Induced tensor and pseudotensor form factors in the reaction $\mu^- + {}^{12}C \to {}^{12}B + \nu_\mu$ †

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Using the experimental data on the capture rate and the recoil polarization in the reaction $\mu^- + {}^{12}C \to {}^{12}B + \nu_\mu$ the weak-magnetism form factor is deduced independently of a specific value of the pseudoscalar form factor $g_P(q^2)$. It is found to be in good agreement with the conserved-vector-current hypothesis. Then we look at the range of compatibility of the induced form factors for the given experimental data.

Muon-capture experiments in ^{12}C have been performed[1,2] and theoretically analyzed by many authors.[3-5] One goal of these experiments is, assuming muon-electron universality, to extract the form factors in muon capture and to compare them to the corresponding coupling constants in β decay. The main difference is the high momentum transfer in the former, implying a higher sensitivity to induced effects.

The weak-magnetism form factor at zero momentum transfer has been determined from the shape factors in β decay of ^{12}B and ^{12}N (Ref. 6) and found to be in agreement with the conserved-vector-current (CVC) hypothesis.[7] Recently Calaprice and Holstein[8] have carefully reanalyzed the weak-magnetism experiment[6] in the $A=12$ system and conclude that it does not support the CVC hypothesis as strongly as previously thought. Considering the impact of their conclusion for the interpretation of second-class-currents experiments we perform here an analysis of the inverse reaction

$$\mu^- + {}^{12}C \to {}^{12}B(\text{g.s.}) + \nu_\mu \tag{1}$$

(where g.s. indicates the ground state) using the experimental values of the capture rate Γ and the recoil polarization P_r and we focus our attention specially on the weak-magnetism and induced pseudotensor form factors. The results are to be taken with the accuracy of the impulse approximation used to relate the axial-vector and the induced tensor from factors and the omission of the momentum-dependent terms in the capture rate. The impulse approximation used here is generally accepted to be valid within a few percent,[9] and the recoil-correction terms in the muon capture considered contribute about 10%. For a complete discussion of these terms we refer to the extended calculations of Foldy and Walecka[3] and of Devanathan *et al.*[10]

In the following, we adopt the theoretical framework and notations of Ref. 11. The form factors for reaction (1) contribute through the combinations

$$G_A = g_A\left(1+\frac{\Delta^2-q^2}{4M^2}\right)+g_T\frac{\Delta}{2M}+g_M\frac{k_0}{2M}\left(1+\frac{\Delta}{2M}\right), \tag{2}$$

$$\begin{aligned}G_P = \frac{k_0}{2E_2}\Big[&g_A\left(1-\frac{m_\mu+2\Delta}{2M}+\frac{2\Delta^2-m_\mu\Delta-2q^2}{4M^2}\right)\\&-g_M\frac{E_2}{M}\left(1+\frac{\Delta}{2M}\right)-g_T\left(1+\frac{m_\mu}{2M}+\frac{\Delta(m_\mu+\Delta)}{4M^2}\right)\\&-g_P\frac{m_\mu}{2M}\left(1+\frac{\Delta}{2M}\right)\left(1-\frac{\Delta}{2M}+\frac{\Delta^2-q^2}{4M^2}\right)\Big].\end{aligned} \tag{3}$$

g_A, g_P, g_M, and g_T are the axial-vector, pseudoscalar, induced tensor (weak-magnetism), and induced pseudotensor (due to second-class currents) form factors, respectively, q is the transfer momentum, $M=\frac{1}{2}(M_{^{12}C}+M_{^{12}B})$, $\Delta = M_{^{12}C}-M_{^{12}B}$, m_μ is the muon mass, k_0 is the neutrino energy, and E_2 is the final nucleus energy.

The usual theoretical analyses[3-5] were based on the only available experimental quantity: the capture rate Γ. Using the β decay $^{12}B(\text{g.s.}) \to {}^{12}C(\text{g.s.}) + e^- + \bar{\nu}_e$ to minimize the nuclear uncertainties and the Goldberger-Treiman[12] relation to fix $g_P(q^2)/g_A(q^2)$, the value of $g_M(q^2)$ was deduced in the absence of second-class currents. Conversely, if $g_M(q^2)$ is supposed to be given by the CVC hypothesis, one could extract $g_P(q^2)/g_A(q^2)$. The recent measurement[13] of the recoil polarization of ^{12}B produced in the capture of polarized muons on ^{12}C provides a new independent experimental parameter clarifying the analysis of the form factors. From Ref. 11 we have

$$\Gamma = \frac{G^2\cos^2\theta_C}{2\pi^2}\left(\frac{Z\alpha m_\mu M_1}{M_1+m_\mu}\right)^3\frac{k_0^2}{1+k_0/E_2}\frac{E_2+M_2}{2E_2}C \times(3G_A^2+2G_AG_P+G_P^2), \tag{4}$$

where

G = universal Fermi constant,

θ_C = Cabibbo angle,

C = correction factor due to finite size of the nucleus ($C = 0.885$),

$M_{1(2)}$ = mass of the initial (final) nucleus,

$$P_r = \frac{2}{3}\,\frac{3G_A{}^2 + 2G_A G_P}{3G_A{}^2 + 2G_A G_P + G_P{}^2}\,. \qquad (5)$$

The experimental values are

$$\Gamma = (6.2 \pm 0.3) \times 10^3\ \text{sec}^{-1}\ \ \text{(Ref. 2)}, \qquad (6)$$

$$P_r = 0.48 \pm 0.10\ \ \text{(Refs. 13, 14)}. \qquad (7)$$

Relations (4) and (5) allow separation of the effective coupling constants G_A and G_P. Now we can analyze the data from two points of view. In the first one we observe that information about $g_M(q^2)$ is essentially given by G_A because it does not contain $g_P(q^2)$ and because the term involving $g_T(q^2)$ may be neglected if $g_T(q^2) \ll 10 g_M(q^2)$. On the other hand, G_P provides information about the whole contribution of the induced terms. From (4) and (5) we have

$$G_A = \pm\frac{1}{3}\left\{\left[\frac{\Gamma}{2K}(2 - 3P_r)\right]^{1/2} \pm \left[\frac{\Gamma}{K}(1 + 3P_r)\right]^{1/2}\right\}, \qquad (8)$$

$$G_P = \mp\left[\frac{\Gamma}{2K}(2 - 3P_r)\right]^{1/2}, \qquad (9)$$

where

$$K \equiv \frac{G^2\cos^2\theta_C}{2\pi^2}\left(\frac{Z\alpha m_\mu M_1}{M_1 + m_\mu}\right)^3 \frac{k_0{}^2}{1 + k_0/E_2}\,\frac{E_2 + M_2}{2E_2}\,C\,.$$

So, combining (2) (with $g_T\Delta/2M$ neglected) and (6), (7), (8), and the relationship

$$\frac{g_A(q^2)}{g_A(0)} \simeq \frac{g_M(q^2)}{g_M(0)}$$

derived in the impulse approximation, we obtain

$$|g_M(q^2 = -0.74 m_\mu{}^2)| = 25 \pm 1.3 \qquad (10)$$

$$\text{if } G_A = \pm\frac{1}{3}\left\{\left[\frac{\Gamma}{2K}(2 - 3P_r)\right]^{1/2} + \left[\frac{\Gamma}{K}(1 + 3P_r)\right]^{1/2}\right\},$$

$$|g_M(q^2 = -0.74 m_\mu{}^2)| = 12 \pm 3.3 \qquad (11)$$

$$\text{if } G_A = \pm\frac{1}{3}\left\{\left[\frac{\Gamma}{2K}(2 - 3P_r)\right]^{1/2} - \left[\frac{\Gamma}{K}(1 + 3P_r)\right]^{1/2}\right\}.$$

Only solution (10) supports comparison with the experimental result $|g^{EM}(q^2 = -0.74 m_\mu{}^2)| = 25 \pm 0.5$ deduced from inelastic electron scattering in ^{12}C.[15] In an earlier analysis Kubodera and Kim[5] analyzed the same problem, but they considered a specific value for $g_P(q^2 = -0.74 m_\mu{}^2)/g_A(q^2 = -0.74 m_\mu{}^2)$ and supposed that second-class currents do not contribute at all.

In principle, another experimental parameter, the longitudinal polarization of ^{12}B,

$$P_L = \frac{2G_A{}^2}{3G_A{}^2 + 2G_A G_P + G_P{}^2}\,,$$

could be used in this kind of analysis.[16] However, experimental data for P_L are not yet available.

In a second point of view, we solve Eqs. (8) and (9) for $g_M(q^2)$ and $g_T(q^2)$ with the aid of Eqs. (2), (3), (6), and (7) and with

$$f_P \equiv \frac{m_\mu}{2MA}\,\frac{g_P(q^2 = -0.74 m_\mu{}^2)}{g_A(q^2 = -0.74 m_\mu{}^2)}$$

considered as a parameter; the Goldberger-Treiman value[12] gives $f_P = 7.1$. Because of the quadratic character of the equations, we obtain two sets of values for $g_M(q^2 = -0.74 m_\mu{}^2)$ and $g_T(q^2 = -0.74 m_\mu{}^2)$. In Fig. 1, we represent the two sets of solutions for $|g_M(q^2 = -0.74 m_\mu{}^2)|$ and $g_T(q^2 = -0.74 m_\mu^2)/A g_A(q^2 = -0.74 m_\mu{}^2)$ as a function of f_P.

Retaining the values of $g_M(q^2 = -0.74 m_\mu{}^2)$ compatible with CVC, we have to consider set 1 of the solutions. [Using Eq. (10) and taking the Goldberger-Treiman[12] value for $g_P(q^2 = -0.74 m_\mu{}^2)$, we ob-

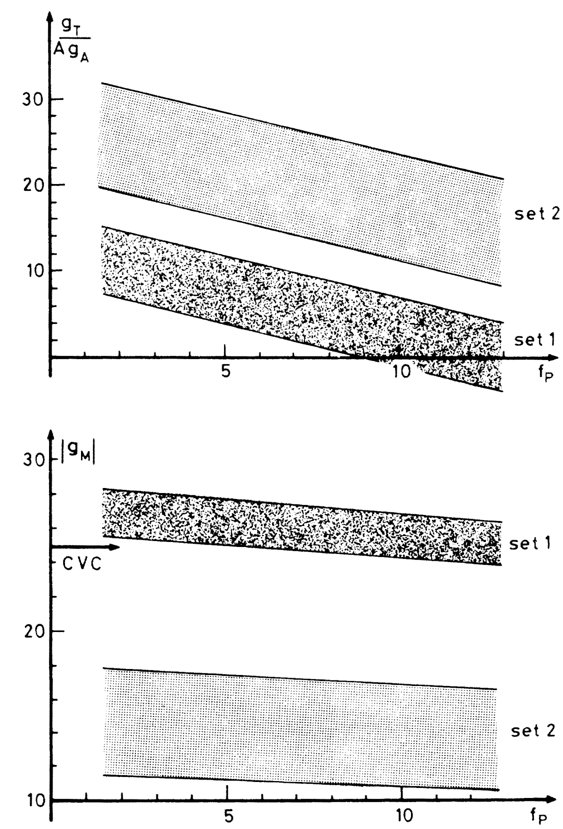

FIG. 1. $(g_T/Ag_A)(q^2 = -0.74 m_\mu{}^2)$ and $|g_M(q^2 = -0.74 m_\mu{}^2)|$ versus f_P obtained from Eqs. (8) and (9).

tain $g_T(q^2=-0.74m_\mu{}^2)/Ag_A(q^2=-0.74m_\mu{}^2)=7\pm 5$.] For $f_P<10$, set 1 of the solutions presents positive values of $g_T(q^2=-0.74m_\mu{}^2)/Ag_A(q^2=-0.74m_\mu{}^2)$. When $f_P>10$, $g_T(q^2=-0.74m_\mu{}^2)/Ag_A(q^2=-0.74m_\mu{}^2)$ may be negative, in agreement with the measurement of Sugimoto *et al.*[17]

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