in using perturbation theory with the operator-product expansion. This program is indeed an extremely important one for weak, electromagnetic, and strong interactions.

Our purpose in this paper is quite modest in comparison to the total program of the above authors. We only wish to point out that there are certain scale-breaking effects that are very simple from a physical point of view and which would seem to be present in any theory susceptible to a parton interpretation. These terms are *a priori* expected to be important for large x, the Bjorken scaling variable. At small x, they do not necessarily dominate from general arguments, and there are many additional effects that could become important. Indeed, the data indicate that the terms under consideration are certainly not dominant there.

These contributions show up first in the twist-6 terms in the language of the operator-product expansion and would thereby be normally neglected. However, they would be expected to be large from physical arguments. While they fall rapidly in q^2, their coefficient is expected to be large. They do not correspond to interference terms between various final-state configurations that tend to populate different regions of the final phase space. If such "trivial" scale-breaking terms are present in the data with their necessarily finite q^2 range, it is certainly important to recognize their effect before asking more fundamental and specific questions of such data since these terms should be present in almost any theory.

These contributions to scale breaking are most easily described in the parton-quark language. The structure functions will be written as a sum over final states in which all the quarks have low transverse momenta except for (a) one quark which recoils with momentum $\approx q$, (b) two quarks that recoil with a total of $\sim q$ but each has a finite fraction of q, (c) three quarks that recoil with a total of $\sim q$, etc. The above classification neglects the coherence between such states and should be applicable for sufficiently large q values where the final configurations become incoherent. The importance of type (b) terms, for example, will be shown to be the fact that while they fall in q^2 at fixed x, they vanish less rapidly than type (a) terms for fixed q^2 as $x \to 1$. We should point out that the partons in our model do not have form factors as used in the extended parton model of Chanowitz and Drell[9] (but our two-quark system does).

A perhaps more physical application of the above classification scheme is to deep-inelastic scattering from the deuteron in which (a) a fragment of one of the baryons recoils with $\sim q$, (b) one baryon recoils with $\sim q$ (quasielastic scattering), and finally (c) both nucleons recoil together (elastic or resonance scattering). This case will be treated in detail in this paper when the neutron structure function is extracted from the data.[10] This extraction will be done using a fully relativistic model for the deuteron, which we do not believe has been done before. As a check on our assumed wave function, the deuteron elastic form factor and structure functions will be considered in some detail and compared to experimental data.[11]

One point worth mentioning is that there are many variables that asymptotically become equal to the Bjorken x, and which make data at small q^2 satisfy scaling to different degrees. One often used is the Bloom-Gilman x' $[=x(1+M^2/2M\nu)^{-1}]$. Most of these improved scaling variables, however, do not have a clear theoretical significance. We shall neglect such effects for the most part, although an estimate of both mass and initial-state effects will be mentioned. Our purpose is to see if one can fit the approach to scaling with terms that have a clearer and more direct physical interpretation.

The paper is organized as follows. In Sec. II we discuss the proton structure function, separating the different contributions according to the different possible final states. We use dimensional-counting rules to get the general form of these terms as a function of x and q^2, and then fit the experimental data. The threshold limit is analyzed in Sec. III (the Drell-Yan-West relation). Sec. IV contains a relativistic description of the deuteron, and explicit expressions for the distribution function of nucleons in the deuteron (essential in inelastic scattering) and for its form factor are given. The parameters in the deuteron wave function are determined by fitting the resulting elastic form factor. In Sec. VI we discuss the deuteron structure function. As was done before for the proton, the different final-state contributions are separated. For large q^2 and/or $x_d < \frac{1}{2}$, we have only inelastic contributions, and by fitting the data for the deuteron in this range, we can extract the neutron structure function. Then we include the quasielastic term (important for low q^2 and x_d around $\frac{1}{2}$, and the possibility of strong final-state interactions between proton and neutron (important for $x_d \to 1$). Finally, some conclusions are presented in Sec. VI.

II. PROTON STRUCTURE FUNCTION

In order to illustrate the physical point that we wish to make without obscuring the issue with algebra, we will treat only the spin-averaged case and hence will neglect the spin of the quarks in the formulation of the model. Following the classification discussed in the Introduction, the contributions to the proton structure function to be considered here are illustrated in Fig. 1. Our analysis is very much in the spirit of the constituent-interchange model (CIM) of hadron collisions,[12] in the sense that it is clearly necessary to consider all possible final states in order to extract those configurations that are expected to dominate in a particular region of phase space. And also as in the CIM, we shall use dimensional counting to predict the behavior of form factors[13] and generalized structure functions.[14]

In Fig. 1(a), one quark absorbs all the momen-

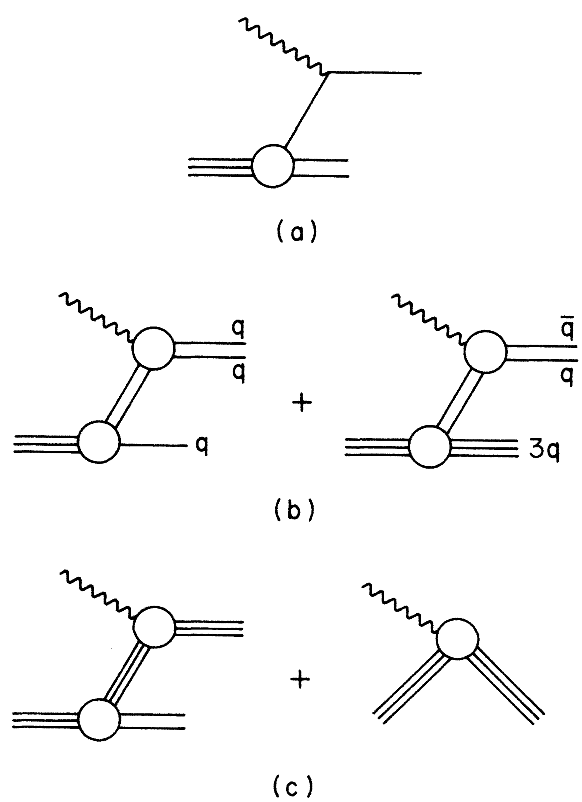

FIG. 1. Contributions to the proton structure function, with one (a), two (b), and three (c) quarks recoiling coherently.

tum q carried by the virtual photon. This is the dominant diagram of the parton model. In Fig. 1(b), the photon is absorbed by a two-quark system which then recoils, each quark having a finite fraction of q. This diquark state need not be thought of necessarily as a bound system, but a photon striking a virtual meson in the target that remains bound will also be of this type. In Fig. 1(c), the photon is absorbed by a triquark, or baryon, system and this obviously involves the form factors for nucleon elastic scattering and resonance production. This latter contribution is very small in the region of interest and will be neglected.

Since the diagram in Fig. 1(a) approximately scales, for the present purposes its contribution to $\nu W_2(x)$, or rather $F_2(x)$, will be written in the form

$$F_{2p}^{s}=A_s(x)(1-x)^3\,, \tag{1}$$

where $A_s(x)$ is a rather slowly varying function of x which is expected to peak such that the most likely quark momentum is near (or less than) $\frac{1}{3}$. $A_s(x)$ may also be a very slowly varying function of q^2. Such slow variations can arise from a fundamental scale breaking, such as QCD, or from the kinematic effects of the binding of the quarks. This latter effect could be called a mass effect, an off-mass-shell effect, or a wave-function effect, as the reader prefers. It has been estimated using a choice for the relativistic bound-state wave function that was successful in other contexts[15] and a version of which will be used in Sec. III to describe the deuteron. We find an effect which goes in the opposite direction from that to be discussed shortly. Such effects will be neglected here but if they were included, it would simply increase the normalization of our explicit nonscaling term. Our object here is to see if the observed scale breaking at moderate and large x values ($x \gtrsim \frac{1}{3}$) can be explained with an $A_s(x)$ that does not depend strongly on q^2.

In the first diagram of Fig. 1(b), the photon is absorbed by a diquark system that has a form factor $f_d(q^2)$ that falls as $1/q^2$ from dimensional counting. The x dependence is quite easy to infer from the graph. This contribution must vanish as $(1-x)$ for $x\to 1$ since there is only one spectator quark. Now the most likely diquark momentum fraction is $\sim\frac{2}{3}$, and this follows automatically if the nonscaling term is chosen to have the form calculated for the valence constituent:

$$F_{2p}^{\rm ns}(x,q^2)=A_{\rm ns}f_d^2(q^2)x^2(1-x)\,. \tag{2}$$

This has all the desired limiting properties if f_d is parametrized as

$$f_d(q^2)=d^2(d^2-q^2)^{-1}\,. \tag{3}$$

Note that if a virtual meson abosrbs the photon and remains bound, as in the second diagram in Fig. 1(b), the structure function will have the same q^2 dependence as above arising from the pion form factor but it will fall as $(1-x)^5$. It is estimated to have a small overall normalization for $x>\frac{1}{3}$. Hence it will be neglected in our fits.

The total structure function in this approximation is

$$F_{2p}=F_{2p}^{s}(x)+F_{2p}^{\rm ns}(x,q^2)\,, \tag{4}$$

and higher terms have been neglected. Fits to the proton data[10] are shown in Fig. 2, and one sees that it is possible to have both a consistent and simple picture of the approach to scaling in this framework for large enough x and $(-q^2)\gtrsim 2$ GeV^2. If scale breaking is to differentiate between specific basic theories, it evidently must be studied at small $x\lesssim 0.3$, not at large x where the observed scale breaking can be simply explained in terms of physically expected effects in any scale-invariant theory with even an approximate parton interpretation. This is not to say that our term necessarily explains all the scale breaking observed in this region, but without prior prejudice and information, it is not possible to decide how much is to be ascribed to the more fundamental (and interesting) properties of the theory under consideration.

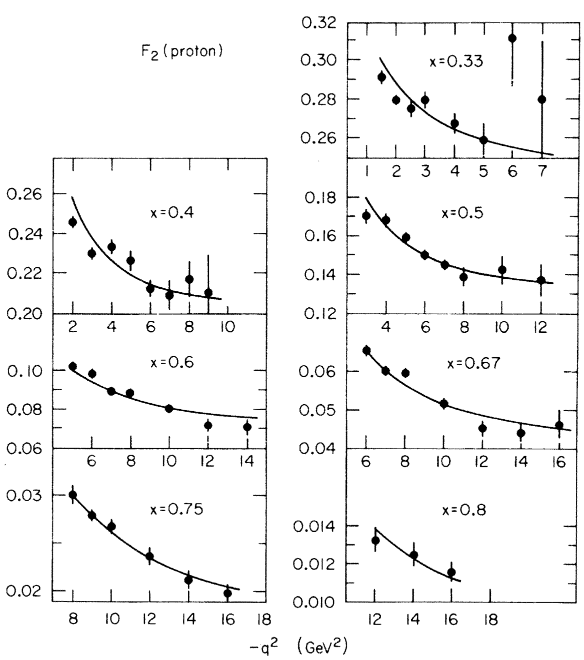

FIG. 2. Fits to the data of the proton structure function $F_{2p}(x,q^2)$, for different values of x, as a function of q^2. See Eq. (4).

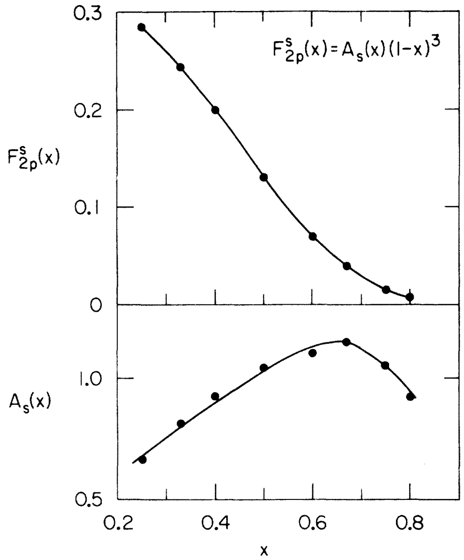

FIG. 3. Scaling part of the proton structure function $[F^s_{2p}(x)]$, and coefficient $A_s(x)$ of this same function.

The parameters used in the above fits are $A_{ns}=2.5$, $d^2=2$. $A_s(x)$ is quite slowly varying in x, with an average value of ~0.9. Graphs of $F^s_{2p}(x)$ and $A_s(x)$ are given in Fig. 3. The value of d^2 we find uncomfortably large, but it is necessary in order to fit the data at small $q^2(\gtrsim 2\ \mathrm{GeV}^2)$. A value of $d^2=1$ fits for $q^2\gtrsim 3\ \mathrm{GeV}^2$. Since $A_s(1)\neq 0$, F_2 satisfies the Drell-Yan-West relation for $x\sim 1$.

While the data[10,16] for F_1 have not been fully analyzed, we have found that the scaling terms in F_1 and F_2 extracted by the above procedure agree better with the Callan-Gross[17] relation $(xF^s_1=F^s_2)$ than the total structure function at low q^2. Since the diquark term is an effectively integer spin object, it could break this relation for the full (unseparated) structure functions in regions where it is important.

III. DRELL-YAN-WEST RELATION

The threshold limit of the structure functions should be smoothly connected, in the sense of Bloom-Gilman duality,[18] to the elastic or resonance form factors $G(q^2)$. According to the Drell-Yan-West relation,[19] as x approaches one from below $[x=1+(m^2-M^2)/q^2$, where m is the missing mass and M is the proton mass], one has

$$(-q^2)G^2_{Mp}(q^2)\cong\int dm^2F_{2p}(x,q^2)\,,$$

$$q^8G^2_{Mp}(q^2)\sim\int dm^2[A_s(1)(m^2-M^2)^3+A_{ns}d^4(m^2-M^2)]\,,\qquad(5)$$

and the integral runs roughly from the nucleon mass M up to the effective threshold for pions.[20] Thus the nonscaling terms contribute to the leading asymptotic behavior of the form factors and for our fit, dominate. The above is clearly not the complete story since there are other contributions, especially interference terms, that become coherent in the limit $x\to 1$ and also contribute to leading order in q^2. This is necessary since G must contain a coherent sum over charges, whereas the contribution to the usual structure functions involves the sum of the squares of the charges of the elementary constituents. The nonscaling terms A_{ns} contain some of the interference effects, but not all. In any case, the relation (5) is approximately satisfied if m is integrated from M to the threshold for two pions, $M+2\mu$.

Finally, we note that if the above connection also holds for the neutron, with the same integration region and $G_p(q^2)/G_n(q^2)=\text{constant}$, then if the scaling term dominates one has $(\mu_n/\mu_p)^2\cong[A^n_s(1)/A^p_s(1)]$, whereas if the nonscaling term dominates, which is the case in our fits, then $(\mu_n/\mu_p)^2=(A^n_{ns}/A^p_{ns})$. Otherwise the value is an intermediate one. Our fit for the first ratio will be shown in a later section to be ~0.40, whereas the ratio for the nonscaling term is ~0.33. Both of these are somewhat below the square of the experimental ratio of magnetic moments (~0.47) but are consistent within the errors of our extraction. This relation is not to be taken too quantitatively because of the coherence problems alluded to above.

IV. THE RELATIVISTIC DEUTERON

In order to describe the deuteron in a relativistic manner, which is necessary for our present purposes, one needs to have to have some knowledge of the Bethe-Salpeter wave function with one particle on-shell. A general relativistic description of nuclear bound states has been given elsewhere,[15] and its connection to the familiar nonrelativistic description was presented in detail.[21] In terms of the deuteron wave function $\psi_a(x,\vec{k}_T)$, the probability function is given by ($a=$ neutron or proton)

$$G_{a/d}(x,\vec{k}_T)=\frac{1}{2(2\pi)^3}\,\frac{x}{(1-x)}\,|\psi_a(x,\vec{k}_T)|^2\,,\qquad(6)$$

and the deuteron form factor at a four-momentum transfer $q^2(=-q_T^2)$ is given by

$$F_d(q^2)=\sum_a F_a(q^2)\int\frac{dx\,d^2k_T}{2(2\pi)^3}\,\frac{x}{(1-x)}\psi^*_a(x,\vec{k}_T+(1-x)\vec{q}_T)\times\psi_a(x,\vec{k}_T)\,,\qquad(7)$$

where $F_a(q^2)$ has been replaced by its on-shell value; the integral multiplying it is then the intrinsic body form factor of the deuteron.

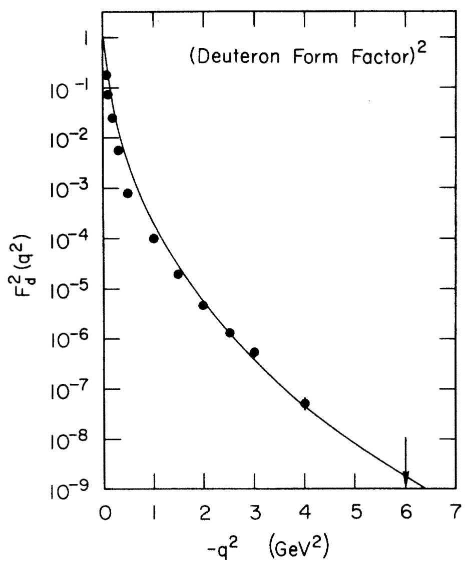

FIG. 4. Fit to the (deuteron form factor) Ref. 2.

In Ref 15 it was shown that a good fit to quasi-elastic scattering processes involving the deuteron could be achieved by choosing

$$\psi(x, \vec{k}_T) = N(x)(1-x)^3\{[k_T^2+M^2(x)][k_T^2+M^2(x)+\delta_1^2] \times [k_T^2+M^2(x)+\delta_2^2]\}^{-1}, \quad M^2(x) = M^2 - x(1-x)M_d^2, \tag{8}$$

where M is the nucleon mass, the deuteron mass is M_d, and $N(x)$ is a slowly varying function of x. Since ψ describes one on-shell and one off-shell particle, neither ψ nor G is necessarily symmetric around $x=\frac{1}{2}$. Isospin symmetry implies that $G_{p/d}(x) = G_{n/d}(x)$, not that $G_{p/d}(x) = G_{n/d}(1-x)$, although the latter relation may be a reasonable approximation in certain circumstances.

Now since $G_{a/d}(x, k_T)$ is the probabiltiy of finding the constituent a in the deuteron with longitudinal momentum fraction x and transverse momentum k_T, it must be related in some way to the square of the nonrelativistic wave function for low momentum. Such a relation follows by writing $x = (M+k_z)/M_d$ and expanding in powers of k_z. The corresponding nonrelativistic probability function is easily seen to be the square of a generalized Hulthén wave function. This approximate connection can be used to estimate the values of the constants δ_1^2 and δ_2^2.

The normalization constant of ψ can be computed by the condition

$$\sum_a \int dx\, d^2k_T\, x G_{a/d}(x, \vec{k}_T) = 1,$$

which expresses the fact that the sum of the fractional momenta of the proton and the neutron is the total (fractional) momentum of the deuteron. Note that if G is symmetric around $x=\frac{1}{2}$, this is equivalent to the condition

$$\int dx\, d^2k_T G_{a/d}(x, \vec{k}_T) = 1,$$

which follows from the fact that the number of particles is fixed and $G_{n/d}(x, k_T) = G_{p/d}(1-x, k_T)$.

The deuteron form factor can now be computed from $\psi(x, k_T)$. A fit that can be achieved for our spinless model is given in Fig. 4 for the values

$$\delta_2^2 = 2\delta_1^2 = 400M\epsilon, \tag{9}$$

where ϵ is the binding energy of the deuteron, and the isoscalar form factor was taken to be equal to the proton form factor for all q^2. The data are from Ref. 11. The fit is not very sensitive to the value of δ_1 and δ_2; for example, the set $\delta_1^2 = \delta_2^2 = 200M\epsilon$ also provides a reasonable fit. If spin were put into the model, and especially if d-state effects were then included, the fit could be made much better since the quadrupole contribution naturally gives a shape that is similar to that of the data points. The form factor has the asymptotic behavior in q^2 given by quark counting.[22]

The deuteron structure function in x is given in terms of

$$xG_{a/d}(x) = \int d^2k_T\, xG_{a/d}(x, \vec{k}_T),$$

which behaves as (for $x \approx 0$ or 1)

$$\sim N^2(x)x^2(1-x)^5.$$

We have chosen $N(x) = N_0x^2$ for the calculation, but setting N = constant has no significant effect on the results given here.

V. THE NEUTRON-STRUCTURE-FUNCTION EXTRACTION

Just as was argued in the proton case, inelastic scattering from the deuteron has several distinct contributions. At very large q^2, on expects that the dominant term has one quark absorbing the photon momentum; as q^2 decreases, more and more components of the deuteron will participate. In order to untangle the many terms that contribute in this case, we shall start our fit at large q^2 and then extend it to lower values by adding in the expected next terms. We shall check at each stage that the fit of the previous stage still holds. We shall work in terms of the natural Bjorken variable for the deuteron,

$$x_d = -q^2/2M_d\nu,$$

which is one-half the x defined in terms of the nucleon mass. The following procedure should be compared with that used by Atwood and West.[23]

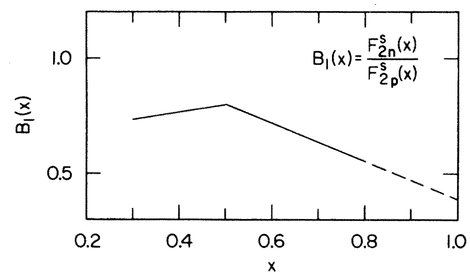

FIG. 5. Ratio of the scaling parts of the neutron and proton structure functions $F^s_{2n}(x)/F^s_{2p}(x)$.

Large q^2. In this limit, the scattering from the constituent nucleons is highly inelastic and the photon momentum is absorbed by one and perhaps two quarks as was discussed in the proton case. The term in which three quarks share the photon q will be considered separately (quasielastic scattering). Thus for large q^2 we can write (neglecting small-k_T^2 effects)

$$F_{2d}(x_d, q^2) = \sum_a \int_{x_d}^1 dy\, F_{2a}(x_d/y, q^2) G_{a/d}(y)\,, \qquad (10)$$

where certain off-shell effects in F_{2a} have been neglected. This formula has been discussed in the scaling limit by Landshoff and Polkinghorne[24] for several types of reactions. Note that since $G_{a/A}(y)$ is strongly peaked at $y \sim \frac{1}{2}$, one has the very approximate relation

$$F_{2d}(x_d, q^2) \sim \sum_a F_{2a}(2x_d, q^2)\theta(\tfrac{1}{2} - x_d)\,,$$

which strictly holds only in the limit of zero binding but has a simply physical interpretation. It turns out that this approximation overestimates the deuteron function by 5–10%.

Since $G_{a/d}(x)$ is known with some accuracy, the more exact relation (10) will be used to extract the neutron structure function $F_{2n}(x, q^2)$ from the large q^2 deuteron data. In order to carry out the fit in a convenient form, define

$$F_{2n}(x, q^2) = F^s_{2n}(x) + F^{ns}_{2n}(x, q^2)\,, \qquad (11)$$

where

$$F^s_{2n}(x) = B_1(x) F^s_{2p}(x)\,,$$

$$F^{ns}_{2n}(x, q^2) = B_2(x) F^{ns}_{2p}(x, q^2)\,.$$

A fit to the data can be achieved with the $B_1(x)$ given in Fig. 5 and with $B_2(x) = \text{constant} = \frac{1}{3}$. We have restricted the extraction to $x_d < \frac{1}{2}$ in order to decrease the sensitivity to the assumed form of $G_{a/d}(y)$. The resultant fit to data[10] in this region is given in Fig. 6. The separated scaling and nonscaling parts (for $q^2 = -8$ GeV^2) of the structure functions for the neutron and proton are given in Fig. 7.

There are several points worth mentioning. The function $B_1(x)$ is slowly varying over the range of

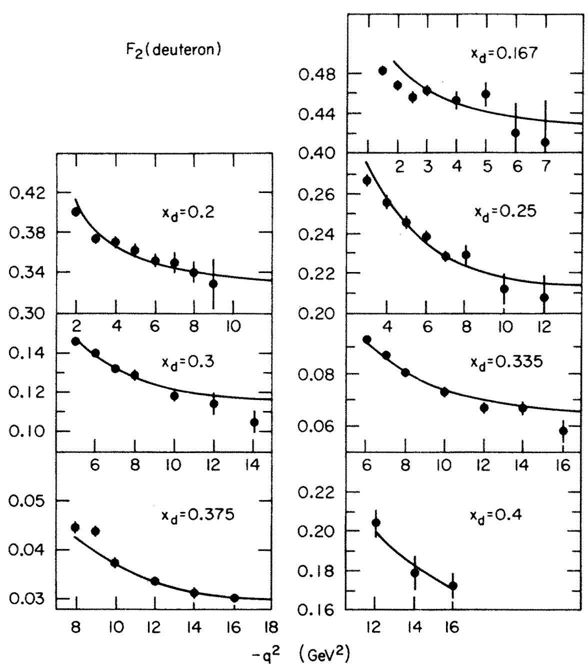

FIG. 6. Fits to the deuteron structure function $F_{2d}(x, q^2)$, for different values of x, as a function of q^2, used in the extraction of the neutron data. *Note added in proof.* The vertical scale for $x_d = 0.4$ should be divided by ten.

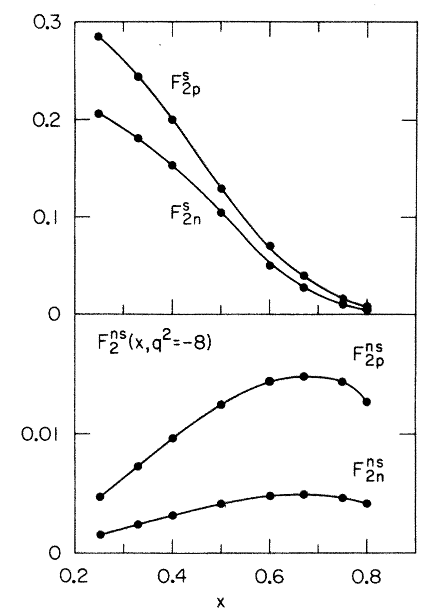

FIG. 7. Comparison of the neutron and proton scaling and nonscaling contributions [for $q^2 = -8$ GeV^2] to the structure functions.

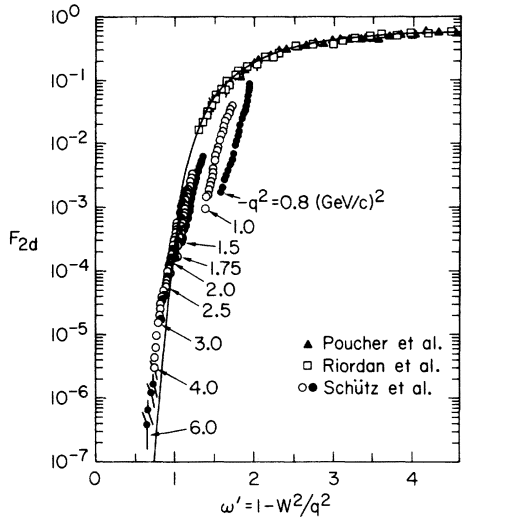

FIG. 8. Prediction for the deuteron structure function for very large q^2. The data are from Refs. 10 and 11.

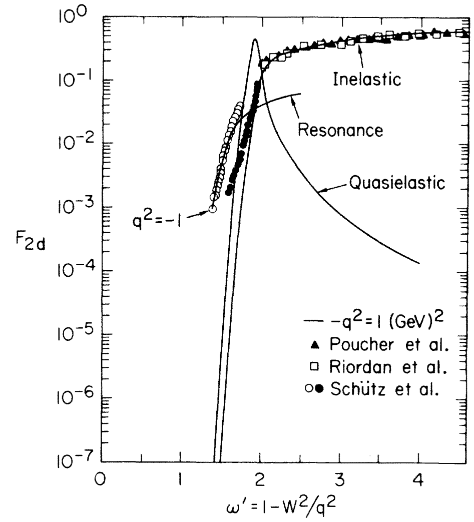

FIG. 10. The same three contributions for $q^2 = -1\ \mathrm{GeV}^2$.

x considered, $x \gtrsim 0.25$. The average value of $B_1(x)$ around the valence peak ($x = \frac{1}{3}$) is roughly consistent with $\frac{2}{3}$, which is the ratio of the sum of the squares of the valence quark charges, neutron/proton $= (\frac{2}{3})/1$. However, at large x, $B_1(x)$ is dropping but still safely extrapolates to be larger than the lower bound of $\frac{1}{4}$ at $x = 1$, which holds in the valence-quark model.[25] The value of $\frac{1}{3}$ found for $B_2(x)$ is the ratio of the sum of the squares of the valence diquark charges, neutron/proton $= (\frac{2}{3})/2$. [A slightly better fit can be obtained by taking $B_2(x)$ to be slowly varying, with an average value

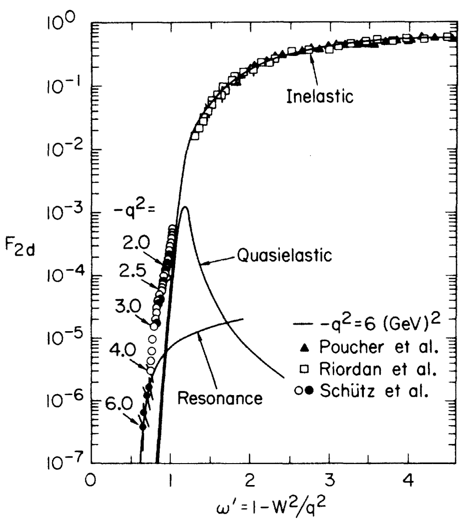

FIG. 9. The three contributions to the deuteron structure function (inelastic, quasielastic, resonance), for $q^2 = -6\ \mathrm{GeV}^2$.

of $\frac{1}{3}$.] These three features of the fit are evidence of the consistency of our interpretation and fit (but certainly not its uniqueness).

Moderate q^2. Using only the above terms, we can now compute $F_{2d}(x_d, q^2)$ for all values of x_d using Eq. (10). The result labeled inelastic is given in Fig. 8 at large q^2 and in Figs. 9 and 10 at moderate q^2 as a function of

$$\omega' = \frac{1}{2x_d} - \frac{M^2}{q^2},$$

which has been used in the presentation of the data of Schütz *et al.*[11] At this stage our curve for the inelastic contribution falls below the present data for $(-q^2) \gtrsim 2\ \mathrm{GeV}^2$ for $1 > \omega' > \frac{1}{2}(x_d - 1)$. This is not surprising since the quasielastic and fully coherent "resonance" contributions have not been included. Quasielastic scattering should be important for $x_d \sim \frac{1}{2}$ and for the lower range of q^2 values.

This contribution which should be added to the $F_{2d}(x_d, q^2)$ given by Eq. (10) is

$$F^{Q}_{2d}(x_d, q^2) = \sum_a x_d G_{a/d}(x_d) F_a{}^2(q^2). \tag{12}$$

It has been plotted separately in Figs. 9 and 10 for $(-q^2) = 6$ and $1\ \mathrm{GeV}^2$, respectively. For the smaller q^2 value there is a clear quasielastic peak which has been suppressed at the larger q^2 value by the nucleon form factor. It would be very interesting to have data in this region that would allow us to explore the properties of the quasielastic peak.

In the region of x_d very close to 1, the data are clearly larger than the sum of the contributions considered so far, even if the experimental resolution is used to smear the prediction. In Ref. 11, the suggestion is made that this could be due to a final-state resonance in which the two nucleons share the momentum of the virtual photon. This contribution can be fitted to the data if written in the form

$$F_{2d}^{R}(x_d, q^2) = (-q^2)F_d{}^2(q^2)10^{3-2x_d{}^2}, \qquad (13)$$

which for $q^2 = -6$ and 1 GeV^2 is shown in Figs. 9 and 10. We are not sure that this is a correct interpretation, but a contribution which roughly has the above structure was predicted by Jankus[26] in scattering from the deuteron near the inelastic threshold. Jankus found a strong localized enhancement in this region that was due to nonresonant (scattering length) final-state interactions. Such an effect was found experimentally.[27] It would be very interesting to compute this effect with a relativistic treatment of the deuteron to check its consistency with the data. A different approach to fitting this data has been described by Frankfurt and Strikman.[28]

VI. CONCLUSIONS

In this paper we have shown that the ordinary parton model, which normally is assumed to scale (except for mass corrections), has physically identifiable terms that do not scale. The final states that were of most interest here were one quark recoiling with the photon momentum and two quarks sharing this momentum. The predicted form of the structure functions and form factors for these terms were shown to provide a reasonable fit to the proton and neutron data for $x \gtrsim \frac{1}{3}$ and $(-q^2) \gtrsim 2$ GeV^2. The ratios between the proton and neutron are as expected in the model. Owing to the uncertainties involved, our parameters should be considered as having "typical" values. The errors are correlated between the parameters of the scaling and nonscaling terms and no systematic error analysis has been made.

Our model and fit is certainly not the only way to understand the nonscaling behavior of the structure function at large x. This behavior is also fitted by using "ξ scaling"[29] plus asymptotic-freedom models.[5,6] However, there should be experimentally measurable differences between this approach and ours. While we do not know precisely what the latter models predict, if our explanation is correct there should be protons in the photon-fragmentation region for large x. The single-quark-recoil or scaling term should tend to decay to mesons [the leading mesons would then have a $(1-x)$ decay-function behavior]. The diquark-recoil term should decay not only to mesons but also should decay strongly to baryons [the leading baryons should also have a $(1-x)$ decay function behavior]. Therefore if our explanation is correct, the proton/pion ratio should follow the ratio of the nonscaling term to the full structure function. The observation of recoil protons arising from a preferred x value of $\frac{2}{3}$ and a q^2 behavior of $(-q^2)^{-2}$ would be confirmation of our general picture. The absence of such protons may be more consistent with asymptotic freedom models. At the present time, the proton/pion ratio cannot be predicted since we do not know the decay probability functions for a diquark system to produce pions and protons. These functions can be measured in principle in several independent ways, however, such as in e^+e^- annihilation and in the target-fragmentation region of deep-inelastic lepton scattering.

The scaling terms in F_1 and F_2 were found to be in reasonable agreement with the Callan-Gross[17] relation. If the diquark system is predominantly spin one, then one expects large asymmetry effects in deep-inelastic lepton scattering with polarized beam and target.[30]

It is clearly possible to ascribe the lack of scaling at large x to either our model or to asymptotic freedom models or to any linear combination. This is not the case at small x. Our model is not able to explain the probable rise in q^2 at small x of the structure function suggested by high-energy μ scattering[31] or the nonscaling behavior at small x seen in neutrino scattering.[32] (A general fit to all these data has been given in Ref. 33.) This behavior is strong evidence for asymptotic freedom and/or the production of new, heavy quarks, and/or Regge-duality effects,[34] but this is unfortunately in a region where it is difficult to make quantitative calculations. However, since the diquark terms can be used to decrease the size of the nonscaling effects due to asymptotic freedom at large x, then there may not be enough rise left at small x to explain the data in such theories.

A relativistic model of the deuteron has been developed and used to extract the neutron structure functions. We do not believe this has been done before. Our method is easily susceptible to a more accurate treatment (especially important here would be the inclusion of spin effects). We have checked our deuteron model by comparing it with the measured elastic form factor and inelastic data for all x_d.

To conclude, we have shown that a simple extension of the parton model, together with dimensional counting, provides a reasonable fit to the nonscaling behavior of the proton and neutron structure functions for x larger than the valence quark peak at $\frac{1}{3}$. The model can be tested by looking at the proton yield in the photon-fragmentation region.

We therefore conclude that if one wants to differentiate between basic theóries of hadrons by studying only the structure functions, it must be done at small x where the above nonscaling terms are probably unimportant. Even in this region of x, however, one is faced with the problem of demonstrating that such effects are indeed small, especially if one is making a quantitative comparison with a particular basic theory.

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