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Gaussian-spindle gravitational wave antenna and single-antenna anticoincidence experiments*

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The stretching modes of a long, thin Gaussian spindle are calculated and compared with those of a long, thin cylinder. The cross sections of these modes for gravitational radiation are calculated and compared. It is found that the fundamental mode of the spindle has between 1.2 and 1.6 times the cross section of the fundamental mode of a comparable cylinder, depending on how the comparison is made. The frequencies of the spindle stretch modes are proportional to the square root of the mode number and only the fundamental mode couples to gravitational waves. This behavior makes the spindle particularly useful for a single-antenna anticoincidence experiment which looks for excitations of the fundamental mode that are not in coincidence with excitations of higher modes. Such an experiment should be quite effective at rejecting nonthermal noise and demonstrating the presence of gravitational waves.

I. INTRODUCTION

Acoustic resonator gravitational-wave antennas are usually cylinders.[1] Cylinders are easy to make, easy to analyze, and easy to couple sensors to. Furthermore, cylinders are compact, an important feature in low-temperature experiments. However, the low-temperature techniques that are now being used in gravitational-wave detection promise a tremendous reduction in random thermal noise. This improvement means more problems from nonthermal noise such as strain-relief processes in the antenna and local disturbances that manage to penetrate the isolation of the antenna. This paper considers a new antenna shape which can discriminate between such spurious signals and genuine gravitational-wave bursts. Figure 1 shows the shape that will be considered: the figure of revolution generated by rotating a Gaussian curve about its asymptote. For lack of a better name, I choose to call it a Gaussian spindle.

The advantage of a Gaussian-spindle antenna is that only its fundamental stretch mode can couple to gravitational waves. Thus, one can do an anticoincidence experiment which monitors several stretch modes and accepts an event in the fundamental mode as a gravitational wave only if it does not coincide with an event in a higher mode. A cylindrical antenna is not very useful for this type of experiment because all of its even-parity modes couple to gravitational waves. Thus, an anticoincidence experiment with a cylinder must either accept all even-parity events as gravitational or else assume a power spectrum for gravitational-wave bursts and use the even-parity modes to sample the power spectrum of each event.

In order to keep the analysis simple, I will consider only the properties of a long, thin antenna and calculate only its gravitational cross sections. It is important to remember that this analysis is only part of the story. The final assessment of an antenna can only be made when the antenna is part of a specific gravitational-wave detector. It is found that a long, thin spindle is just as easy to analyze as a long, thin cylinder and has between 1.2 and 1.6 times the cross section of a comparable cylinder, depending on how the comparison is made. This slight improvement in cross section comes about only because all of the antenna response is concentrated in the fundamental mode. A similar improvement was noted by Bonazzola and Chevreton for antennas similar in shape to the Gaussian spindle.[2] The increased cross section is more than compensated for by the fact that the spindle antenna is sensitive at only one frequency while the cylinder is sensitive at many frequencies. Thus one would be rather foolish to build a spindle antenna solely to obtain an increased cross section. The only good reason for building a spindle is to do a clean anticoincidence experiment.

The Gaussian-spindle shape results from the requirement that there exist a stretch mode with uniform strain. This requirement guarantees that only one mode will couple to gravitational waves and is the key to the anticoincidence experiment. It is this requirement, rather than the exact Gaussian-spindle shape, that would govern the design

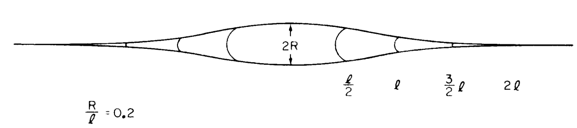

FIG. 1. A Gaussian spindle with shape factor 0.2. The end-face locations for various truncated spindles are shown.

of a realistic anticoincidence-type gravitational-wave detector.

I find it convenient to use the notation and results given in the text by Misner, Thorne, and Wheeler.[3] I also find it convenient to use Planck units defined by setting $G=c=h/2\pi=1$.[4]

Section II considers the stretching modes of long, thin solids and derives the Gaussian-spindle shape from the uniform strain requirement. The normal modes of long, thin spindles are derived and compared to those of long, thin cylinders. The gravitational-wave cross sections of these modes are calculated and compared in Sec. III. Single-antenna anticoincidence experiments are discussed in Sec. IV.

II. STRETCH MODES OF LONG, THIN SPINDLES AND CYLINDERS

Consider a long, thin solid oriented along the x axis of a Cartesian coordinate system. All that is assumed about the shape of this solid is that it has a normal stretch mode which can be approximated by displacements $x\rightarrow x+\xi$ in the x direction alone, where ξ is independent of y and z. The displacement function ξ is then governed by the Lagrangian

$$L=\tfrac{1}{2}\int\lambda(\dot{\xi}^2-v^2\xi'^2)dx\ ,$$

where λ is the mass per unit length, v is the sound speed, overdots denote time derivatives, and primes denote derivatives with respect to x. This Lagrangian yields the acoustic wave equation

$$-\ddot{\xi}+v^2\lambda^{-1}(\lambda\xi')'=0\ . \tag{1}$$

Now impose the requirement that the normal stretch mode have uniform strain by substituting $\xi=Kx\exp(i\omega t)$ into the wave equation. The result of this substitution is a condition on the mass per unit length λ,

$$(\ln\lambda)'=-(\omega/v)^2x\ .$$

Thus, the mass per unit length of a solid which has a uniform strain mode is required to have the form

$$\lambda=\lambda_0\exp(-\alpha x^2)\ , \tag{2}$$

where λ_0 and α are constants.

An axially symmetric solid which satisfies Eq. (2) is defined by $r\le R\exp(-x^2/l^2)$, where $r=(y^2+z^2)^{1/2}$. I choose to call this solid a Gaussian spindle of radius R and length parameter l. Its stretch modes are described by Eq. (1) with $\lambda=\rho\pi R^2\exp(-2x^2/l^2)$ and by the condition that the total energy

$$E=\tfrac{1}{2}\int\lambda(\dot{\xi}^2+v^2\xi'^2)dx \tag{3}$$

be finite. These modes will be compared with those of a long, thin cylinder which are described by Eq. (1) with $\lambda=\rho\pi R^2$ and by the boundary conditions $\xi'(l)=\xi'(-l)=0$.

The normal-mode solutions of Eq. (1) are of the form

$$\xi_n(x,t)=\mathrm{Re}[u_n(x)B_n\exp(-i\omega_n t)]\ , \tag{4}$$

where B_n is a complex constant and u_n obeys the equation

$$\lambda^{-1}(\lambda u_n')'+(\omega_n/v)^2u_n=0\ . \tag{5}$$

These modes will be normalized so that

$$\int\lambda u_n u_m dx=M\delta_{nm}\ , \tag{6}$$

where M is the total mass of the solid. The total energy in mode n is then

$$E_n=\tfrac{1}{2}M|B_n|^2\omega_n^{\,2}\ . \tag{7}$$

For the cylinder of radius R and overall length l, the normal modes that satisfy Eq. (5) and the vanishing strain boundary condition as well as the normalization condition of Eq. (6) are

$$u_n^C=\begin{cases}\sqrt{2}\sin\dfrac{n\pi x}{2l}\ , & n \text{ odd (even parity)}\\[2ex] \sqrt{2}\cos\dfrac{n\pi x}{2l}\ , & n \text{ even (odd parity)}\end{cases} \tag{8}$$

with

$$l\omega_n^C=\tfrac{1}{2}\pi n v\ . \tag{9}$$

For the spindle of radius R and length parameter l, notice that Eq. (5) is a form of the Hermite equation[5] whose square-integrable (i.e., finite-energy) solutions normalized according to Eq. (6) are

$$u_n^S(x)=(2^n n!)^{-1/2}H_n(2^{1/2}x/l)\ , \tag{10}$$

with

$$l\omega_n^S=2n^{1/2}v\ . \tag{11}$$

III. GRAVITATIONAL-WAVE CROSS SECTIONS

It is now a straightforward matter to calculate the ways in which spindles and cylinders couple to gravitational waves. I will follow the MTW procedure[6] of first calculating the Einstein A coefficients

$$A_{\mathrm{GW}}=P_{\mathrm{GW}}/E\ , \tag{12}$$

$$A_{\mathrm{diss}}=P_{\mathrm{diss}}/E\ , \tag{13}$$

where P_{GW} is the gravitational-wave power radiated by a mode with energy E and P_{diss} is the corresponding nongravitational dissipation power. These coefficients can be used to compute a variety of things, only some of which will appear here.

To find A_{GW}, the basic geometrical quantity that must be calculated is the moment-of-inertia factor for each mode:

$$I_{(n)jk}=\int \lambda(u_n^j x^k + u_n^k x^j)dx \ ,$$

where $x^1=x$, $x^2=y$, $x^3=z$, and $u_n^1=u_n$. For the stretch modes considered here, $u_n^2 \simeq u_n^3 \simeq 0$ so that the component

$$I_{(n)11}=2\int \lambda u_n x\, dx \tag{14}$$

is dominant. One can easily show that the other components are all much less than $(R/l)^2 I_{(n)11}$ and may therefore be neglected. For the cylinder, one substitutes Eq. (8) into Eq. (14), looks the resulting integral up in a table, and finds that

$$I^C_{(n)11}=(-1)^n 2^{7/2}\pi^{-2} M\, l n^{-2} \text{ for } n \text{ odd} . \tag{15}$$

The even-numbered modes give zero by symmetry. For the spindle, the calculation is greatly simplified if one notices that, from Eq. (10), $x=\frac{1}{2}lu_1^S(x)$. One substitutes this result into Eq. (14) and uses the orthonormality relation, Eq. (6), to obtain

$$I^S_{(n)11}=Ml\delta_{1n} . \tag{16}$$

Notice the Kronecker δ that appears here. This factor is of crucial importance for the anticoincidence performance of the spindle antenna. It arises solely from the fact that the spindle has a uniform strain mode.

In terms of the moment-of-inertia factor for the nth mode, A_{GW} for that mode is given by[7]

$$A_{GW}=\tfrac{2}{15} I_{(n)11}{}^2 M^{-1}\omega_n{}^4 . \tag{17}$$

For the nth mode of a cylinder, Eqs. (9) and (15) then imply

$$A^C_{GW}=\tfrac{16}{15}Ml^{-2}v^4 . \tag{18}$$

For the nth mode of a spindle, Eqs. (11) and (16) imply

$$A^S_{GW}=\tfrac{32}{15}Ml^{-2}v^4\delta_{1n} . \tag{19}$$

The coefficient A_{GW} alone can be used to calculate the frequency integral of the direction- and polarization-averaged cross section[8]:

$$\bar{\sigma}=\int \langle\sigma\rangle d\nu=\tfrac{1}{2}\pi\,\omega^{-2}A_{GW} . \tag{20}$$

From Eqs. (9) and (18), the frequency-integrated average cross section of the nth stretch mode of a cylinder for gravitational-wave absorption is

$$\bar{\sigma}^C_n=\tfrac{32}{15}\pi^{-1}Mv^2n^{-2} \quad (n \text{ odd}) , \tag{21}$$

a result obtained earlier by Wheeler and Ruffini.[9]

The corresponding result for the nth stretch mode of a spindle follows from Eqs. (11) and (19):

$$\bar{\sigma}^S_n=\tfrac{4}{15}\pi M v^2\delta_{1n} . \tag{22}$$

Notice that the Kronecker δ factor that originated with the uniform strain property causes all of the higher modes to have zero-integrated cross section for gravitational waves.

The frequency-integrated average cross sections of the fundamental modes are particularly easy to compare because they depend only on the mass and sound speed of each antenna and not at all on the ratio R/l, the quality factor, or the resonant frequency. For a cylinder and a spindle of the same mass and sound speed, Eqs. (21) and (22) yield

$$\bar{\sigma}^S_1/\bar{\sigma}^C_1=\pi^2/8\simeq 1.23 . \tag{23}$$

other comparisons are also possible. For example, one can write the cross section in terms of the resonant frequency ω and the shape factor R/l. For a cylinder and a spindle with the same shape factor and resonant frequency one finds that

$$\bar{\sigma}^S_1/\bar{\sigma}^C_1=2^{3/2}\pi^{-1/2}\simeq 1.60 . \tag{24}$$

In order to go further with this comparison of cylinders and spindles, one must estimate the internal damping rate A_{diss}. This estimate is a treacherous business because the damping mechanisms for very long wavelength sound waves are not completely understood and because A_{diss} includes the dissipation in the sensors that couple the antenna to the detector. I will assume that weakly coupled or perhaps nondissipative sensors (based on a SQUID, for example[10]) are used so that most of the dissipation is in the antenna. Two different models of the dissipation will be used:

I. Dissipation is a volume effect proportional to the squared rate of change of the strain so that

$$P_{diss}=\eta\int \rho\langle \dot{\xi}'^2\rangle_{\text{time average}}\, d^3x , \tag{25}$$

where η is a strain-viscosity constant.

II. Dissipation is a surface effect proportional to the squared rate of change of the strain so that the volume integral in Eq. (25) is replaced by a surface integral. Model I is probably valid for all metal antennas and for single-crystal antennas with uniformly distributed imperfections. It is also valid for a perfect crystal subject only to Landau damping (phonon viscosity). Model II might apply to a nearly perfect crystal with a lot of surface imperfections caused by cutting and polishing.

For the cylinder with volume dissipation, Eq. (25) together with Eqs. (4) and (8) give

$$A^{\mathrm{CI}}_{\mathrm{diss}} = \tfrac{1}{4}\pi^2\eta n^2 l^{-2}\,. \tag{26}$$

The corresponding result for a spindle with volume dissipation is easily obtained by using the properties of Hermite polynomials,

$$A^{\mathrm{SI}}_{\mathrm{diss}} = 4\eta n l^{-2}\,. \tag{27}$$

A more familiar quantity to compare is the quality factor $Q = \omega/A_{\mathrm{diss}}$. By using Eqs. (9) and (11) to eliminate the length parameter l, one finds the Q of the nth mode for each antenna to be

$$Q^{\mathrm{I}}_n = v^2\eta^{-1}\omega_n^{-1}\,. \tag{28}$$

Thus, for a given resonant frequency, one gets the same Q with any mode of either antenna. If one now computes the gravitational-wave cross section of the fundamental mode at resonance, then one finds that, for a given antenna mass and fundamental frequency, the ratio of the spindle cross section to the cylinder cross section is given by Eq. (23). This is the ratio calculated by Bonazzola for a shape very similar to the spindle. He found a 20% improvement over the cylinder so that the 23% improvement found here is reasonable.

For the cylinder with only surface dissipation the Q is found to be

$$Q^{\mathrm{CII}}_n = \tfrac{1}{2}v^2\eta^{-1}R\omega_n^{-1}\,. \tag{29}$$

For the spindle, one obtains

$$Q^{\mathrm{SII}}_n = \tfrac{1}{2}k(n)v^2\eta^{-1}R\omega_n^{-1}\,, \tag{30}$$

where the first few values of $k(n)$ are $k(1) = 2^{-1/2}$, $k(2) = \frac{1}{2}2^{-1/2}$, $k(3) = \frac{2}{9}2^{-1/2}$. One can see that the spindle modes are much more vulnerable to surface dissipation effects than the cylinder modes. The ratio of resonant gravitational-wave cross sections for the fundamental modes of a cylinder and a spindle of the same mass and resonant frequency now becomes

$$\sigma^{\mathrm{SII}}_{\mathrm{res}}/\sigma^{\mathrm{CII}}_{\mathrm{res}} = \pi^2/(8\sqrt{2}) \simeq 0.87\,.$$

Thus, when surface dissipation dominates, the spindle shows a 13% loss in resonant cross section in comparison with a cylinder.

IV. SINGLE-ANTENNA ANTICOINCIDENCE EXPERIMENTS

An anticoincidence gravitational-wave detector would monitor several different modes of the antenna and accept an event as a gravitational-wave burst only if it appears solely in the fundamental mode. This arrangement is designed to discriminate against spurious events that drive the antenna according to the equation

$$-\ddot{\xi} + v^2\xi'' = f(x)\,\delta(t)$$

for the cylinder and according to a similar equation for the spindle. Here f could be the impulse-field exerted by the sudden release of a locked-in thermal strain in an antenna which has been cooled to a low temperature. It could also be due to an acoustic or electromagnetic shock that has defeated the antenna shielding. In terms of the inner products $(u_n, f) = \int \lambda u_n f\,dx$, the normal modes of either the cylinder or the spindle are driven according to the equation

$$-M\ddot{\xi}_n + M\omega_n^{\,2}\xi_n = (u_n, f)\,\delta(t)\,. \tag{31}$$

Thus, each mode acts like a harmonic oscillator that has received a hammer-blow impulse of (u_n, f) and thus receives energy $(u_n, f)^2/(2M)$. The total energy of the spurious event is then divided more or less evenly among all of the normal modes of the antenna whose wavelengths exceed the characteristic size of the region where f is nonzero. The exact distribution of energy depends, of course, on the nature of the spurious event which determines the overlap integrals. The unique value of the Gaussian spindle antenna is that there is one and only one type of broad-spectrum event that puts all of its energy into the fundamental mode, namely a gravitational-wave burst.

Before going on to discuss anticoincidence experiments done with spindle antennas, I will first describe the consequences of attempting such an experiment with a cylinder. Only the odd-parity modes of the cylinder are insensitive to gravitational waves. But an anticoincidence experiment which monitors only the odd-parity higher modes will interpret any approximately-even-parity event as a gravitational-wave burst. If the experiment is working near the thermal-noise limit, an unacceptably large fraction of spurious events could get through. Because the anticoincidence technique necessarily produces a slight degradation of the overall signal-to-thermal-noise performance of the detector, the result could easily be a net loss in sensitivity or a gain that is too small to be worth the effort. A modified anticoincidence experiment which monitors both the odd- and the even-parity modes of a cylinder is possible if one is willing to use sophisticated data analysis. For example, one could monitor the first three modes. A gravitational-wave burst will excite modes 1 and 3 but not mode 2. If the burst has a flat power spectrum then Eq. (21) implies that mode 1 will receive nine times as much energy as mode 3. If, on the other hand, the burst has a square-law power spectrum, then it will put exactly the same amount of energy into both modes and be indistinguishable from a spurious noise signal. If one has some reason to believe that the gravitational waves have a nearly flat power spectrum, then one can make a succession of such power-spectrum hypotheses and calculate the evidence for each.[11]

However, it is easy to invent sources of gravitational-wave burst which produce rapidly rising power spectra. For example, a decaying black-hole binary system will produce a burst that rises in frequency as it rises in intensity. A cylinder-based anticoincidence detector could not distinguish such bursts from spurious noise.

The Gaussian-spindle antenna has one fundamental advantage over the cylinder: If one sees an event in any spindle mode other than the fundamental, then one knows that the event is not gravitational radiation. This property makes it possible to discriminate between any sort of gravitational-wave burst and broad-spectrum spurious noise bursts. No assumptions about the gravitational-wave power spectrum are needed. Furthermore, because the spindle resonant frequencies go as the square root of the mode number instead of linearly, they are much closer together than those of the cylinder. Having the frequencies closer together makes it more difficult for a narrow-bandwidth noise burst to masquerade as a gravitational wave.

For detection schemes that monitor the end-face displacement of the antenna, the spindle has a considerable practical advantage because, for a given mode energy, the end-faces of a truncated spindle move further than the end-faces of a cylinder. In the coordinate system that I am using to describe the spindle, it is reasonable to truncate it by requiring $-2l \leq x \leq 2l$. For the cylinder, one finds that the end-face displacement produced by an energy E in the fundamental mode is given by $(\Delta x_1^{\mathrm{C}})^2 = 4EM^{-1}(\omega_1^{\mathrm{C}})^{-2}$. For the truncated spindle with the same mass and fundamental resonant frequency, the squared displacement is eight times as large as this. Of particular interest for anticoincidence experiments is the fact that, for mode 3, the squared displacement of the spindle is found to be 225.33 times that of the cylinder.

V. DISCUSSION

In terms of cross sections and quality factors the Gaussian spindle is just about as good a gravitational-wave antenna as a cylinder. The improvements of 23% or 60% that the spindle offers under certain assumptions are not, I think, of any great significance. Similarly, the loss of 13% that can occur when surface losses are dominant is also rather insignificant. The Gaussian spindle becomes a viable gravitational-wave antenna only in a detector that has beaten the thermal-noise problem by cooling the antenna to a very low temperature but continues to be plagued by bursts of nonthermal noise. In that situation, it offers a very large improvement in real sensitivity through the anticoincidence technique. Furthermore, if gravitational-wave bursts are actually detected in the fundamental mode of a Gaussian spindle and not in higher modes, then a convincing case for their identity can be made.

I consider the detailed design of a gravitational-wave detector to be outside the scope of this paper. However, in order to avoid leaving an over-optimistic impression of the Gaussian-spindle antenna, I offer a few remarks about detection schemes. The central consideration in the design of any detector that uses a Gaussian-spindle antenna is to preserve the uniform strain property of the fundamental mode. Unfortunately, this property is rather fragile. Sensors that couple strongly to the antenna will change the boundary conditions and can destroy the uniform strain property. Thus, a detector with strongly coupled sensors must be designed carefully with the shape of the antenna adapted to the sensor locations and sensor properties so as to achieve uniform strain. Another difficulty with strongly coupled sensors is that they can introduce dissipative coupling between antenna modes, thus destroying the anticoincidence properties of the antenna.

The antenna considered in this paper is obviously highly idealized. The infinitely long "whips" on its ends are clearly impossible to construct. A real antenna would be truncated. A partial remedy for the fact that a truncated spindle has vanishing strain at its ends might be to attach a nonresonant flat plate to each end-face in order to simulate the effective mass of the missing "whips." An alternative procedure for simulating the missing "whips" is to couple the end-face movement to a large inductance through a capacitative or inductive transducer.

Note added in proof. The anticoincidence detection scheme has been discussed for spherical antennas in a talk presented by R. V. Wagoner and H. J. Paik at the Accademia Nazionale dei Lincei International Symposium on Experimental Gravitation, Pavia, Italy, 1976 (unpublished). It is also discussed for disk antennas by H. J. Paik, Phys. Rev. D 15, 409 (1977).

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[1]J. Weber, Phys. Rev. 117, 306 (1960); *General Relativity and Gravitational Waves* (Interscience, New York, 1961); in *Relativity, Groups, and Topology*, edited by C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1964), pp. 865–880; W. M. Fairbank, S. P. Boughn, H. J. Paik, M. S. McAshan, J. E. Opfer, R. C. Taber, W. O. Hamilton, B. Pipes, T. Bernat, and J. M. Reynolds, in *Experimental Gravitation*, edited by B. Bertotti (Academic, New York, 1974), pp. 294–308.

[2]S. Bonazzola and M. Chevreton, Phys. Rev. D 8, 359 (1973).

[3]C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[4]In these units, all physical quantities are dimensionless numbers. The reader who wishes to use my results to compute quantities in cgs units will find it helpful to know that $\log_{10}(\text{cm}) = 32.791\,56$, $\log_{10}(\text{sec}) = 43.268\,33$, $\log_{10}(\text{g}) = 4.662\,14$, $\log_{10}(\text{erg/sec}) = -59.559\,73$, $\log_{10}(^\circ\text{K}) = -32.151\,33$. My experience with calculating gravitational effects is that one encounters such large powers of ten that the usual scientific notation is cumbersome and it is much more convenient to work with logarithms. The constants given here were calculated from the constants given in Misner, Thorne, and Wheeler, Ref. 3, and are correct to within the limits of our knowledge of the gravitational constant. Linear combinations of logarithms are known much more accurately, for example, $\log_{10}(\text{cm}) - \log_{10}(\text{sec}) = -10.472\,682\,070$.

[5]See the following standard references: R. Courant and D. Hilbert, *Methods of Mathematical Physics* (Interscience, New York, 1953), Vol. 1, pp. 91–93; P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill, New York, 1953), Vol. 1, pp. 786 and 787.

[6]C. W. Misner *et al.*, Ref. 3. The procedure that I use is described on pp. 1028–1036.

[7]Reference 3, box 37.4, pp. 1031–1033. Use Eqs. (5) and (12).

[8]Reference 3, pp. 1031–1033, Eq. (10b).

[9]R. Ruffini and J. A. Wheeler, in *Proceedings of the Conference on Space Physics* (European Space Research Organization, Paris, 1971), pp. 45–74.

[10]H. Paik, in *Experimental Gravitation*, edited by B. Bertotti (Academic, New York, 1975), pp. 515–524.

[11]The techniques needed for such a program may be found in G. H. B. Rydbeck, Ph.D. thesis, University of Maryland, 1976 [University of Maryland Report No. UM PP-76-248 (unpublished)].