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Speculation on cosmological bounce*

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Two problems are considered. First it is argued that cosmological bounce cannot arise from ordinary electromagnetic effects. Second, it is conjectured that the attractive nature of the shadow potential used in electrodynamics to obtain a stable point electron could be used also to obtain cosmological bounce, provided it is attractive between all particles. To ensure this a modified shadow potential is formulated which has mass instead of charge as its source. This then leads to a negative static interaction energy proportional to T^6 (T the temperature), providing the screening parameter is independent of T. For large enough densities (i.e., temperature) this negative energy provides a mechanism that can turn the universe around.

I. INTRODUCTION

In the standard Friedmann models of the universe, there is implicit a singularity in the past and/or in the future, a singularity characterized by the radius R of the universe approaching zero. In this paper we consider two topics associated with this singularity that could possibly prevent the collapse. If the collapse does not occur we call it "cosmological bounce" in analogy with the same expression used for charged spheres[1] or charged spherical shells[2] of finite radius.

First we discuss the analogy of these charged objects: Could electromagnetic effects prevent collapse to a point? Could a charge occur at all? In Sec. III various arguments indicate that electromagnetism will not prevent collapse, and that in many cosmological models a net charge cannot exist. The reason basically is that the electromagnetic energy density is positive, whereas what is needed to prevent collapse is a negative (attractive) energy density or pressure; this is a rather paradoxical-sounding but well-known[3] prescription. Positive energy contributes to gravitational mass, which promotes collapse; to prevent collapse one needs negative energy or pressure.

In Sec. IV of the paper we speculate that such an attraction can possibly be found in the as-yet-unknown forces which bind the electron. The idea is that the potential inside the electron that binds it must be very short-ranged, but, for large enough densities of the universe, the potential could reach out beyond the electron and effect an attraction between particles. The potential must act predominantly as an attraction and not as an attraction to like charges and a repulsion to unlike charges; otherwise no net attraction would occur.

To pursue the idea we employ a prototype of such an effect, the "shadow potential,"[4,5] used recently in quantum and classical electrodynamics to avoid singularities in the theory of a point electron. By treating the screening parameter as a universal constant and making the potential attractive, we find the energy density associated with this potential to be negative and proportional to T^6 (T is the temperature), which for large enough temperature (i.e., density) can overcome the T^4 term associated with kinetic and radiant energies, and thereby cause the universe to turn around at some small enough radius.

In Appendix B, a scalar potential is used. The procedure is the same, but the point electron will not transform as a Lorentz 4-vector if the potential is just scalar. Nevertheless bounce can occur in much the same way.

Section II contains a brief review of the standard Friedmann models, Appendix A briefly derives the standard T^4 terms in these models, and Sec. V contains a short discussion of the results.

II. BASIC MODELS

The closed cosmological models in this paper start from the metric

$$\begin{aligned} ds^2 &= g_{ij}dx^i dx^j \\ &= d\tau^2 - R(\tau)^2[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta\, d\varphi^2)] \end{aligned} \tag{2.1}$$

in hyperspherical coordinates. The expansion or contraction is described in comoving coordinates by a velocity 4-vector $u^i = (0, 0, 0, 1)$.

The Einstein field equations then lead to

$$3R^{-2}(\dot{R}^2 + 1) = \kappa\rho\,, \quad \dot{R} = dR/d\tau c \tag{2.2}$$

$$\begin{aligned} 6\ddot{R} &= -\kappa R(T^0_0 - T^1_1 - T^2_2 - T^3_3) \\ &= -\kappa R(\rho + 3p) \end{aligned} \tag{2.3}$$

for a perfect fluid, where ρ is the energy density and p is the pressure. Equation (2.3) contains the general form in terms of the energy-momentum tensor $T_i{}^j$ as well as the special case of a perfect fluid.

From Eq. (2.3) it is immediately seen that if ρ and p are positive then $\ddot{R}$ is always negative, which implies a singularity ($R=0$) at least sometime in the past.

Equations (2.2) and (2.3) also lead to

$$\dot{\rho}+3(\rho+p)\dot{R}/R=0\,, \tag{2.4}$$

which also comes from $T^{i}{}_{k;i}=0$ for $k=0$.

Equations (2.2)–(2.4) are two independent equations in three unknowns, ρ, p, and R. For a complete solution an equation of state is needed.

We shall concern ourselves primarily with the epoch between 10^{10} and 10^{12} degrees Kelvin, where, it was thought, electrons, positrons, photons, and neutrinos existed in equilibrium with one another.[6] As an example of the use of the cosmological equations, consider the equation of state, reasonable at this epoch (see Appendix A),

$$\rho=aT^4\,,\quad p=aT^4/3\,. \tag{2.5}$$

Here a is a constant, and T is the temperature.

Instead of the three unknowns ρ, p, and R, there are now two: T and R. Equations (2.2) and (2.4) become

$$3R^{-2}(\dot{R}^2+1)=\kappa aT^4\,, \tag{2.6}$$

$$\dot{T}/T=-\dot{R}/R\,. \tag{2.7}$$

Equation (2.7) is solved immediately;

$$T=K_0/R\,, \tag{2.8}$$

where K_0 is a constant of integration. Substitution into Eq. (2.6) gives

$$\dot{R}^2=-1+\kappa aK_0{}^4R^{-2}\,. \tag{2.9}$$

The turning points are determined by $\dot{R}=0$. They can occur for only one positive R. Furthermore, this R is an outer turning point, since the right-hand side of Eq. (2.9) is positive only for small R. Thus, a singularity at $R=0$ is inevitable. Of course, this calculation is only schematic, since as R increases the equation of state will change. However, the general behavior of an outer turning point is characteristic of closed Friedmann universes.

III. UNIFORM CHARGE DISTRIBUTION

It might be expected that a uniform charge density σ in a cosmological model could give rise to "gravitational bounce," just as in the case of a finite 3-dimensional charged sphere[1] or a finite 2-dimensional spherical surface.[2] In this way, the singularity of the big bang could be avoided.

However, it can immediately be seen from Eq. (2.3) that the effect of charge on the cosmological model will be different. In the symmetry assumed, homogeneity and isotropy, the electromagnetic part of the energy-momentum tensor S^{ij} will satisfy $S_1^1=S_2^2=S_3^3$, just as does the matter part, which was the isotropic pressure p in Eq. (2.3). However, $S_i^i=0$ for electromagnetism. Thus the combination in Eq. (2.3) has an electromagnetic contribution

$$S_0^0-S_1^1-S_2^2-S_3^3=2S_0^0\,. \tag{3.1}$$

But S_0^0 is the electromagnetic energy density, which is proportional to E^2+H^2, where E and H are the electric and magnetic fields, respectively. Thus the electromagnetic contribution to the right-hand side of Eq. (2.3) is negative, increasing the curvature magnitude $-\ddot{R}$.

The fact that the energy density is positive for an excess charge can also be easily seen from summing the self-energy of each particle plus the Coulomb energies e^2/r_{ij} between the excess charges. All the terms are positive. One can avoid retardation if the Coulomb gauge is used.

Since the energy density associated with the excess charges is positive, their effect will be to promote collapse, not prevent it.

Let there then be no excess charge, but rather charge neutrality. The compensating positive and negative charges will also give a positive field energy $\sim E^2+H^2$. On the other hand, if one does not use the field form, but computes the energy on the basis of particle interactions and self-interactions, it seems at first glance that the energy could go negative.

For simplicity consider a static situation. The electromagnetic energy of each charge (the self-energy) is, say, m. For N particles in a volume V, the self-energy density is $mn\sim R_0{}^{-3}$, where $n=N/V$ is the number density ($\sim R_0{}^{-3}$) and R_0 is the interparticle distance.

The interaction energy between one charge and another charge summed over the charges is of the form $-A/R_0$ per particle, where A is a positive constant. For a static ionic lattice, A is the Madelung constant; in an electron gas, a term of this type corresponds to the exchange energy. The energy density is then $-An/R_0$, which is proportional to $R_0{}^{-4}$. For small enough R_0, this negative interaction energy $-An/R_0$ will overtake the positive self-energy, mn of the previous paragraph, making the total electromagnetic energy density turn negative.

This is in obvious disagreement with the field view that the energy density is proportional to E^2+H^2 and is always positive. The difference could be important if cosmological bounce is contemplated. One way to avoid the problem is to notice that the (positive) kinetic energy also goes as R_0^{-4} for small enough R_0 (just as does $-An/R_0$), as seen in Eq. (A7) of Appendix A, and the coefficient for this contribution is orders of magnitude larger than for the Madelung or exchange term. Thus the negative Coulomb-interaction effect could never win out.

However, this argument does not explain why the two views of electromagnetic energy can give different results. The difficulty lies in the self-energy, which was taken to be m per particle, where m is some constant. In fact, the electromagnetic self-energy is known to diverge for a point charge. For a finite extended particle of radius R', it will depend on the radius R'. An individual particle characterization is legitimate only if the radius R' is much smaller than R_0. In the process of cosmological collapse, however, R_0 will go to zero. As it does so, the self-energy will have to refer to an R' which is going to zero faster than (or at least at the same rate as) R_0, in terms of a limiting process. For this reason, the positive self-energy will dominate the negative interaction energy, and cannot be represented by a constant m, as R_0 continues to get smaller and smaller.

But, if a renormalized mass m *is* used for a point charge, then it is taking into account the electromagnetic infinity plus a cancellation of that infinity by some other interaction. And the negative Madelung type of term *will* eventually dominate, for small enough R_0, the mass term, although it will not dominate the kinetic energy. A cancellation of this type could come from the shadow potential discussed in the next section.

Thus the difference between an essentially positive energy density $\sim E^2+H^2$ and a possible negative one dominated by the exchange or Madelung term lies in the fact that in the latter the renormalization of energy implies additional nonelectromagnetic interactions which help cancel the (positive) electromagnetic infinities, making the total energy density less positive.

As has been shown above, a net charge of one sign will not tend to prevent collapse. It is interesting to note that in many cosmological models a net charge cannot exist. For example, in a homogeneous isotropic universe, finite or infinite (this includes all the Friedmann models), it must be that $\vec{E}=\vec{H}=0$, otherwise isotropy is violated.[7] From Maxwell's equations, $\operatorname{div}\vec{E}=4\pi\sigma$, which implies that the charge density σ is also zero.

It could perhaps be argued that although the average of $\vec{E}$ is zero over every macroscopic volume, the average of E^2 is not zero, and a net charge could result from this. However, the charge Q even in curved spaces can be written[8] as an integral over the surface enclosing the volume V containing the charge:

$$Q\sim\int_S E_{\text{normal}}\,d^2x\,. \tag{3.2}$$

The integral in Eq. (3.2) is either zero or it has a small value as a result of the fluctuations of E not averaging to zero on the particular surface S involved. If the surface is displaced a bit the integral must go from negative to positive etc., otherwise E would not average to zero over every macroscopic volume. Thus in the immediate vicinity of the original surface S there is another one S', for which the charge Q' enclosed is zero. Thus any macroscopic volume V can be deformed slightly to a volume V' which contains no charge. Since the universe is supposed to be macroscopically homogeneous, the only conclusion is that there is no overall charge density.

If the universe is closed and unbounded, then independent of the symmetry there also can be no charge.[9] Imagine a 2-dimensional universe in a 3-space, say, the surface of an ellipsoid. Draw a closed loop on the ellipsoid and call it $C(+)$ or $C(-)$, depending on whether it is traversed clockwise or counterclockwise. On one side is enclosed a surface S_1, on the other a surface S_2. Stokes's law applied to C and S_1 and then to C and S_2 gives

$$\int_{C(+)}A_l d_l=\int_{S_1}(\operatorname{curl}A)_n\,dS_n\,, \tag{3.3}$$

$$\int_{C(-)}A_l d_l=\int_{S_2}(\operatorname{curl}A)_n\,dS_n \tag{3.4}$$

for a vector $\vec{A}$. Here the subscript n refers to the outward normal. Adding gives

$$0=\int_{S_1+S_2}(\operatorname{curl}A)_n\,dS_n\,. \tag{3.5}$$

The integral on the right is the analog of a charge integral, and it is zero.

To apply the argument to Gauss's law in a 3-dimensional closed universe, choose a hypersurface S $S(+)$ or $S(-)$ depending on the direction of the normal), enclosing a volume V_1 on one "side" and V_2 on the other. We can write Gauss's law twice, as in Eqs. (3.3) and (3.4), and then add. The right-hand sides represent the charge Q, and the result will be zero. The fact that Gauss's law can be written as in Eq. (3.2) implies that the result is valid for general curved spaces.

If the universe is not homogeneous or isotropic on the one hand, nor finite and unbounded on the other, the proof of zero net charge is difficult in terms of Maxwell's equations. A finite bounded universe could perhaps utilize Gauss's law, if one goes beyond the occupied region to evaluate the integrals.[10] But infinite nonsymmetric universes cannot be managed. However, if one adopts the

Faraday picture that if a line of force starts on a positive charge it must end on a negative charge, then all universes will have zero net charge. This idea cannot be described by Gauss's law because the idea is not uniformly converging. That is, given any volume of an infinite universe, no matter how large, more lines may be emerging than entering and the net charge within may not be zero. Thus one cannot apply Gauss's law and then let the volume become infinite. But one could still say with Faraday that every line that starts on a positive charge cannot just end abruptly in space, even curved space, but must sooner or later find its way to a negative charge. In this sense, the lines of force notion is stronger than the Maxwell equations.

IV. COSMOLOGICAL BOUNCE: A MODIFIED SHADOW POTENTIAL

In this section we speculate on what could conceivably prevent the universe from collapsing to a point [i.e., prevent R from becoming zero in Eqs. (2.2)–(2.4)]. From Eq. (2.3) it is clear that what is needed is either a negative ρ or a negative p or both. What could supply such terms?

In this connection, notice that any mechanism that holds the electron together (Poincare stress) would be contributing energy and pressure of the correct sign for cosmological bounce. A perhaps extreme example can be seen in the classical model of an extended electron in general relativity proposed a few years ago[11] in which the matter density in the electron was found to be negative. By imagining that the positive self-energy due to Coulomb interactions gets canceled out when R_0 becomes so small that electrons and positron overlap, one could conclude that for small enough R, there appear actual negative energy density and pressure from the interior electron-positron matter.

Rather than pursue this rather drastic point of view, we shall consider a slightly less drastic modification of the "shadow-potential" idea introduced in quantum electrodynamics[4] and applied by Chiang[5] to describe a classical electron without self-energy divergence difficulties.

The shadow potential is a vector field which provides a Poincaré stress. The ordinary Coulomb plus the shadow potential from a point charge is[4,5]

$$\varphi = (e/r)[1-\exp(-r/\Lambda)] \tag{4.1}$$

in terms of a screening parameter Λ (Chiang's M is Λ^{-1}) which must be at least as small as the classical electron radius $R_{c1}=e^2/mc^2$. One of the possibilities for Λ is that it is a universal constant. We shall adopt this view. The second (shadow) term of Eq. (4.1) represents the vestige of an internal binding force that holds the electron together. It is not necessary for us that this term actually be the shadow potential. In fact it could be gravitational or something else in origin, possibly arising from the negative matter density effects mentioned above.

The essential ingredients of the shadow potential for us are the order of magnitude and the range of the binding forces in the electron that it gives. In addition we shall require that it be predominantly attractive when acting and that the range Λ be a universal constant, or at least not a strong function of temperature.

However, we wish to push further the explicit form of the shadow potential as in Eq. (4.1) and the references cited, since it already provides for a stable electron in a simple way. To do this, the Lagrangian must be modified to avoid repulsion. The simplest such modification would be to make the source the mass current[12] $j_k^{(m)}$ with the positive coupling constant f:

$$f=|e/m|\,. \tag{4.2}$$

Here e/m is the electron charge to mass ratio. Thus the complete Lagrangian is

$$\begin{aligned} L &= L_{\rm em}+\tilde{L} \\ &= -(16\pi)^{-1}F_{ik}F^{ik}+(16\pi)^{-1}(\tilde{F}_{ik}\tilde{F}^{ik}-2\Lambda^{-2}\tilde{A}_k\tilde{A}^k) \\ &\quad -j_kA^k-f\,j_k^{(m)}\tilde{A}^k\,. \end{aligned} \tag{4.3}$$

Here $\tilde{A}_k$ will be called the "modified shadow potential." A_k is the usual electromagnetic 4-potential, $F_{ik}=A_{k,i}-A_{i,k}$ and similarly for $\tilde{F}_{ik}$.

The Lagrange equation for A_k is then in the Lorentz gauge

$$(\Box-\Lambda^{-2})\tilde{A}_k=4\pi f j_k^{(m)}\,,\quad \Box=\nabla^2-\partial_t^{\,2}\,. \tag{4.4}$$

In a static situation the solution is

$$\begin{aligned}\tilde{A}_k(x) = -f\int d^3x\,|\vec{x}-\vec{x}'|^{-1} \\ \times\exp(-\Lambda^{-1}|\vec{x}-\vec{x}'|)j_k^{(m)}(x')\,.\end{aligned} \tag{4.5}$$

In the Coulomb gauge, Eq. (4.5) is generally valid for $k=0$. Equation (4.5) for a point mass m gives back the shadow term of Eq. (4.1).

The energy-momentum tensor defined from

$$\delta\int L(-g)^{1/2}d^4x=\tfrac{1}{2}\int T_{ik}\delta g^{ik}(-g)^{1/2}d^4x$$

contains the usual electromagnetic term constructed from the A_k plus the shadow term $\tilde{T}_{ik}$ where

$$\begin{aligned}\tilde{T}_i^k=(4\pi)^{-1}[\tilde{F}_{in}\tilde{F}^{kn}-\Lambda^{-2}\tilde{A}_i\tilde{A}^k \\ +\delta_i^k(-\tfrac{1}{4}\tilde{F}_{pq}\tilde{F}^{pq}+\tfrac{1}{2}\Lambda^{-2}\tilde{A}_n\tilde{A}^n)]\,.\end{aligned} \tag{4.6}$$

Thus the basic equations are essentially as given by Chiang, and his results will ensue, namely, that in the limit of a point electron (or positron) the object will transform as a Lorentz 4-vector and will have finite (renormalized) self-energy.

Our interest is to use Eq. (4.6) to take into account interactions between electrons, between positrons, and between electrons and positrons. For simplicity, we shall confine the calculation to a static situation:

$$\tilde{T}_0^0 = -(8\pi)^{-1}[(\nabla\tilde{A})^2 + \Lambda^{-2}\tilde{A}^2], \qquad (4.7)$$

where $\tilde{A} = \tilde{A}_0 = \tilde{A}^0$. From this the total energy in a volume is

$$T_0^0 = -(8\pi)^{-1}\int d^3x\,[-\tilde{A}(\nabla^2 - \Lambda^{-2})\tilde{A} + \nabla\cdot(\tilde{A}\nabla\tilde{A})]$$
$$= -\tfrac{1}{2}f^2 \iint d^3x\, d^3x' \frac{\exp(-\Lambda^{-1}|\vec{x}-\vec{x}'|)}{|\vec{x}-\vec{x}'|} \times \rho^{(m)}(x)\rho^{(m)}(x'). \qquad (4.8)$$

The second form emerges here when the surface term is neglected and Eq. (4.4) is used in the others. We have in mind a uniform distribution of charges so that the $\tilde{A}_0$ of Eq. (4.5) is a constant $(-4\pi\Lambda^2 fmn)$. Thus not only can the surface term be set equal to zero in Eq. (4.8) but also the $\nabla^2\tilde{A}$ term. However, the second form in Eq. (4.8) has a familiar look: it is just like an ordinary electrostatic Coulomb interaction, except that it is screened and negative. Here $\rho^{(m)}$ is the rest mass density.

The energy in Eq. (4.8) divided by the volume is what we shall use as the shadow energy density. We select out the contributions that correspond to interactions between electrons, between positrons, and between electrons and positrons, but not the self-energy terms. We have in mind the epoch of the early universe when only electrons, positrons, and neutrinos in equilibrium are thought to have existed.[6] Equation (4.8) becomes

$$\rho_{SP} = V^{-1}\int \tilde{T}_0^0\, d^3x$$
$$= -(n/2)\sum_j f^2 m^2 r_{ij}^{-1}\exp(-r_{ij}\Lambda^{-1}). \qquad (4.9)$$

Here n is the total density of particles $[= n_+ + n_-$, where n_- (n_+) is the number density of electrons (positrons)] and r_{ij} is the distance between particles i and j. The sum over j is assumed to be independent of i and is performed by making the approximation of a uniform density n of particles:

$$\rho_{SP} \cong -\tfrac{1}{2}n^2 f^2 m^2 \int d^3x\, x^{-1}\exp(-x\Lambda^{-1})$$
$$= -2\pi n^2 f^2 m^2 \Lambda^2$$
$$= -8\pi n_+^2 e^2 \Lambda^2\,. \qquad (4.10)$$

Associated with this energy density is a negative pressure p_{SP} computed from

$$p_{SP} = -\left(\frac{dU_{SP}}{dV}\right)_{N_+, N_-}, \qquad (4.11)$$

where $U_{SP} = V\rho_{SP}$. Using the definition of interparticle distance R_0 in Eq. (A3) of Appendix A and the fact that $V \sim R_0^3$ if N_+ $(\equiv n_+ V)$ and N_- are constant, we get from Eq. (4.11) that[13]

$$p_{SP} = \rho_{SP}\,. \qquad (4.12)$$

To continue, we need now to put the shadow terms alongside the ordinary terms (subscript 0) of the type mentioned in Eq. (2.9). In sum we have

$$\rho_0 = aT^4\,, \quad p_0 = \tfrac{1}{3}aT^4\,,$$
$$\rho_{SP} = -bT^6\,, \quad p_{SP} = -bT^6\,. \qquad (4.13)$$

The T^6 behavior in the shadow terms comes from the factor n_+^2 in Eq. (4.10) using Eq. (A1) of Appendix A. T is an abbreviation for T_9, the temperature in units of 10^9 degrees Kelvin. The quantity a is found from Eq. (A7); b is found from Eq. (4.10) using Eqs. (A3) and (A4):

$$a \sim 10^{33}\ \mathrm{eV/cm^3}\,; \qquad (4.14)$$

$$b \sim 10^{26}(\Lambda/R_{cl})^2\ \mathrm{eV/cm^3}\,. \qquad (4.15)$$

Notice that at $T_9 = 10^3$, ρ_{SP} is approaching ρ_0 in magnitude if Λ is of the order of R_{cl}.

Substitution of Eq. (4.13) into Eq. (2.2) and (2.4) gives

$$3R^{-2}(\dot{R}^2 + 1) = \kappa(aT^4 - bT^6)\,, \qquad (4.16)$$

$$\dot{T}/T = -\dot{R}/R\,, \qquad (4.17)$$

respectively. Equation (4.17) can be integrated immediately;

$$T = K_0/R\,, \qquad (4.18)$$

where K_0 is a constant of integration. Substitution into Eq. (4.16) gives a final equation for R:

$$\dot{R}^2 = -1 + AR^{-2} - BR^{-4}\,, \qquad (4.19)$$

where

$$A = \tfrac{1}{3}\kappa a K_0^4\,, \qquad (4.20)$$

$$B = \tfrac{1}{3}\kappa b K_0^6\,. \qquad (4.21)$$

From Eq. (4.19) we shall only extract the fact that there are *two* turning points, obtained by setting $\dot{R} = 0$. They are

$$R_\pm^2 = \tfrac{1}{2}A[1 \pm (1 - 4BA^{-2})^{1/2}]\,. \qquad (4.22)$$

Real positive roots occur when

$$1 > 4BA^{-2} = 12K_0^{-2}ba^{-2}\kappa^{-1}\,. \qquad (4.23)$$

If Eq. (4.23) is violated, the right hand side of Eq. (4.19) is negative for *all* R. Thus all physical solutions of Eq. (4.19) correspond to an oscillating universe between the radii R_+ and R_-.

The conclusion is that if the modified shadow potential is assumed to exist, then the universe can bounce back before collapsing to a point.

With the modifed shadow potential proportional to mass, the radiant energy will also contribute to the interaction energy, indicating that the turning point of Eq. (4.22) is a low estimate.

V. DISCUSSION

In this paper we have considered two possible sources for the prevention of cosmological collapse: ordinary electromagnetism and the modified shadow potential. The former does not tend to prevent collapse. Its effects are thus different from what they are in the collapse of stars. Further, there can be no overall charge of one sign, in homogeneous isotropic models.

On the other hand, if the shadow potential is modified to be attractive between all particles, then its effect is in the right direction for the prevention of collapse. The idea here is that the force that prevents the explosion of the electron (taken here to be the shadow potential) might also serve to prevent the collapse of the cosmos to a point.

The two types of interaction (ordinary electromagnetic and shadow) may be contrasted by noticing that although the electric field E_i and the shadow field $\tilde{E}_i$ (computed from $\tilde{A}$) must both average to zero in homogeneous isotropic models, the consequences of this fact are different. From $E_i = 0$ it follows from Gauss's law that the average charge σ must be zero. But from $\tilde{E}_i = 0$, the average "shadow charge" need not be zero, as can be seen from Eq. (4.4). The term $\Lambda^{-2}\tilde{A}_0$ there does not average to zero.

The reason that the effect of the shadow potential becomes so important at small R is because its interaction energy is proportional to n^2 (n the density), as is seen in Eq. (4.10). This itself stems from the assumption that the screening length Λ is a constant independent of R or the temperature T.

The calculation of this paper is schematic in the sense that the nonshadow energy density and pressure of Eq. (4.13) are not even qualitatively correct when R gets sufficiently large. However, the characteristic feature, in closed Friedmann universes, of an outer turning point is contained in them, and the additional effect of the shadow terms is to provide the inner turning point. Both together give a universe oscillating between outer and inner turning points. For open universes there would occur just one inner bounce.

Notice that at the inner turning point the total energy density $aT^4 - bT^6$ cannot be negative. Substituting R_- from Eq. (4.22) into the condition $\rho_{\text{total}} \geq 0$ gives

$$(1-v)^{1/2} \geq 1 - v/2\,, \tag{4.24}$$

where $v = 4BA^{-2}$. With $v \leq 1$, this inequality is always satisfied. It can be seen by the same method that the total pressure is always negative at the inner turning point.

Note added in proof. An article by A. Das and P. Agrawal, Gen. Relativ. Gravit. 5, 359 (1974), has recently come to my attention. It surveys a number of wave fields in Friedmann cosmologies, and bears on, although does not much overlap, the problem as discussed in the present paper.

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APPENDIX A: DENSITIES IN THE ULTRARELATIVISTIC LIMIT

The densities $n_\pm$ of electrons and positrons are obtained in the epoch considered in this paper from equations of the type[6]

$$\begin{aligned} n_+ = n_- &= V^{-1} 2 (V/8\pi^3)\hbar^{-3} \int d^3p\, f_p \\ &= 16 \times 10^{27} T_9{}^3 I_n \text{ cm}^{-3}\,, \end{aligned} \tag{A1}$$

where

$$I_n = (8\pi)^{-1} \int d^3x\, f \cong 1\,. \tag{A2}$$

In the relativistic limit, the particle energies are taken as $E \cong cp$, where p is the momentum. Here $x = cp/kT$, and T_9 is the temperature in units of 10^9 degrees Kelvin. The Fermi function f contains a zero chemical potential, since the number of positrons is assumed equal to the number of electrons. The factor of 2 in Eq. (A1) is from spin.

The interparticle distance R_0 is defined by

$$n = 2n_+ = 3/(4\pi R_0{}^3)\,, \tag{A3}$$

and comes out to be

$$R_0 = 2 \times 10^{-10} T_9{}^{-1} I_n{}^{-1/3} \text{ cm}\,, \tag{A4}$$

which lies between the Compton radius R_{comp} and the classical electron radius R_{cl} in the epoch considered:

$$R_{\rm comp} = h/mc = 2.5 \times 10^{-10}\ \text{cm}\,, \tag{A5}$$

$$R_{\rm cl} = e^2/mc^2 = 3.0 \times 10^{-13}\ \text{cm}\,. \tag{A6}$$

The energy density is computed from

$$\rho_0 = V^{-1} 2(V/8\pi^3)\hbar^{-3} \int d^3p E(p) f_p$$
$$\cong 5 \times 10^{33} T_9{}^4 I_{\rm kin}\ \text{eV/cm}\,, \tag{A7}$$

where $I_{\rm kin}$ is normalized to about 1 in the ultrarelativistic region:

$$I_{\rm kin} = (24\pi)^{-1} \int d^3x [(mc^2/kT)^2 + x^2]^{1/2} f$$
$$\cong 1\,. \tag{A8}$$

The pressure associated with this is $\rho_0/3$ from the definition in Eq. (4.11).

APPENDIX B: A SCALAR POTENTIAL

From the point of view of obtaining a binding force in the electron, one could consider a scalar potential φ rather than the vector shadow potential. However, not all the nice properties will ensue that occur when the vector potential is used. We briefly describe here what happens.

The scalar Lagrangian has the form (to within an overall sign)

$$\tilde{L} = (8\pi)^{-1}(\eta^{ik}\varphi_{,i}\varphi_{,k} - \Lambda^{-2}\varphi^2) - f\rho^{(m)}\varphi\,, \tag{B1}$$

where η^{ik} is the Minkowski metric. The Lagrange equation of motion is

$$\Box\varphi - \Lambda^{-2}\varphi = 4\pi f \rho^{(m)}\,. \tag{B2}$$

The energy-momentum tensor is

$$\tilde{T}^i_k = (4\pi)^{-1}[\varphi^{,i}\varphi_{,k} + \tfrac{1}{2}\delta^i_k(-\varphi^{,n}\varphi_{,n} + \Lambda^{-2}\varphi^2)]$$
$$+ \delta^i_k f\rho^{(m)}\varphi\,. \tag{B3}$$

For a static situation

$$\tilde{T}^i_k = (4\pi)^{-1}\varphi^{,i}\varphi_{,k} + (8\pi)^{-1}\delta^i_k \nabla\cdot(\varphi\nabla\varphi)$$
$$+ \tfrac{1}{2}\delta^i_k f \rho^{(m)}\varphi\,. \tag{B4}$$

And the integral of this is

$$\int d^3x \tilde{T}^i_k = (4\pi)^{-1}\int d^3x \varphi^{,i}\varphi_{,k} - \tfrac{1}{2} f^2 \delta^i_k \iint d^3x\, d^3x' \rho^{(m)}(x)\rho^{(m)}(x')|\vec{x} - \vec{x}'|^{-1}\exp(-\Lambda^{-1}|\vec{x} - \vec{x}'|)\,. \tag{B5}$$

The ordinary electromagnetic Lagrangian $-(16\pi)^{-1}F_{ik}F^{ik}$ gives, with $A = A_0 = A^0$,

$$\int d^3x T^i_k = -(4\pi)^{-1}\int d^3x A^{,i}A_{,k} + (\delta^i_0\delta^0_0 - \tfrac{1}{2}\delta^i_k)\iint d^3x\, d^3x' \sigma(x)\sigma(x')|\vec{x} - \vec{x}'|^{-1}\,; \tag{B6}$$

where σ is the charge density.

The problem is this: the sum of Eqs. (B7) and (B5) does not get rid of the singularity for a point electron in both the space diagonal components T^k_k ($k = 1, 2, 3$) and the time diagonal component T^0_0. By choosing the overall sign of $\tilde{L}$ one way or the other, either the kk components ($k = 1, 2, 3$) or the 00 component of the sum becomes finite, but not both. With one of the choices, the scalar potential will lead to cosmological bounce, just as in Sec. IV of the paper.

Thus from the point of view of cosmological bounce, either the scalar or vector potential will work. But only the latter provides for a stable point electron with energy and momentum transforming as a Lorentz 4-vector.

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