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Cherenkov radiation in a charge-separated magnetic plasma as a possible source for radio emission in pulsars

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The Cherenkov radiation from relativistic charged particles moving through a charge-separated magnetic plasma is discussed. Several features of the radiation resemble the observed radio emission from the Crab pulsar.

I. INTRODUCTION

A neutral plasma in a strong magnetic field has dispersion properties resembling those of a uniaxial crystal, the direction of the magnetic field corresponding to the optic axis.[1] Attempts have been made to explain the nonthermal radio emission from the sun as being due to Cherenkov radiation from jets of electrons traveling from the sunspots through the sun's atmosphere.[2] However, the emitted radiation will encounter stop bands on its way out of the atmosphere, and the Cherenkov effect has to be ruled out as the emission process.[3]

Heintzmann *et al.*[4] have discussed the propagation of radio waves through a completely charge-separated plasma moving in a strong magnetic field, i.e., a plasma of the kind believed to exist in pulsar magnetospheres. The dispersion properties of such a plasma differ radically from those of a neutral magnetic plasma at rest. Radio waves can propagate freely in all directions, and Cherenkov radiation should be considered a possible candidate for the pulsar radio emission.

II. THE EMISSION PROCESS

We consider a completely charge-separated plasma in a magnetic field B. The plasma particles of mass m, charge e, and number density N are moving parallel to the field direction with velocity β and energy γ (in units of the vacuum velocity of light and the rest energy, respectively). The dispersion equation for an electromagnetic wave of frequency ω, propagating through the plasma at an angle θ with respect to the field direction, is given by[4]

$$\hat{\omega}^2(n^2-1)[n^2\cos^2\theta - 1 - (n^2-1)\tilde{\omega}^2\gamma^2\nu^2] = \nu^2(1-\tilde{\omega}^2\gamma^2\nu^2)[1+(n^2-1)\tilde{\omega}^2]^2 . \quad (1)$$

We have used the abbreviations $\tilde{\omega}=\omega/\omega_p$ and $\hat{\omega} = \omega\omega_B/\omega_p^2$ where $\omega_p=(4\pi Ne^2/m\gamma)^{1/2}$ is the relativistic transverse plasma frequency and $\omega_B = eB/m\gamma$ is the relativistic gyro frequency. Further, $\nu = 1 - n\beta\cos\theta$, where n is the plasma index of refraction. Letting $n\to\infty$ and $n\to 0$, respectively, we find no resonances and cutoffs at the three frequencies ω_p^2/ω_B, ω_p/γ, and ω_B.

In the magnetosphere of a pulsar the plasma is believed to fulfill the relation $\omega_p^2 = 2\Omega\omega_B$, Ω being the pulsar rotation frequency. In this case we have $\hat{\omega}=\omega/2\Omega$, which is a very large number when we consider waves at radio frequencies. We thus limit our discussion to the asymptotic case of $\hat{\omega}\to\infty$. The dispersion equation (1) for the ordinary mode of propagation then becomes

$$n^2\cos^2\theta - 1 - \tilde{\omega}^2\gamma^2(n^2-1)(1-n\beta\cos\theta)^2 = 0 \quad (2)$$

corresponding to a wave polarization in the plane through the direction of propagation and the direction of the magnetic field.

Assume that a charged particle is moving through the plasma in the same direction as the plasma particles, but with different velocity β_0 and energy γ_0. This particle will emit Cherenkov radiation when the condition

$$n\beta_0\cos\theta = 1 \quad (3)$$

is fulfilled. A simultaneous solution of this condition and the dispersion equation will give the index of refraction n_C and the emission angle θ_C as functions of ω.

If we apply the asymptotic dispersion equation (2), we find the simple results

$$n_C^2 = 1 + (\omega^*/\omega)^2 , \quad (4)$$

$$\sin^{-2}\theta_C \approx 1 + (\omega/\omega^*)^2 , \quad (5)$$

where ω^* is a characteristic frequency given by

$$\omega^* = \omega_p|\gamma_0\gamma(\beta_0-\beta)|^{-1} . \quad (6)$$

We have assumed that the radiating particle is extremely relativistic and we have considered "low" wave frequencies only:

$$\gamma_0 \gg 1, \quad \omega \ll \gamma_0\omega^* . \quad (7)$$

A criterion for the validity of the asymptotic approximation can be established as follows: We first assume $(n^2-1)\tilde{\omega}^2 \ll 1$ and thus approximate the expression in square brackets on the right-hand side of Eq. (1) by unity. In this case we can find a simultaneous solution of Eqs. (1) and (3) which reduces to the results above when

$$\omega_p^3/\gamma\,\omega_B \ll \omega^{*2} \ll \omega_p^2 . \tag{8}$$

The second inequality of Eq. (8) is necessary to ensure the assumed smallness of the term $(n^2-1)\tilde{\omega}^2$ and also restricts the validity of the results to cases when γ and γ_0 are of different orders of magnitude.

The emitted energy per unit time per frequency interval is given by[5]

$$\frac{dW}{dt\,d\omega} = (e^2\beta_0\omega/2c)\sin^2\theta_C\left[\frac{d(\cos\theta - n^{-1}\beta_0^{-1})}{d(\cos\theta)}\right]_C^{-1},$$

where the derivative is to be calculated at the Cherenkov angle by means of the dispersion equation. With the approximations of Eqs. (7) and (8) we get the result

$$\frac{dW}{dt\,d\omega} = \frac{e^2\gamma_0^2}{2c}\,\frac{\omega}{[1+(\omega/\omega^*)^2]^2} . \tag{9}$$

The total energy emitted per unit time is found to be

$$\frac{dW}{dt} = (e^2/4c)\gamma_0^2\omega^{*2} . \tag{10}$$

Note that the energy of the plasma particles enters the formulas above through ω^* only.

III. APPLICATION TO THE CRAB PULSAR

The magnetosphere of a pulsar is generally believed to consist of a completely charge-separated plasma[6] streaming along the magnetic field lines. Near the polar cap of the Crab pulsar, assuming the plasma is made up of electrons, the relativistic plasma frequency ω_p and gyro frequency ω_B are given by

$$\omega_p \sim 10^{11}\gamma^{-1/2}\ \mathrm{sec}^{-1},$$

$$\omega_B \sim 10^{19}\gamma^{-1}\ \mathrm{sec}^{-1}.$$

Assume that some relativistic charges are emitted from the polar cap and move along the field lines through the plasma, emitting Cherenkov radiation along the way. What energies γ and γ_0 are necessary to give a radio spectrum resembling the observed one? The critical frequency ω^* must then have a value of $\omega^* \sim 10^9\ \mathrm{sec}^{-1}$. With the restrictions imposed by Eq. (8), we then find that the plasma electrons can have energies in the range $1<\gamma \lesssim 10^3$. The corresponding values for the energy of the radiating particles lie in the interval $10^2 \lesssim \gamma_0 \lesssim 10^4$. None of these values seems unreasonably high.

If the Cherenkov emission begins close to the surface, radiation of frequency ω is emitted at an angle $\theta_{C0}(\omega)$ given by Eq. (5) with the surface value for ω^*. As the emitting particle moves outwards through the magnetosphere of decreasing density, the plasma frequency, and thereby ω^*, decreases. This in turn implies a decrease in the emission angle and in the emitted power. Radiation of frequency ω will thus be emitted within a hollow cone of opening angle $\theta_{C0}(\omega)$ around the trajectory of the emitting particle, assumed to be along a field line close to the magnetic axis. The width of the cone decreases with increasing frequency ω. The radiation is linearly polarized in a plane through the direction of propagation and the magnetic axis. We also note from Eq. (9) that the emission spectrum falls off like ω^{-3} when $\omega \gg \omega^*$. All of these features are qualitatively consistent with observations.[6]

We make no attempt to explain the total radio luminosity of the Crab pulsar. As for all other emission mechanisms, this requires the introduction of some bunching mechanism for the emitting particles to account for the high brightness temperatures observed. We note, however, that with the pulsar parameters applied here, the emission, and consequently the bunching, must take place near the pulsar surface.

Since the magnetic field lines are curved, charged particles leaving the polar cap will also emit curvature radiation. The total power emitted in this way is given by

$$\frac{dW'}{dt} = (2e^2/3c)(\gamma_0^2 c/\rho)^2 , \tag{11}$$

the spectrum extending to a maximum frequency of $\omega_{max} \approx \gamma_0^3 c\rho^{-1}$. Here ρ is the curvature radius of the magnetic field lines, assumed to be of the order of $\rho \approx 10^7$ cm near the pole of the Crab pulsar. When $\gamma_0 \lesssim 10^4$ as above, we find that the Cherenkov mechanism totally dominates as the generator of radio waves in our case.

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