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Absorption corrections in the inclusive production of the Δ(1236) in the triple-Regge region

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The inclusive production of the Δ(1236) is considered in a parameter-free Mueller-Regge model with absorption corrections. The inclusive cross sections and the density matrix elements are obtained and are compared with the experimental data.

I. INTRODUCTION

The improvement in the quality of the single-particle-inclusive production data has indicated the need for some form of cut corrections to the Mueller-Regge model.[1] Indeed, the importance of cuts had been foreseen through theoretical[2] and phenomenological[3] considerations long before the data became available to substantiate their existence. In this paper we apply a parameter-free Mueller-Regge model with cut corrections[4] to study the inclusive production of the Δ(1236). Following an earlier study of Δ(1236) production in this model with only simple poles,[5] it was found that while the qualitative features of the inclusive cross sections were reasonably well accounted for, the decay density matrices and the normalization of the cross sections were inadequately predicted by the model.

The formalism for our absorbed Mueller-Regge model is given in Sec. II. In Sec. III the results of the absorption model are compared with the data. Both the inclusive cross sections and the density matrix elements of the decaying Δ(1236) are presented. The final form of the absorbed Mueller-Regge amplitudes is given in the Appendix.

II. FORMALISM

The Mueller generalized optical theorem relates the one-particle-inclusive process to the forward $3\to 3$ amplitudes[6] [see Fig. 1(a)], giving

$$\sum_X \int \prod_{i=1}^{N_X} \frac{d^3q_i}{(2\pi)^3 2E_i}(2\pi)^4\delta^4\left(\sum_{i=1}^{N_X} q_i + p_c - p_a - p_b\right) \times \sum_K f_{\lambda_c K;\lambda_a\lambda_b} f^*_{\lambda_c' K;\lambda_a'\lambda_b'} = \frac{1}{2i}\,\mathrm{Disc}_{M^2} f_{\lambda_c'\lambda_b'\lambda_a';\lambda_c\lambda_b\lambda_a}\,,$$

where N_X is the number of particles in the M^2 state, λ denotes the helicity of the appropriate single-particle state, and K denotes the helicity of the composite M^2 state. For the single-particle-inclusive production of the Δ(1236) in the triple-Regge region with the assumption of M^2 duality, the corresponding Mueller-Regge limit is shown in Fig. 1(b).

The currents at the $(\frac{1}{2}^+,\frac{3}{2}^+,R)$ vertices are taken

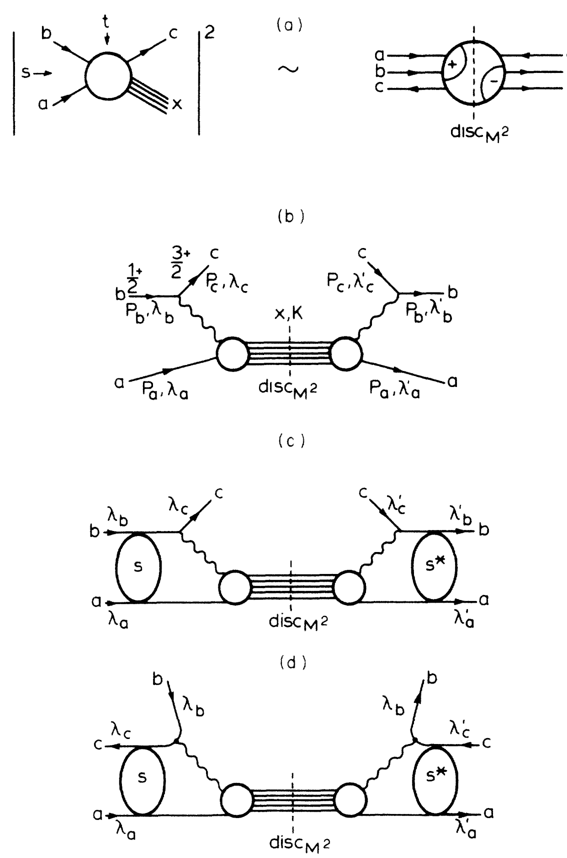

FIG. 1. (a) Schematic representation of Mueller's generalized optical theorem. (b) The Mueller-Regge diagram for the triple-Regge region with the condition M^2 large relaxed. (c) Elastic rescattering correction in the s_{ab} channel. (d) Elastic rescattering correction in the $s_{a\bar{c}}$ channel.

to be[7]

$$J_5 = \frac{g}{m_B} G Q_\lambda (\bar{D}_\lambda N) ,$$

and

$$J_\mu = -\frac{g}{2m_B^2} G \epsilon_{\mu\nu\kappa\lambda} P_\nu Q_\kappa (\bar{D}_\lambda N) ,$$

for, respectively, pseudoscalar and vector exchanges, where g is the $\Delta \to p\pi$ coupling constant, m_B is the average of the proton and the Δ masses, G denotes the Clebsch-Gordan coefficient, P and Q denote, respectively, the sum and the difference of the incoming and the outgoing momenta at the $(\frac{1}{2}^+, \frac{3}{2}^+, R)$ vertex, and D_λ and N are the $\frac{3}{2}^+$ and the $\frac{1}{2}^+$ wave functions. If R represents a π, a ρ, or an A_2 coupling to a p-Δ^{++} vertex, G has the value $-\sqrt{2}$.

The Mueller-Regge helicity amplitudes corresponding to Fig. 1(b) are given by

$$H^{\lambda_c\lambda_b, \lambda_c'\lambda_b'}_{\lambda_a K, \lambda_a' K} = J_5^{\lambda_b \lambda_c} \Delta \Gamma_5^{\lambda_a K} (J_5^{\lambda_b' \lambda_c'} \Delta \Gamma_5^{\lambda_a' K})^\dagger ,$$

and

$$H^{\lambda_c\lambda_b, \lambda_c'\lambda_b'}_{\lambda_a K, \lambda_a' K} = J_\mu^{\lambda_b \lambda_c} \Delta_{\mu\nu} \Gamma_\nu^{\lambda_a K} (J_{\mu'}^{\lambda_b' \lambda_c'} \Delta_{\mu'\nu'} \Gamma_{\nu'}^{\lambda_a' K})^\dagger ,$$

for, respectively, pseudoscalar and vector exchanges. We have denoted the Reggeized propagator and structure function at the inclusive vertices by, respectively, Δ and Γ. The structure functions, after averaging and summing over, respectively, λ_a and K assume the form

$$\sum_{\lambda_a, K} \Gamma_5^{\lambda_a} (\Gamma_5^{\lambda_a K})^\dagger = 2\Delta^{1/2}(M^2, t, m_a^2) \sigma_{\text{tot}}(\pi a \to \pi a) ,$$

for pseudoscalar exchange, while, in the case of vector exchange, we have[8]

$$\sum_{\lambda_a, K} \Gamma_\nu^{\lambda_a K} (\Gamma_{\nu'}^{\lambda_a K})^\dagger = p_{1\nu} p_{1\nu'} V ,$$

with

$$V = \frac{8 m_\rho^2}{M^2} \sigma_{\text{tot}}(\rho a \to \rho a) .$$

After averaging and summing over λ_a and K the helicity amplitudes are

$$\sum_{\lambda_a, K} H^{\lambda_c\lambda_b, \lambda_c'\lambda_b'}_{\lambda_a K, \lambda_a K} = H^{\lambda_c\lambda_b, \lambda_c'\lambda_b'} .$$

Duality[9] is assumed for the M^2 channel and the exoticity criterion of Chan *et al.*[10] is adopted and is taken into account in the model by the form of the σ_{tot} as given in the Appendix.

Absorption corrections are included by considering elastic rescattering in the initial state [see Fig. 1(c)]. We work in impact-parameter space and obtain the standard form for the absorbed helicity amplitudes of Fig. 1(c)

$$H^{\lambda_c\lambda_b, \lambda_c'\lambda_b'}_{abs}(b, b') = S^{1/2}(b) H^{\lambda_c\lambda_b, \lambda_c'\lambda_b'}(b, b') \times S^{1/2*}(b') ,$$

where b is the impact parameter, and S is the elastic-scattering S matrix, which is given by

$$S(b) = 1 - C \exp(-\lambda b^2) ,$$

with C and λ denoting the opacity and the inverse of the interaction radius squared.

We now make an approximation for the rescattering effect in the $\bar{c}a$ channel [see Fig. 1(d)]. In the kinematic region under investigation $s_{ab} \simeq s_{\bar{c}a}$, and by making the simplifying assumption that the elastic scattering of ab and $a\bar{c}$ is approximately equal, $S_{ab} \simeq S_{\bar{c}a}$, then to first order we have

$$S_{ab}^{1/2} S_{\bar{c}a}^{1/2} \simeq \tfrac{1}{2}(S_{ab} + S_{\bar{c}a}) \simeq S ,$$

and

$$H^{\lambda_c\lambda_b, \lambda_c'\lambda_b'}_{abs}(b, b') = S(b) H^{\lambda_c\lambda_b, \lambda_c'\lambda_b'}(b, b') S^*(b') .$$

The values of C and λ are then taken to be $C = 0.7$ and $\lambda = 0.068\ (\text{GeV}/c)^2$.[11]

Inverting the amplitudes, we obtain

$$H^{\lambda_c\lambda_b, \lambda_c'\lambda_b'}_{abs}(s, \tau, M^2) = \int_0^\infty \tau' d\tau' \int_0^\infty \tau_1' d\tau_1' \int_0^\infty b\, db\, J_\nu(b\tau) J_\nu(b\tau') S(b) \times \int_0^\infty b' db' J_{\nu'}(b\tau') J_{\nu'}(b'\tau_1') S^*(b') H^{\lambda_c\lambda_b, \lambda_c'\lambda_b'}(s, \tau', \tau_1', M^2) .$$

This in turn yields

$$H^{\lambda_c\lambda_b, \lambda_c'\lambda_b'}_{abs}(s, \tau, M^2) = \int_0^\infty \tau' d\tau' \int_0^\infty \tau_1' d\tau_1' H^{\lambda_c\lambda_b, \lambda_c'\lambda_b'}(s, \tau', \tau_1', M^2) \times \left\{ \frac{1}{\tau} \delta(\tau - \tau') - \frac{C}{2\lambda} \exp\left[-\frac{\tau^2 + \tau'^2}{4\lambda} I_\nu\left(\frac{\tau\tau'}{2\lambda}\right)\right]\right\} \times \left\{ \frac{1}{\tau} \delta(\tau - \tau_1') - \frac{C}{2\lambda} \exp\left[-\frac{\tau^2 + \tau_1'^2}{4\lambda} I_{\nu'}\left(\frac{\tau\tau_1'}{2\lambda}\right)\right]\right\} .$$

The variable τ, the conjugate variable of b in the Fourier-Bessel transform is given by

$$q\tau^2/k = t_{\min} - t ,$$

where k and q are the center-of-mass initial- and final-state three-momenta. The assumption that the inclusive vertex is primarily non-flip is made, giving us $\nu = |\lambda_c - \lambda_b|$ and $\nu' = |\lambda_c' - \lambda_b'|$, where ν and ν' are the total helicity flip on each side of Disc_{M^2}.

For the inclusive cross section and the decay density matrix of the $\Delta(1236)$ we average over the helicity of particle b, for an unpolarized target, and obtain

$$\bar{\sum_{\lambda_b}} H^{\lambda_c\lambda_b,\lambda_c'\lambda_b} = \frac{1}{(2S_b+1)} H^{\lambda_c\lambda_c'} ,$$

where S_b is the spin of particle b. The resulting Mueller-Regge amplitudes are given in the Appendix.

The Regge parameters required for the Reggeized propagators are obtained through the assumption of strong exchange degeneracy for the particle pairs π-B, ρ-A_2, and ω-f and the requirement that the trajectories pass through the respective particles on the Chew-Frautschi plot. The trajectory parameters are given in Table I.

TABLE I. Regge trajectories.

Trajectory	α_0	α' [(GeV/c)$^{-2}$]
π,B	-0.013	0.665
ρ,A_2	0.470	0.905
ω,f	0.386	1.017

The one-particle-inclusive cross section is given by

$$\frac{s}{\pi}\frac{d^2\sigma}{dt\,dM^2} = \frac{1}{64\pi^2k^2}\frac{1}{(2S_b+1)}\sum_i\sum_{\lambda_c} H_{abs}^{\lambda_c\lambda_c} ,$$

where the summation over i signifies the summation over all possible exchanges.

The decay density matrix elements for the decay of the $\Delta(1236)$ are given by[12]

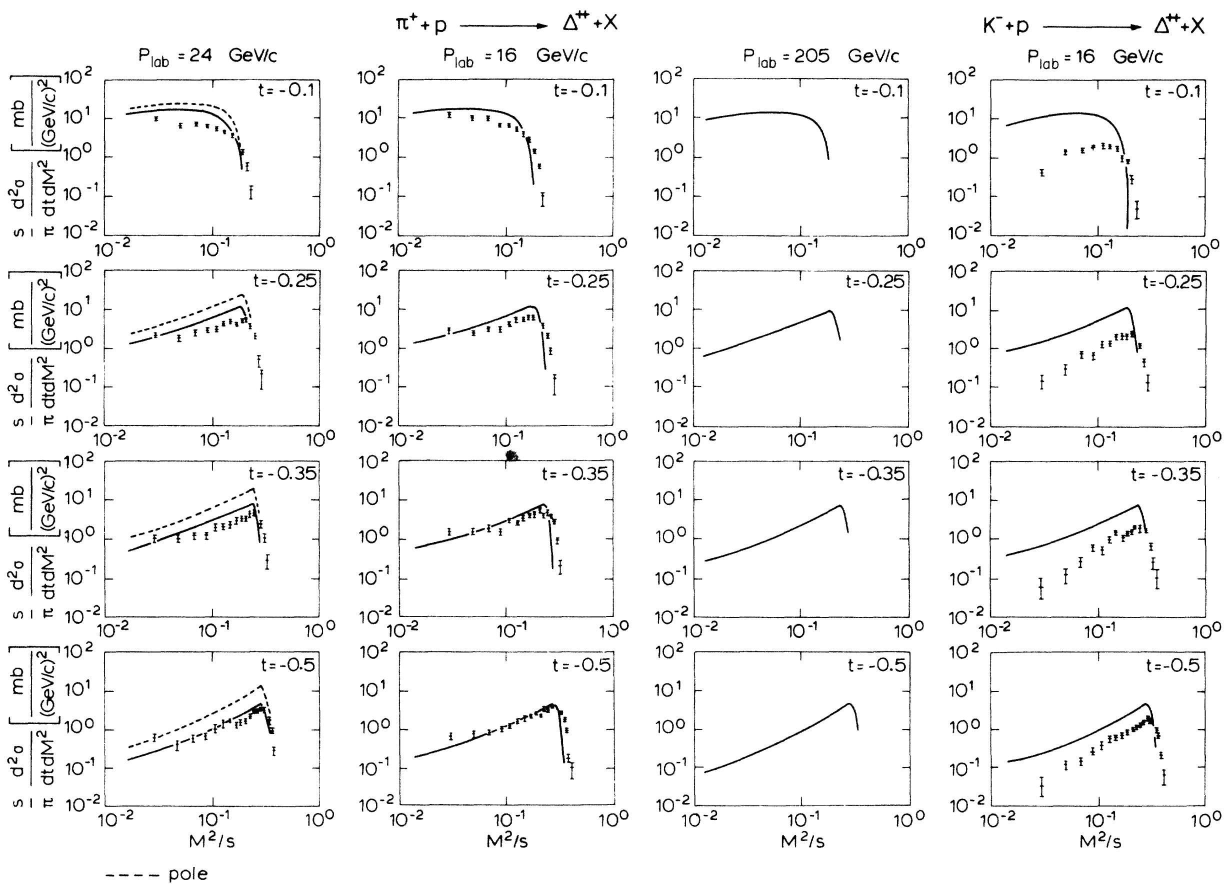

FIG. 2. The inclusive cross sections for ($p \xrightarrow{a} \Delta^{++}$) for fixed t [in (GeV/c)^2] against M^2/s for $a=\pi^+$ at $P_{\mathrm{lab}}=24$, 16, and 205 GeV/c and $a=K^-$ at $P_{\mathrm{lab}}=16$ GeV/c. The contribution from the pole exchange alone is also shown for $a=\pi^+$ at 24 GeV/c. Data are from Refs. 15 and 18.

$$\rho'_{\mu\mu} = \frac{\sum_i H^{\mu'\mu}_{abs}}{\sum_i \sum_\mu H^{\mu\mu}_{abs}}.$$

We present the density matrix elements in the Gottfried-Jackson frame where they are given by

$$\rho_{m'm} = \sum_\mu d^{3/2}_{m'\mu'}(\psi) d^{3/2}_{m\mu}(\psi) \rho_{\mu'\mu},$$

where ψ is the angle between the directions of particle b and the M^2 cluster as seen in the rest frame of the decaying $\Delta(1236)$.

All the calculations of differential cross sections, decay density matrices, and total cross sections were carried out numerically.[13] All the figures were, for accuracy, plotted by computer.[14]

III. DISCUSSION

From the various reports of the experimental studies of the single-particle-inclusive production of the $\Delta(1236)$ (Refs. 15–18) a picture of the production mechanism is emerging. Qualitatively the triple-Regge model is reasonably effective in describing the data at least as far as the inclusive cross section is concerned and when the effective Regge trajectories are arbitrarily parametrized with free parameters which are obtained from a fit to the data. However, when the polarization of the $\Delta(1236)$ is considered and/or when the Regge parameters are constrained to the values obtained in two-body phenomenology, it appears that some form of absorption correction is needed as we shall show and as has also been pointed out by Gotsman[19] and Barish *et al.*[1] The question of M^2 duality is substantiated by the suppression of the exotic M^2 channel processes, e.g., $(p \xrightarrow{\pi^-} \Delta^{++})$ and $(p \xrightarrow{K^+} \Delta^{++})$ and also the early scaling of these processes although, at present, the energy range is still limited. However, the indication is that the hypothesis of Chan *et al.*[10] is valid.

The results of the model calculation are presented, together with the data, in Figs. 2–4. It can be seen that the inclusive cross sections are well accounted for, and in the cases where the unabsorbed (pole) calculation is shown, e.g., Fig. 2 at 24 GeV/c and Figs. 3(a) and 3(b), it

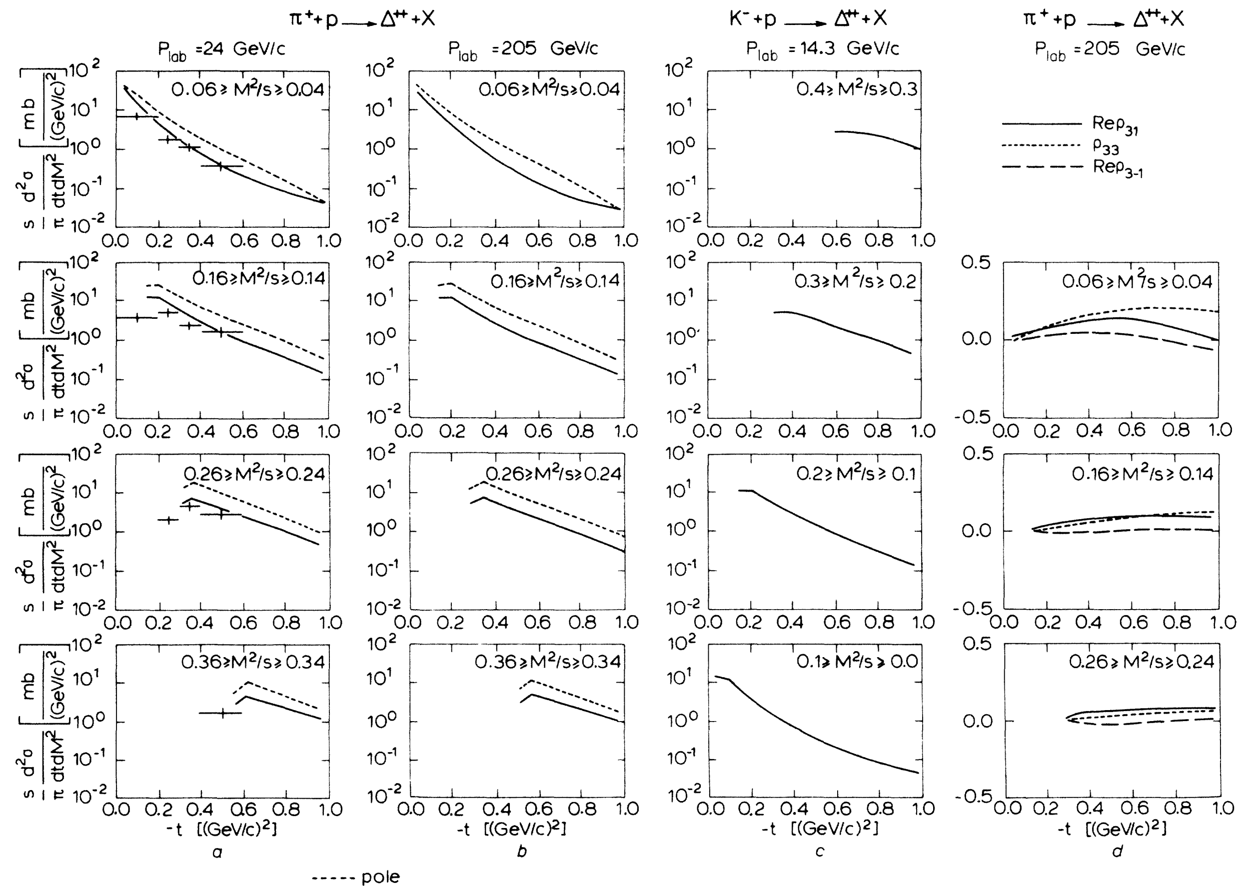

FIG. 3. The inclusive cross sections for fixed M^2 against t for $(p \xrightarrow{a} \Delta^{++})$ with $a = \pi^+$ and K^- at 24, 205, and 14.3 GeV/c and the density matrix elements for the Δ^{++} produced from a π^+ beam at 205 GeV/c. In (a) and (b) the pole contribution is shown for comparison. Data are from Ref. 15.

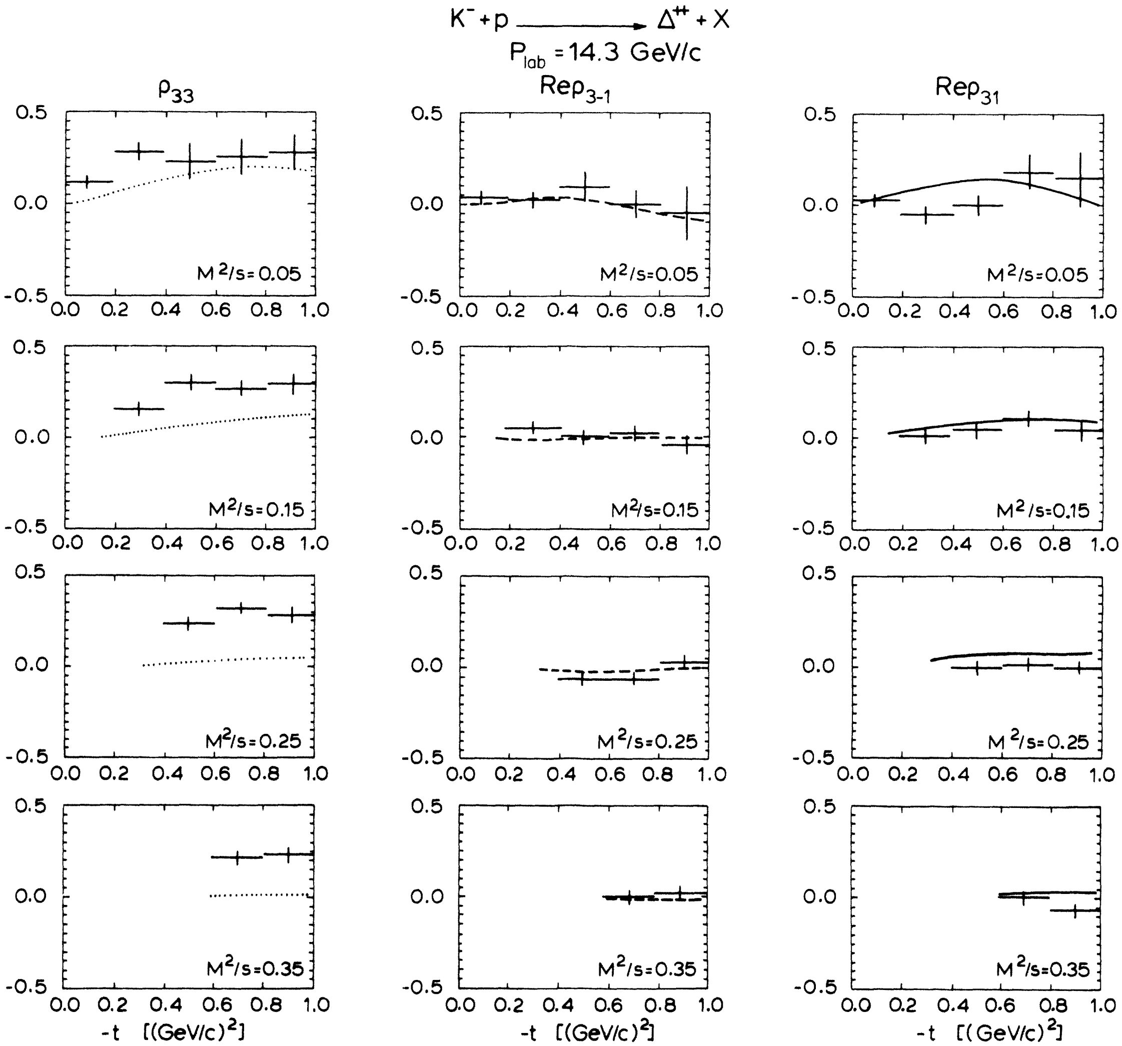

FIG. 4. The density matrix elements for Δ^{++} decay in $(p \xrightarrow{K^-} \Delta^{++})$ at 14.3 GeV/c. Data are from Ref. 17.

is evident that the normalization and the slope are improved after absorption. For the exotic M^2 channel of $(p \xrightarrow{K^-} \Delta^{++})$ the data[18] indicate a substantial suppression of the inclusive cross section relative to the $(p \xrightarrow{K^+} \Delta^{++})$ at the same energy where the M^2 channel is nonexotic. Although our model takes into account exoticity (see the Appendix), it seems that the suppression is somewhat greater than the suppression given by the model.

TABLE II. Parameters for the Γ-function approximation [obtained by a fit in the range $0 \le |t| \le 2$ (GeV/c)^2].

Exchange	A_1	B_1	A_2	B_2
π,B	−59.891	41.12	−14.143	3.398
ρ,A_2	0.8914	2.96	0.778	−0.072
ω,f	0.8076	2.75	0.645	−0.266

The density matrix elements of the decaying Δ^{++} are given in Fig. 4 with the $(p \xrightarrow{K^-} \Delta^{++})$ data of Paler *et al.*[17] Chliapnikov *et al.*[16] have presented compatible data for the density matrices for $(p \xrightarrow{K^+} \Delta^{++})$ at 16 GeV/c, where $0.1 \le \rho_{33} \le 0.2$, Re ρ_{31} is slightly negative, and Re$\rho_{3,-1}$ is compatible with zero. Dao *et al.*[1] and Barish *et al.*[1] have also given the density matrices for $(p \xrightarrow{p} \Delta^{++})$ at 303 and 205 GeV/c, respectively. Here the density matrices are in large M^2 bins and $\rho_{33} \sim 0.1$ and Reρ_{31} and Re$\rho_{3,-1}$ are both compatible with zero. It can be seen that in the small M^2/s region the density matrix elements are well accounted for by the model. Taking into account the data of Chliapnikov *et al.*,[16] where ρ_{33} is lower, it can be said that the model is very reasonable. Note that the absorption effect falls off with increasing M^2.

From the present study and in a previous analysis of single-particle-inclusive vector-meson production we have found that the model with absorption corrections goes a long way in accommodating the data. Taking into account the parameter-free nature of the model this is indeed very encouraging. Recently Craigie *et al.*[20] have looked at other forms of absorption which may be relevant in the triple-Regge region. Further high-quality data will of course indicate whether these further corrections are significant and necessary.

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APPENDIX

In this Appendix we outline the kinematics and the evaluation of the absorbed Mueller-Regge helicity amplitudes. We work in the c.m. frame with the initial- and final-state 3-momenta given by k and q, respectively. The beam direction defines the z axis and the reaction plane, after integrating over the angle ϕ for the M^2 cluster, is taken to be the x-z plane. The direction of the produced particle from the z axis defines the c.m. scattering angle θ.

We cast the Mueller-Regge helicity amplitudes in the form

$$H_i^{\lambda_c \lambda_b, \lambda'_c \lambda'_b} = (\Psi_i^{\lambda_c \lambda_b}) W_i (\Psi_i^{\lambda'_c \lambda'_b})^* ,$$

where i denotes the exchange, for evaluation of the absorption integrals. We have introduced

$$W_i = \begin{cases} \left(g \dfrac{G}{m_B}\right)^2 2\Delta^{1/2}(M^2, m_2{}^2, t)\sigma_{\text{tot}}(\pi a), & \text{for } i = \pi \text{ exchange} \\ \left(-\dfrac{g}{2m_B{}^2} G\right)^2 V, & \text{for } i = \text{vector exchange} \end{cases}$$

and

$$\Psi_i^{\lambda_c \lambda_b} = \alpha'_i \, \frac{\{1 + \xi \exp[-i\pi\alpha_i(t)]\}}{2} \, \Gamma_i(f(t)) \left(\frac{s}{M^2}\right)^{\alpha_i(t) - J_i} a_i^{\lambda_c \lambda_b} ,$$

where the standard Regge notation has been used, J_i denotes the spin of the exchange i, and $\Gamma_i(f(t))$ are the Euler Γ functions given by

$$\Gamma_i(f(t)) = \begin{cases} -\Gamma(-\alpha_i(t)), & \text{for } i = \text{unnatural-parity exchange} \\ \Gamma(1 - \alpha_i(t)), & \text{for } i = \text{natural-parity exchange}. \end{cases}$$

To obtain analytic solutions of the absorption integrals the Γ functions are parametrized in the form

$$\Gamma(f(t)) = A_1 \exp(B_1 t) + A_2 \exp(B_2 t) ,$$

with the values of the parameters for the various relevant exchanges given in Table II.

The following variables are introduced:

$$E = [(E_2 + m_2)(E_4 + m_4)]^{1/2}, \quad C_\pm = E^2 \pm kq ,$$

$$D = (1/E)k \sin\theta , \quad F = (1/E)[2kq(E_1 + E_2) \sin\theta] .$$

Then the $a_i^{\lambda_c \lambda_b}$ are given for pseudoscalar exchange by

$$a_s^{3,1} = -\frac{1}{\sqrt{2}} DC_- \cos\frac{\theta}{2} ,$$

$$a_s^{1,1} = \frac{1}{\sqrt{6}} DC_+ \sin\frac{\theta}{2} + \left(\frac{2}{3}\right)^{1/2} \frac{(qE_2 - kE_4 \cos\theta)}{m_4 E} C_- \cos\frac{\theta}{2} ,$$

$$a_s^{-1,1} = \frac{1}{\sqrt{6}} DC_- \cos\frac{\theta}{2} - \left(\frac{2}{3}\right)^{1/2} \frac{(qE_2 - kE_4 \cos\theta)}{m_4 E} C_+ \sin\frac{\theta}{2} ,$$

$$a_s^{-3,1} = -\frac{1}{\sqrt{2}} DC_+ \sin\frac{\theta}{2} ,$$

and

$$a_s^{3,-1} = a_s^{-3,1}, \quad a_s^{1,-1} = -a_s^{-1,1},$$
$$a_s^{-1,-1} = a_s^{1,1}, \quad a_s^{-3,-1} = -a_s^{3,1}.$$

For vector exchange we obtain

$$a_v^{3,1} = -\frac{1}{\sqrt{2}} FC_- \cos\frac{\theta}{2},$$
$$a_v^{1,1} = \frac{1}{\sqrt{6}} FC_+ \sin\frac{\theta}{2},$$
$$a_v^{-1,1} = -\frac{1}{\sqrt{6}} FC_- \cos\frac{\theta}{2},$$
$$a_v^{-3,1} = \frac{1}{\sqrt{2}} FC_+ \sin\frac{\theta}{2},$$

with

$$a_v^{3,-1} = -a_v^{-3,1}, \quad a_v^{1,-1} = a_v^{-1,1},$$
$$a_v^{-1,-1} = -a_v^{1,1}, \quad a_v^{-3,-1} = a_v^{3,1}.$$

Note that for conciseness we have written down only the top half of the half-integer helicity labels.

Writing $H^{\lambda_c\lambda_b,\lambda_c'\lambda_b'}$, $\Psi^{\lambda_c\lambda_b}$, and $(\Psi^{\lambda_c'\lambda_b'})^*$ in terms of the variable τ, using $\tau = 2k\sin(\theta/2)$, and suppressing the M^2 and s labels for the present, we obtain

$$H^{\lambda_c\lambda_b,\lambda_c'\lambda_b'}(\tau',\tau_1') = \Psi^{\lambda_c\lambda_b}(\tau')W(\Psi^{\lambda_c'\lambda_b'}(\tau_1'))^*.$$

Then after substituting this in the absorption integral we have

$$H_{abs}^{\lambda_c\lambda_b,\lambda_c'\lambda_b'}(\tau) = \left[\Psi^{\lambda_c\lambda_b}(\tau) - \frac{C}{2\lambda}\exp\left(-\frac{\tau^2}{4\lambda}\right)\int_0^\infty \tau' d\tau'\, \Psi^{\lambda_c\lambda_b}(\tau')\exp\left(-\frac{\tau'^2}{4\lambda}\right) I_\nu\left(\frac{\tau\tau'}{2\lambda}\right)\right]$$
$$\times W\left[(\Psi^{\lambda_c'\lambda_b'})^* - \frac{C}{2\lambda}\exp\left(-\frac{\tau^2}{4\lambda}\right)\int_0^\infty \tau_1' d\tau_1' (\Psi^{\lambda_c'\lambda_b'}(\tau_1'))^* \exp\left(-\frac{\tau_1'^2}{4\lambda}\right) I_\nu\left(\frac{\tau\tau_1'}{2\lambda}\right)\right].$$

We can then cast the amplitudes in the form

$$H_{abs}^{\lambda_c\lambda_b,\lambda_c'\lambda_b'} = (\Psi^{\lambda_c\lambda_b} - \mathfrak{C}^{\lambda_c\lambda_b})W((\Psi^{\lambda_c\lambda_b})^* - (\mathfrak{C}^{\lambda_c\lambda_b})^*).$$

We define the variables

$$\phi_i = \sum_{j=1}^{2} A_i^j \exp(B_i^j t_{\min})\left(\frac{\alpha_i'}{2}\right)\left(\frac{s}{M^2}\right)^{(\alpha_{0_i}+\alpha_i' t_{\min} - J_i)},$$

$$\zeta_i = \sum_{j=1}^{2} \frac{1}{4\lambda} + \left[B_i^j + \alpha_i' \ln\left(\frac{s}{M^2}\right)\right]\frac{q}{k},$$

$$\zeta_i' = \zeta_i - i\pi\alpha_i'\left(\frac{q}{k}\right),$$

$$\chi_i = \frac{\tau^2}{16\lambda^2\zeta_i},$$

$$\chi_i' = \frac{\tau^2}{16\lambda^2\zeta_i'},$$

$$\eta_i = \xi_i \exp[-i\pi(\alpha_{0_i} + \alpha_i' t_{\min})],$$

$$\gamma = \frac{C}{2\lambda}\exp\left(-\frac{\tau^2}{2\lambda}\right).$$

Then we obtain for pseudoscalar exchange

$$\mathfrak{C}_s^{3,1} = -\frac{kC_-}{\sqrt{2}E}\gamma\phi_i\frac{\tau}{2k\lambda}\left\{\left[\frac{\exp(\chi_i)}{\zeta_i^2}\left(1-\frac{(\chi_i+2)}{4k^2\zeta_i}\right)\right] + \eta_i[\chi_i\to\chi_i', \zeta_i\to\zeta_i']\right\},$$

$$\mathfrak{C}_s^{1,1} = \frac{1}{\sqrt{3}E}\gamma\phi_i\left(\frac{C_+}{\sqrt{2}}\frac{1}{4k}\left\{\left[\frac{\exp(\chi_i)}{\zeta_i^2}\left((\chi_i+1) - \frac{[(\chi_i+3)(\chi_i+1)-1]}{8k^2\zeta_i}\right)\right] + \eta_i[\chi_i\to\chi_i', \zeta_i\to\zeta_i']\right\}\right.$$
$$+\frac{\sqrt{2}}{m_4}qE_2C_-\frac{1}{2}\left\{\left[\frac{\exp(\chi_i)}{\zeta_i}\left(1-\frac{(\chi_i+1)}{8k^2\zeta_i}\right)\right] + \eta_i[\chi_i\to\chi_i', \zeta_i\to\zeta_i']\right\}$$
$$\left.-\frac{\sqrt{2}}{m_4}kE_4C_-\frac{1}{2}\left\{\left[\frac{\exp(\chi_i)}{\zeta_i}\left(1-\frac{(\chi_i+1)}{16k^2\zeta_i}\right)\right] + \eta_i[\chi_i\to\chi_i', \zeta_i\to\zeta_i']\right\}\right),$$

$$\mathcal{C}_s^{-1,1}=\frac{1}{\sqrt{3}E}\,\gamma\phi_i\,\frac{\tau}{8k\lambda}\Bigg(\frac{C_-}{\sqrt{2}}\,k\left\{\left[\frac{\exp(\chi_i)}{\zeta_i^{\,2}}\left(1-\frac{(\chi_i+2)}{4k^2\zeta_i}\right)\right]+\eta_i\,[\chi_i\to\chi_i',\zeta_i\to\zeta_i']\right\}$$

$$-\frac{\sqrt{2}}{m_4}\,qE_2C_+\frac{1}{2}\left[\frac{\exp(\chi_i)}{\zeta_i^{\,2}}+\eta_i\,\frac{\exp(\chi_i')}{\zeta_i'}\right]$$

$$+\frac{\sqrt{2}}{m_4}\,kE_4C_+\frac{1}{2}\left\{\left[\frac{\exp(\chi_i)}{\zeta_i^{\,2}}\left(1-\frac{(\chi_i+2)}{2k^2\zeta_i}\right)\right]+\eta_i\,[\chi_i\to\chi_i',\zeta_i\to\zeta_i']\right\}\Bigg),$$

$$\mathcal{C}_s^{-3,1}=-\frac{kC_+}{\sqrt{2}E}\,\gamma\phi_i\,\frac{\tau^2}{64k^2\lambda^2}\left\{\left[\frac{\exp(\chi_i)}{\zeta_i^{\,3}}\left(1-\frac{(\chi_i+3)}{8k^2\zeta_i}\right)\right]+\eta_i[\chi_i\to\chi_i',\zeta_i\to\zeta_i']\right\},$$

with

$$\mathcal{C}_s^{3,-1}=\mathcal{C}_s^{-3,1}\,,\quad \mathcal{C}_s^{1,-1}=-\,\mathcal{C}_s^{-1,1}\,,$$

$$\mathcal{C}_s^{-1,-1}=\mathcal{C}_s^{1,1}\,,\quad \mathcal{C}_s^{-3,-1}=-\,\mathcal{C}_s^{3,1}\,.$$

For vector exchange we have

$$\mathcal{C}_v^{3,1}=-\frac{\sqrt{2}\,kq(E_1+E_2)C_-}{E}\,\gamma_i\phi_i\,\frac{\tau}{8k\lambda}\left\{\left[\frac{\exp(\chi_i)}{\zeta_i^{\,2}}\left(1-\frac{(\chi_i+2)}{4k^2\zeta_i}\right)\right]+\eta_i[\chi_i\to\chi_i',\zeta_i\to\zeta_i']\right\},$$

$$\mathcal{C}_v^{1,1}=\frac{2kq(E_1+E_2)}{\sqrt{6}E}\,C_+\gamma_i\,\phi_i\,\frac{1}{4k^2}\left\{\left[\frac{\exp(\chi_i)}{\zeta_i^{2}}\left((\chi_i+1)-\frac{[(\chi_i+3)(\chi_i+1)-1]}{8k^2\zeta_i}\right)\right]+\eta_i[\chi_i\to\chi_i',\zeta_i\to\zeta_i']\right\},$$

$$\mathcal{C}_v^{-1,1}=-\frac{2kq(E_1+E_2)}{\sqrt{6}E}\,C_-\gamma_i\phi_i\frac{\tau}{8k\lambda}\left\{\left[\frac{\exp(\chi_i)}{\zeta_i^{2}}\left(1-\frac{(\chi_i+2)}{4k^2\zeta_i}\right)\right]+\eta_i[\chi_i\to\chi_i',\zeta_i\to\zeta_i']\right\},$$

$$\mathcal{C}_v^{-3,1}=\frac{\sqrt{2}kq(E_1+E_2)C_+}{E}\,\gamma_i\phi_i\frac{\tau^2}{64k^2\lambda^2}\left\{\left[\frac{\exp(\chi_i)}{\zeta_i^{3}}\left(1-\frac{(\chi_i+3)}{8k^2\zeta_i}\right)\right]+\eta_i[\chi_i\to\chi_i',\zeta_i\to\zeta_i']\right\},$$

and

$$\mathcal{C}_v^{3,-1}=-\,\mathcal{C}_v^{-3,1}\,,\quad \mathcal{C}_v^{1,-1}=\mathcal{C}_v^{-1,1}\,,\quad \mathcal{C}_v^{-1,-1}=-\,\mathcal{C}_v^{1,1}\,,\quad \mathcal{C}_v^{-3,-1}=\mathcal{C}_v^{3,1}\,.$$

Finally, the required σ_{tot} are given by[21,22]

$$\sigma_{\text{tot}}(\pi p)=23.4+\frac{8.3}{M}\ \text{mb},$$

and

$$\sigma_{\text{tot}}(\rho p)=0.27\left(98.6+\frac{64.9}{M}\right)\frac{0.65}{(1-t/m_\rho^{\,2})^2}\ \text{mb}.$$

We assume the simple quark-counting rule to relate $\sigma_{\text{tot}}(ip)$ to $\sigma_{\text{tot}}(ia)$ obtaining

$$\sigma_{\text{tot}}(ip)=\tfrac{3}{2}\sigma_{\text{tot}}(im)\,,$$

where m denotes a meson.

The criterion for the exoticity of the M^2 channel of Chan *et al.*[10] is adopted and for M^2 exotic the M-dependent term in the expression for σ_{tot} is dropped.

Note added in proof. After completion of this work we received notice of an absorption-model calculation for the reaction $pp\to\Delta^{++}+X$ (Ref. 23). This paper also points out the need for including absorption corrections in the triple-Regge region.

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