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Interaction of electric and magnetic charges

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It is shown classically that in a head-on collision between an electric and a magnetic charge a repulsive polarization force of the form r^{-5} results (where r is the distance between the charges), if one (both) charge(s) is (are) assigned a finite spherical size. This force leads to a minimum distance of approach and prevents one particle from going through the other, and thus guards against the violation of the conservation of angular momentum. This polarization force is a manifestation of the diamagnetism (diaelectricity) of extended electric (magnetic) charges.

Thomson[1,2] was fond of the system consisting of an electric charge e and a magnetic charge g. He showed that the angular momentum $\vec{L}$ of the electromagnetic field of the charges is given by $(eg/c)\hat{r}$, where $\hat{r}$ is a unit vector in the direction $\vec{r}$ and $\vec{r}$ is the position vector of the magnetic charge relative to the electric charge. He also proved by a torque argument that the *total* angular momentum $\vec{J}$ of the system is conserved. $\vec{J}$ is the sum of the field angular momentum $\vec{L}$ and the orbital momentum $\vec{l}$ of the charges. Dirac[3] considered the wave function of this system and arrived at the charge-quantization condition, $eg/c = n\hbar/2$, where n is an integer. Many experimental and theoretical papers on magnetic poles have followed. It has long been recognized that this monopoles system suffers from inherent fundamental difficulties. For if the monopoles are point charges, then in a head-on collision (zero impact parameter) one charge will go through the other, the field angular momentum $\vec{L}$ is reversed while $\vec{l}$ remains zero, and the total angular momentum is not conserved in this collision. It was also shown by Rosenbaum[4] that no classical action integral can be defined for this system if the monopoles should coincide. In quantum mechanics, the wave function for the relative motion of the monopoles vanishes at the origin,[5] and thus the probability of finding the monopoles a small distance ϵ apart is negligible. In classical mechanics, no satisfactory argument has been given to rule out the coincidence of the monopoles. In this note we show by classical methods that the monopoles cannot coincide. The resolution of the difficulty lies in recognizing the fact that the monopoles must be charges of finite extent and not point charges.[6] For monopoles of finite spherical size, the *field angular momentum is no longer constant but depends slightly on the distance between the monopoles.*[7] As the monopoles approach each other (in a head-on collision) the Lorentz force sets them into rotation to conserve angular momentum. A magnetic dipole is induced in the electric charge, which is repelled by the magnetic field of the magnetic pole. Similarly, an electric dipole is induced in the magnetic charge which is repelled by the electric charge. This repulsive polarization force leads to a minimum distance of approach and prevents the overlap of the particles, and thus guards against the violation of the conservation of angular momentum. We do not dwell on the detailed structure of the particles, a subject which is fraught with difficulties and pitfalls, but invoke only the general principles of *classical electrodynamics* and *classical nonrelativistic mechanics.*

Before we consider the general case in which both charges are finite spheres, let us consider the simpler case in which only the electric charge e is finite. Let this charge of mass m, and radius R, approach a magnetic point charge g fixed at the origin. Let the electric charge move along the negative x axis with velocity $v\hat{x}$. The Lorentz force on the elements of the electric charge produces circulating currents which are equivalent to a magnetic dipole of moment $\vec{\mu}$ pointing in the positive x direction, and the charge e also acquires an angular momentum $\vec{l}$ in the same direction. The field angular momentum $\vec{L}$ is given by[7]

$$\vec{L} = (eg/c)(1 - \alpha R^2 x^{-2})\hat{x}\,, \tag{1}$$

where α depends on the details of the distribution of the electric charge, for example, $\alpha = \frac{1}{5}$ for a uniform spherical volume distribution and $\alpha = \frac{1}{3}$ for a uniform surface charge distribution. Conservation of angular momentum requires that

$$\vec{l} = \alpha(eg/c) R^2 x^{-2}\hat{x}\,. \tag{2}$$

If we assume the *classical* result, $\vec{\mu} = (e/2mc)\vec{l}$, then the induced magnetic moment of the electric charge is given by

$$\mu = \frac{\alpha e^2 g}{2mc^2}\,\frac{R^2}{x^2}\,. \tag{3}$$

The magnetic pole repels this magnetic dipole with the force $f=\mu(\partial B/\partial x)$, namely,

$$\vec{f}=\frac{\alpha e^2g^2}{mc^2}\frac{R^2}{x^5}\hat{x}. \tag{4}$$

Equating f to $-\partial V/\partial x$ we can define a potential energy $V(x)$ for the system by

$$V(x)=\frac{\alpha e^2g^2}{mc^2}\frac{R^2}{4x^4}. \tag{5}$$

In addition, the electric charge has a rotational energy $E_r=l^2/(2I)$, where I is the moment of inertia which is of order mR^2, or

$$E_r=\frac{\alpha^2e^2g^2}{2Ic^2}\frac{R^4}{x^4}. \tag{6}$$

The conservation-of-energy equation now reads

$$\tfrac{1}{2}mv^2+\frac{\alpha e^2g^2}{2mc^2}\frac{R^2}{x^4}\left(\frac{1}{2}+\frac{\alpha mR^2}{I}\right)=\tfrac{1}{2}mv_0^{\,2}, \tag{7}$$

where $v_0\hat{x}$ is the initial velocity of the electric charge as it enters the field of the pole at $x=-\infty$. The distance of closest approach x_0 is given by (7) with $v=0$, namely,

$$x_0=\left(\alpha'\frac{eg}{mc}\frac{R}{v_0}\right)^{1/2},\qquad \alpha'^2\equiv\alpha\left(\frac{1}{2}+\frac{\alpha mR^2}{I}\right). \tag{8}$$

If

$$eg/c=\hbar/2,\qquad x_0=\left(\frac{\alpha'}{2}\frac{\hbar}{mv_0}R\right)^{1/2}=(\tfrac{1}{2}\alpha'\lambda\!\!\!^{-}R)^{1/2}, \tag{9}$$

which is the geometrical mean of the de Broglie wavelength/2π and $\frac{1}{2}\alpha'R$, the latter is a structure length for the particle.

Now we endow the magnetic pole with structure. We have now the masses m_e, m_g, the radii R_e, R_g, and the moments of inertia I_e, I_g. The electric charge acquires as before an angular momentum $\vec{l}_e$ and a magnetic dipole moment $\vec{\mu}_e$, while the magnetic charge acquires an angular momentum $\vec{l}_g$, and an electric dipole moment $\vec{p}_g=-(g/2m_gc)\vec{l}_g$. Since the force with which the charge e repels p_g must equal the repulsive force on μ_e due to the charge g, we have the relation

$$l_g/m_g=l_e/m_e. \tag{10}$$

Since[7]

$$L=\left(\frac{eg}{c}\right)\left(1-\alpha\frac{R_e^{\,2}+R_g^{\,2}}{x^2}\right), \tag{11}$$

we have

$$l_g+l_e=\alpha\left(\frac{eg}{c}\right)\frac{R_e^{\,2}+R_g^{\,2}}{x^2}, \tag{12}$$

and by (10)

$$l_e=\alpha\left(\frac{eg}{c}\right)\frac{m_e}{m_e+m_g}\frac{R_e^{\,2}+R_g^{\,2}}{x^2}, \tag{13}$$

$$l_g=\alpha\left(\frac{eg}{c}\right)\frac{m_g}{m_e+m_g}\frac{R_e^{\,2}+R_g^{\,2}}{x^2}. \tag{14}$$

The mutual repulsive $|\vec{f}|$ is given by

$$|\vec{f}|=\alpha\frac{e^2g^2}{c^2}\frac{1}{m_e+m_g}\frac{R_e^{\,2}+R_g^{\,2}}{|x^5|}, \tag{15}$$

and the potential energy of the system is

$$V(x)=\alpha\frac{e^2g^2}{c^2}\frac{R_e^{\,2}+R_g^{\,2}}{m_e+m_g}\frac{1}{4x^4}. \tag{16}$$

The rotational energy is now

$$E_r=\alpha^2\frac{e^2g^2}{c^2}\left(\frac{R_e^{\,2}+R_g^{\,2}}{m_e+m_g}\right)^2\left(\frac{m_e^{\,2}}{2I_e}+\frac{m_g^{\,2}}{2I_g}\right)\frac{1}{x^4}, \tag{17}$$

and the energy conservation in the center-of-momentum system now reads

$$\tfrac{1}{2}Mv^2+\frac{1}{2}\left(\frac{e^2g^2}{c^2}\right)\left(\frac{R_e^{\,2}+R_g^{\,2}}{m_e+m_g}\right)\times\alpha\left[\tfrac{1}{2}+\alpha\left(\frac{R_e^{\,2}+R_g^{\,2}}{m_e+m_g}\right)\left(\frac{m_e^{\,2}}{I_e}+\frac{m_g^{\,2}}{I_g}\right)\right]x^{-4}=\tfrac{1}{2}Mv_0^{\,2}, \tag{18}$$

with M the reduced mass equal to $m_em_g/(m_e+m_g)$. The distance of closest approach is

$$x_0=\left[\alpha''\frac{eg}{v_0c}\left(\frac{R_e^{\,2}+R_g^{\,2}}{m_e+m_g}\right)^{1/2}\right]^{1/2},\qquad \alpha''^2=\alpha\left[\tfrac{1}{2}+\alpha\frac{R_e^{\,2}+R_g^{\,2}}{m_e+m_g}\left(\frac{m_e^{\,2}}{I_e}+\frac{m_g^{\,2}}{I_g}\right)\right], \tag{19}$$

which is similar to Eq. (8) with obvious modifications.

It is interesting to observe that the potential and rotational energies as given by Eqs. (5) and (6) and by Eqs. (16) and (17) have the same distance dependence, x^{-4}, and combine simply by adding the coefficients to form an effective potential for point charges in the energy equations (7) and (18).

The polarization repulsion can be looked upon as a manifestation of the "diamagnetic" behavior of the extended electric charge and the "diaelectric" behavior of the extended magnetic charge. For the dipoles induced are proportional but opposite in direction to the inducing fields which vary as x^{-2}, and the repulsive force is proportional to the gradient of the field which varies as x^{-3} times the induced dipole moment giving the x^{-5} dependence.[8] To its many remarkable properties this system adds the distinction of offering the first model for diaelectricity in physics.

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[1]J. J. Thomson, *Elements of the Mathematical Theory of Electricity and Magnetism*, 4th ed. (Cambridge Univ. Press, Cambridge, 1909), p. 532.

[2]J. J. Thomson, *Recollections and Reflections* (Macmillan, New York, 1937), p. 370.

[3]P. A. M. Dirac, Proc. R. Soc. London A133, 60 (1931); see also a later paper by Dirac, Phys. Rev. 74, 817 (1948).

[4]D. Rosenbaum, Phys. Rev. 147, 891 (1966).

[5]See, e.g., H. J. Lipkin, W. I. Weisberger, and M. Peshkin, Ann. Phys. (N.Y.) 53, 203 (1969).

[6]Perhaps the fact that the integral giving the field angular momentum converges makes us oblivious of our experience with electrostatics where the electron radius is introduced early to render the electrostatic energy finite.

[7]I. Adawi, Am. J. Phys. 44, 762 (1976).

[8]The diamagnetic and diaelectric polarizabilities of the system are $\alpha e^2(R_e{}^2+R_g{}^2)/[2(m_e+m_g)c^2]$ and $\alpha g^2(R_e{}^2+R_g{}^2)/[2(m_e+m_g)c^2]$, respectively, as can be read from Eqs. (13) and (14).