Comments on spontaneous CP violation in a gauge model with spontaneous P violation

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The various versions of a simple P-conserving extension of the Weinberg-Salam model are reviewed. It is pointed out that only one version, discussed previously by the author, allows for both spontaneous parity and spontaneous CP violation. It is shown how the CP violation, and the mass of one Higgs boson, can be generated together in the one-loop approximation. The CP violation does not arise in the tree approximation as asserted by the author in earlier work.

I. INTRODUCTION

The least reworking of the $SU(2)\times U(1)$ Weinberg-Salam (W-S) model[1] which accommodates spontaneous parity violation requires fermion $SU(2)$-weak doublets with both helicities present, not just left-handed ones as in the W-S model, and companion singlets with both helicities present, not just right-handed ones. A pseudoscalar doublet, in addition to the scalar of the W-S model, is also needed to support the spontaneous parity violation. A version of this kind of scheme was first presented by Fayet.[2] McKay and Munczek[3] studied a different version in which neutral Higgs particles couple to e^+e^- and in which *charged* heavy leptons are present, in contrast with Fayet's scheme. Herbert[4] analyzed a variation which has the same electron- and muon-multiplet choice as Ref. 3, but a more complicated Higgs structure and a different quark structure. Subsequently, I pointed out that CP could also be violated spontaneously in this kind of extended Weinberg-Salam model,[5] since the necessary minimum of two complex neutral spin-zero fields is present,[6] and I pointed out that in the McKay-Munczek version a small ratio of electron mass to heavy-electron mass requires that the CP-violation parameter be small.

In this comment I point out that the variant studied by McKay and Munczek is the only one which accommodates *both* spontaneous P and CP violation with the simplest spinless-field choices. The spontaneous CP violation is shown to emerge at the one-loop level, from fermion loops, rather than in the tree approximation as I claimed in Appendix A of Ref. 5. It is interesting that one of the Higgs particles acquires mass at this one-loop level as well.

II. FOUR VERSIONS OF THE $SU(2)\times U(1)$ PARITY-CONSERVING SCHEME

As in the W-S model, one has an $SU(2)$-weak fermion doublet, D, and singlet, S, associated with the electron and its neutrino, and likewise for the muon and its neutrino. Every field undergoes the gauged $U(1)$ phase transformation. Yukawa couplings of the fermions to the spin-zero fields provide, after spontaneous breakdown, the masses for fermions. These couplings can be restricted to prevent the neutrinos from acquiring mass by imposition of a global γ_5 symmetry.

In addition to the fermions and the usual gauge fields, we need two spin-zero doublets, denoted ϕ_L and ϕ_R, where under the parity transformation, P,

$$P\phi_L P^{-1}=\phi_R\,.$$

At least two doublets are needed in order to support spontaneous parity violation.

The global transformations on the fermions and spin-zero fields are designated as follows:

$$D'=e^{i\alpha\gamma_5}D,\quad S'=e^{i\beta\gamma_5}S\,,\qquad \phi'_L=e^{i\delta}\phi_L,\quad \phi'_R=e^{i\gamma}\phi_R\,. \tag{1a}$$

The transformations of left- and right-handed components of the fermion fields can be displayed separately as

$$D'_L=e^{-i\alpha}D_L,\quad D'_R=e^{i\alpha}D_R\qquad S'_L=e^{-i\beta}S_L,\quad S'_R=e^{i\beta}S'_R\,. \tag{1b}$$

Given the same gauge-group-transformation assignments as in the usual W-S model, all the versions which include spontaneous parity violation and massless neutrinos can be characterized by the choices of $\alpha,\beta,\gamma,\delta$.

TABLE I. Summary of the different cases of global-γ_5-symmetry assignments. The corresponding multiplet assignments and allowed Yukawa couplings to the spin-zero doublets are indicated.

Case	Leptons	Yukawa	Mass terms
I: $\alpha,\beta,\gamma\neq 0$	$\begin{pmatrix}D^0\\D^-\end{pmatrix}, S^-$	$A_{\rm I}(\bar{S}_R\phi_L^\dagger D_L+\bar{S}_L\phi_R^\dagger D_R)+\text{H.c.}$	0
II: $\beta,\gamma\neq 0$	$\begin{pmatrix}D^0\\D^-\end{pmatrix}, S^0$	$A_{\rm II}(\bar{S}_R\phi_L^\dagger D_L+\bar{S}_L\phi_R^\dagger D_R)+\text{H.c.}$	$M_D\bar{D}D$
III: $\alpha,\gamma\neq 0$	$\begin{pmatrix}D^0\\D^-\end{pmatrix}, S^-$	$A_{\rm III}(\bar{S}_R\phi_L^\dagger D_L+\bar{S}_L\phi_R^\dagger D_R)+\text{H.c.}$	$M_S\bar{S}S$
IV: $\alpha,\beta\neq 0$	$\begin{pmatrix}D^0\\D^-\end{pmatrix}, S^-$	$A_{\rm IV}(\bar{S}_R\phi_L^\dagger D_L+\bar{S}_L\phi_R^\dagger D_R)+\text{H.c.}$ $B_{\rm IV}(\bar{S}_R\phi_R^\dagger D_L+\bar{S}_L\phi_L^\dagger D_R)+\text{H.c.}$	0

Consider the parity- and gauge-symmetric Yukawa coupling

$$L\ (\text{Yukawa})=A(\bar{S}_R\phi_L^\dagger D_L+\bar{S}_L\phi_R^\dagger D_R)+\text{H.c.} \tag{2}$$

Under the global transformation, these terms become

$$(\bar{S}_R\phi_L^\dagger D_L)'=e^{i(-\beta-\alpha-\delta)}(\bar{S}_R\phi_L^\dagger D_L)$$

and

$$(\bar{S}_L\phi_R^\dagger D_R)'=e^{i(\beta+\alpha-\gamma)}(\bar{S}_L\phi_R^\dagger D_R)\,.$$

Invariance requires that

$$-\delta=\gamma=\alpha+\beta\,. \tag{3}$$

Keeping in mind that either $\alpha\neq 0$ or $\beta\neq 0$ is necessary to ensure massless neutrinos, one of the defining features of the model, one can allow the four cases

$$\begin{aligned}&\text{I: } \alpha\neq 0,\ \beta\neq 0,\ \gamma=\alpha+\beta\,;\\ &\text{II: } \alpha=0,\ \beta\neq 0,\ \gamma=+\beta\ (\text{Fayet})\,;\\ &\text{III: } \alpha\neq 0,\ \beta=0,\ \gamma=+\alpha\,;\\ &\text{IV: } \alpha=-\beta\neq 0,\ \gamma=0\ (\text{McKay and Munczek})\,.\end{aligned} \tag{4}$$

In cases II and III, mass terms for the doublet and singlet, respectively, can be added to Eq. (2). In case IV, an additional Yukawa coupling with ϕ_L and ϕ_R switched is allowed. The situation is summarized in Table I.

III. THE EFFECTIVE POTENTIALS

A. Tree approximation

The minima of the effective potential determine the possible broken-symmetry vacuums of each case. Finding the absolute minimum of the spinless fields potential in the tree approximation does not always settle the question of CP violation.[7,8,9] This is what happens in the models under consideration here, as discussed below, and I will subsequently turn to a brief discussion of the consequences of including the one-loop terms in the effective potential in Sec. III B.

A look at the spinless-field potential, U, for cases I, II, and III,

$$\begin{aligned}U(\phi_L,\phi_R)=&A(\phi_L^\dagger\phi_L+\phi_R^\dagger\phi_R)+B[(\phi_L^\dagger\phi_L)^2+(\phi_R^\dagger\phi_R)^2]\\&+C(\phi_L^\dagger\phi_L)(\phi_R^\dagger\phi_R)+D(\phi_L^\dagger\phi_R)(\phi_R^\dagger\phi_L)\,,\end{aligned} \tag{5}$$

reveals that there is no possibility of spontaneous-CP-violation support in the tree approximation.[10] One can see this most simply by noting that U does not depend upon the relative phase, θ, between $\langle\phi_L\rangle_0$ and $\langle\phi_R\rangle_0$, the vacuum expectation values of the spinless fields. Following Lee's[6] triangular-molecule analogy, the triangle energy is degenerate not only under rotation of the whole system, but also under rotation of the individual sides independently. There is no relative orientation "spring constant" of the sides. One can express this trivially in terms of the minimum of the potential (5), namely,

$$\frac{\partial U}{\partial\theta}=0 \text{ and } \frac{\partial^2 U}{\partial\theta^2}=0\,. \tag{6}$$

The extra zero-mass mode is a true Goldstone mode associated with the global γ_5 symmetry, about which I will say more in considering the one-loop contributions. Therefore, the stable-symmetry-breaking solution is the type discussed by Fayet,[2] where either $\langle\phi_L\rangle_0=0$ or $\langle\phi_R\rangle_0=0$, and no spontaneous CP violation emerges in cases I, II, and III. In other words, spontaneous CP violation would require $\langle\phi_L\rangle\neq 0$ *and* $\langle\phi_R\rangle\neq 0$ in order that a vacuum phase be defined, but such a solution is not the stable vacuum in cases I, II, and III since it is a point of inflection, or Goldstone mode. The (parity-violating) *minimum* occurs when $\langle\phi_L\rangle_0=0$ or $\langle\phi_R\rangle=0$.

In case IV, the spinless fields do not transform under the global group and the less-restricted potential *can* depend on the relative phase between ϕ_L and ϕ_R, so there may be the possibility of spontaneous breakdown of *CP* in this case. The potential reads

$$U_{\text{IV}} = U(\phi_L, \phi_R) + A'(\phi_L^\dagger\phi_R + \phi_R^\dagger\phi_L) + B'[(\phi_L^\dagger\phi_R)^2 + (\phi_R^\dagger\phi_L)^2] + F[(\phi_L^\dagger\phi_L)(\phi_R^\dagger\phi_L) + (\phi_R^\dagger\phi_R)(\phi_L^\dagger\phi_R)] + F^*[(\phi_L^\dagger\phi_L)(\phi_L^\dagger\phi_R) + (\phi_R^\dagger\phi_R)(\phi_R^\dagger\phi_L)], \quad (7)$$

where $U(\phi_L, \phi_R)$ was defined in Eq. (5).[10] Explicit T invariance requires that $F = F^*$. The analysis is simplified if U_{IV} is expressed in terms of s and t fields which have definite C, T, *and* P assignments, since then one field can be chosen to have a real vacuum expectation value and the *CP*-violating phase can be ascribed to the other. We have

$$U_{\text{IV}}(s,t) = \tilde{A}_s s^\dagger s + \tilde{A}_t t^\dagger t + \tilde{B}_s(s^\dagger s)^2 + \tilde{B}_t(t^\dagger t)^2 + \tilde{C}(t^\dagger t)(s^\dagger s) + \tilde{D}(t^\dagger s)(s^\dagger t) + \tilde{F}[(s^\dagger t)^2 + (t^\dagger s)^2], \quad (8)$$

where $\langle s\rangle_0 \equiv \lambda_s$ and $\langle t_0\rangle \equiv \lambda_t e^{i\tilde{\theta}}$, with λ_s and λ_t real. To ensure T invariance, $\tilde{F}$ is assumed real in Eq. (8). The value of the relative $s - t$ phase, $\tilde{\theta}$, for which U_{IV} is a minimum is determined by the condition

$$\frac{\partial U_{\text{IV}}}{\partial \tilde{\theta}} = 0 = \lambda_s^2 \lambda_t^2 \tilde{F} \sin 2\tilde{\theta}. \quad (9)$$

The restriction has three obvious solutions and corresponding consequences:

(a) $\lambda_s = 0$ (or $\lambda_t = 0$)	No *CP* violation, degenerate *E-e* masses, $\tilde{A}_t < 0$ (or $\tilde{A}_s < 0$).	
(b) $\tilde{\theta} = 0$	No *CP* violation at this level	(10)
(c) $\tilde{F} = 0$	An extra zero-mass scalar and no *CP* violation apparent at this level.	

A summary of the *CP* possibilities for the four cases is given in Table II.

In case IV, as in cases I, II, and III, no spontaneous *CP* violation emerges in tree approximation. The *CP* issue must be settled in higher order, and I next consider the one-loop modifications to the effective potential.

B. One-loop corrections

In cases I, II, and III, the tree-approximation minimum at $\langle\phi_L\rangle_0 \neq 0$, $\langle\phi_R\rangle_0 \neq 0$ introduces a zero-mass field which is a true Goldstone boson associated with the global γ_5 transformation. This field, therefore, remains massless to all orders. It is easy to verify that the one-loop effective potential generated by the fermion, vector-meson, and spin-zero self-interactions contains no dependence on the relative phase between ϕ_L and ϕ_R, which ensures that Eq. (6) remains valid in loop approximation as it must. The model with γ_5-transformation cases I, II, and III can never acquire a *CP*-violating phase in any order.[9]

The situation in case IV, studied in Refs. 3 and 5, is different because the spinless fields do not transform under the γ_5 group, and the Goldstone phenomenon does not occur for this group. As pointed out in discussion of the tree approximation, the extremum condition with respect to the relative $s - t$ field phases can be satisfied by: (a) $\langle\phi_L\rangle_0 = 0$, no *CP* violation, which is uninteresting because of the degeneracy between left- and right-handed fermion field components; (b) $\langle\phi_L\rangle_0 \neq 0$, $\langle\phi_R\rangle_0 \neq 0$, and $\tilde{\theta} = 0$ in tree approximation; and (c) $\tilde{F} = 0$, but $\langle\phi_L\rangle_0 \neq 0$, $\langle\phi_R\rangle_0 \neq 0$, and $\tilde{\theta}$ undetermined in tree approximation. Of the interesting possibilities (b) and (c), the later allows the simpler treatment in the one-loop correction to the effective potential.[11]

I wish to consider the consequences of setting $\tilde{F} = 0$ in the potential $U_{\text{IV}}(s,t)$. This restriction is not natural in the general or technical sense,[12] but makes the one-loop *CP* effects transparent and leads to an interesting consequence. With $\tilde{F} = 0$, $\tilde{U}_{\text{IV}}$ has a symmetry under independent phase transformations on the s and t fields, whose kinetic-energy terms in the Lagrangian also have this symmetry. The Yukawa couplings do not share the invariance just mentioned, and it is the Yukawa couplings whose loop contributions to the effective potential support the *CP*-violating phase and the mass of the Higgs field $-\text{Re}t_2 \sin\tilde{\theta} + \text{Im}t_2 \cos\tilde{\theta}$, which is massless in the tree approximation owing to the artificial symmetry imposed

TABLE II. Summary of possibilities for spontaneous *CP* violation arising in tree or loop approximation. The "?" means undetermined in the sense of Ref. 7 and 9.

CP violation	I	II	III	IVa	IVb	IVc
Tree approximation	No	No	No	No	?	No
Loop approximation	No	No	No	No	Yes	No

on U_{IV}. The extra global symmetry shared by the potential U_{IV} and the kinetic-energy term $(D_\mu s)^\dagger D^\mu s + (D_\mu t)^\dagger D^\mu t$ ensures that loops involving spinless fields and gauge fields contribute neither to the mass of the $-\mathrm{Re}t_2 \sin\tilde\theta + \mathrm{Im}t_2 \cos\tilde\theta$ field combination nor to the CP-violating phase.

The one-loop effective-potential contribution of the electron part of the fermion section is[13]

$$V_f(s,t) = -\frac{1}{16\pi^2}\left[M_+^2 \ln\left(\frac{M_+}{M_0^2}\right) + M_-^2 \ln\left(\frac{M_-}{M_0^2}\right)\right], \tag{11}$$

where M_0 is an arbitrary renormalization mass, and

$$M_\pm(s,t) = A_e^2 s^\dagger s + B_e^2 t^\dagger t \pm 2A_eB_e(s^\dagger t + t^\dagger s), \tag{12a}$$

$$M_+(\langle s\rangle_0, \langle t\rangle_0) = M_E^2 = A_e\lambda_s^2 + B_e\lambda_t^2 + 2A_eB_e\lambda_s\lambda_t\cos\tilde\theta, \tag{12b}$$

$$M_-(\langle s\rangle_0, \langle t\rangle_0) = M_e^2 = A_e\lambda_s^2 + B_e\lambda_t^2 - 2A_eB_e\lambda_s\lambda_t\cos\tilde\theta, \tag{12c}$$

with

$$L\ (\text{Yukawa}) = A_e\bar S_e s^\dagger D_e + B_e \bar S_e t^\dagger \gamma_5 D_e + \text{H.c.}$$

The heavy- and light-electron-mass conventions are those of Ref. 5. As I mentioned above, $V_f(s,t)$ is the only loop term which can depend upon the relative $s \leftrightarrow t$ phase $\tilde\theta$. Therefore I restrict my attention to the minimization of V_f with respect to $\tilde\theta$. We have

$$\left.\frac{\partial V_f}{\partial\tilde\theta}\right|_{\langle s\rangle=\lambda_s, \langle t\rangle = \lambda_t e^{i\tilde\theta}} = 0 = -\frac{1}{16\pi^2}\left[M_E^2 2\ln\left(\frac{M_E^2}{M_0^2}\right) - M_e^2 2\ln\left(\frac{M_e^2}{M_0^2}\right) + M_E^2 - M_e^2\right] 4A_eB_e\lambda_s\lambda_t\sin\tilde\theta \tag{13}$$

as a necessary condition. Clearly $\sin\tilde\theta = 0$ is one solution, but there is the more interesting solution

$$2M_E^2\ln\left(\frac{M_E^2}{M_0^2}\right) + M_E^2 - 2M_e^2\ln\left(\frac{M_e^2}{M_0^2}\right) - M_e^2 = 0, \tag{14}$$

which can be satisfied if

$$0 \le \frac{M_e}{M_0} \le e^{-3/2}$$

and

$$e^{-3/2} \le \frac{M_E}{M_0} \le e^{-1/2}.$$

The solutions with $M_e/M_E \ll 1$ correspond to $\tilde\theta \ll 1$ as discussed in Ref. 5. One can readily verify that $\partial^2 V_f/\partial\tilde\theta^2 > 0$ for M_E, M_e values which satisfy (15), so that the solution is indeed a minimum of the effective potential and a mass develops for the field which "lost" its mass by my restriction $\tilde F = 0$. This minimum can be chosen to be lower than the symmetric minimum, $V=0$, so long as the lepton masses are small compared to gauge-boson masses.

IV. DISCUSSION

There have been two points made in this comment. The first is that for the simplest extension of the Weinberg-Salam[1] scheme which allows P violation to arise spontaneously, there is only one transformation on the spinless fields under the global γ_5 group which ensures a massless neutrino to all orders (a ground rule of the W-S model) and which also admits a spontaneous CP violation. The second, related, point is that this CP violation must arise through loop corrections to the effective potential. An example of how this can occur was presented with a slightly restricted potential. The CP violation and the mass of one of the Higgs particles both developed from the one-fermion-loop term in the effective potential.

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[2] P. Fayet, Nucl. Phys. B74, 14 (1974). A supergauge extension is discussed by Fayet [Nucl. Phys. B90, 104 (1975)]. T invariance holds in the spontaneously broken theory in this supergauge version.

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[4] J. M. Herbert, Nucl. Phys. B97, 426 (1975).

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[6]T. D. Lee, Phys. Rev. D 8, 1226 (1973). Lee considers a version of the W-S scheme which allows spontaneous *CP* violation, but where *P* violation is still explicit. It is the additional, explicit, *P*-symmetry restrictions which prohibit a *CP*-violating phase from emerging in tree approximation in the models discussed in the present paper. The distinctions between real and complex representation examples are drawn by T. D. Lee, Phys. Rep. 96, 143 (1974).

[7]A. Zee, Phys. Rev. D 9, 1772 (1974).

[8]R. N. Mohapatra, Phys. Rev. D 9, 3461 (1974).

[9]H. Georgi and A. Pais, Phys. Rev. D 10, 1246 (1974).

[10]Expressed in terms of fields s and t with definite parity assignments, the potential U reads

$$\begin{aligned}U(s,t) = 2A\,(s^\dagger s + t^\dagger t) &+ (2B + C + D)\;[(s^\dagger s)^2 + (t^\dagger t)^2]\\ &+ (2B - C - D)\;[(t^\dagger s)^2 + (s^\dagger t)^2]\\ &+ (8B - 2C + 2D)\,(t^\dagger s)(s^\dagger t)\\ &+ (t^\dagger t)(s^\dagger s)(4B + 2C - 2D),\end{aligned}$$

where $\phi_L \equiv s - t$ and $\phi_R \equiv s + t$. The conditions $\partial U/\partial\phi_L^\dagger\,(\phi_L, \phi_R) = 0$ and $\partial U/\partial\phi_R^\dagger = 0$ imply that $2B-C-D = 0$ and $A + 2B(\lambda_R{}^2 + \lambda_L{}^2) = 0$.

[11]Case (b) appears to satisfy the conditions of perturbative breaking discussed by Georgi and Pais (Ref. 9). No zero-mass field is present when $\tilde{\theta} = 0$, and one therefore expects that $\tilde{\theta} = 0$ will still hold even after loop corrections are made.

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