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Meson masses and electromagnetic decays in SU(4) six-quark models

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The phenomenological consequences of SU(4) six-quark models are investigated including the general features of the vector and pseudoscalar meson multiplets and their masses and radiative decay widths. Various experimental tests, some of which are entirely unequivocal, are pointed out. Certain similarities and differences with other six-quark models as well as the standard SU(4) four-quark model are also briefly discussed.

I. INTRODUCTION

Recently there have been a number of experimental indications that there might be some new kind of quantum number.[1] One attractive possibility is a single new flavor, charm,[2] in which case the hadron symmetry group is extended from SU(3) to SU(4). In terms of quarks the standard SU(4) model[3] involves four quarks $\mathcal{P}, \mathfrak{N}, \lambda, \mathcal{P}'$ (or u, d, s, c) belonging to the fundamental four-dimensional representation $(1,0,0)$ of SU(4).

However, there is an alternative possible SU(4) charm model[4,5] which involves six quarks $\mathcal{P}, \mathfrak{N}, \lambda, x, y, z$ belonging to the fundamental six-dimensional representation $(0,1,0)$ of SU(4). In this six-quark model bosons and fermions will belong to the class-0 and class-2 representations of SU(4), respectively; the class-1 and class-3 representations, e.g., $4, \overline{4}, \ldots,$ are not used at all. The underlying symmetry group is actually $SU(4)/Z(2)$, which is isomorphic to O(6). Here we examine some of the consequences of this possible model and point out some experimental tests.

Various other six-quark models have been discussed in the literature,[6-12] each having certain attractive aspects and most sharing some common features. In the following discussion some comparisons between these models and the SU(4) six-quark model will be made, assuming that all of the new quark composites lie at currently accessible energies. That is, we identify the new vector mesons $J/\psi \equiv \psi$, ψ', and ψ'' with quark-antiquark $^3S_1$ ground states of the new quarks in contrast to the standard model where ψ' and ψ'' are radial excitations.[13] We will not consider the possibility here that any of the new quarks are so massive as to be of no practical importance at present energies.

The most significant experimental tests which distinguish between the various six-quark models, as well as the standard four-quark model, are the different SU(3)-multiplet structures of the new particles composed of the charmed quarks. These multiplet predictions are clear and can serve to rule out any of the models. Although the mass spectra and radiative decay widths in each of the models are qualitatively different, they provide only experimental restrictions on the models since clear quantitative distinctions are blurred by mixing arising from symmetry breaking. We therefore will present in detail these features only for the SU(4) six-quark models.

We begin the discussion with the quark composition of the pseudoscalar- and vector-meson states including the mixing arising from breaking of SU(3) and higher symmetries. The lepton-pair decay rates of the vector mesons ρ, ω, ϕ, ψ, ψ', and ψ'' are then calculated and found to be in good agreement with the data. Nine mass formulas are obtained, three of which agree with the existing data, while the remaining six predict the masses of the as yet undiscovered charmed vector mesons. Next we discuss the quark composition of the pseudoscalar mesons and the mixing of the four isoscalars η, η', η_c, and η_c' which turns out to be somewhat more complex than the corresponding mixing of the four isoscalar vector mesons ω, ϕ, ψ, and ψ'. The radiative decays $P\to\gamma+\gamma$, $V\to P+\gamma$, and $P\to V+\gamma$ are considered, emphasizing their importance for determining the various mixing angles. The quark-antiquark orbital excitations, particularly the P states, are discussed, and we remark on possible radial excitations.

II. QUARK COMPOSITION AND MIXING IN SU(4)/Z(2) MODELS

Since the group SU(4) contains two invariant subgroups, $Z(2)$ and $Z(4)$, it is possible to formulate hadron symmetries[4] based on $SU(4)/Z(2)$ or $SU(4)/Z(4)$ as well as the full group SU(4). For the hadron symmetry group $SU(4)/Z(2)$, which is isomorphic to O(6), there are six quarks $(\mathcal{P}, \mathcal{N}, \lambda, x, y, z)$ belonging to the fundamental six-dimensional representation (0, 1, 0) just as the four quarks $\mathcal{P}$, $\mathcal{N}$, λ, and $\mathcal{P}'$ belong to the four-dimensional representation (1, 0, 0) in the standard model[3]; however, only one new quantum number, charm, is introduced, in contrast with other six-quark models[6-12] based on SU(6), which requires three new quantum numbers.

Under $SU(3)\times U(1)$ the six quarks decompose into a $\underline{3}$ $(\mathcal{P}, \mathcal{N}, \lambda)$ and a $\underline{\bar{3}}$ (x, y, z), which differ by one unit of charm as shown in Fig. 1. The usual Gell-Mann–Zweig quarks[14] $(\mathcal{P}, \mathcal{N}, \lambda)$ are assigned charm $C=0$, while the new, presumably much heavier, quarks (x, y, z) are assigned $C=-1$.

The 36 mesons composed of quark-antiquark pairs belong to the SU(4) representations

$$\underline{6}\times\underline{6}=\underline{1}+\underline{15}+\underline{20} \tag{2.1}$$

or, in terms of their $SU(3)\times U(1)$ decompositions,

$$\begin{pmatrix}\underline{3}\\ \\ \underline{\bar{3}}\end{pmatrix}\times\begin{pmatrix}\underline{3}\\ \\ \underline{\bar{3}}\end{pmatrix}=(\underline{1})+\begin{pmatrix}\underline{\bar{3}}\\ \underline{1}+\underline{8}\\ \underline{3}\end{pmatrix}+\begin{pmatrix}\underline{6}\\ \underline{8}\\ \underline{\bar{6}}\end{pmatrix}. \tag{2.2}$$

The nine charmed $C=+1$ mesons are composed of the light quarks $(\mathcal{P}, \mathcal{N}, \lambda)$ and the heavy antiquarks $(\bar{z}, \bar{y}, \bar{x})$, and, of course, the nine $C=-1$ mesons are composed of $\bar{\lambda}, \bar{\mathcal{N}}, \bar{\mathcal{P}}$ and x, y, z. The uncharmed eighteen $C=0$ states consist of linear combinations of light-quark pairs and heavy-quark pairs.

Since the $SU(4)/Z(2)$ symmetry is badly broken, worse than SU(3), in addition to large mass splittings one expects a great deal of mixing among SU(4) states in analogy to singlet-octet mixing in SU(3). Among the 36 mesons there are four isoscalars whose mixing in general is described by six real parameters. For purposes of illustration, it will be sufficient to represent the mixing by a restricted form involving only three angles. We shall assume the SU(3) singlets in the SU(4) multiplets $\underline{1}$ and $\underline{15}$ mix with an angle ϕ_1, while the SU(3) octets in $\underline{15}$ and $\underline{20}$ mix with an angle ϕ_8. At the SU(3) level the mixing between the singlet and octet, which are primarily composed of light quarks, will be described by an angle θ, while the heavy-quark singlet and octet are mixed by Θ.

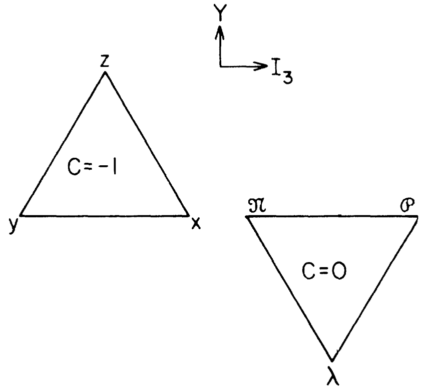

FIG. 1. The six-dimensional quark representation (0, 1, 0) of $SU(4)/Z(2)$.

For the vector mesons, we shall assume that the SU(4) mixing is ideal in that the light mesons (ρ, ω, ϕ) consist entirely of light quarks in agreement with the familiar SU(3) model, while the heavy mesons (ψ, ψ', ψ'') are composed entirely of heavy quarks. Ideal mixing implies that $\phi_1^V = \phi_8^V = \pi/4$ as will be seen in Sec. III.

For the pseudoscalar mesons, a certain amount of nonideal mixing will be required to account for the π, η, η' mass spectrum. For this purpose we shall take $\phi_8^P=\pi/4$ but choose $\phi_1^P \neq \pi/4$ as discussed below in Sec. IV.

Explicitly, the quark composition of the uncharmed SU(4) meson states $|D_4, D_3, D_2\rangle$, which have $I_3=0$ and belong to the SU(4), SU(3), and SU(2) representations with dimensions D_4, D_3, and D_2, respectively, are the following:

$$|1,1,1\rangle=\frac{1}{\sqrt{6}}(\mathcal{P}\bar{\mathcal{P}}+\mathcal{N}\bar{\mathcal{N}}+\lambda\bar{\lambda}+x\bar{x}+y\bar{y}+z\bar{z}), \tag{2.3a}$$

$$|15,1,1\rangle=\frac{1}{\sqrt{6}}(-\mathcal{P}\bar{\mathcal{P}}-\mathcal{N}\bar{\mathcal{N}}-\lambda\bar{\lambda}+x\bar{x}+y\bar{y}+z\bar{z}), \tag{2.3b}$$

$$|15,8,1\rangle=\frac{1}{2\sqrt{3}}(\mathcal{P}\bar{\mathcal{P}}+\mathcal{N}\bar{\mathcal{N}}-2\lambda\bar{\lambda}-x\bar{x}-y\bar{y}+2z\bar{z}), \tag{2.3c}$$

$$|20,8,1\rangle=\frac{-1}{2\sqrt{3}}(\mathcal{P}\bar{\mathcal{P}}+\mathcal{N}\bar{\mathcal{N}}-2\lambda\bar{\lambda}+x\bar{x}+y\bar{y}-2z\bar{z}), \tag{2.3d}$$

$$|15,8,3\rangle=\tfrac{1}{2}(\mathcal{P}\bar{\mathcal{P}}-\mathcal{N}\bar{\mathcal{N}}+x\bar{x}-y\bar{y}), \tag{2.3e}$$

$$|20,8,3\rangle=\tfrac{1}{2}(-\mathcal{P}\bar{\mathcal{P}}+\mathcal{N}\bar{\mathcal{N}}+x\bar{x}-y\bar{y}). \tag{2.3f}$$

III. VECTOR-MESON MASSES AND LEPTON-PAIR WIDTHS

We shall assume the SU(4) mixing of the vector mesons is ideal, in which case ρ, ω, and ϕ are composed only of the Gell-Mann–Zweig quarks[14] $\mathcal{P}, \mathcal{N}, \lambda$, while ψ, ψ', and ψ'' are composed only of the charmed quarks x, y, z. However, we allow for an arbitrary mixing between the light singlet and octet described by the ω-ϕ mixing angle θ. In addition there is an independent ψ-ψ' angle Θ describing the mixing between the heavy singlet and octet. The wave function of the neutral $C=0$ vector mesons can then be written in terms of the SU(4) states $|D_4, D_3, D_2\rangle$ given above as follows:

$$|\rho^0\rangle = [\,|15,8,3\rangle - |20,8,3\rangle\,]/\sqrt{2}\ , \tag{3.1a}$$

$$|\omega\rangle = \cos\theta[\,|1,1,1\rangle - |15,1,1\rangle\,]/\sqrt{2} + \sin\theta[\,|15,8,1\rangle - |20,8,1\rangle\,]/\sqrt{2}\ , \tag{3.1b}$$

$$|\phi\rangle = \sin\theta[\,|1,1,1\rangle - |15,1,1\rangle\,]/\sqrt{2} - \cos\theta[\,|15,8,1\rangle - |20,8,1\rangle\,]/\sqrt{2}\ , \tag{3.1c}$$

$$|\psi\rangle = \cos\theta[\,|1,1,1\rangle + |15,1,1\rangle\,]/\sqrt{2} + \sin\Theta[\,|15,8,1\rangle + |20,8,1\rangle\,]/\sqrt{2}\ , \tag{3.1d}$$

$$|\psi'\rangle = \sin\Theta[\,|1,1,1\rangle + |15,1,1\rangle\,]/\sqrt{2} - \cos\Theta[\,|15,8,1\rangle + |20,8,1\rangle\,]/\sqrt{2}\ , \tag{3.1e}$$

$$|\psi''\rangle = [\,|15,8,3\rangle + |20,8,3\rangle\,]/\sqrt{2}\ . \tag{3.1f}$$

The vector-meson lepton-pair partial widths provide an important test of any six-quark model.[15] We consider two possible choices of electric charges for the six quarks:

$$\text{(A)}\ Q_{\mathcal{P}} = \tfrac{2}{3},\ Q_{\mathfrak{N}} = Q_\lambda = Q_x = Q_z = -\tfrac{1}{3},\ Q_y = -\tfrac{4}{3} \tag{3.2a}$$

and

$$\text{(B)}\ Q_{\mathcal{P}} = Q_x = Q_z = \tfrac{2}{3},\ Q_{\mathfrak{N}} = Q_\lambda = Q_y = -\tfrac{1}{3}\ . \tag{3.2b}$$

Possibility A arises in the six-quark gauge model of weak and electromagnetic interactions we have previously proposed[5] and B has first been discussed by Harari.[7] For these charges the ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ is 8 for model A and 5 for model B. While model B agrees better with data on R at present energies, we shall see below that the lepton-pair widths of the vector mesons which result from this charge assignment are in very significant disagreement with the experimentally known widths. On the other hand, unless R increases at higher energy and approaches 8 asymptotically, model A is untenable also.

In model A the electromagnetic current can be expressed in terms of quarks or SU(4) currents as follows:

$$J_\mu = \tfrac{1}{3}(2\bar{\mathcal{P}}\gamma_\mu\mathcal{P} - \bar{\mathfrak{N}}\gamma_\mu\mathfrak{N} - \bar{\lambda}\gamma_\mu\lambda - \bar{z}\gamma_\mu z - \bar{x}\gamma_\mu x - 4\bar{y}\gamma_\mu y)$$
$$= V_\mu^3 + \frac{1}{\sqrt{3}}V_\mu^8 - \left(\frac{2}{3}\right)^{1/2}V_\mu^{15} - \left(\frac{2}{3}\right)^{1/2}V_\mu^0\ , \tag{3.3a}$$

while for model B

$$J_\mu = \tfrac{1}{3}(2\bar{\mathcal{P}}\gamma_\mu\mathcal{P} - \bar{\mathfrak{N}}\gamma_\mu\mathfrak{N} - \bar{\lambda}\gamma_\mu\lambda + 2\bar{z}\gamma_\mu z + 2\bar{x}\gamma_\mu x - \bar{y}\gamma_\mu y)$$
$$= V_\mu^3 + \frac{1}{\sqrt{3}}V_\mu^8 + \frac{1}{\sqrt{6}}V_\mu^{15} + \frac{1}{\sqrt{6}}V_\mu^0\ . \tag{3.3b}$$

The lepton-pair partial width of the vector meson V is given by

$$\Gamma = \frac{4\pi\alpha}{3m_V^{\,3}}|\langle 0|\epsilon\cdot J|V\rangle|^2\ , \tag{3.4}$$

where the amplitude $\langle 0|\epsilon\cdot J|V\rangle$ is evidently a linear combination of the matrix elements $\langle 0|\epsilon\cdot V^\alpha|D_4, D_3, D_2\rangle$. The only such nonvanishing matrix elements can be written in terms of a single form factor

$$g(k^2) = \langle 0|\epsilon\cdot V^0|1,1,1\rangle = \langle 0|\epsilon\cdot V^{15}|15,1,1\rangle = \langle 0|\epsilon\cdot V^8|15,8,1\rangle = \langle 0|\epsilon\cdot V^3|15,8,3\rangle\ , \tag{3.5}$$

assuming the hadron symmetry is U(4) in the equal-mass limit. Although the physical masses are not equal, the different coupling constants that result when the form factor $g(k^2)$ is evaluated at $k^2 = m_V^{\,2}$ for each of the six vector mesons are related by the Weinberg first-spectral-function sum rule.[16]

Weinberg's first-spectral-function sum rule, extended to the 16 currents of U(4), is

$$\int dm^2 \frac{\rho_{\alpha\beta}(m^2)}{m^2} = S\delta_{\alpha\beta}\quad (\alpha,\beta = 0,3,8,15), \tag{3.6}$$

where $\rho_{\alpha\beta}(m^2)$ is the spin-1 spectral function occurring in the Källén-Lehmann representation of the current propagator $\langle 0|TV_\mu^\alpha(x)V_\nu^\beta(0)|0\rangle$. Saturating Eq. (3.6) with the six vector mesons ρ, ω, ϕ, ψ, ψ', and ψ'' requires that the ratio $g^2(m_V^{\,2})/m_V^{\,2}$ be independent of the vector-meson mass. Then independent of the ω-ϕ and ψ-ψ' mixing angles θ and Θ there are three relations among the lepton-pair widths. For model A one finds

$$m_\rho\Gamma_\rho = 3(m_\omega\Gamma_\omega + m_\phi\Gamma_\phi) = \tfrac{1}{3}(m_\psi\Gamma_\psi + m_{\psi'}\Gamma_{\psi'}) = m_{\psi''}\Gamma_{\psi''}\ , \tag{3.7a}$$

while for model B

$$m_\rho\Gamma_\rho = 3(m_\omega\Gamma_\omega + m_\phi\Gamma_\phi) = m_\psi\Gamma_\psi + m_{\psi'}\Gamma_{\psi'} = m_{\psi''}\Gamma_{\psi''}\ . \tag{3.7b}$$

In addition, in each model there are two relations depending on the mixing angles. In both models

$$\frac{m_\omega\Gamma_\omega}{m_\phi\Gamma_\phi} = \tan^2\theta, \tag{3.8}$$

and in model A

$$\frac{m_\psi\Gamma_\psi}{m_{\psi'}\Gamma_{\psi'}} = \left(\frac{2\sqrt{2} - \tan\Theta}{1 + 2\sqrt{2}\tan\Theta}\right)^2 \tag{3.9a}$$

while in model B

$$\frac{m_\psi \Gamma_\psi}{m_{\psi'} \Gamma_{\psi'}} = \left(\frac{\sqrt{2} + \tan\Theta}{1 - \sqrt{2}\tan\Theta}\right)^2 . \tag{3.9b}$$

These results can also be obtained from the naive-quark-model amplitude, Fig. 2, using the wave functions given in Eqs. (2.3) and (3.1).

From the experimental widths,[17] we find

$$m_\rho \Gamma_\rho : 3(m_\omega \Gamma_\omega + m_\phi \Gamma_\phi) : \tfrac{1}{3}(m_\psi \Gamma_\psi + m_{\psi'} \Gamma_{\psi'}) : m_{\psi''}\Gamma_{\psi''}$$
$$= 4.97 \pm 0.67 : 5.90 \pm 0.73 : 7.65 \pm 1.36 : \; ? \tag{3.10}$$

in rather good agreement with the prediction of model A, Eq. (3.7a), but in poor agreement with model B, Eq. (3.7b). Therefore, in the following we shall restrict our attention to the charges of model A, Eq. (3.2a). The lepton-pair width of the isovector ψ'' is predicted from Eq. (3.10) to be

$$\Gamma_{\psi''} \simeq 1.5 \text{ keV}. \tag{3.11}$$

From the data, Eqs. (3.8) and (3.9a) imply $\tan\theta = 0.66 \pm 0.08$ and $\tan\Theta = 0.31 \pm 0.08$. These empirical angles suggest the magic-mixing hypothesis for which

$$\tan\theta = 1/\sqrt{2} \simeq 0.71 \tag{3.12}$$

and

$$\tan\Theta = 1/2\sqrt{2} \simeq 0.35. \tag{3.13}$$

With magic mixing, ω and ϕ are composed of purely nonstrange and strange quarks, respectively, while ψ and ψ' transform purely as octet and singlet components, respectively, under the SU(3) subgroup whose multiplets lie in the planes normal to the direction of strangeness plus charm.[4] Under the usual SU(3), whose multiplets are normal to the direction of charm, ψ is predominantly a singlet and ψ' is predominantly an octet component.

For these magic-mixing angles, the lepton-pair widths are in the ratios

$$m_\rho \Gamma_\rho : m_\omega \Gamma_\omega : m_\phi \Gamma_\phi : m_\psi \Gamma_\psi : m_{\psi'} \Gamma_{\psi'} : m_{\psi''} \Gamma_{\psi''}$$
$$:: 9 : 1 : 2 : \tfrac{49}{3} : \tfrac{32}{3} : 9. \tag{3.14}$$

Experimentally, one has[17]

$$\frac{m_\rho \Gamma_\rho}{9} : \frac{m_\omega \Gamma_\omega}{1} : \frac{m_\phi \Gamma_\phi}{2} : \frac{3 m_\psi \Gamma_\psi}{49} : \frac{3 m_{\psi'} \Gamma_{\psi'}}{32} : \frac{m_{\psi''} \Gamma_{\psi''}}{9}$$
$$= 0.55 \pm 0.07 : 0.60 \pm 0.14 : 0.69 \pm 0.05 : 0.91 \pm 0.11 : 0.76 \pm 0.21 : \; ? \; . \tag{3.15}$$

The quark wave functions for the neutral vector mesons can be found from Eqs. (2.3) and (3.1) for the magic mixing angles of Eq. (3.12) and (3.13):

$$|\rho^0\rangle = (\mathcal{P}\bar{\mathcal{P}} - \mathcal{N}\bar{\mathcal{N}})/\sqrt{2} \; , \tag{3.16a}$$

$$|\omega\rangle = (\mathcal{P}\bar{\mathcal{P}} + \mathcal{N}\bar{\mathcal{N}})/\sqrt{2} \; , \tag{3.16b}$$

$$|\phi\rangle = \lambda\bar{\lambda} \; , \tag{3.16c}$$

$$|\psi\rangle = (x\bar{x} + y\bar{y} + 2z\bar{z})/\sqrt{6} \; , \tag{3.16d}$$

$$|\psi'\rangle = (x\bar{x} + y\bar{y} - z\bar{z})/\sqrt{3} \; , \tag{3.16e}$$

$$|\psi''\rangle = (x\bar{x} - y\bar{y})/\sqrt{2} \; . \tag{3.16f}$$

In the naive quark model, these neutral-vector-meson wave functions lead to two mass formulas: the well-known relation

$$m_\rho^2 = m_\omega^2 \simeq (0.783 \text{ GeV})^2 \tag{3.17}$$

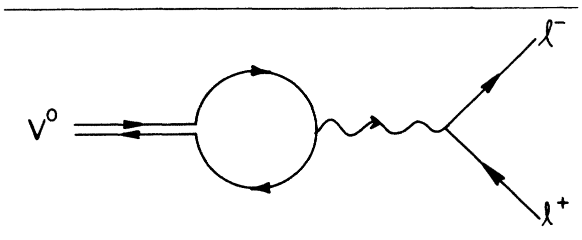

FIG. 2. Quark diagram for vector-meson decay into a lepton pair.

and the new relation

$$m_{\psi''}^2 = 2m_{\psi'}^2 - m_\psi^2 \simeq (4.19 \text{ GeV})^2 \; , \tag{3.18}$$

which predicts the mass of the $I = 1$ vector meson whose neutral component is presumably one of the peaks seen in the recent SPEAR data.[13]

We can extend these naive-quark-model arguments to the strange and charmed $^3S_1$ $q\bar{q}$ ground states that occur in the product $\underline{6} \times \underline{6}$, and calculate the masses of all the 36 vector mesons in the SU(4) six-quark model. These are two charm-0, strangeness-(+1) isodoublets K^* and K_c^* and their masses are

$$m_{K^*}^2 = \tfrac{1}{2}(m_\omega^2 + m_\phi^2) \simeq (0.909 \text{ GeV})^2 \tag{3.19}$$

and

$$m_{K_c^*}^2 = \tfrac{1}{2}(m_\psi^2 + m_{\psi'}^2) \simeq (3.40 \text{ GeV})^2 \; . \tag{3.20}$$

The K^* mass agrees well with the observed value, $m_{K^*} = 0.892$ GeV, and Eq. (3.20) is a prediction.

The singlet σ^{*+} and triplet τ^{*++}, τ^{*+}, τ^{*0} vector mesons have $C = 1$, $S = 0$ and their masses are

$$m_{\sigma^*}^2 = m_{\tau^*}^2 = \tfrac{1}{2}(m_\rho^2 + m_{\psi''}^2) \simeq (2.99 \text{ GeV})^2 \; . \tag{3.21}$$

There are two $C = 1$, $S = -1$ vector-meson doublets ξ^{*+}, ξ^{*0} and ζ^{*+}, ζ^{*0} with masses

$$m_{\xi^*}^2 = m_{\zeta^*}^2 = \tfrac{1}{4}(m_\omega^2 + m_\phi^2 + m_\psi^2 + m_{\psi'}^2) \simeq (2.49\ \mathrm{GeV})^2\ . \tag{3.22}$$

Lastly, the mass of the $C=+1$, $S=-2$ singlet vector meson θ^{*0} is

$$m_{\theta^*}^2 = \tfrac{1}{2}(m_\omega^2 + m_\phi^2 + m_\psi^2 + m_{\psi'}^2 - m_\rho^2 - m_{\psi''}^2) \simeq (1.87\ \mathrm{GeV})^2\ . \tag{3.23}$$

Observe that $\tau^*, \zeta^*, \theta^*$ which belong to an SU(3) $\underline{6}$ are equally spaced, and that their splitting is equal to that of σ^* and ξ^* which belong to an SU(3) $\underline{\bar{3}}$.

Altogether, nine mass formulas have been obtained, three of which can be compared with existing data, and the agreement is excellent. The other six relations predict the masses of the strange and charmed vector mesons that remain to be discovered. The resulting vector-meson mass spectrum is shown in Fig. 3.

In the other six-quark models the vector-meson spectrum is similar although the quantum numbers of the vector mesons differ due to different quark quantum numbers. For example, in Barnett's model[6] the $C=1$ states form an SU(3) $\underline{1}+\underline{8}$, while in the vector model[9-12] the $C=1$ states transform like $\underline{\bar{3}}+\underline{\bar{3}}+\underline{\bar{3}}$, and for Harari's SU(6) model[7] they belong to $\underline{\bar{3}}+\underline{6}$ as in our SU(4) model.[5] The exact masses will differ due to different mixing, and the lepton-pair widths will depend on the quark charges. All the six-quark models having Harari's charges [Eq. (3.2b)] disagree with the observed lepton-pair widths of the new vector mesons.

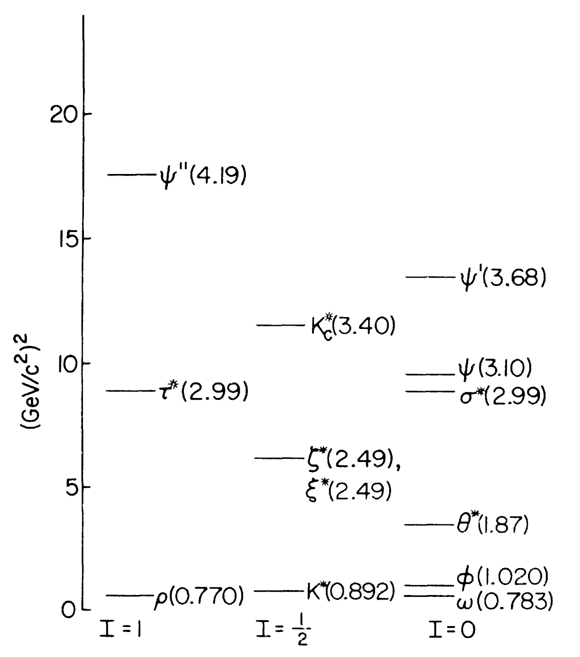

FIG. 3. Vector-meson mass spectrum predicted in the SU(4) six-quark model with magic mixing.

IV. PSEUDOSCALAR-MESON MASSES AND MIXING ANGLES

The rather large masses of the $\eta(549)$ and $\eta'(958)$ relative to the pion and kaon masses suggest that the isospin-singlet mesons may contain an admixture of heavy quarks.[19] On the other hand, the very small mass of the pion suggests that it is composed of purely light quarks. These observations suggest that the SU(3) singlets contained in the SU(4) multiplets $\underline{1}$ and $\underline{15}$ are not ideally mixed in the case of the pseudoscalars ($\phi_1^P \neq \pi/4$) although the octets in the $\underline{15}$ and $\underline{20}$ are ideally mixed ($\phi_8^P = \pi/4$). With $\phi_1^P = \phi$, $\phi_8^P = \pi/4$, but arbitrary singlet-octet mixing angles θ and Θ, the quark wave functions of the neutral $C=0$ pseudoscalar mesons can be written

$$|\pi^0\rangle = (\mathcal{P}\bar{\mathcal{P}} - \mathcal{N}\bar{\mathcal{N}})/\sqrt{2}\ , \tag{4.1a}$$

$$\begin{aligned}|\eta\rangle = &-\cos\theta(\mathcal{P}\bar{\mathcal{P}} + \mathcal{N}\bar{\mathcal{N}} - 2\lambda\bar{\lambda})/\sqrt{6} \\ &+ \sin\theta(\cos\phi + \sin\phi)(\mathcal{P}\bar{\mathcal{P}} + \mathcal{N}\bar{\mathcal{N}} + \lambda\bar{\lambda})/\sqrt{6} \\ &+ \sin\theta(\cos\phi - \sin\phi)(x\bar{x} + y\bar{y} + z\bar{z})/\sqrt{6}\ ,\end{aligned} \tag{4.1b}$$

$$\begin{aligned}|\eta'\rangle = &\cos\theta(\cos\phi + \sin\phi)(\mathcal{P}\bar{\mathcal{P}} + \mathcal{N}\bar{\mathcal{N}} + \lambda\bar{\lambda})/\sqrt{6} \\ &+ \cos\theta(\cos\phi - \sin\phi)(x\bar{x} + y\bar{y} + z\bar{z})/\sqrt{6} \\ &+ \sin\theta(\mathcal{P}\bar{\mathcal{P}} + \mathcal{N}\bar{\mathcal{N}} - 2\lambda\bar{\lambda})/\sqrt{6}\ ,\end{aligned} \tag{4.1c}$$

$$\begin{aligned}|\eta_c\rangle = &\cos\Theta(x\bar{x} + y\bar{y} - 2z\bar{z})/\sqrt{6} \\ &+ \sin\Theta(\sin\phi - \cos\phi)(\mathcal{P}\bar{\mathcal{P}} + \mathcal{N}\bar{\mathcal{N}} + \lambda\bar{\lambda})/\sqrt{6} \\ &+ \sin\Theta(\sin\phi + \cos\phi)(x\bar{x} + y\bar{y} + z\bar{z})/\sqrt{6}\ ,\end{aligned} \tag{4.1d}$$

$$\begin{aligned}|\eta_c'\rangle = &\cos\Theta(\sin\phi - \cos\phi)(\mathcal{P}\bar{\mathcal{P}} + \mathcal{N}\bar{\mathcal{N}} + \lambda\bar{\lambda})/\sqrt{6} \\ &+ \cos\Theta(\sin\phi + \cos\phi)(x\bar{x} + y\bar{y} + z\bar{z})/\sqrt{6} \\ &- \sin\Theta(x\bar{x} + y\bar{y} - 2z\bar{z})/\sqrt{6}\ ,\end{aligned} \tag{4.1e}$$

$$|\pi_c^0\rangle = (x\bar{x} - y\bar{y})/\sqrt{2}\ . \tag{4.1f}$$

Note that with the singlet-octet mixing angle θ small so that η is nearly a pure octet component, the heavy quarks can contribute significantly to η' if ϕ is sufficiently different from $\pi/4$, with the result that the η-η' mass splitting can be made rather large as observed.

It is a straightforward matter to explicitly write out the quark wave functions of the strange and charmed pseudoscalar mesons as was done for the vector mesons (Sec. III). One then obtains the following six mass formulas independent of the mixing angles:

$$m_\sigma^2 = m_\tau^2 = \tfrac{1}{2}(m_\pi^2 + m_{\pi_c}^2)\ , \tag{4.2}$$

$$m_\xi^2 = m_\zeta^2 = \tfrac{1}{4}(m_{\eta'}^2 + m_\eta^2 + m_{\eta_c}^2 + m_{\eta_c'}^2)$$
$$= \tfrac{1}{2}(m_\tau^2 + m_\theta^2)$$
$$= \tfrac{1}{2}(m_K^2 + m_{K_c}^2)\ . \tag{4.3}$$

With four independent quark masses ($m_{\mathcal{P}} = m_{\mathfrak{N}}$, m_λ, $m_x = m_y$, m_z) and thirteen pseudoscalar-meson masses, one obtains three additional relations which involve the three undetermined angles ϕ, θ, and Θ. We choose to write

$$\sin 2\phi = \frac{4(m_\xi^2 + m_K^2) - (m_\theta^2 + m_\pi^2) - 3(m_\eta^2 + m_{\eta'}^2)}{4m_\xi^2 - m_\theta^2 - 2m_K^2 - m_\pi^2}\ , \tag{4.4}$$

$$m_\eta^2 = \tfrac{1}{3}\cos^2\theta(4m_K^2 - m_\pi^2) + \tfrac{4}{3}\sin\theta\cos\theta(\cos\phi + \sin\phi)(m_K^2 - m_\pi^2)$$
$$+ \tfrac{1}{6}\sin^2\theta(\cos\phi + \sin\phi)^2(2m_K^2 + m_\pi^2) + \tfrac{1}{6}\sin^2\theta(\cos\phi - \sin\phi)^2(8m_\xi^2 - 2m_\theta^2 - 2m_K^2 - m_\pi^2)\ , \tag{4.5}$$

and

$$m_{\eta_c}^2 = \tfrac{1}{3}\cos^2\Theta(4m_\xi^2 + 2m_\theta^2 - 4m_K^2 - m_\pi^2) + \tfrac{4}{3}\sin\Theta\cos\Theta(\cos\phi + \sin\phi)(2m_\xi^2 - 2m_\theta^2 + m_K^2 - m_\pi^2)$$
$$+ \tfrac{1}{6}\sin^2\Theta(\cos\phi - \sin\phi)^2(2m_K^2 + m_\pi^2) + \tfrac{1}{6}\sin^2\Theta(\cos\phi + \sin\phi)^2(8m_\xi^2 - 2m_\theta^2 - 2m_K^2 - m_\pi^2)\ . \tag{4.6}$$

In the following, it will prove convenient to select a set of values for the masses of the charmed pseudoscalars θ, ξ, and ζ. The light-heavy quark mixing in the unitarity singlet is then determined in terms of ϕ by Eq. (4.4). From Eq. (4.5) one can then compute the singlet-octet mixing angle θ for the light pseudoscalars η and η'. By choice of the singlet-octet mixing angle Θ for the heavy pseudoscalars all the remaining masses are determined.

To illustrate various possibilities for the mass spectrum consistent with what is known at present, we shall give three numerical examples. In each case the masses of the established pseudoscalar nonet $\pi(135)$, $K(496)$, $\eta(549)$, and $\eta'(958)$ are input. We further require that $m_\sigma = m_\tau > m_\xi = m_\zeta > m_\theta$ corresponding to the strangeness-(+1) quark z being lighter than the nonstrange quark doublet (x, y), in agreement with the vector-meson mass spectrum.

In addition, we impose the following conditions: Since $\psi'(3.7)$ is quite narrow, we require that $m \geq 1.85$ GeV to prohibit the strong decay $\psi' \to \theta + \bar{\theta}$. On the other hand, the charmed mesons must not be too heavy if we are to identify at least some of the structure in R observed at SPEAR[18] beginning at about 4 GeV with charmed-meson pair production. We have also chosen to present examples for which one of the new $I = 0$ pseudoscalars, either η_c or η_c', has a mass of about 2.75 GeV as suggested by experiments at DESY.[20] Finally, in all examples we have assumed the singlet-octet mixing among the heavy quarks is magic so that

$$\Theta = \arctan(1/\sqrt{2}) \simeq 35.3° \tag{4.7}$$

In Table I we present the resulting masses and mixing angles for three examples consistent with the above thinking. The choice of ideal mixing angle Θ in Eq. (4.7) tends to maximize the separation between the η_c and η_c' masses since the η_c' is composed mostly of z quarks, while the η_c contains primarily the heavier x and y quarks. In each example either η_c or η_c' is around 2.75 GeV while the other one occurs either (a) below 2.75 GeV, (b) in the 3.5 GeV region among the P states,[21] or (c) higher still at 4.5 GeV. The mass spectra are presented in Figs. 4(a), 4(b), and 4(c) to illustrate the implications of Eqs. (4.2)–(4.3).

In case (a) the charmed mesons θ, ξ, ζ, τ, and σ lie very close together and all can be pair-produced in the 4–5 GeV region at SPEAR. In cases (b) and (c) only the θ meson can be pair-produced near the ψ''. For each case, the singlet-octet mixing angle θ for the light mesons is small (−3°), which implies that the η is mostly an octet member while the η' is mostly a singlet.[22]

The light-heavy quark mixing in the SU(3)-singlet

TABLE I. Pseudoscalar mass spectra and mixing angles for illustrative cases (a), (b), and (c) discussed in Sec. IV.

Meson	Mass (GeV/c^2) (a)	(b)	(c)
θ (input)	1.90	2.00	1.85
$\xi = \zeta$ (input)	1.96	2.30	2.70
$\tau = \sigma$	2.01	2.57	3.34
π_c	2.84	3.63	4.72
K_c	2.72	3.21	3.79
η_c	2.76	3.49	4.48
η_c'	2.54	2.78	2.79
	Mixing angle (degrees) (a)	(b)	(c)
Θ (input)	35.3	35.3	35.3
θ	−3.1	−3.1	−3.0
ϕ	26.8	30.2	33.3

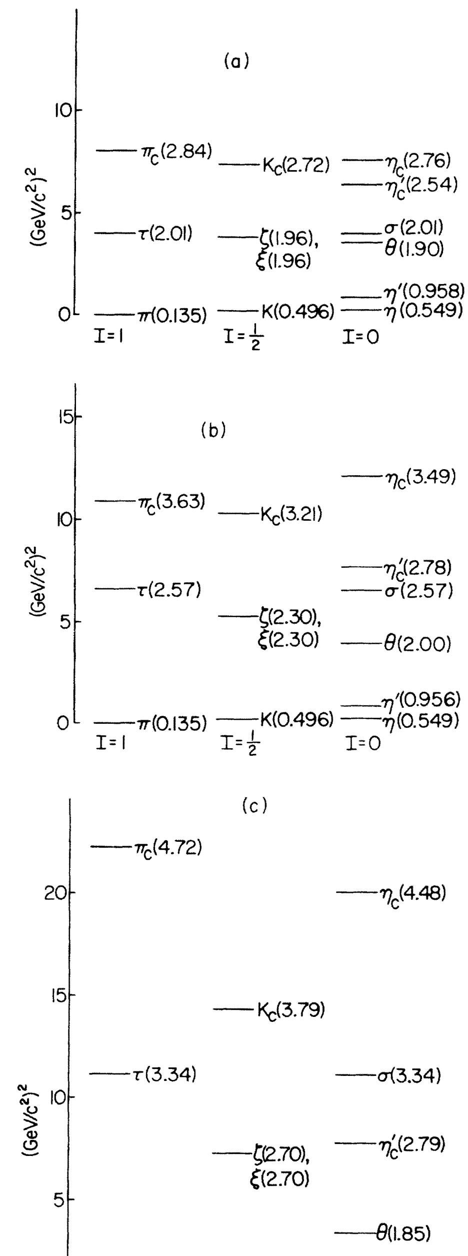

FIG. 4. Pseudoscalar-meson mass spectrum predicted in the SU(4) six-quark model for cases (a), (b), and (c), respectively.

states, on the other hand, is substantial as ϕ departs significantly from 45° (up to 15°). Note that as the mass of η_c increases, a smaller amount of light-heavy quark mixing is required to produce the 400 MeV separation between the η and η' masses. Since η remains nearly a pure octet

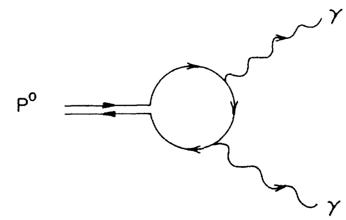

FIG. 5. Quark diagram for the $\gamma+\gamma$ decay mode of a pseudoscalar meson.

member, only the singlet η' contains an appreciable heavy-quark admixture. This will have a significant effect on radiative decay widths as discussed in the next section.

In cases (a) and (b) the pseudoscalar masses are generally lighter than the vector masses found above in Sec. III. However, for case (c) the pseudoscalars are generally heavier than their vector counterparts. As will become apparent in Sec. V, the radiative transitions $P\to\gamma+\gamma$ and $V\to P+\gamma$ are quite different in the three cases and serve to test critically the six-quark models discussed here.

V. RADIATIVE DECAYS

In the preceding sections the quark wave functions and masses of the pseudoscalar and vector mesons have been discussed. In the context of the SU(4) six-quark model[5] we next discuss the radiative decays of these mesons.

In the naive quark model the decays of the pseudoscalar mesons into two photons proceed as shown in Fig. 5. The decay amplitudes, up to an overall normalization, can be readily calculated from the quark wave functions of the pseudoscalar mesons and the quark charges. Including the phase-space corrections the partial widths are given by

$$\Gamma(P\to\gamma+\gamma)=|A(P\to\gamma+\gamma)|^2 m_P^{\,3}\,, \tag{5.1}$$

where $A(P\to\gamma+\gamma)$ is the amplitude given by the naive quark model.

In Table II we present the partial widths calculated for the same three examples of pseudoscalar mass spectra considered in Sec. IV. The experimental value[17] for the partial width $\Gamma(\pi^0\to\gamma+\gamma)=7.8$ eV has been used to determine the absolute rates. There is very little difference in value obtained for the partial width $\Gamma(\eta\to\gamma+\gamma)$ in the three examples and the standard SU(3) calculation[23] since the η remains nearly a pure octet mixture of the light quarks $\mathcal{P}, \mathfrak{N}, \lambda$ in each case. However, in the case of $\eta'(958)$ there is a significant amount of the heavy quarks, x, y, z, admixed with the approximately pure SU(3)-singlet combination of light quarks. In the three examples

TABLE II. $P \to \gamma + \gamma$ partial widths.

Decay	Partial width (keV) (a)	(b)	(c)	Experiment
$\pi \to \gamma\gamma$	0.0078	0.0078	0.0078	0.0078 ± 0.009
$\eta \to \gamma\gamma$	0.291	0.278	0.268	0.329 ± 0.096
$\eta' \to \gamma\gamma$	25.9	21.8	18.6	
$\eta_c' \to \gamma\gamma$	165	250	280	
$\eta_c \to \gamma\gamma$	1500	3160	6930	
$\pi_c \to \gamma\gamma$	1810	3780	8340	

$\Gamma(\eta' \to \gamma + \gamma)$ decreases towards the SU(3)-quark-model value as the masses of η_c and η_c' increase so as to cause the light-heavy mixing angle ϕ to approach the ideal value $\phi = \pi/4$. In any event the agreement with the experimental value is reasonably good in all cases and as a result the experimental value of their partial width does not distinguish between the three cases.

The partial widths of the heavy pseudoscalars η_c, η_c', and π_c are generally quite large compared to the η, η', and π widths. This is due primarily to the dramatic increase in phase space, but in addition the large widths reflect the large magnitude of the y-quark electric charge $Q_y = -\frac{4}{3}$, especially the η_c and π_c widths.

We identify the possible state P_c reported by DESY[20] at about 2.75 GeV with the heavy $I = 0$ pseudoscalar meson at that mass. In case (a) η_c has a mass of 2.76 GeV and the rather large partial width $\Gamma(\eta_c \to \gamma\gamma) = 1500$ keV arises because η_c is approximately composed of only x, y quarks and $Q_y = -\frac{4}{3}$ is large. However, in cases (b) and (c) it is η_c' whose mass is close to 2.75 GeV, and since η_c' is approximately composed of only z quarks and $Q_z = -\frac{1}{3}$ the resulting partial width is considerably smaller.

Unfortunately there is no experimental value available at present for the partial width for $P_c(2.8) \to \gamma + \gamma$; nor are there any candidates for the second heavy isoscalar or the heavy isovector. Such data would not only provide a basis for distinguishing between the four- and six-quark models, but also would differentiate among the six-quark variations.

In the naive quark model the radiative decays of the ${}^3S_1$ $q\bar{q}$ vector mesons into the ${}^1S_0$ $q\bar{q}$ pseudoscalar meson states with the emission of a photon is an M1-quark spin-flip transition. The process is shown in Fig. 6. The partial decay width is given by

$$\Gamma(V \to P + \gamma) = |A(V \to P + \gamma)|^2 (m_V^2 - m_P^2)^3 / m_V^3 , \tag{5.2}$$

where the amplitudes $A(V \to P + \gamma)$ can be calculated using the quark wave function as indicated in Fig. 6. The M1 transition operator depends inversely on the quark mass, and we have attempted to include corrections for the vastly different effective quark masses in the various decays by taking the M1 amplitude $A(V \to P + \gamma)$ proportional to $(m_V + m_P)^{-1}$ in each decay $V \to P + \gamma$. This means of taking into account mass differences is crude, nevertheless it is in the spirit of the naive-quark calculations.

In Table III we give the resulting partial widths for the radiative decays $V \to P + \gamma$ for the three examples of possible pseudoscalar mass spectra discussed above. The vector-meson quark wave functions found in Sec. III were used. The experimental rate[17] $\Gamma(\omega \to \pi^0 + \gamma) = 870$ keV was taken to fix the overall normalization. In the last column of Table III the experimental data are given for comparison, where available. It is evident that in the case of transitions involving only the old mesons the results are essentially the same as the predictions of the standard SU(3) quark model.

For the decays of the new vector mesons the calculated partial widths are generally too large, just as in the case of the standard four-quark SU(4) model.[24] The rates for the decays $\psi \to \eta' + \gamma$ and $\psi' \to \eta' + \gamma$ are particularly large owing to the substantial admixture of heavy quark in the η' required to increase its mass up to 958 MeV from the SU(3)-singlet mass of $(2m_K^2 + m_\pi^2)/3 \simeq (413\ \text{MeV})^2$. This seems to be an unavoidable consequence of assuming that the large η' mass is solely due to a heavy-quark admixture and it occurs in the standard four-quark model, also.[19] We note, however, that the calculated rates have been overestimated in these simple quark-model

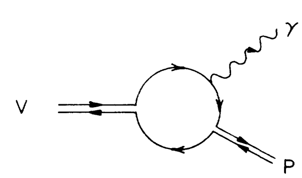

FIG. 6. Quark diagram for the radiative decay of a vector meson into a pseudoscalar meson.

TABLE III. $V \to P + \gamma$ partial widths.

Decay	Partial width (keV) (a)	(b)	(c)	Experiment
$\rho \to \pi\gamma$	94.3	94.3	94.3	35 ± 10
$\to \eta\gamma$	20.0	20.0	20.0	<160
$\omega \to \pi\gamma$	870	870	870	870 ± 80
$\to \eta\gamma$	2.53	2.53	2.53	<50
$\phi \to \pi\gamma$	0	0	0	5.9 ± 2.1
$\to \eta\gamma$	70.8	70.8	70.8	65 ± 15
$\psi \to \eta\gamma$	1.20	0.770	0.494	
$\to \eta'\gamma$	263	176	116	
$\to \eta_c'\gamma$	34.6	6.57	6.05	
$\to \eta_c\gamma$	5.91	...	...	
$\to \pi_c\gamma$	1.91	...	...	
$\psi' \to \eta\gamma$	1.01	0.647	0.415	
$\to \eta'\gamma$	242	162	107	
$\to \eta_c'\gamma$	0.931	0.702	0.857	
$\to \eta_c\gamma$	325	3.42	...	
$\to \pi_c\gamma$	88.3	0.033	...	
$K_c^{*+} \to K_c^{+}\gamma$	910	19.9	...	
$K^{*0} \to K_c^{0}\gamma$	196	3.19	...	

estimates since a perfect overlap of initial and final wave functions has been assumed. Correcting for the more realistic imperfect overlap would result in a large reduction of the calculated widths for transitions between the new vector mesons and the old pseudoscalar mesons since the energy difference is so large.

In Table IV we give the partial widths for the interesting radiative decays $P \to V + \gamma$ that are energetically allowed. The general features are similar to the $V \to P + \gamma$ decays already discussed.

While these naive calculations should not be taken too seriously, the *relative* magnitudes of the partial widths for processes involving similar masses are probably significant. For example, from Table III it is seen that the pseudoscalar around 2.75 GeV is fed much more readily by the $\psi(3.1)$ than the $\psi'(3.7)$ in all three examples in agreement with the data.[20]

TABLE IV. $P \to V + \gamma$ partial widths.

Decay	Partial widths (keV) (a)	(b)	(c)
$\eta_c' \to \rho\gamma$	79.6	53.4	35.3
$\to \omega\gamma$	8.68	5.82	3.85
$\to \phi\gamma$	12.0	8.07	5.34
$\eta_c \to \rho\gamma$	47.2	31.7	21.0
$\to \omega\gamma$	5.17	3.47	2.29
$\to \phi\gamma$	7.52	5.04	3.34
$\to \psi\gamma$	...	23.7	572
$\to \psi'\gamma$	...	...	455
$\pi_c \to \psi\gamma$	...	35.1	535
$\to \psi'\gamma$	...	...	299
$K^+ \to K_c^{*+}\gamma$	...	...	4000
$K^0 \to K_c^{*0}\gamma$	...	...	641

Although it is probably not worthwhile improving the radiative decay rate calculations at this point, we emphasize that these rates are very sensitive to the mixing angle and thus provide an excellent means to determine phenomenologically the quark compositions of the pseudoscalar mesons.

VI. ORBITAL AND RADIAL EXCITATIONS

In addition to the quark-antiquark ground states, of course, one expects both orbital and radial excitations in all the six-quark models just as in the standard SU(4) four-quark model.[13] In particular, there should be $\underline{6} \times \overline{6}$ mesons having spin-parity-(charge conjugation) $J^{PC} = 1^{+-}, 0^{++}, 1^{++}, 2^{++}$ corresponding to the quark-antiquark $^1P_1$, $^3P_0$, $^3P_1$, and $^3P_2$ states, respectively. Presumably, some of these have been seen[21] in the energy region between $\psi(3.1)$ and $\psi'(3.7)$ which are the $l = 1$ levels of the $\psi(3.1)$ combination of quarks in the six-quark models. Above the $\psi'(3.7)$ and $\psi''(4.1)$ one also expects to find P states corresponding to the $l = 1$ excitations of the quarks that make up the $\psi'(3.7)$ and $\psi''(4.1)$. We emphasize that these latter states are radial ground states in the six-quark model whereas in the four-quark model such P states would also be radial excitations. (Recall that in the standard model $\psi(3.7)$ and $\psi(4.1)$ are the first and second radial excitations of $\psi(3.1)$, while all three are $^3S_1$ orbital ground states.)

The spectroscopy is somewhat complicated by the fact that the orbital-excitation energies are smaller than the SU(4) mass splittings. That is, the orbital excitations of $\psi(3.1)$ lie lower than the $\psi'(3.7)$ and $\psi''(4.1)$ members of the ground-state multiplet. However, the spin-orbit splittings seem to be quite small.

In general the P states are expected to decay radiatively into the quark ground states. The decay can proceed via electric dipole emission and the signature is a monoenergetic photon.

The radially excited states in the six-quark SU(4) model are expected to occur at excitation energies somewhat larger than the orbital excitations. All the six-quark models predict $\underline{6} \times \overline{6}$ pseudoscalar and vector mesons which are the radial excitations of the $^1S_0$ and $^3S_1$ orbital ground states. Although there is at present no firm evidence for these states, the $^3S_1$ states should be seen in e^+e^- annihilation, perhaps as weaker and broader peaks much like the $\rho(1600)$ which is presumably the first radial excitation of the $\rho(770)$.

Possibly the broad structure[18] appearing just above 4 GeV in e^+e^- annihilation is due in part to radially excited $^3S_1$ quark states.

VII. SUMMARY

We have investigated phenomenological six-quark models based on SU(4) in some detail with the purpose of determining the spectra, masses, and electromagnetic decay widths of the new heavy vector and pseudoscalar mesons in such schemes. SU(4) six-quark models have the attractive feature that while three new heavy quarks are introduced, only one new quantum number, charm, is required. Such models are therefore interesting to consider as potential alternatives to the admittedly attractive standard SU(4) four-quark model, and should be tested experimentally.

The most unequivocal test of all six-quark models lies in their predicted multiplet structure for the charmed and the heavy uncharmed particles. In particular, in the framework of our SU(4) model as well as in Harari's SU(6) model, the $C=0$ members belong to two sets of $\underline{1}+\underline{8}$, while $C=1\,(-1)$ members belong to $\underline{6}+\underline{\bar{3}}$ $(\underline{\bar{6}}+\underline{3})$ SU(3) multiplets. In other proposed six-quark models, the $C=+1$ mesons belong to either $\underline{1}+\underline{8}$ or $\underline{\bar{3}}+\underline{\bar{3}}+\underline{\bar{3}}$. For the situation of interest to us here, where all three heavy quarks are assumed to be excited at SPEAR energies, the $\psi''(4.1)$ must be an isospin triplet while the lightest charmed mesons (θ and θ^*) in the SU(4) scheme are isospin singlets. In contrast, the standard SU(4) four-quark model places the $C=0$ mesons into $\underline{1}+\underline{1}+\underline{8}$ multiplets and the $C=1\,(-1)$ members into $\underline{3}$ $(\underline{\bar{3}})$ SU(3) multiplets. Although the multiplet structure is simpler in this model, the lightest mesons (D and D^*) are isospin doublets.

In the standard charm model, above charm-pair thresholds the ratio R in e^+e^- annihilation should approach $\frac{10}{3}$ in the absence of heavy-lepton production. On the other hand, in the six-quark SU(4) models it should reach 5 with Harari's choice of charges and 8 for the charge assignment proposed by us. Other charge assignments are more exotic and lead to still higher values of R. Unfortunately, it is not clear just how far above charm threshold the value of R should reach its asymptotic value. At present the largest nonresonant value of R reached at SPEAR is about 5. However, this probably includes the contribution from a heavy lepton, so the standard-model R prediction appears to be in better agreement than any of the six-quark models. Certainly unless R increases at not too much higher energies and approaches 8, our SU(4) six-quark model must be abandoned.

The masses of the $\underline{6}\times\underline{\bar{6}}$ vector mesons which are $^3S_1$ $q\bar{q}$ ground states in the SU(4) quark model have been calculated and are given in Fig. 3. A charm $C=0$ isovector state ψ'' is predicted at about 4.2 GeV and its lepton-pair width was found to be about 1.5 keV. Based on experience with analogous calculations in SU(3) this prediction should be reasonably firm, and therefore poses a real challenge for the model. Comparing the lepton pair widths of the known vector mesons ρ, ω, ϕ, ψ', and ψ'' with the data distinctly favors the electric charge assignment $\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, -\frac{4}{3}$ for the quarks $(\mathcal{P}, \mathfrak{N}, \lambda, z, x, y)$ over the alternative $(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, \frac{2}{3}, -\frac{1}{3})$, although at present energies R is in better agreement with the latter.[25]

The masses and partial widths of the heavy pseudoscalar mesons are somewhat ambiguous since multiplet mixing occurs due to symmetry breakdown. Three possibilities, each consistent with present information, for the mass spectrum of the $\underline{6}\times\underline{\bar{6}}$ $^1S_0$ quark-antiquark ground states have been given [Figs. 4(a), 4(b), and 4(c)] to illustrate the features of the pseudoscalar masses expected in the SU(4) six-quark model.

The partial widths have been given for the electromagnetic decays $P\to\gamma+\gamma$ (Table II), $V\to P+\gamma$ (Table III), and $P\to V+\gamma$ (Table IV) based on the predicted vector-meson spectrum (Fig. 3) for all three examples of pseudoscalar masses [Figs. 4(a), 4(b), and 4(c)]. The radiative widths for the $V\to P+\gamma$ and $P\to V+\gamma$ decay modes are somewhat less reliable than the lepton-pair widths of the vector mesons calculated with Weinberg's first spectral-function sum rule. However, relative magnitudes probably give a reasonably good indication of which decay channels should have the largest branching ratios and therefore restrict the model, but are not very sensitive tests. The $P\to\gamma+\gamma$ widths are the least reliable of all, just as in the case for the analogous SU(3) calculations.

Note added. Since this investigation was completed, recent data from SPEAR indicate the appearance of two narrow peaks at 1.865 GeV and 1.87 GeV in the ($K^-\pi^+$ and $K^-\pi^+\pi^+\pi^-$) and ($K^-\pi^+\pi^+$) channels, respectively. It is tempting to interpret these two states to be isospin-doublet partners D^0 and D^+ in the standard four-quark charm model. These closely spaced peaks cannot be understood in the framework of the six-quark SU(4) models if they are indeed the lightest charmed mesons, since the lowest-lying member should be an isospin singlet.

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[1] J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974); J.-E. Augustin *et al.*, *ibid.* 33, 1406 (1974); C. Bacci *et al.*, *ibid.* 33, 1408 (1974); G. S. Abrams *et al.*, *ibid.* 33, 1453 (1974); J.-E. Augustin *et al.*, *ibid.* 34, 764 (1975); A. Benvenuti *et al.*, *ibid.* 33, 984 (1974); 34, 419 (1975); 34, 597 (1975); 35, 1199 (1975); 35, 1203 (1975); 35, 1249 (1975); J. von Krogh *et al.*, *ibid.* 36, 710 (1976); J. Blietschau *et al.*, Phys. Lett. 60B, 207 (1976).
[2] J. D. Bjorken and S. L. Glashow, Phys. Lett. 11, 255 (1964); D. Amati, H. Bacry, J. Nuyts, and J. Prentki, Nuovo Cimento 34, 1732 (1964); S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).
[3] For a comprehensive exposition of the standard SU(4) model, see M. K. Gaillard, B. W. Lee, and J. Rosner, Rev. Mod. Phys. 47, 277 (1975).
[4] J.-P. Antoine, D. Speiser, and R. J. Oakes, Phys. Rev. 141, 1542 (1966). Mathematical details can be found in this reference.
[5] C. H. Albright and R. J. Oakes, Fermilab Reports Nos. FERMILAB-Pub-75/53-THY (unpublished) and FERMILAB-Pub-75/65-THY (unpublished); R. J. Oakes, in *Proceedings of the ZGS Conference on New Directions in Hadron Spectroscopy,* edited by E. Berger and S. Kramer, (ANL, Argonne, 1975), p. 264. Some aspects of this model have independently been discussed by F. E. Close and E. W. Colglazier, CERN Report No. TH-2065 (unpublished). As a gauge theory this model seems to give too large neutral strangness-changing amplitudes, e.g. $K_L \to \mu^+\mu^-$ is of order αG; however, these calculations are not entirely unambiguous. In any event we will regard the model as purely phenomenological in the present discussion of mass spectra and radiative decays.
[6] R. M. Barnett, Phys. Rev. Lett. 34, 41 (1975); Phys. Rev. D 11, 3246 (1975). See also the review talk presented at the 6th Hawaii Topical Conference in Particle Physics, University of Hawaii, 1975 [Fermilab Report No. FERMILAB-Conf-75/71-THY (unpublished)].
[7] H. Harari, Phys. Lett. 57B, 265 (1975); Ann. Phys. (N.Y.) 94, 391 (1975).
[8] F. Gürsey, P. Ramond, and P. Sikivie, Phys. Rev. D 12, 2166 (1975).
[9] F. A. Wilczek, A. Zee, R. L. Kingsley, and S. B. Treiman, Phys. Rev. 12, 2768 (1975).
[10] A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 3589 (1975).
[11] H. Fritzsch, M. Gell-Mann, and P. Minkowski, Phys. Lett. 59B, 256 (1975).
[12] S. Pakvasa, W. A. Simmons, and S. F. Tuan, Phys. Rev. Lett. 35, 702 (1975).
[13] C. G. Callan, R. L. Kingsley, S. B. Treiman, F. Wilczek, and A. Zee, Phys. Rev. Lett. 34, 52 (1975); B. J. Harrington, S. Y. Park, and A. Yildiz, *ibid.* 34, 168 (1975); 34, 296(E) (1975); 34, 706(C) (1975); T. Appelquist, A. De Rújula, H. D. Politzer, and S. L. Glashow, *ibid.* 34, 365 (1975); E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K. D. Lane, and T.-M. Yan, *ibid.* 34, 369 (1975).
[14] M. Gell-Mann, Phys. Lett. 8, 214 (1964); G. Zweig (unpublished).
[15] K. J. Barnes, E. W. Colglazier, A. J. G. Hey, and R. K. P. Zia, Acta Phys. Polon. B6, 823 (1975).
[16] S. Weinberg, Phys. Rev. Lett. 18, 507 (1967).
[17] Particle Data Group, Rev. Mod. Phys. 47, 535 (1975); Phys. Lett. 50B, 1 (1974).
[18] The have been recently reviewed by R. F. Schwitters, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California*, edited by W. T. Kirk (SLAC, Stanford, 1976), p. 5.
[19] In the context of the standard model such mixing has been discussed by H. Harari, Phys. Lett. 60B, 172 (1976).
[20] B. H. Wiik, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California*, edited by W. T. Kirk (SLAC, Stanford, 1976), p. 69.
[21] G. Feldman, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California*, edited by W. T. Kirk (SLAC, Stanford, 1976), p. 39.
[22] See, for example, M. Chanowitz, Phys. Rev. Lett. 35, 977 (1975).
[23] See, for example, B. T. Feld, *Models of Elementary Particles* (Blaisdell, Waltham, 1969).
[24] For example, for the analogous $^3S_1 \to {}^1S_0$ transition in the standard model one finds $\Gamma(\psi(3.1) \to \eta_c(2.8) + \gamma) \simeq 15$ keV.
[25] For charge assignments (A) and (B) the asymptotic values of R are 8 and 5, respectively.