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A dependence of large-momentum-transfer inclusive reactions*

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The large-momentum-transfer inclusive cross section for production of pions, protons, and kaons from nulcei is discussed in the framework of a multiple-scattering theory. We find that the A dependence seen in recent experiments can be explained by considering higher-order large-momentum-transfer reactions between the projectile and the nucleons. The theory has one parameter, which can be interpreted as the value of of $p_\perp$ at which the exponential falloff of the elementary cross section changes to a more-slowly-falling form. It is found that the data can be fitted with first- and second-order terms only.

I. INTRODUCTION

In this paper we attempt to understand quantitatively some aspects of recent experimental data[1] on single-particle inclusive production of large-transverse-momentum secondaries in the collisions of protons with various nuclear targets; in particular, we focus on the A dependence of the results and what they tell us about the fundamental p-p process. We work in the framework of a Glauber-type multiple-scattering picture of the nuclear scattering. The experiments referred to above used projectile of momentum from 200 to 400 GeV/c and detected $\pi^\pm$, $K^\pm$, p, $\bar{p}$, and d particles at $\approx 90°$ in the center-of-mass system with a range of momentum $p_\perp$ (perpendicular to the beam axis) from 1 to 7 GeV/c; targets were Be, Ti, and W. While the results do not encompass all possible configurations of the above combinations, they do allow a rather extensive survey of A dependence.

It has been known for some time that inclusive cross sections show qualitative changes when p increases beyond 2 or 3 GeV/c, or in other words as one penetrates the hadronic center to less than 10^{-15} cm. In particular, several experiments[2-4] on hydrogen targets demonstrated that the familiar nearly s-independent $\exp(-bp^2)$ behavior is replaced for the range of $p_\perp$ by a much flatter behavior. This behavior can be characterized at, say, $90°$ in terms of s and $p_\perp$ or a perpendicular scaling variable $x_\perp = 2p_\perp/\sqrt{s}$ to be[3]

$$d\sigma \sim p_\perp^{-n}\exp(-bx_\perp) , \qquad (1.1)$$

where n and b are dependent on the detected particle; for example, for pions $n\approx 8$ and $b\approx 13$. This power law is of course rather striking and is very evident for $p_\perp > 3$ GeV/c. In addition, the rise in $d\sigma$ with s for fixed $p_\perp$ is clearly distinguished from the behavior in the smaller-$p_\perp$ region. Of course the small-$p_\perp$ behavior still dominates the integrated cross section, since for $p_\perp > 3$ GeV/c, $d\sigma$ has already fallen many orders of magnitude from its value at $p_\perp = 0$.

Many theoretical models have been proposed[5] to account for the flattening of the $p_\perp$ distribution, all with successes and failures. In particular, these models predict in more or less detail inclusive cross sections at $90°$ in the form $(\sqrt{s})^{-n}f(x_\perp)$, as in Eq. (1.1).

By using nuclear targets to increase the cross section, the authors of Ref. 1 were able to extend the $p_\perp$ range of inclusive measurement as well as to study the A dependence of the results. Their results show a clear empirical behavior with A which allows extrapolation to hydrogen. The extrapolated behavior is consistent with the results of earlier experiments[2-4] in the common part of the $p_\perp$ and s spectrum; however, for larger values of $p_\perp$ the extrapolated cross section seems to show a somewhat steeper falloff. In particular, for $x_\perp > 0.4$ to 0.45 (depending on the detected particle) and fixed s the data are once again consistent with exponential behavior in $p_\perp$. More generally, for pions and kaons the nuclear data follow the functional form $(\sqrt{s})^{-n}e^{-ax_\perp}$, with a and n constants for $x_\perp > 0.4$ to 0.45.

The A dependence itself is also different from the A dependence of low-$p_\perp$ cross sections. The low-$p_\perp$ data are proportional to $A^{2/3}$, corresponding to linear dependence on $A_{\rm eff} = \sigma_{\rm abs}/\sigma_p$, where $\sigma_{\rm abs}$ is the absorption cross section in the nucleus under consideration and σ_p is the p-p total cross section. This is explicable using Glauber theory in a standard fashion.[6,7] The data of Ref. 1 show A dependence of the form $A^{1.1}$ to $A^{1.45}$, the exponent depending on the detected particle and rising with rising $p_\perp$.

In Ref. 8 it was shown that an atomic-number dependence greater than $A^{1.0}$ is possible only in the case where more than one "hard" (as opposed to "soft") collision occurs. The hard reactions are rare types, such as large-$p_\perp$ reactions, and are treated on the basis of their rarity as occurring only a finite number of times in the nucleus. The soft reactions are those common reactions comprising the bulk of the inelastic cross section, such as small-$p_\perp$ reactions; there are allowed to occur any number of times in the nucleus. If m hard collisions and an arbitrary number of soft inelastic and elastic collisions are allowed within the nucleus, then to order $\sigma_{\rm hard}/\sigma_{\rm soft}$, the effects of the elastic and soft inelastic collisions cancel out, and one is left with the classical result one finds if the m hard collisions alone are allowed. This resulting inclusive cross section is proportional to $A^{(m+2)/3}$.

The data in Ref. 1 are consistent in their simple A dependence with a combination of single ($\sim A^1$) and double ($\sim A^{4/3}$) hard, or large-$p_\perp$, scattering. Higher-order hard-scattering terms, such as triple order, are estimated and found to be negligible at the values of momentum transfer we are investigating. In any theoretical treatment of these data one must account not only for the observed A dependence, but for other aspects of the cross section. In addition to discussion of the theorem on rare collisions, Ref. 8 briefly treated the $p_\perp$ dependence of double scattering in an asymptotic regime and with certain assumptions about the behavior of the large-$p_\perp$ cross section.

The purpose of this paper is to make a more careful phenomenological examination of the data of Ref. 1, using the multiple-hard-collision scheme[8] discussed above. We have found a parametrization scheme which satisfactorily fits the experimental data. The parameter, κ, with dimensions of momentum, is the cutoff point which separates the hard from the soft collisions.

By adjusting the value of κ for each particle measured in Ref. 1, ($\pi^\pm, K, p, \bar{p}$), we were able to obtain a cross section agreeing with the observed $p_\perp$ and A dependence. The values of κ vary slightly depending on which particle is being measured, but generally have a value of about 2 GeV/c.

We have not committed ourselves to any particular theory of the primary mechanism of large-$p_\perp$ inclusive production in this paper. However, the reader should note that whatever the mechanism, it should be one which is repeatable in a short time scale ($<10^{-23}$ sec), as for example a quark collision process, or at least contains a rare ingredient of a type enhanced by counting advantages in the nuclear medium.[9] As far as we know, the constituent-interchange model[10] is not a candidate which fits into these categories.

In what follows it should be kept in mind that we are not proposing a fundamental theory for large-$p_\perp$ reactions on nuclei, but simply pointing out that the main features of the data, which have puzzled theorists up to this point, can be explained in terms of a small admixture of double-hard-scattering terms in the cross section. In keeping with this goal, a number of assumptions have been made in order to allow us to carry out the calculation. These are discussed in the text, but the most important one is this: We neglect any degradation in energy suffered by the projectile as it traverses the nucleus undergoing both soft and hard scatterings. This assumption will be discussed in Sec. III.

The plan of the remainder of the paper is as follows: For Sec. II we review the hard-collision theory and present our calculation and results; in Sec. III we discuss the physical implications of our results and make some predictions about higher-momentum-transfer experiments.

II. DERIVATIONS AND RESULTS

We have used the extension of the Glauber theory to the case of inclusive reactions[7] in our work; a brief review is in order. Following Ref. 8, we visualize a reaction as in Fig. 1. The projectile enters from the left and is allowed to scatter elastically any number of times (including zero) before making its first inelastic collision at the position z_1 (z axis along the beam direction). At z_1 an inelastic reaction occurs which excites either the target nucleon or the projectile (or both) to some state. This excited state then propagates to the point z_2, scattering elastically any number of times. At z_2 another inelastic event occurs and the process repeats to the point z_3, etc. The total cross section for N inelastic scatterings and any number of elastic ones can be written[8] as

$$\sigma_N^A = \prod_{i=1}^{N} \left(\frac{\Gamma_{i,i+1}}{\sigma_{\rm in}} \right) \frac{1}{N!} \int d^2B [t(B)]^N e^{-t(B)} , \qquad (2.1)$$

where

$$t(B) = A\sigma_{\rm in} \int_{-\infty}^{\infty} \rho(\vec{B}, z) dz , \qquad (2.2)$$

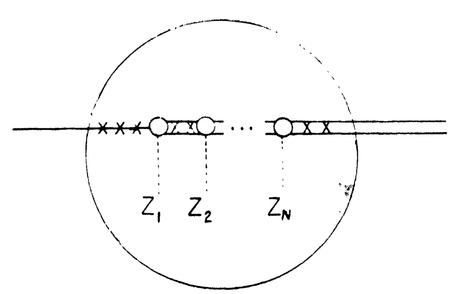

FIG. 1. A typical production process in a nucleus. The × represents elastic scatterings, and the inelastic scatterings occur at the vertices labeled z_1, z_2, etc.

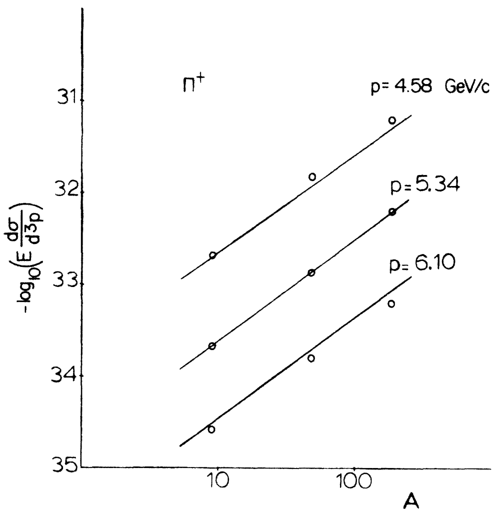

FIG. 2. Logarithm of the invariant cross section of π^+ in cm^2/GeV^2 versus atomic number A. Data points are from Ref. 1. Calculated points (straight lines) are given in Table I.

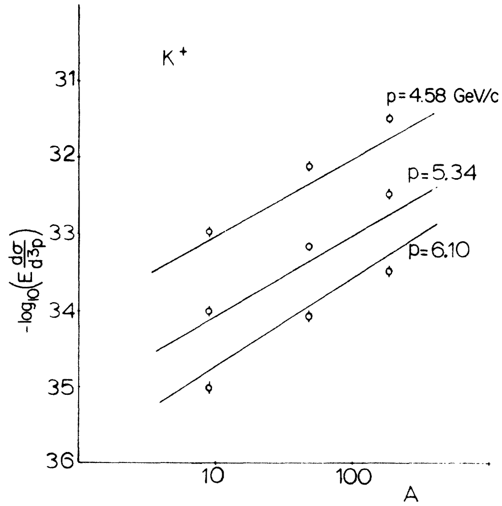

FIG. 4. Logarithm of the invariant cross section of K^+ in cm^2/GeV^2 versus atomic number A. Data points are from Ref. 1. Calculated points (straight lines) are given in Table III.

ρ is the nuclear density, normalized to unity, and

$$\Gamma_{i,i+1}=\int dx_{i+1}d^2k_{i+1}\frac{d^3\sigma_{i,i+1}}{d^2k_{i+1}dx_{i+1}} \tag{2.3}$$

is the inclusive inelastic cross section for the transition from state i to state $i+1$. The above sum includes all types of inelastic collisions. If one were looking for an inclusive cross section of a certain type, then one would not complete the appropriate integrations in Eq. (2.3) or integrate the variables, so that the multiple-scattering expression of Eq. (3.1) yields the final variables of interest.

In particular, let us look at an inclusive nuclear cross section in a rare dynamical region. We suppose this is due to constituent-constituent reactions of the corresponding hard type and expand to finite order in these collisions while allowing any number

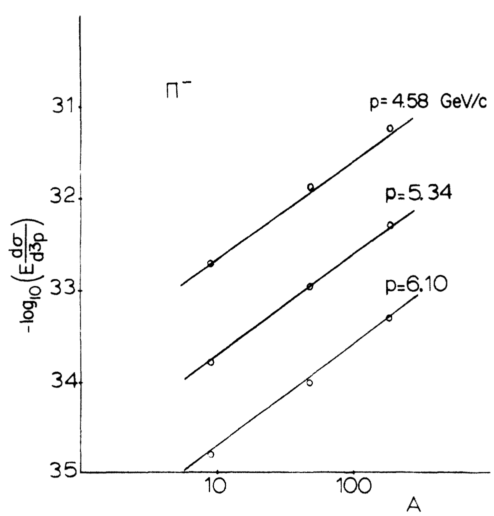

FIG. 3. Logarithm of the invariant cross section of π^- in cm^2/GeV^2 versus atomic number A. Data points are from Ref. 1. Calculated points (straight lines) are given in Table II.

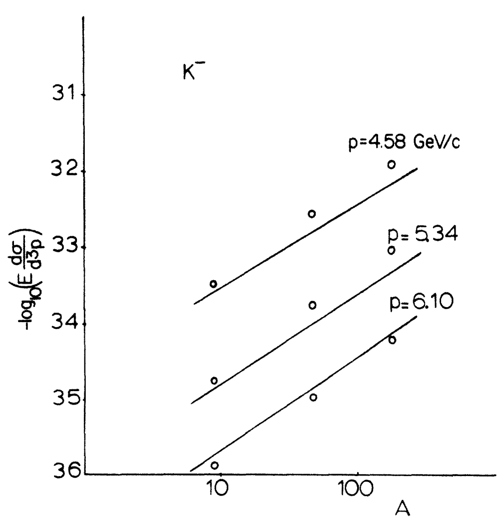

FIG. 5. Logarithm of the invariant cross section of K^- in cm^2/GeV^2 versus atomic number A. Data points are from Ref. 1. Calculated points (straight lines) are given in Table IV.

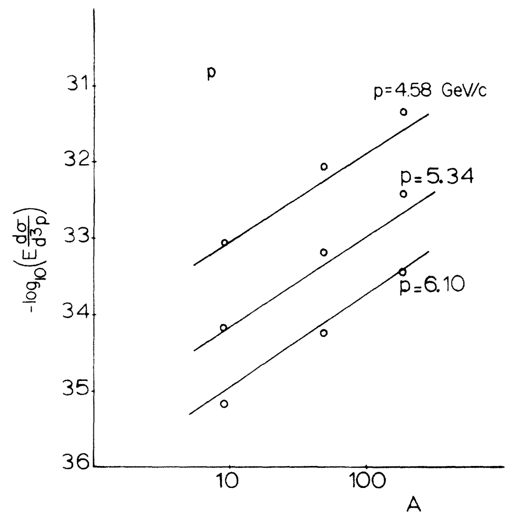

FIG. 6. Logarithm of the invariant cross section of p in cm^2/GeV^2 versus atomic number A. Data points are from Ref. 1. Calculated points (straight lines) are given in Table V.

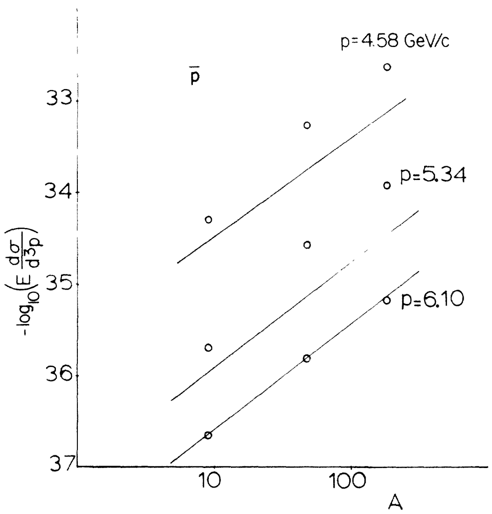

FIG. 7. Logarithm of the invariant cross section of $\bar{p}$ in cm^2/GeV^2 versus atomic number A. Data points are from Ref. 1. Calculated points (straight lines) are given in Table VI.

of soft collisions. The soft cross sections are integrated to allow any (soft) intermediate state, i.e.,

$$\Gamma^{\text{soft}}_{i,\,i+1} = \sigma_{\text{in}} - \Gamma'_h \,, \tag{2.4}$$

where

$$\Gamma'_h = \int dx \int_\kappa^\infty d^2k \, \frac{d^3\sigma_{\text{hard}}}{d^2k\,dx} \,. \tag{2.5}$$

The hard collisions are constrained to give the observed final state; thus in accordance with the experimental measurement[1] the single-collision term is of the form

$$\Gamma_h = \frac{d^3\sigma_{\text{hard}}}{d^2k\,dx}\bigg|_{x=0} \,, \tag{2.6}$$

while the double-hard-collision term is of the form $\Gamma_h * \Gamma_h$, a convolution of two expressions of the type (2.6).

The major result of Ref. 8 is that up to corrections of $O(\Gamma'_h/\sigma_{\text{in}})$ [this correction is due to Eq. (2.4)] the effect of summing all possible numbers of soft collisions is to just cancel all terms due to $t(B)$ in Eq. (2.1), leaving only hard-collision terms. Namely,

$$\sigma_1^A = \Gamma_h A - \Gamma_h \Gamma'_h A^2 \int d^2B\,dz\,dz'\rho(\vec{B},z)\rho(\vec{B},z') \,, \tag{2.7a}$$

$$\sigma_2^A = \Gamma_h * \Gamma_h A^2 \int d^2B\,dz\,dz'\rho(\vec{B},z)\rho(B,z') + O(A^3) \,, \tag{2.7b}$$

$$\sigma_3^A = \Gamma_h * \Gamma_h * \Gamma_h A^3 \times \int d^2B\,dz\,dz'dz''\rho(\vec{B},z)\rho(\vec{B},z')\rho(\vec{B},z'') \,, \tag{2.7c}$$

TABLE I. Calculated cross section for π^+, $\kappa = 2.2$ GeV/c. Also listed are the exponents from $I = I_1 A^\chi$, where I is our calculated cross section and I_1 is the extrapolated cross section for $A=1$ from Ref. 1. Errors of χ are estimated $\approx \pm 0.02$.

	Be		Ti		W	
$p_\perp$ (GeV/c)	$E\frac{d\sigma}{d^3p}\left(\frac{cm^2}{GeV^2}\right)$	χ	$E\frac{d\sigma}{d^3p}\left(\frac{cm^2}{GeV^2}\right)$	χ	$E\frac{d\sigma}{d^3p}\left(\frac{cm^2}{GeV^2}\right)$	χ
4.58	2.00×10^{-33}	1.088	1.16×10^{-32}	1.072	5.11×10^{-32}	1.080
5.34	2.22×10^{-34}	1.164	1.37×10^{-33}	1.131	6.49×10^{-33}	1.138
6.10	3.13×10^{-35}	1.138	1.89×10^{-34}	1.110	8.78×10^{-34}	1.119

TABLE II. Calculated cross section for π^-, $\kappa = 2.1$ GeV/c. Errors of χ are estimated ≈ 0.02.

	Be		Ti		W	
$p_\perp$ (GeV/c)	$E\frac{d\sigma}{d^3p}\left(\frac{\text{cm}^2}{\text{GeV}^2}\right)$	χ	$E\frac{d\sigma}{d^3p}\left(\frac{\text{cm}^2}{\text{GeV}^2}\right)$	χ	$E\frac{d\sigma}{d^3p}\left(\frac{\text{cm}^2}{\text{GeV}^2}\right)$	χ
4.58	1.88×10^{-33}	1.107	1.11×10^{-32}	1.087	5.00×10^{-32}	1.096
5.34	1.75×10^{-34}	1.159	1.08×10^{-33}	1.128	5.10×10^{-33}	1.135
6.10	1.81×10^{-35}	1.213	1.15×10^{-34}	1.166	5.67×10^{-34}	1.171

where now the subscript on σ refers to the number of hard collisions considered.

Let us now turn to more details of our calculation. Since all values of the Feynman (longitudinal) variable x enter into the convolution, we must assume that the large-$p_\perp$ cross section obeys Feynman scaling in a multiplicative way, so that for Γ_h we have

$$\Gamma_h = \tilde{f}(x)I(\vec{p}_\perp) \;; \tag{2.8}$$

$\tilde{f}(x)$ will be taken as a constant function whose value is determined by the value of the extrapolated cross section ($x=0$) measured in Ref. 1. Thus

$$\Gamma_h = I(\vec{p}_\perp) \;, \tag{2.9}$$

where $I(p_\perp)$ is the measured[1] cross section extrapolated to $A=1$. We fit this function, for $s=23.8$ (GeV/c)^2, to

$$I(\vec{p}_\perp) = \frac{\text{const}}{(\sqrt{s})^n}\, e^{-ax_\perp}, \quad x_\perp > 0.4 \tag{2.10a}$$

where n and a are given in Ref. 1, and to

$$I(\vec{p}_\perp) = \frac{f(p_\perp)}{p_\perp^{\,8}}, \quad \frac{2\kappa}{\sqrt{s}} < x_\perp < 0.4 \;. \tag{2.10b}$$

$f(p_\perp)$ is a species-dependent polynomial which we derived from a match of the data of Ref. 1 in this region of $x_\perp$ to the power-law function suggested in Refs. 2 and 3.

The double-scattering term is given by convolution to be

$$\Gamma_h * \Gamma_h = \int d^2k\, d^2k' \times \int dx\, dx'\, \delta(x+x') I(\vec{k}) I(\vec{k}')\, \delta(\vec{k}+\vec{k}'-\vec{p}_\perp) = \int_{x_{\min}}^{x_{\max}} dx \int d^2k\, I(\vec{k}) I(\vec{p}_\perp - \vec{k}) \,. \tag{2.11}$$

The integral in Eq. (2.11) is evaluated only for $|\vec{p}_\perp - \vec{k}|$ and $|\vec{k}| > k$, thus ensuring the convolution contains no soft inelastic collisions.

We also require the wave functions for the nuclei in question. For Be (Ref. 11) we take a ground-state density corresponding to a harmonic-oscillator wave function

$$\rho(r) = \frac{2k^3}{\pi^{3/2}a^3(2+3\alpha)}(1+\alpha k^2y^2)\exp(-k^2y^2) \;,$$

where $y = r/a$, $a \approx 2.2$ f, $\alpha \approx \frac{2}{3}$, $k \approx \sqrt{2}$. For Ti (Ref. 11) we take

$$\rho(r) = R_1\left[\exp\left(\frac{r-c}{z}\right)+1\right]^{-1} \;,$$

where $c \approx 3.9\times10^{-13}$ cm, $z \approx 2.4\times10^{-13}$ cm, and R_1 is a normalization constant. Finally, for W we take[11,12]

$$\rho_d(r) = R_2[\rho_0(r) + \rho_2(r)P_2(\cos\gamma)] \,,$$

where R_2 is a normalization constant, P_2 is the Legendre polynomial of second order, γ is the angle measuring the departure from the spherical shape of the nucleus, $\rho_0(r) = \rho(r) + \frac{1}{10}\alpha^2 r^2 \rho''(r)$, $\rho_2(r) = \alpha r \rho'(r) + \frac{1}{7}\alpha^2 r^2 \rho''(r)$, $\rho(r)$ is a distribution

TABLE III. Calculated cross section for K^+, $\kappa = 1.1$ GeV/c. Errors of χ are estimated ≈ 0.03.

	Be		Ti		W	
$p_\perp$ (GeV/c)	$E\frac{d\sigma}{d^3p}\left(\frac{\text{cm}^2}{\text{GeV}^2}\right)$	χ	$E\frac{d\sigma}{d^3p}\left(\frac{\text{cm}^2}{\text{GeV}^2}\right)$	χ	$E\frac{d\sigma}{d^3p}\left(\frac{\text{cm}^2}{\text{GeV}^2}\right)$	χ
4.58	8.41×10^{-34}	1.010	4.54×10^{-33}	1.009	177×10^{-32}	1.010
5.34	7.58×10^{-35}	1.070	4.34×10^{-34}	1.058	1.87×10^{-33}	1.065
6.10	1.63×10^{-35}	1.154	1.11×10^{-34}	1.122	5.86×10^{-34}	1.130

TABLE IV. Calculated cross section for K^-, $\kappa = 1.7$ GeV/c. Errors of χ are estimated ≈ 0.03.

	Be		Ti		W	
$p_\perp$ (GeV/c)	$E\frac{d\sigma}{d^3p}\left(\frac{\text{cm}^2}{\text{GeV}^2}\right)$	χ	$E\frac{d\sigma}{d^3p}\left(\frac{\text{cm}^2}{\text{GeV}^2}\right)$	χ	$E\frac{d\sigma}{d^3p}\left(\frac{\text{cm}^2}{\text{GeV}^2}\right)$	χ
4.58	2.51×10^{-34}	1.119	1.50×10^{-33}	1.097	6.84×10^{-33}	1.105
5.34	1.43×10^{-35}	1.230	9.22×10^{-35}	1.179	4.60×10^{-34}	1.184
6.10	1.91×10^{-36}	1.485	1.36×10^{-35}	1.350	7.57×10^{-35}	1.331

such as the above for Ti (primes denote differentiation with respect to r), $\alpha = 0.19$, and $\lambda = 1.08$.

We first used the normalization conditions for the densities to find the constants R_1, R_2 and then evaluated the density integrals in Eq. (2.7). These integrals were found to be

$$\phi_{\text{Be}} = 4.8675\times10^{-2}, \quad \phi_{\text{Ti}} = 1.3882\times10^{-2},$$

$$\phi_{\text{W}} = 5.8\times10^{-3}\ (\text{cm}^{-2}) .$$

We restrict our attention to terms up to third order in the hard scattering. This is partly because this is the highest order required (for pions only second order is required). Because of the computing time involved, the fits to third-order scattering are rather expensive and we have in places sacrificed accuracy.

By variation of the cutoff parameter, we made "eyeball" fits to the data for each final-state particle. Our hope that the value of κ would fall between 1 and 3 GeV/c was fulfilled in each case. We also found that satisfactory fits could be made using only single and double hard scattering, which is fortunate because our estimates of the contributions from the triple-scattering term show it to be quite small. Figures 2–7 and Tables I–VI show the final best fits to the data of Ref. 1.

A few words are in order about these fits. First, it is obvious that the theory does not enjoy uniform success in fitting the data. The agreement is much better for pions than for the other particles. This may be partly due to the fact that the ratios K^-/π^-, p/π^+, and $\bar{p}/\pi^-$ do not appear to scale for large $x_\perp$, as was pointed out in Ref. 1. Since our choice of parameters for the elementary scatterings assumes scaling for $x_\perp > 0.45$, it may be that this introduces discrepancies. However, in the absence of direct data on elementary collisions at large $x_\perp$ we are somewhat restricted in our choice of parameters.

We estimated the triple-scattering correction to these fits in the case of the π^+. With a cutoff value of 2.2 GeV/c, we found it to be less than 1% of the theoretical cross section. Even at $p_\perp$ as large as 11 GeV/c, we found this correction to be quite small. We feel confident, therefore, that with the double-scattering term which we retain our results would not be affected by adding contributions from higher-order terms. While it also should be obvious that the ratio of triple to double scattering is fairly sensitive to the cutoff, we felt satisfied that the triple-scattering terms were unimportant within a reasonable range of cutoff. See Sec. III for further discussion.

With these reservations in mind, however, it is still obvious that the Glauber theory does an adequate job of explaining both the A dependence and the $p_\perp$ dependence for the data reported in Ref. 1.

III. CONCLUSIONS AND DISCUSSION

Summarizing this work, the inclusion of double-hard-scattering corrections in the formulas derived in Ref. 7 and 8 enables us to explain the general features of the behavior of the data of Ref. 1 at high values of $p_\perp$. We considered one adjustable parameter for each particle species which came out to be the same for all. This is the cutoff for hard collisions to occur.

TABLE V. Calculated cross section for p, $\kappa = 1.8$ GeV/c. Errors of χ are estimated ≈ 0.03.

	Be		Ti		W	
$p_\perp$ (GeV/c)	$E\frac{d\sigma}{d^3p}\left(\frac{\text{cm}^2}{\text{GeV}^2}\right)$	χ	$E\frac{d\sigma}{d^3p}\left(\frac{\text{cm}^2}{\text{GeV}^2}\right)$	χ	$E\frac{d\sigma}{d^3p}\left(\frac{\text{cm}^2}{\text{GeV}^2}\right)$	χ
4.58	8.11×10^{-34}	1.257	5.30×10^{-33}	1.198	2.69×10^{-32}	1.201
5.34	6.32×10^{-35}	1.303	4.22×10^{-34}	1.230	2.19×10^{-33}	1.229
6.10	1.04×10^{-35}	1.470	7.40×10^{-35}	1.341	4.09×10^{-34}	1.324

TABLE VI. Calculated cross section for $\bar{p}$, κ 1.9 GeV/c. Errors of χ are estimated ≈ 0.03.

	Be		Ti		W	
$p_\perp$ (GeV/c)	$E\frac{d\sigma}{d^3p}\left(\frac{\text{cm}^2}{\text{GeV}^2}\right)$	χ	$E\frac{d\sigma}{d^3p}\left(\frac{\text{cm}^2}{\text{GeV}^2}\right)$	χ	$E\frac{d\sigma}{d^3p}\left(\frac{\text{cm}^2}{\text{GeV}^2}\right)$	χ
4.58	2.92×10^{-35}	1.094	1.71×10^{-34}	1.077	7.55×10^{-34}	1.085
5.34	1.14×10^{-36}	1.197	7.20×10^{-36}	1.155	3.51×10^{-35}	1.161
6.10	2.41×10^{-37}	1.190	1.51×10^{-36}	1.149	7.34×10^{-36}	1.156

For pions, we found that third-order collisions are not necessary, and we predict that they will not be up to momentum transfers of approximately 15 GeV/c. To check this we estimated the third-order correction:

$$\sigma_3 = \Gamma_h * \Gamma_h * \Gamma_h A^3 \int d^2B\left[\int dz\,\rho(B,z)\right]^3 .$$

Taking

$$\rho(B,z) = \frac{1}{(\sqrt{\pi}R)^3}\, e^{-(z^2+B^2)/R^2}$$

$$R = 1.2\times10^{-13}A^{1/3}\ \text{cm}\,,$$

we find

$$\sigma_3 \approx 5\times10^{50}A^{5/3}\Gamma_h * \Gamma_h * \Gamma_h\ (\text{cm}^2)\,.$$

The integral $\Gamma_h * \Gamma_h * \Gamma_h$ is calculated the same way as the $\Gamma_h * \Gamma_h$ term of the double–hard-scattering term:

$$\Gamma_h * \Gamma_h * \Gamma_h = \int_\kappa^\infty d^2k_1 d^2k_2 f(|\vec{k}_1|)f(|\vec{k}_2|) \times f(|\vec{p} - \vec{k}_1 - \vec{k}_2|)\,.$$

This leads us into a discussion of the physical significance of the cutoff parameter κ. This parameter obviously plays an important role in double- and higher-order collisions of the type we have discussed here. Roughly speaking, we can think of it as the transition point between regular inelastic and hard collisions, and hence argue that it should be comparable to the momentum transfers at which exponential behavior in scattering amplitudes crosses over into a power law. However, the precise definition of κ as we have used it in our theoretical development does not depend on the momentum-transfer dependence of a scattering amplitude, but rather on whether the processes were rare enough to be singled out for an expansion in a multiple-scattering series. In the particular problem we discussed, it was the momentum transfer at which we could consider the collision to be "hard," as opposed to the "soft" collisions which we included in the coherent excitation terms.

However, whatever the physical source of this cutoff, we may study the analytic behavior of double and higher scattering in the asymptotic retion. In particular, if the true asymptotic behavior is a power law, then the asymptotic behavior of the double-scattering term is

$$\int_\kappa d^2k\,\frac{1}{k^m}\,\frac{1}{(p_\perp - k)^m} \underset{p_\perp\to\infty}{\sim} \frac{1}{p_\perp{}^m}\,, \quad \kappa \text{ const}\,.$$

This analytic behavior in $p_\perp$ holds true for nth-order hard scattering. (These higher-order terms have more complicated dependence on the other parameters.) We may contrast this with the case of exponential behavior in $p_\perp$, where the higher-order terms fall slower than the single-scattering term.

We conclude with a caveat about the physical assumptions of this model. We believe that for the short longitudinal distances involved in high-energy collisions within the nucleus the inelastic and elastic collisions are those of nonasymptotic hadronic "matter." For such matter it is reasonable that the energy of this matter is not degraded in an inelastic collision as it would be for asymptotic matter in the sense that a substantial portion of the energy goes into particle production, i.e., the collision of a hadron is highly (50%) inelastic. If the nonasymptotic nuclear matter were somehow highly inelastic in this sense, the effect on multiple large-$p_\perp$ scattering would be important, because the elementary large-$p_\perp$ production is such a strong function of s; we would then expect the importance of double scattering to decrease.

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