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# Difficulty of the pure $\rho$-Regge-pole exchange model in forward $\pi^- p \rightarrow \pi^0 n$ at 20–200 GeV/$c$ [*]

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The combination of new data for $(d\sigma/dt)_{t=0}^{\mathrm{CEX}}$ with those for $\Delta\sigma_T(\pi^\pm p)$ at 20–200 GeV/$c$ shows that the pure $\rho$-Regge-pole exchange model is not sufficient to explain the $\pi^- p \rightarrow \pi^0 n$ mechanism.

There have been some investigations of the correlation between the differential cross section for $\pi^- p \rightarrow \pi^0 n$ charge exchange (CEX) in the forward direction $(d\sigma/dt)_{t=0}^{\mathrm{CEX}}$ and the difference of the total cross section for pion-proton interactions $\Delta\sigma \equiv \sigma(\pi^- p) - \sigma(\pi^+ p)$ in the high-energy region above 5 GeV. They can be broadly classified into 3 types of models: pure $\rho$-Regge-pole exchange model,[1] $\rho + \rho'$ exchange (with or without absorptive cuts) model,[2,3] and normal Regge-pole exchange model with a new contribution of some non-Regge terms.[4,5] So far these phenomenological models have been discussed in comparison with data where the laboratory momentum of incident negative pions $p$ is below 100 GeV/$c$.

In this note, the pure $\rho$-Regge-pole exchange model is examined with new data for both $(d\sigma/dt)_{t=0}^{\mathrm{cex}}$ (Ref. 6) and $\Delta\sigma_T(\pi^\pm p)$ (Ref. 7) at $p = 23$–$240$ GeV/$c$.

The preliminary Fermilab data reported in 1974 for $(d\sigma/dt)_{t=0}^{\mathrm{cex}}$ (Ref. 8) were very exciting because they were quite different from Serpukhov data[9] between 20 and 50 GeV/$c$, and because the Fermilab data showed that the data point at 101 GeV/$c$ was slightly but clearly higher than expected from the other data points at lower momenta. Then, this phenomenon might introduce a new model at very high energies.

On the other hand, the new Fermilab data published in 1976 for $(d\sigma/dt)_{t=0}^{\mathrm{cex}}$ (Ref. 6) are not so low as the old ones, and all 6 data points between 20.8 and 199.3 GeV/$c$ are almost exactly on a straight line in log-log scales. Since there is no description in the 1976 version about the great difference between the old and new $(d\sigma/dt)_{t=0}^{\mathrm{cex}}$ data, here we discuss only the 1976 Fermilab data on $(d\sigma/dt)_{t=0}^{\mathrm{cex}}$.

The data of Barnes *et al.*[6] can be parametrized as

$$(d\sigma/dt)_{t=0}^{\mathrm{cex}}[\mu\mathrm{b}/(\mathrm{GeV}/c)^2] = 2340\,p^{-1.038}\,[\mathrm{GeV}/c]. \tag{1}$$

The differential cross section for CEX in the forward direction

$$(d\sigma/dt)_{t=0}^{\mathrm{cex}} = \frac{|A'_0|^2}{8\pi p^2} \tag{2}$$

and the combination of optical theorem with isospin invariance for pion-proton scattering

$$\mathrm{Im}A'_0 = \frac{p}{2}\Delta\sigma_T(\pi^\pm p) \tag{3}$$

give a simple theoretical relation as follows, if the pure $\rho$-Regge-pole exchange model is employed:

$$\Delta\sigma = k[(d\sigma/dt)_0]^{1/2} \quad (\hbar = c = 1), \tag{4}$$

where

$$k = [32\pi/(1+r^2)]^{1/2},$$

$$r = \tan\tfrac{1}{2}\pi\alpha(0)$$

$$= \mathrm{Re}A'_0/\mathrm{Im}A'_0,$$

and $\alpha(0)$ represents the $\rho$ trajectory at $t = 0$.

The crucial point is to check the quantity $k$ in Eq. (4) at various energies theoretically and experimentally. Figure 1 shows the data plot of $\Delta\sigma$ (Refs. 7 and 10) vs $[(d\sigma/dt)_0]^{1/2}$ (Ref. 6), where $p$ is a momentum acting as a parameter. In this figure, Eq. (4) should be linear in the case of $t = 0$. The $[(d\sigma/dt)_0]^{1/2}$ value for each $p$ between 23 and 240 GeV/$c$ was calculated by Eq. (1). The best fitted straight line was explored by the least-squares method.

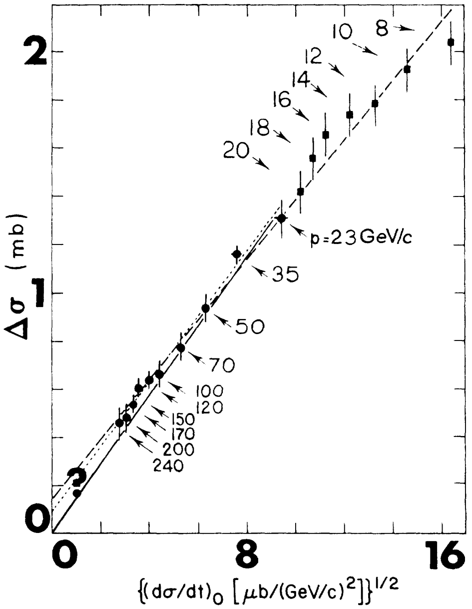

FIG. 1. Data plots of $\Delta\sigma_T(\pi^\pm p)$ vs $[(d\sigma/dt)_{t=0}^{\mathrm{cex}}]^{1/2}$ as a parameter of $p_{\mathrm{lab}}^{\pi^-}$, where ● are from Refs. 6 and 7 and ■ are from Refs. 7 and 10. The solid, dashed, and dotted lines represent Eqs. (8), (7), and (5), respectively.

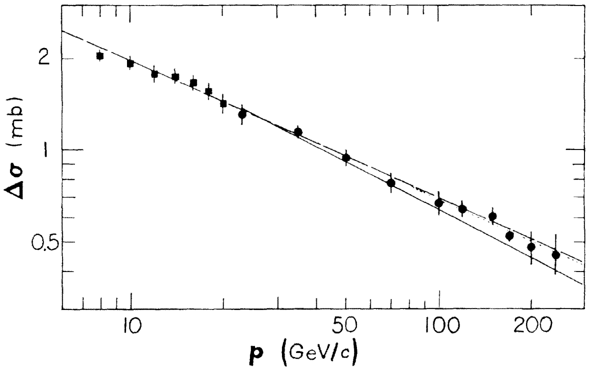

FIG. 2. Data plots of $\Delta\sigma_T(\pi^\pm p)$ vs $p_{\mathrm{lab}}^{\pi^-}$, where ● are from Carroll *et al.* (Ref. 7) and ■ are from Foley *et al.* (Ref. 10). The solid, dashed, and dotted lines represent Eqs. (9), (11), and (10), respectively.

The best fit to these 10 data points at 23–240 GeV/*c* (Refs. 6 and 7) can be represented by

$$\Delta\sigma\,[\mu\mathrm{b}] = 135.6\{(d\sigma/dt)_0\,[\mu\mathrm{b}/(\mathrm{GeV}/c)^2]\}^{1/2} + 90.5 \tag{5}$$

if the experimental error for each $\Delta\sigma$ is respected. If it is assumed to be the same throughout the whole region,

$$\Delta\sigma\,[\mu\mathrm{b}] = 146.5\{(d\sigma/dt)_0\,[\mu\mathrm{b}/(\mathrm{GeV}/c)^2]\}^{1/2} + 24.4 \tag{6}$$

can be obtained at 23–240 GeV/*c*. If 7 data points of Foley *et al.*[10] and 10 data points of Carroll *et al*[7] for $\Delta\sigma$ are included under an assumption of the same experimental error, the 17 data points at 8–240 GeV/*c* can be connected by

$$\Delta\sigma\,[\mu\mathrm{b}] = 124.3\{(d\sigma/dt)_0[\mu\mathrm{b}/(\mathrm{GeV}/c)^2]\}^{1/2} + 141.4 \tag{7}$$

Therefore it is concluded that as far as these data are concerned any linear fit does not pass through the origin of the graph within the experimental error. Since $\Delta\sigma$ is not in proportion to $[(d\sigma/dt)_0]^{1/2}$, the application of any type of pure $\rho$-exchange model to this analysis cannot be successful. It is not known where the empirical $\Delta\sigma$ vs $[(d\sigma/dt)_0]^{1/2}$ curve tends at $p\to\infty$. The empirical $p$ dependence of $r$ (=0.6 to 1.0) or $\alpha(0)$ (=0.34 to 0.51) at 23–240 GeV/*c* cannot be accepted by this pure $\rho$ model.

From the best fit to the effective trajectory in CEX analysis, Barnes *et al.* obtain $\alpha(0)=0.481$, resulting in $r=0.942$ and

$$\Delta\sigma\,[\mu\mathrm{b}] = 144.0\{(d\sigma/dt)_0\,[\mu\mathrm{b}/(\mathrm{GeV}/c)^2]\}^{1/2}, \tag{8}$$

which is, however, not successful, as shown in Fig. 1. Eventually these data and the pure $\rho$-exchange model will predict at 20–200 GeV/*c*

$$\Delta\sigma\,[\mu\mathrm{b}] = 6967p^{-0.519} \tag{9}$$

for the total cross-section difference, where $p$ is in GeV/*c*. Here we must turn to the new data for $\Delta\sigma_T(\pi^\pm p)$ (Ref. 7) for comparison.

Those 10 data points of Carroll *et al.* can be fitted by

$$\Delta\sigma\,[\mu\mathrm{b}] = 5793\,p^{-0.463} \tag{10}$$

if the weight for each experimental error is respected. If these 10 data points and another 7 data points of Foley *et al.* are included with their experimental errors, at 8–240 GeV/*c*

$$\Delta\sigma\,[\mu\mathrm{b}] = 5606p^{-0.455} \tag{11}$$

is obtained. Hendrick *et al.*[1] get

$$\Delta\sigma\,[\mu\mathrm{b}] = 5240p^{-0.43} \tag{12}$$

below 200 GeV/*c*. Białkowski *et al.*[4] get

$$\Delta\sigma\,[\mu\mathrm{b}] = 5898p^{-0.460} \tag{13}$$

between 1 and 200 GeV/*c*. Equations (9), (10) and (11) are shown in Fig. 2.

Equations (12) and (13) are obtained from old data. These exponents (−0.43 and −0.460) are very close to those in Eqs. (10) and (11), which are obtained from the new data ranging up to 240 GeV/*c*. However, the exponent of Eq. (9) is far from the others. This is reflected by the pure $\rho$-exchange model. Although Barnes *et al.*[6] say that their prediction, Eq. (9), agrees very well with the data of Carroll *et al.*, Eq. (10) or Eq. (11), in Fig. 2, evidently the pure $\rho$-exchange model does not show a good fit for the $(d\sigma/dt)_{t=0}^{\mathrm{CEX}}$ vs $|t|$ plot at 199.3 GeV/*c* (see Fig. 1 of Ref. 6), aside from the polarization. From the point of view of the $A_0'$ amplitude, we need a model where the ratio $r$ should gradually decrease with increasing $p$. In conclusion, we still have some reasons to search for another suitable model which will show a good fit at $t\neq 0$ as well as at $t=0$.[11]

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