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D and D^* in the purely algebraic approach to broken SU(4)*

H. Hallock, S. Oneda, and Milton D. Slaughter†
Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
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Several simple intra- and inter-16-plet boson mass relations in broken SU(4) [including broken SU(3) and SU(2)] are derived from the chiral SU(4) $\otimes$ SU(4) charge algebra, by using the hypothesis of asymptotic SU(4). The mass formulas are exact except for the neglect of inter- (but not intra-) 16-plet mixings and take into account the presence of the quark-line-type selection rules inherent in the theory. They can predict the masses of the $D^*(1^{--})$, $D^{**}(2^{++})$,... and $F(0^{-+})$, $F^*(1^{--})$, $F^{**}(2^{++})$,... as well as the K^*, K^{**},..., once the masses of the K, D, and pion members of 16-plets are given. The result is compatible with the recently observed D-D^* mass splitting. Implications on the productions and decays of the D and D^* are also discussed.

In the charm[1] scheme of SU(4), we denote[2] a 15 $\oplus$ 1-plet boson by $\alpha_r(\pi_r,\ K_r,\ \eta_r,\ \eta_{cr},\ D_r,\ F_r$ and $\eta_r')$, where r stands for J^{PC} and other quantum numbers. There have been numerous works[2] which give mass relations based on certain assumed SU(4) [or SU(8)] transformation properties of the hadron mass operator, although from the perturbation-theoretic point of view the large masses of new resonances imply that SU(4) [or SU(8)] is a badly broken and perhaps not a very useful symmetry. However, if the underlying quark dynamics is favorable one can derive[3] SU(4)- and SU(8)-like mass relations *without* subscribing to perturbation-theoretic arguments.

The purpose of this paper is to discuss, in this approach, some characteristics of the charmed-meson masses, production, and decays in light of the discovery[4] of the new state at 1.865 GeV.

The basic assumption is the hypothesis[3] of asymptotic SU(4). One can trace the multiplet classification, made in our *asymptotic* limit, in the real world through the chiral SU(4)$\otimes$SU(4) *charge* algebras which are valid in *broken* SU(4). Thanks to asymptotic SU(4), the inherent SU(4) particle mixings can be treated[3] in our $\vec{k}\to\infty$ limit among the physical creation or annihilation operator $a_{\alpha,r}(\vec{k},\lambda)$ of a 16-plet r and the hypothetical representation operator $a_{j,t}(\vec{k},\lambda)$ $(j=0,1,\ldots,15)$, where r and t belong to the same J^{PC} or J^P. Then the matrix elements of the vector and axial-vector charges, $\langle\alpha_r|V_i|\beta_t(\vec{k})\rangle$ and $\langle\alpha_r|A_i|\beta_t(\vec{k})\rangle$, etc., can be parametrized in terms of a few *reduced* matrix elements using the conventional prescription of exact SU(4) plus mixings but only in the limit $\vec{k}\to\infty$. However, in this paper we consider only the full SU(4) mixings [including broken SU(3) and SU(2)] in the *same* 16-plet r. Since SU(4) is more broken than SU(3), the *inter-16-plet* SU(4) mixings neglected may turn out to be important.[3] The test of our predictions among charmed mesons is thus instructive also from this point.[5] The usual mechanism of SU(4) breaking is expressed[3] algebraically by the presence of the exotic commutation relations $[\dot{V}_\alpha, V_\beta]=[\dot{V}_\alpha,A_\beta]=0$ with $\dot{V}_\alpha=(d/dt)V_\alpha$, where (α,β) stands for the *exotic* combination of the physical SU(4) indices.[3] The Gell-Mann–Okubo–mass–formula–type (but exact) SU(4) constraints, involving the *squared* masses and the η_r-η_{cr}-η_r'-π_r^0 mixing angles, follow for any 16-plet r from $[\dot{V}_\alpha, V_\beta]=0$. $[\dot{V}_\alpha,A_\beta]=0$ also yields several intermultiplet as well as intramultiplet mass-mixing angle sum rules. Assuming *exact* SU(2), Takasugi and Oneda[3] used these sets of sum rules to determine the ω-ϕ-ψ mixing angles and subsequently the D^* and F^* masses, when the mass of the ψ (J) was given by experiment. Combined with the intermultiplet mass relations, the masses of D, F, η_c, etc. were then computed.[6]

However, there are several reasons which indicate that this route is subject to considerable error ($\gtrsim 10\%$), *unless* the effect of SU(2) breaking is taken into account. (In fact, the predicted mass[6] of $D\simeq 2.11$ GeV is larger than the mass value of $\simeq 1.865\pm 0.015$ GeV of the new particle[4] which may be identified with the D.) First, for the *ideal* 16-plets one of the mixing angles vanishes.[3] Thus the η_r-π_r^0, η_r'-π_r^0, and η_{cr}-π_r^0 mixings arising through SU(2) breaking cannot be neglected. Second, for the *ideal* 16-plet r the axial-vector matrix elements involving the η_r and η_{cr} (which become a pure $s\bar{s}$ and $c\bar{c}$ state, respectively) vanish,[3] i.e., we have $\langle\eta_r|A_{\pi^-}|\pi_u^+(\vec{k})\rangle=\langle\eta_{cr}|A_{\pi^-}|\pi_u^+(\vec{k})\rangle=0$ and their SU(4) counterparts (selection rules), where $\vec{k}\to\infty$ and u is arbitrary provided that $C_rC_u=1$. Then the neglect of the G-forbidden matrix elements such as $\langle\pi_r^0|A_{\pi^-}|\pi_u^+(\vec{k})\rangle$ produces appreciable error. Third, for the *nonideal* 16-plet 0^{-+}, the π^0-η, π^0-η', and π^0-η_c mixings cannot be negligible, since the η-η' mixing angle [which arises through the SU(3) breaking] itself is small. In SU(3) but in the *same* theoretical framework, Slaughter and Oneda recently found[7] that the SU(2)-breaking effect is indeed nontrivial especially for the prediction on the ninth 0^{-+} meson mass. For the G-forbidden matrix element $\langle \rho^0 | A_{\pi^-} | \rho^+(\vec{k})\rangle$, they also found $\langle \rho^0 | A_{\pi^-} | \rho^+(\vec{k})\rangle \simeq 0.25\langle \phi | A_{\pi^-} | \rho^+(\vec{k})\rangle$. Since the rate of $\psi \to \rho\pi$ is extremely small, this implies $\langle \psi | A_{\pi^-} | \rho^+\rangle \lesssim \langle \rho^0 | A_{\pi^-} | \rho^+\rangle$. Therefore, the importance of the inclusion of SU(2) breaking in the usual route[6] of SU(4) for predicting the masses of D^*, D, etc. is apparent.

However, it turns out that some particular intermultiplet as well as intramultiplet mass relations can be obtained even in the *presence* of SU(2) breaking and they retain simple forms without involving mixing parameters, *as long as we are able to neglect the inter-16-plet SU(4) mixings*. The derivation of these sum rules is straightforward,[8] although we have to deal with six mixing angles. From $[\dot{V}_\alpha, V_\beta] = 0$ we obtain for *any* r

$$F_r^{+2} - D_r^{+2} = K_r^{+2} - \pi_r^{+2}, \quad F_r^{+2} - D_r^{02} = K_r^{02} - \pi_r^{+2}, \quad (1)$$

which also implies for *any* r

$$D_r^{+2} - D_r^{02} = K_r^{02} - K_r^{+2}. \quad (2)$$

By realizing the $[\dot{V}_\alpha, A_\beta] = 0$ between the 16-plet r and u with $C_r C_u = -1$ in our asymptotic limit and combining the result in a hybrid way, the following SU(8)-like (but more general) inter-16-plet mass constraints are obtained, where r and t are *arbitrary*:

$$K_r^{02} - \pi_r^{+2} = K_t^{02} - \pi_t^{+2}, \quad K_r^{+2} - \pi_r^{+2} = K_t^{+2} - \pi_t^{+2}, \quad (3)$$

$$D_r^{02} - \pi_r^{+2} = D_t^{02} - \pi_t^{+2}, \quad D_r^{+2} - \pi_r^{+2} = D_t^{+2} - \pi_t^{+2}. \quad (4)$$

Equations (1)–(4) imply the simple relation

$$K_r^{02} - K_t^{02} = K_r^{+2} - K_t^{+2} = D_r^{02} - D_t^{02} = D_r^{+2} - D_t^{+2} = F_r^{+2} - F_t^{+2} = \pi_r^{+2} - \pi_t^{+2}. \quad (5)$$

It is interesting to see to what degree the *inter-16-plet* mixings neglected affect[5] these simple predictions. The ordering of the masses and the mass spacings among the members of $D(0^{-+})$, $D^*(1^{--})$, $D^{**}(2^{++})$... and $F(0^{-+})$, $F^*(1^{--})$, $F^{**}(2^{++})$... as well as $K(0^{-+})$, $K^*(1^{--})$, $K^{**}(2^{++})$... are fixed, once the ordering and the spacings of the masses of the pion members of 16-plets, $\pi(0^{-+})$, $\pi(1^{--})$, $\pi(2^{++})$, ... are fixed. Equation (5) implies that the lowest-lying charmed meson should be the $D(0^{-+})$. It is not very easy to pinpoint the central mass value[9] of broad resonances. At present, the most relevant predictions are $D^{02} - \pi^{+2} = D^{*02} - \rho^{+2} = D^{**02} - A_2^{+2} = \cdots$ and $D^{+2} - D^{02} = D^{*+2} - D^{*02} = D^{**+2} - D^{**02} = K^{02} - K^{+2} = \cdots$. With the input of $D^0 = 1.865$ GeV and values for K^0, K^+, and π^+ from the Particle Data Group, we find from Eqs. (1) and (2) $D^+ = 1.866$ and $F^+ = 1.925$ GeV. In the framework of broken SU(3) plus SU(2), Slaughter and Oneda found[7] that $\rho^+ \simeq 751.79$, $\rho^0 = 751.76$, $K^{*+} = 888.52$, and $K^{*0} = 890.74$, when $\omega = 782.66$ and $\phi = 1019.69$ MeV were *input*. If SU(4) breaking does not appreciably affect[3] the mass spectrum obtained[7] in the usual SU(3) sector, we can determine D^{*0}, D^{*+}, F^{*+}, D^{**0}, F^{**+}, etc. from Eqs. (1)–(4), since ρ^+ is known.[9] The results based on this ρ^+ mass are presented in Table I. For the purpose of comparison, we also list the results for the value[10] $\rho^+ \simeq \rho^0 = 773$ MeV from the Particle Data Group. The predicted mass values of D^{*-}, in the range of 2.007–2.015 GeV (depending on the choice of input ρ mass), enable us to identify the D^* with the system of mass 2.01 ± 0.02 GeV, which is produced in association with a new state[4] of mass 1876 ± 15 MeV.

TABLE I. Charmed-meson mass spectrum as predicted by asymptotic SU(4). [Inter-16-plet SU(4) mixings are neglected.] We choose $A_2^+ \simeq 1.310$ GeV.

$r \equiv J^{PC}$	D_r^0 (GeV)	D_r^+ (GeV)	F_r^+ (GeV)
0^{-+}	1.865 (input)	1.866	1.925
1^{--}	2.006 [a] 2.014 [b]	2.007 [a] 2.015 [b]	2.062 [a] 2.070 [b]
2^{++}	2.275	2.276	2.325

[a] See Ref. 7, $\rho^+ = 751.70$ MeV.
[b] ρ^+ is taken to be 773 MeV.

As will be discussed later (Table II) the branching ratios

$$R(D^{*0}) = \Gamma(D^{*0} \to D^+\pi^-)/\Gamma(D^{*0} \to D^0\pi^0)$$

and

$$R(D^{*+}) = \Gamma(D^{*+} \to D^0\pi^+)/\Gamma(D^{*+} \to D^+\pi^0)$$

will be significantly smaller from their SU(2) value 2, especially if the ρ mass is in the range of 750–760 MeV.

We have presented clear, simple, general, and the most reliable (in our formulation) predictions which are *independent* of the values of the ρ^0-ω-ϕ-ψ and π^0-η-η'-η_c mixing parameters or of the actual $q\bar{q}$ composition of the mesons in the *presence* of SU(2) breaking. Since the masses of the D and also of the D^* (as computed in this paper) are sharp, comparison with experiment is certainly interesting. The only small uncertainty involved in predicting the masses of the D^*, F^*, F, etc. is the slight ambiguity associated with the precise mass of the ρ^+ or K^{*+} (if we replace the ρ^+ mass by the K^{*+} mass using our relation $K^{*+2} - \rho^{+2} = K^{+2} - \pi^{+2}$). Our prediction of the small SU(2) mass splittings of D, D^*, etc. is also different from other works.[11]

At this point we address the question of whether our mass values of D and D^* can be compatible with the mass of ψ. The compatible fitting of the D^* and ψ with the ρ^0-ω-ϕ-ψ mixing parameters will

TABLE II. Some charmed-meson partial widths and Q values are predicted by asymptotic SU(4). [Inter-16-plet SU(4) mixings are neglected.] We choose $A_2^+ \simeq 1.310$ GeV. For $\rho^+ = 751.79$ MeV the mode $D^{*0} \to D^+\pi^-$ is greatly suppressed.

Decay	Partial width (keV)		Q value (MeV)	
	$\rho^+=751.79$ MeV [a]	$\rho^+\simeq 773$ MeV	$\rho^+=751.79$ MeV [a]	$\rho^+\simeq 773$ MeV
$D^{*+}\to D^0\pi^+$	28.7	243.2	2.4	10.4
$D^{*+}\to D^+\pi^0$	53.1	182.4	5.9	13.9
$D^{*+}\to D^+\gamma$	4.4	5.2	140.9	148.9
$D^{*0}\to D^0\pi^0$	54.5	182.4	6.0	14.0
$D^{*0}\to D^+\pi^-$	1.4	167.2	0.3	8.3
$D^{*0}\to D^0\gamma$	71.1	83.5	141.0	149.0
$D^{**+}\to (D\pi)^+$	11.1 (MeV)	11.1 (MeV)	273	273
$D^{**0}\to (D\pi)^0$	10.9 (MeV)	10.9 (MeV)	272	272
$D^{**+}\to (D^*\pi)^+$	5.7 (MeV)	5.5 (MeV)	132	124
$D^{**0}\to (D^*\pi)^0$	5.8 (MeV)	5.3 (MeV)	131	124

[a] See Ref. 7.

be relatively easy to achieve, since it amounts to finding theoretical mass values of octet vector mesons which are consistent with all our theoretical constraints. However, the central mass values and SU(2) splittings of the vector mesons are *not* precisely known. A considerable portion of the 200-MeV difference between the previous prediction[3] on the D^* mass [with exact SU(2)] and our present one could come from the assignment of the input octet vector-meson masses in Ref. 3. In fact, a different (but reasonable) choice of the octet vector-meson masses in Ref. 3 can produce a substantially lower D^* mass, which in turn (via intermultiplet mass relation) predicts the D mass closer to the experimental value. For the pseudoscalar mesons, the masses of π, K, and η are well known but there are also the following complications: (i) The problem of the η' (X or E?). (ii) $X(2.8)$ is not well established. (iii) The width of the η_c could be very large (Ref. 3). If we input $\eta' \equiv E(1420) \simeq 1.416$ GeV and $\eta_c \equiv X(2.8) \simeq 2.8$ GeV in the pseudoscalar *intramultiplet* mass equation of Ref. 3 [which, however, neglects the SU(2) breaking] we predict a D mass of 1.91 GeV. On the other hand, the choice of $\eta' \equiv X(958)$ yields a D mass of 2.18 GeV. Therefore, the experimentally established value of the D mass and the presently available possible mass value of $\eta_c \simeq 2.8$ GeV are not inconsistent with the result of Ref. 7, which predicts that the η' should be the $E(1420)$ in broken SU(3) and SU(2). The study of the mass of η_c [in terms of the masses of the D and $\eta' \equiv E(1420)$] including broken SU(2) is being undertaken.

Strong and radiative decays of D^{*0}, D^{*+}, etc. Neglecting now SU(2) breaking, we note the *broken* SU(4) relation

$$\langle D^{*0}|A_{\pi^-}|D^+(\vec{k})\rangle = (1/\sqrt{2})\langle \rho^0|A_{\pi^-}|\pi^+(\vec{k})\rangle$$

for $\vec{k}\to\infty$. Using partial conservation of axial-vector current (PCAC) for A_{π^-}, $\Gamma(\rho^0\to\pi^+\pi^-)\simeq 152$ MeV, and the masses of D^* and D listed in Table I, one can compute the rates of $D^*\to D\pi$ decays.[12] Also estimates of the rates of the $D^{**}\to D\pi$ and $D^{**}\to D^*\pi$ are made in a similar way,[12] by comparing these decays with their SU(4) counterparts, $K^{**}\to K\pi$ and $A_2\to\rho\pi$, respectively. The results are displayed in Table II for $\rho^+=751.79$ MeV and $\rho^+ = 773$ MeV. The quark charge assignment and asymptotic SU(4) roughly predict[13] that

$$g_{D^{*0}D^0\gamma} : g_{D^{*+}D^+\gamma} : g_{\omega\pi\gamma} = 4:1:3. \quad (6)$$

Thus with the input $\Gamma(\omega\to\pi\gamma)\simeq 870$ keV,[14] the $D^*\to D\gamma$ widths can be computed. They are listed in Table II again for the two chosen values of ρ^+. From Eq. (6) we first note that the ratio of the rates of the production processes $e^+ + e^- \to (\gamma) \to D^{*+} + D^-, D^{*-} + D^+$ versus $D^{*0} + \bar{D}^0, \bar{D}^{*0} + D^0$ is greatly suppressed, i.e., $\simeq \frac{1}{16}$ for both values of ρ^+. The produced D^*'s in these reactions decay quickly into either $D+\pi$ or $D+\gamma$. According to Table II the ρ^+ mass value plays a significant role for the branching ratios of the D^* decays and, therefore, for the ratio of the number of $D^\pm$'s to D^0's observed in the reactions. If the above considered production processes *dominate* over other processes (present experiment[4] is consistent with the production scheme $e^+ + e^- \to D^{*0} + \bar{D}^0, \bar{D}^{*0} + D^0$), our result predicts $N(D^+,D^-)/N(D^0,\bar{D}^0)\simeq 6\%$ for the choice $\rho^+ = 751.79$ MeV and $\simeq 29\%$ for $\rho^+ = 773$ MeV. To the approximation (no inter-16-plet mixings) adopted in this paper, Eq. (6) also holds for the virtual-photon processes. For energy E in the e^+e^- annihilation larger than 4.012 or 4.028 GeV (depending on the choice of ρ^+), the channel $e^+e^-\to D^* + \bar{D}^*$ becomes open and $D^\pm$ will be produced more easily. We also find that about 57% of the D^0's and $\bar{D}^0$'s observed should be accompanied by a photon for $\rho^+ = 751.79$ MeV as opposed to about 19% for the case $\rho^+ = 773$ MeV. The search for this monochromatic photon (with energy $\simeq 140$ MeV in the rest frame of the parent D^*) will certainly be enlightening.

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†Now at Univ. of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544.

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[5]The predicted rate of the $\psi \to e^+ e^-$ by using asymptotic SU(4) seems to be about a factor 2 off from the experimental rate. See Ref. 3.

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[11]For example, see A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. 37, 398 (1976). Our result on the branching ratios of the decays of D^{*0} and D^{*+} is significantly different from the result obtained in the above paper.

[12]We neglect the effect of the off-mass-shell extrapolation $m_\pi{}^2 \to 0$ in the coupling. The effect of SU(4) breaking appears as $g_{D^*D\pi}/g_{\rho\pi\pi} \simeq$ SU(4) factor $\times (D^*/\rho)$, $g_{D^{**}D\pi}/g_{K^{**}K\pi} \simeq$ SU(4) factor $\times (D^{**}/K^{**})$ and $g_{D^{**}D\pi}/g_{A_2\rho\pi} \simeq$ SU(4) factor (D^{**}/A_2), when we use the intermultiplet mass relation, Eq. (5).

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