Adler's sum rule in source theory

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(Received 7 October 1976)

The neutrino-nucleon structure functions suggested by source theory are used to test Adler's sum rule; the numerical result argues for its correctness. An independent source-theoretic derivation is then produced, with the emphasis transferred from hadronic currents to the vector field of a partially U_2-invariant unified theory of electromagnetic and weak interactions.

The equal-time commutation relations of current algebra provide a ready supply of sum rules, as in the deep-inelastic form of Adler's neutrino sum rule[1]

$$I=\frac{1}{2}\int_0^1 \frac{dx}{x}\,[F_2^{\nu n}(x)-F_2^{\nu p}(x)]=1\,, \tag{1}$$

but offer no detailed information about the values of the individual scaling functions $F_2^{\nu n}(x), F_2^{\nu p}(x)$. Source theory,[2] in contrast, has been prolific in producing quantitative accounts of just such functions,[3] but has had relatively little to say about sum rules. (A gratifying exception is the new sum rule encountered for the scattering of polarized particles.[4]) A useful overlap of the two approaches appears on applying the physical functions suggested by source-theoretic considerations to test the sum rules proclaimed by current algebra, particularly for the situation of the Adler neutrino sum rule, where experimental data are not yet of high accuracy. The specific forms used are[3,5]

$$F_2^{\nu n}(x)-F_2^{\nu p}(x)=\frac{x^{1/2}}{1-\frac{1}{2}x^{1/2}}\,F_2(x)\,,$$
$$F_2(x)=\frac{2}{\pi}\frac{(1-x)^3}{(1+0.2x)^4}\times\left[\frac{1+4.52x}{1-0.75x}+(1+0.24x^{1/2})^2\left(\frac{1+0.2x}{1-0.1x}\right)^4\right], \tag{2}$$

and the numerical result obtained is

$$I=0.97\,, \tag{3}$$

which is more than sufficiently near unity to promote a bias in favor of the correctness of Adler's sum rule. Here, then, is a challenge to source theory: Supply a physical basis for Adler's sum rule without using the inapplicable machinery of operator-valued field and currents.

The hadronic system is not the principal character in our response. That role is played by the vector field of a unified electromagnetic-weak interaction theory, for which we use a partially U_2-symmetric version[6] that is related to, but quite distinct from, Weinberg's conception.[7] The starting point is a mixed action expression involving nucleon fields ψ and vector fields $A^{\mu}_{ab}(A, W^{\pm}, Z)$, where ψ unites the incoming and outgoing plane-wave fields of physical nucleons, while the A^{μ}_{ab} obey the nonlinear inhomogeneous field equations that are implied by this (incompletely stated) action:

$$W=\int\,[ej^{\mu}_{ba}(\psi)A_{\mu ab}+\mathcal{L}(A)-\tfrac{1}{2}A^{\mu}M^2A_{\mu}]+\int e^2\tfrac{1}{2}\psi\gamma^0A^{\mu}G_{\mu\nu}A^{\nu}\psi\,. \tag{4}$$

Here, the j^{μ}_{ba} are the nucleonic currents appropriate to electromagnetic and weak interactions. $\mathcal{L}(A)$ is the non-Abelian Lagrange function that, without the indicated physical mass term, is invariant under the infinitesimal gauge transformation (matrix notation)

$$\delta A_{\mu}=\partial_{\mu}\delta\lambda-ie[A_{\mu},\delta\lambda]\,, \tag{5}$$

while the last term of (4) symbolizes the physical process of Compton scattering. The stationary action property implies that (omitting terms more than bilinear in A and $\delta\lambda$)

$$0=\int\{-e\partial_{\mu}j^{\mu}_{ba}\delta\lambda_{ab}-ie^2j^{\mu}_{ba}[A_{\mu},\delta\lambda]_{ab}+\delta\lambda M^2\partial_{\mu}A^{\mu}\}+\int e^2\tfrac{1}{2}\psi\gamma^0(A^{\mu}G_{\mu\nu}\partial^{\nu}\delta\lambda+\partial^{\mu}\delta\lambda G_{\mu\nu}A^{\nu})\psi\,. \tag{6}$$

In the following we apply this relation to forward Compton scattering of a virtual vector particle of momentum q by a nucleon with momentum p. Since the nucleon momentum does not change, we have

$$\partial_{\mu}j^{\mu}_{ba}=0\,, \tag{7}$$

without reference to detailed conservation. In addition, the vectorial aspect of j^{μ} is fixed by the momentum p, which we convey by writing

$$j^{\mu}_{ba}=2p^{\mu}d\omega_pT_{ba}\,, \tag{8}$$

where

$$d\omega_p = \frac{(d\vec{p})}{(2\pi)^3}\frac{1}{2p^0} \tag{9}$$

is an invariant momentum measure and T_{ba} represents the appropriate numerical magnitude. (In our application, all that enters is $T_{11}-T_{22}=\frac{1}{2}\tau_3$, with $\tau_3=+1$ and -1 for proton and neutron, respectively.) We further specialize to the charged W sector, picking out the coefficient of $\delta\lambda_{ab}(q)$ and $A^{\mu}_{ba}(-q)$, where $ab=12$ or 21, to get

$$0 = 2p^{\mu}\tfrac{1}{2}\tau_3 t_3 + 2G^{\mu\nu}(p,q)q_{\nu}\,, \tag{10}$$

in which $t_3=\pm 1$ is the W charge and $G^{\mu\nu}(p,q)$ is also a linear function of $\frac{1}{2}\tau_3 t_3$. The M^2 contribution, proportional to q^{μ}, has been omitted in (10), in anticipation of our preoccupation solely with the coefficients of the vector p^{μ}. The latter constraint also means, with regard to the tensor structure of $G^{\mu\nu}(p,q)$,

$$G^{\mu\nu}(p,q) = p^{\mu}p^{\nu}B(qp,\, q^2) + p^{\mu}q^{\nu}C(qp,q^2) + \cdots\,, \tag{11}$$

that only the indicated terms need be considered. We thus infer the scalar relation

$$-\tfrac{1}{2}\tau_3 t_3 = qp\, B(qp,q^2) + q^2 C(qp,q^2)\,. \tag{12}$$

Concerning the crossing symmetry of $G^{\mu\nu}(p,q)$, which is invariant under the substitutions $q\to -q$, $\mu \leftrightarrow \nu$, and $t_3 \to -t_3$, we learn from (11) that the scalar function B is invariant under $qp\to -qp$ and $t_3\to -t_3$, while (12) informs us that C must then change sign.

The functions $B(qp,q^2)$ and $C(qp,q^2)$ will be represented by single spectral forms of the type

$$F(qp,q^2) = \int dM^2\left[\frac{1}{(p+q)^2+M^2} \pm \frac{1}{(p-q)^2+M^2}\right]F_1(M^2,q^2) + \tfrac{1}{2}\tau_3 t_3 \int dM^2\left[\frac{1}{(p+q)^2+M^2} \mp \frac{1}{(p-q)^2+M^2}\right]F_2(M^2,q^2)\,, \tag{13}$$

where the upper and lower signs are appropriate to B and C, respectively. We now observe that

$$\begin{aligned} qp\,B(qp,q^2) = &-\int dM^2\left[\frac{1}{(p+q)^2+M^2} - \frac{1}{(p-q)^2+M^2}\right]\tfrac{1}{2}(q^2+M^2-m^2)B_1(M^2,q^2) \\ &-\tfrac{1}{2}\tau_3 t_3\int dM^2\left[\frac{1}{(p+q)^2+M^2} + \frac{1}{(p-q)^2+M^2}\right]\tfrac{1}{2}(q^2+M^2-m^2)B_2(M^2,q^2) + \tfrac{1}{2}\tau_3 t_3\int dM^2 B_2(M^2,q^2)\,. \end{aligned} \tag{14}$$

Accordingly, Eq. (12) yields the relations

$$\tfrac{1}{2}(q^2+M^2-m^2)B_{1,2}(M^2,q^2) = q^2 C_{1,2}(M^2,q^2) \tag{15}$$

and also

$$-\int dM^2 B_2(M^2,q^2) = 1\,, \tag{16}$$

which is the desired sum rule, or, rather, its more general form that holds for arbitrary q^2.

One immediately identifies the integrand of (16) in terms of the spectral form for B as

$$\begin{aligned} &-B_2(M^2,q^2) = \frac{1}{\pi}\,\mathrm{Im}[B^{\nu n}(qp,q^2) - B^{\nu p}(qp,q^2)]\,, \\ &(p+q)^2 + M^2 = 0\,. \end{aligned} \tag{17}$$

A comparison of the forward scattering amplitude implied by (4),

$$1 + ie^2 d\omega_p 2A^{\mu}(-q)G_{\mu\nu}(p,q)A^{\nu}(q)\,, \tag{18}$$

with that presented by DeRaad, Milton, and Tsai[8] yields the relation

$$(2/q^2)B(qp,q^2) = H_2(qp,q^2)\,, \tag{19}$$

or

$$\mathrm{Im}B = \frac{1}{2}\frac{1}{2m\nu}f_2(\omega,q^2)\,, \tag{20}$$

where

$$\begin{aligned} &2m\nu q^2\,\mathrm{Im}H_2(qp,q^2) = f_2(\omega,q^2)\,, \\ &2m\nu = \omega q^2 = M^2 - m^2 + q^2\,. \end{aligned} \tag{21}$$

The sum rule (16) then reads

$$\frac{1}{2\pi}\int_1^{\infty}\frac{d\omega}{\omega}(f_2^{\nu n} - f_2^{\nu p})(\omega,q^2) = 1\,, \tag{22}$$

and, in the ideal deep-inelastic limit where the q^2 dependence disappears from the integrand of (22), we do indeed recover (1), with

$$F_2(x) = (1/\pi)f_2(\omega)\,,\quad x = 1/\omega\,. \tag{23}$$

I am grateful to Wu-Yang Tsai for his independent numerical evaluation of the integral.

Note added in proof. Although the validity of the sum rule is not affected, this derivation is incomplete in one important respect. It was overlooked that the quantity $M^2\partial_{\mu}A^{\mu}$, as it enters in Eq. (6), is also dynamically determined, and thus has a contribution of the form

$$M^2\partial_\mu A^\mu \to \int e^2 \tfrac{1}{2}\psi\gamma^0 A^\mu G_\mu \psi \ .$$

This adds to (10) a vector structure of the type

$$p^\mu D(qp, q^2) + q^\mu E(qp, q^2)$$

so that Eq. (12) acquires the additional term $\frac{1}{2}D(qp, q^2)$. The right-hand side of (15) is then supplemented by $\frac{1}{2}D_{1,2}(M^2, q^2)$, but the sum rule emerges as before.

*Work supported in part by the National Science Foundation.

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