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$X^0(958)$ or $E(1420)$: Which one in broken SU(3) and SU(2) symmetry?

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General nonet mass relations are derived for bosons in broken SU(3) plus SU(2) symmetry. The formulas predict the mass of the ninth pseudoscalar meson to be very close to $E(1420)$.

In agreement with the $q\bar{q}$ description, bosons seem to appear always as nonets in SU(3), i.e., $0^{-+}(\pi, K, \eta, \eta')$, $1^{--}(\rho, K^*, \varphi, \omega)$, etc. We denote them as $\alpha_r(\pi_r, K_r, \eta_r, \eta_r')$ where r stands for J^{PC}. The 1^{--}, 2^{++}, etc., are known to follow approximately the ideal[1,2] nonet pattern, whereas the 0^{-+} deviates significantly from the ideal structure.

It has been noted[2] that one can study the hadron mass spectra and interactions in a purely algebraic way, using neither the notion of exact SU(3) nor SU(6). The theoretical frameworks are (i) asymptotic SU(3), (ii) chiral SU(3)$\otimes$SU(3) algebra of the charges V_α and $A_\alpha(\alpha=\pi, K, 8)$, (iii) the simple mechanism of symmetry breaking expressed algebraically by the presence of the *exotic* commutation relations (C.R.'s) involving the time derivative of the SU(3) charge, i.e., $[\dot{V}_\alpha, V_\beta]=0$ and $[\dot{V}_\alpha, A_\beta]=0$, where (α,β) stands for the *exotic* combination of SU(3) indices such as (K°, K°), (K°, π^-), etc., and (iv) the hypothesis[2,3] of level realization of asymptotic SU(3) in the C. R., $[A_\alpha, A_\beta]=if_{\alpha\beta\gamma}V_\gamma$. We then find[2,3] the following: (a) The quadratic Gell-Mann–Okubo (GMO) mass formula involving the $\eta_r-\eta_r'$ mixing is an exact constraint. (b) The quadratic mass spacing $K_r^2-\pi_r^2$ is universal. (c) An intimate (intermultiplet as well as intramultiplet) interplay among the masses, mixing angles, and axial-vector matrix elements is inherent. This interplay permits the existence of particular nonets called ideal nonets which satisfy the ideal nonet constraints[2] or Okubo's nonet ansatz.[1] (d) Every nonet, *ideal or not*, satisfies the general nonet mass relation

$$(3\eta_r'^2+\pi_r^2-4K_r^2)(3\eta_r^2+\pi_r^2-4K_r^2)=-8(K_r^2-\pi_r^2)^2 . \tag{1}$$

Equation (1) is found to coincide with the nonet formulas first introduced by Schwinger[4] and often rediscovered in various different approaches. The possible general nature of Eq. (1) is demonstrated by the fact that the *approximately* ideal 1^{--} and 2^{++} nonets satisfy Eq. (1) better than the ideal nonet mass constraints, $\pi_r^2=\eta_r'^2$ and $\eta_r^2-K_r^2=K_r^2-\pi_r^2$. However, for the 0^{-+} nonet Eq. (1) predicts the ninth meson mass to be around 1600 MeV in contrast with the possible candidates $X(958)$ and $E(1420)$.

In this paper we consider the effect of SU(2) breaking and derive general sum rules valid in broken SU(3) plus SU(2) symmetry. Contrary to the usual expectation, the effect is significant: First, for an *ideal* nonet r a selection rule for the axial-vector matrix elements involving η_r arises, i.e., $\langle\eta_r|A_{\pi^-}|\pi_u^+(\vec{k})\rangle=0$ for $\vec{k}\to\infty$ and u is arbitrary provided $C_rC_u=1$. In the ideal configuration, the η_r becomes a pure $s\bar{s}$ state. Thus, in dealing with an *approximately ideal* nonet r, the neglect of the *G-forbidden* axial-vector matrix element $\langle\pi_r^0|A_{\pi^-}|\pi_u^+(\vec{k})\rangle$ introduces an appreciable error, since the G-allowed matrix element $\langle\eta_r|A_{\pi^-}|\pi_u^+\rangle$ (which *vanishes* in the ideal limit) is of the same order of magnitude. Second, for a nonideal nonet such as the 0^{-+}, the π_r^0-η_r and π_r^0-η_r' mixings arising through the SU(2) breakings cannot be neglected, since the η_r-η_r' mixing arising from SU(3) breaking is small.

We find, in particular for the 0^{-+} nonet, that the inclusion of SU(2) breaking in our theoretical framework[2,3] brings the mass of η' very close to $E(1420)$. Therefore, the assignment $\eta'\equiv E(1420)$ is strongly favored. Our other results obtained for the 0^{-+}, 1^{--}, and 2^{++} also agree with present experiment reasonably well.

In broken SU(3) plus SU(2), the π_r^0, η_r, and η_r' can mix. According to asymptotic SU(3), the creation and annihilation operators of physical particles transform *linearly* under SU(3) [including SU(2)] but only in the limit of infinite momentum. Thus the mixing parameters are defined among the annihilation operators $a_{\pi_r^0}(\vec{k})$, $a_{\eta_r}(\vec{k})$, and $a_{\eta_r'}(\vec{k})$ and the hypothetical representation operators a_{3r}, a_{8r}, and a_{0r} in the limit $\vec{k}\to\infty$ by (suppressing the helicity indices)

$$\begin{pmatrix} a_{\pi_r^0}(\vec{k}) \\ a_{\eta_r}(\vec{k}) \\ a_{\eta_r'}(\vec{k}) \end{pmatrix} = \begin{pmatrix} \lambda_r & \sigma_r & \mu_r \\ \lambda_r' & \sigma_r' & \mu_r' \\ \lambda_r'' & \sigma_r'' & \mu_r'' \end{pmatrix} \begin{pmatrix} a_{3r}(\vec{k}) \\ a_{8r}(\vec{k}) \\ a_{0r}(\vec{k}) \end{pmatrix}, \quad \vec{k}\to\infty . \tag{2}$$

We can also parametrize[5] λ_r, σ_r, etc., in terms of the Euler angles $(\omega_r, \theta_r, \phi_r)$. Now imposing the C. R.'s $[V_\alpha, V_\beta] = if_{\alpha\beta\gamma}V_\gamma$ and $[V_\alpha, A_\beta] = if_{\alpha\beta\gamma}A_\gamma$ and realizing them in the $\vec{k}\to\infty$ limit using asymptotic SU(3) and SU(2), significant simplifications arise for the vector and axial-vector matrix elements evaluated at $\vec{k}\to\infty$. Namely, $f^{(r)}_{\alpha\beta} \equiv \langle\alpha_r|V_\gamma|\beta_r(\vec{k})\rangle$ and $\langle\alpha_r|\beta_t\rangle \equiv \langle\alpha_r|A_\gamma|\beta_t(\vec{k})\rangle$ with $\vec{k}\to\infty$, can be parametrized in broken SU(3) [and SU(2)] by the usual prescription of *exact* SU(3) and SU(2) *plus* mixing. Here $\alpha, \beta = \pi, K, \eta, \eta'$. The $f^{(r)}_{\alpha\beta}$'s involving the mixing angles $(\omega_r, \theta_r, \phi_r)$ are given, for example, by[6]

$$f^{(r)}_{\pi^+\pi^0} = f^{(r)}_{\pi^-\pi^0} = \sqrt{2}\,\lambda_r\,,$$
$$f^{(r)}_{\pi^+\eta} = f^{(r)}_{\pi^-\eta} = \sqrt{2}\,\lambda'_r\,, \qquad (3)$$
$$f^{(r)}_{\pi^+\eta'} = f^{(r)}_{\pi^-\eta'} = \sqrt{2}\,\lambda''\,, \text{ etc.,}$$

whereas the f's which are independent of mixing angles are, according to our sign convention, $f^{(r)}_{K^0K^+} = f^{(r)}_{K^0\pi^-} = -f^{(r)}_{K^0\pi^+} = 1$, etc., for any r. Depending on whether $C_rC_t = 1$ or -1, $\langle\alpha_r|\beta_t\rangle$ become the so-called D type or F type coupling and they can be expressed in terms of a few reduced matrix elements and mixing parameters. Then the imposition of our theoretical constraints (iii), $[\mathring{V}_\alpha, V_\beta] = 0$ and $[\dot{V}_\alpha, A_\beta] = 0$, and (iv) lead to several mass-mixing angle sum rules, after we eliminate the axial-vector couplings involved. Upon eliminating further the mixing angles, we obtain *two* nonet mass relations together with the simple intermultiplet mass relations.

I. CONSTRAINTS FROM $[\dot{V}_\alpha, V_\beta] = 0$

With SU(2) breaking (including the possible u_3 terms), we need to consider the exotic C.R.'s with $(\alpha, \beta) = (\pi^+\pi^+)$, (K^0, π^-), (K^+, π^+), (K^0, K^0), etc. The asymptotic realization of these C.R.'s yields[7] three independent exact mass-mixing angle constraints with the help of Eq. (3):

$$[\dot{V}_{\pi^+}, V_{\pi^+}] = 0 \to \lambda_r^2\pi_r^{0\,2} + \lambda_r'^2\eta_r^2 + \lambda_r''^2\eta_r'^2 = \pi_r^{+\,2}\,, \qquad (4)$$

$$[\dot{V}_{K^0}, V_{K^0}] = 0$$
$$\to \sigma_r^2\pi_r^{0\,2} + \sigma_r'^2\eta_r^2 + \sigma_r''^2\eta_r'^2 = -\tfrac{1}{3}\pi_r^{+\,2} + \tfrac{2}{3}(K_r^{+\,2} + K_r^{0\,2})\,, \qquad (5)$$

$$[\dot{V}_{K^0}, V_{\pi^-}] = 0$$
$$\to \sqrt{3}\,(\lambda_r\sigma_r\pi_r^{0\,2} + \lambda_r'\sigma_r'\eta_r + \lambda_r''\sigma_r''\eta_r'^2) = K_r^{+\,2} - K_r^{0\,2}\,. \qquad (6)$$

Equations (5) and (6) are the generalization of the GMO and the η-π^0 transition mass formulas, respectively, in the presence of the η'_r and the SU(2) breaking. Although usually overlooked, Eq. (4) should be treated on an equal footing with Eqs. (5) and (6). We give a derivation of Eq. (6), since the procedure used is convenient for deriving other sum rules later. Equation (6) is obtained from

$$\sum_\alpha \langle K_r^0|\dot{V}_{K^0}|\alpha_r\rangle\langle\alpha_r|V_{\pi^-}|\pi_r^+(\vec{k})\rangle = \langle K_r^0|V_{\pi^-}|K_r^+\rangle\langle K_r^+|\dot{V}_{K^0}|\pi_r^+(\vec{k})\rangle$$

with $\vec{k}\to\infty$ and $\alpha = \pi^0, \eta^0, \eta'^0$. This yields

$$\sum_\alpha (K_r^{0\,2} - \alpha_r^2) f^{(r)}_{K^0\alpha} f^{(r)}_{\alpha\pi^+} = (K_r^+ - \pi_r^{+\,2}) f^{(r)}_{K^0K^+} f^{(r)}_{K^+\pi^+}$$
$$= -(K_r^{+\,2} - \pi_r^{+\,2})\,. \qquad (7)$$

With Eqs. (3) and (4), Eq. (7) reduces to Eq. (6). We can solve Eqs. (4), (5), and (6) for ω_r, θ_r, and ϕ_r in terms of the masses of π_r^+, π_r^0, η_r, η_r', K_r^0, and K_r^+. In general, we obtain 16 formal solutions of which only 2 will satisfy *all* of the constraints we will impose later. Precisely which of the 16 possibilities are solutions is determined by the mass spectrum of the s nonet under consideration.

II. INTERMULTIPLET MASS RELATIONS FROM $[\dot{V}_\alpha, A_\beta] = 0$

We assume that $[\dot{V}_\alpha, A_\beta] = 0$ together with $[\dot{V}_\alpha, V_\beta] = 0$. We now prove that in broken SU(3) plus SU(2) symmetry the following simple intermultiplet mass relations are valid:

$$K_r^{+\,2} - \pi_r^{+\,2} = K_t^{+\,2} - \pi_t^{+\,2} \quad (r \text{ and } t \text{ are arbitrary})\,, \qquad (8)$$

$$K_r^{0\,2} - \pi_r^{+\,2} = K_t^{0\,2} - \pi_t^{+\,2} \quad (r \text{ and } t \text{ are arbitrary})\,, \qquad (9)$$

which also implies[8] $K_r^{0\,2} - K_r^{+\,2} = K_t^{0\,2} - K_t^{+\,2}$. ($K^{0\,2} - K^{+\,2} = K^{*0\,2} - K^{*+\,2}$ is not in contradiction with experiment.) The simplest way to derive Eq. (8) is perhaps as follows. From $[\dot{V}_{K^0}, A_{\pi^-}] = 0$,

$$\sum_\alpha \langle K_r^0|\dot{V}_{K^0}|\alpha_r\rangle\langle\alpha_r|A_{\pi^-}|\pi_t^+(\vec{k})\rangle = \langle K_r^0|A_{\pi^-}|K_t^+\rangle\langle K_t^+|\dot{V}_{K^0}|\pi_t^+(\vec{k})\rangle$$

with $\vec{k}\to\infty$, i.e.,

$$\sum_\alpha (K_r^0 - \alpha_r^2) f^{(r)}_{K^0\alpha}\langle\alpha_r|\pi_t^+\rangle = (K_t^{+\,2} - \pi_t^{+\,2}) f^{(t)}_{K^+\pi^+}\langle K_r^0|K_t^+\rangle\,,$$

where $\alpha = \pi^0, \eta, \eta'$. The important point to note is that if $C_rC_t = -1$, $\langle\alpha_r|\pi_t^+\rangle$ and $\langle K_r^0|K_t^+\rangle$ are F-type couplings and

$$\langle\alpha_r|\pi_t^+\rangle / f^{(r)}_{\alpha\pi^+} = \langle K_r^0|K_t^+\rangle / f^{(r)}_{K^0K^+}\,.$$

Then compared with Eq. (7), the above equation yields $K_r^{+\,2} - \pi_r^{+\,2} = K_t^{+\,2} - \pi_t^{+\,2}$ $[(C_rC_t) = -1]$. By using this result in a hybrid way we arrive at Eq. (8).

In exactly the same way we obtain Eq. (9) from $[\dot{V}_{K^+}, A_{\pi^+}]=0$.

III. NONET MASS RELATION FROM $[\dot{V}_\alpha, A_\beta]=0$

Consider $\langle K_r^0|[V_{K^0}\,(\text{and }\dot{V}_{K^0}),\, A_{\pi^-}]|\pi_t^+(\vec{k})\rangle=0$ with $\vec{k}\to\infty$. With $\alpha=\pi^0,\ \eta,\ \eta'$ and $\epsilon\equiv C_rC_t$ we obtain

$$\langle K_r^0|K_t^+\rangle+\sum_\alpha f^{(r)}_{K^0\alpha}\langle\alpha_r^0|\pi_t^+\rangle=0\ , \tag{10}$$

$$(K_r^{+2}-\pi_r^{+2})\langle K_r^0|K_t^+\rangle+\sum_\alpha(K_r^0-\alpha_r^{\,2})f^{(r)}_{K^0\alpha}\langle\alpha_r^0|\pi_t^+\rangle=0 \tag{11}$$

and similarly from $\langle K_r^+|[V_{K^+}\ (\text{and }\dot{V}_{K^+}), A_{\pi^+}]|\pi_t^-(\vec{k})\rangle=0$,

$$\langle K_r^+|K_t^0\rangle+\epsilon\sum_\alpha f^{(r)}_{K^+\alpha}\langle\alpha_r^0|\pi_t^+\rangle=0\ , \tag{12}$$

$$(K_r^{0\,2}-\pi_r^{+2})\langle K_r^+|K_t^0\rangle+\epsilon\sum_\alpha(K_r^{+2}-\alpha_r^{\,2})f^{(r)}_{K^+\alpha}\langle\alpha_r^0|\pi_t^+\rangle=0\ . \tag{13}$$

In Eqs. (11) and (13) we have replaced $(K_t^{+2}-\pi_t^{+2})$ and $(K_t^{0\,2}-\pi_t^{+2})$ by $(K_r^{+2}-\pi_r^{+2})$ and $(K_r^{0\,2}-\pi_r^{+2})$, respectively, thanks to Eqs. (8) and (9)—a crucial step. Equations (10) and (12) are nothing but the asymptotic SU(3) plus SU(2) parametrization of axial-vector matrix elements. However, Eq. (1) (obtained from $[\dot{V}_{K^0}, A_{\pi^-}]=0$) now demands an intimate *interplay* among the masses, mixing angles, and the axial-vector matrix elements. From Eqs. (10), (11), and (12) we can express the axial-vector matrix element $\langle\alpha_r|\beta_t\rangle$ under consideration in terms of $\langle\eta_r|\pi_t^+\rangle$, i.e.,

$$\frac{\langle\pi_r^0|\pi_t^+\rangle}{\langle\eta_r|\pi_t^+\rangle}\equiv M_3,\quad \frac{\langle\eta_r'|\pi_t^+\rangle}{\langle\eta_r|\pi_t^+\rangle}\equiv M_1M_3+M_2,\quad \frac{\langle K_r^0|K_t^+\rangle}{\langle\eta_r|\pi_t^+\rangle}\equiv M_4\ , \tag{14}$$

where M_i $(i=1,2,3,4)$ depends *only* on s (once $\epsilon=C_rC_t$ is fixed) and is given by

$$M_1=-(f^{(r)}_{K^0\pi^0})(f^{(r)}_{K^0\eta'})^{-1}[(K_r^{+2}-K_r^{0\,2})-(\pi_r^{+2}-\pi_r^{02})]\times[(K_r^{+2}-K_r^{02})-(\pi_r^{+2}-\eta_r'^2)]^{-1}\ ,$$

$$M_2=-(f^{(r)}_{K^0\eta})(f^{(r)}_{K^0\eta'})^{-1}[(K_r^{+2}-K_r^{0\,2})-(\pi_r^{+2}-\eta_r^{\,2})]\times[(K_r^{+2}-K_r^{02})-(\pi_r^{+2}-\eta_r'^2)]^{-1},$$

$$M_3=-[(\epsilon f^{(r)}_{K^+\eta}-f^{(r)}_{K^0\eta})+(\epsilon f^{(r)}_{K^+\eta'}-f^{(r)}_{K^0\eta'})M_2]\times[(\epsilon f^{(r)}_{K^+\pi^0}-f^{(r)}_{K^0\pi^0})+(\epsilon f^{(r)}_{K^+\eta'}-f^{(r)}_{K^0\eta'})M_1]^{-1}\ ,$$

and

$$M_4=-[f^{(r)}_{K^0\pi^0}M_3+f^{(r)}_{K^0\eta}+f^{(r)}_{K^0\eta'}(M_1M_3+M_2)]\ .$$

We stress the general nature of Eq. (14), i.e., r and t are arbitrary. In exact SU(2) the second equation of Eq. (14) recovers the SU(3) result[2,3]

$$\langle\eta_r|\pi_t^+\rangle/\langle\eta_r'|\pi_t^+\rangle=\tan\omega(\eta_r'^2-\pi_r^{\,2})(\eta_r^{\,2}-\pi_r^{\,2})^{-1}$$

for $C_rC_t=1$, where ω is the SU(3) η_r-η_r' mixing angle. For the ideal nonet r satisfying the mass constraint $\eta_r'^2=\pi_r^{\,2}$, the remarkable selection rule $\langle\eta_r|\pi_t^+\rangle=0$ thus follows. Substituting Eq. (14) into Eq. (13) we obtain a nonet mass relation,

$$f^{(r)}_{K^+\pi^0}M_3(K_r^{+2}-\pi_r^{0\,2})+f^{(r)}_{K^+\eta}(K_r^{+2}-\eta_r^{\,2})+f^{(r)}_{K^+\eta'}(M_1M_3+M_2)(K_r^{+2}-\eta_r'^2)+\epsilon M_4(K_r^{02}-\pi_r^{+2})=0\ , \tag{15}$$

which is a genuine broken-SU(2) nonet mass formula when combined with Eqs. (4), (5), and (6) [it reduces to an identity in exact SU(2)], and is, therefore, sensitive to the accuracy of input SU(2) mass differences.

IV. MODIFIED SCHWINGER'S NONET MASS FORMULA

Another nonet mass relation is obtained by using the hypothesis[2,3] of level realization of asymptotic SU(3) in the C. R., $[A_\mu, A_\nu]=if_{\mu\nu\sigma}V_\sigma$. Insert this C. R. between the states $\langle\alpha_r(\vec{k},\lambda)|$ and $|\beta_r(\vec{k},\lambda)\rangle$ belonging to the same nonet r with helicity λ and $\vec{k}\to\infty$. Then the right-hand side of this equation produces a definite pure number $g^\sigma_{\alpha\beta}$ according to asymptotic SU(3) and SU(2) and the ratios of $g^\sigma_{\alpha\beta}$'s (for different α, β, and σ) are fixed. Write the left-hand side as the sum over the single-particle intermediate state χ_t inserted between the charges A_α and A_{β^0}. We now assume that hadrons constitute levels (such as the levels of $l=0,1,\ldots$ of the quark model) $R_0, R_1,\ldots$ and among the intermediate states χ_t each level R_i separately realizes the ratios of $g^\sigma_{\alpha\beta}$'s. Actually the states χ_t with $C_rC_t=-1$ automatically realize the ratios under consideration so that our realization procedure yields constraints only for the matrix elements $\langle\alpha_r|A_\mu|\chi_t\rangle$, etc., with $C_rC_t=1$. This hypothesis opens a way to replace (with the introduction of the concept of levels) the usual brute-force imposition of higher symmetry which is significantly broken and is able to produce[3] the good results of SU(6) without producing the bad ones. We now, for example, choose $\alpha_r=\beta_r=\pi_r^+$ and $\alpha_r=\beta_r=K_r^+$ for the C. R., $[A_{\pi^+}, A_{\pi^-}]=2V_{\pi^0}$. Then the χ_t will be π_t^0, η_t, η_t', and K_t^0. For our present purpose, consideration of the realization by the ground state R_0 (i.e., $t=0^{-+}$ and 1^{--} and $\lambda=0$ or $\lambda=\pm1$) is sufficient and the realization condition becomes,

$$|\langle\pi_r^+|\pi_t^0\rangle|^2+\langle\pi_r^+|\eta_t\rangle|^2+|\langle\pi_r^+|\eta_t'\rangle|^2-2|\langle K_r^+|K_t^0\rangle|^2=0\,,$$

where $t=0^{-+}$ for $C_r=1$ and $t=1^{--}$ for $C_r=-1$. By using Eq. (14) this equation provides another mass-mixing angle constraint valid for any nonet r

$$M_3{}^2+1+(M_1M_3+M_2)^2-2M_4{}^2=0\ ; \qquad (16)$$

Eq. (16) is a modified Schwinger's nonet mass formula when combined with Eqs. (4), (5), and (6) in broken SU(3) plus SU(2) and reduces to Eq. (1), if SU(2) is exact.

V. NUMERICAL RESULTS

We now have five mass-mixing angle constraints, i.e., Eqs. (4), (5), (6), (15), and (16). Expressing the mixing angles $(\omega_r, \theta_r, \phi_r)$ in terms of the masses from Eqs. (4), (5), and (6), Eqs. (15) and (16) become the two genuine nonet mass formulas. Therefore, out of the six masses $(\pi_r^+, \pi_r^0, K_r^+, K_r^0, \eta_r, \eta_r')$ four will be predicted from Eqs. (8), (9), (15), and (16) once the other two are input (for $r=0^{-+}$, four masses must be input). However, Eq. (15) especially requires an accurate input SU(2) mass differences as mentioned before. In fact, for the 0^{-+} Eq. (15) is sensitive even to the errors of the masses of π^+, π^0, K^+, K^0 as listed by the Particle Data Group. However, the experimental value of η is certainly in the range of the solution of Eq. (15) and the adoption of the experimental value of η predicts the mass of η' at $\eta' = 1413.49$ MeV. We obtain two solutions which differ only by the signs of the mixing angles and both predict the same mass for η'. Therefore, $E(1420)$ is strongly favored as the ninth 0^{-+} meson. The mixing angles are given by $\omega=\pm 6.2280°$, $\theta=-0.6249°$, and $\phi=\mp 1.4285°$ and they may be regarded as the SU(3), SU(3)-SU(2) interference, and SU(2) mixing angles, respectively. The importance of SU(2) mixing is indicated by the magnitude of ϕ. Our solution, in fact, automatically singles out the favored solutions[7] (which can explain the violation of the $|\Delta \vec{I}|=\frac{1}{2}$ rule in the K_{l3} decays) out of 16 possibilities obtained when we consider only the constraints from $[\mathring{V}_\alpha, V_\beta]=0$, i.e., Eqs. (4), (5), and (6).

Since the widths of 1^{--} mesons are broad and the center mass values are not well known, we proceed as follows. We input only the relatively better known masses of ω and ϕ (we take $\omega = 782.66$ MeV and $\phi = 1019.69$ MeV) and predict the others from Eqs. (4), (5), (6), (8), (9), (15), and (16). We again obtain only two solutions which differ essentially only in the signs of the mixing angles. The predicted masses are the same for both solutions. They are $\rho^+=751.79$, $\rho^0=751.76$, $K^{*+}=888.52$, and $K^{*0}=890.74$ MeV. The SU(3), SU(3)-SU(2) interference, and SU(2) mixing angles are given by $\omega=\pm 39.5148°$, $\theta=+0.4458°$, and $\phi=\pm 1.0687°$, respectively. We also find $\langle\rho^0|\rho^+\rangle = 0.259\langle\phi|\rho^+\rangle$ which implies that the consistent neglect of the G-forbidden axial-vector matrix element $\langle\rho^0|\rho^+\rangle$ as compared with $\langle\phi|\rho^+\rangle$ (which is usually assumed to vanish by the quark-line rule) may lead to erroneous results. For the suppression of $\langle\phi|\rho^+\rangle$ we predict $\langle\phi|\rho^+\rangle/\langle\omega|\rho^+\rangle \simeq 0.074$, without invoking any assumption, which is consistent with the observed rate of $\phi\to 3\pi$ decay.[3]

It is interesting to note that our predicted values for the ρ^+ and ρ^0 (only a 30-keV mass difference) are some 20 MeV lower than the Particle Data Group values. We feel that this is in line with the observation by Lichtenberg and others[9] that the mass of a broad resonance and its width as determined by the position of the pole are smaller than the mass of the resonance and the width which enter the phenomenological Breit-Wigner parametrization. A similar situation is, in fact, experienced[9,10] for the $\Delta(1232)$.

The interplay between masses, mixing angles, and axial-vector matrix elements given by Eq. (14) enables us to compute many important branching ratios using partial conservation of axial-vector current for A_π. For example, with $s=1^{--}$ and $t=0^{-+}$, $\langle\omega/\pi^+\rangle/\langle\rho^0|\pi^+\rangle = M_1^{(r)}+M_2^{(r)}/M_3^{(r)} \simeq -0.019$.

We then predict $\Gamma(\omega\to\pi^+\pi^-)/\Gamma(\rho^0\to\pi^+\pi^-)\simeq 4.00\times 10^{-4}$ [experimental value $\simeq (8.64\pm 2.10)\times 10^{-4}$]. For $s=0^{-+}$ and $t=2^{++}$ we predict $\langle K^0|K^{+**}\rangle/\langle\eta|A_2^+\rangle = M_4^{(r)} \simeq -1.068$ which yields $\Gamma(K^{+**}\to(K\pi)^+)/\Gamma(A_2^+\to\eta\pi^+)\simeq 2.75$ (experimental value $\simeq 3.62 \pm 0.61$). If $\eta'\equiv E$, then the two-body decay $\Gamma(E\to K^*\bar{K})$ is small, $\simeq 1$ MeV, and $E\to K\bar{K}\pi$ should take place through *virtual* processes like $E\to\delta(0^{++})+\pi$, $\kappa(0^{-+})+\bar{K}$, $K^{**}+\bar{K}$, etc. $\to K\bar{K}\pi$. If $\eta'\equiv X$, the ratio $R=\Gamma(A_2\to X\pi)/\Gamma(A_2\to\eta\pi)$ is appreciable[3] (even from the naive quark-counting argument), whereas present experiment indicates $R<7\%$.

VI. OUTLOOK IN SU(4)

In the framework of SU(4), the implication of the assignment $\eta'\equiv E$ has been discussed by many authors.[11,12]

As to the question of whether SU(4) preserves our present result (especially the assignment $\eta'\equiv E$) obtained in SU(3) or not, we hold a rather optimistic expectation. First, in the similar theoretical framework extended to SU(4) [i.e., in the framework of asymptotic SU(4) and the chiral SU(4) $\times$ SU(4) charge algebra plus exotic C.R.'s, but without *imposing* the hypothesis of level realization of SU(4) in the C.R., $[A_\mu, A_\nu]=if_{\mu\nu\sigma}V_\sigma$], Takasugi and Oneda have demonstrated[13] that the Schwinger's nonet mass relation emerges, for nonet mesons belonging to any 16-plet of SU(4), in the interesting limit $\eta_{cr}{}^2\to\infty$ and $\gamma_r\to 0$ but $\eta_{cr}{}^2\gamma_r\to 0$. η_{cr} denotes the mass of the heaviest member (predominantly a $c\bar{c}$ state) of a 16-plet r and γ_r essentially represents the coupling between the η_{cr} and the octet members of the 16-plet r. Therefore, if this limit is close to reality [which is suggested by the heavy mass of η_{cr} and the narrow widths of the decays of η_{cr} into old SU(3) particles] Schwinger's nonet formula again emerges as a nonet mass relation even in the charm scheme of SU(4), favoring the assignment $\eta' \equiv E$. Second, if we input $\eta' \equiv E \simeq 1.416$ GeV and $\eta_c \equiv X(2.8) \simeq 2.8$ GeV in the pseudoscalar intramultiplet mass equation obtained[11] in this theoretical framework [which, however, *neglects* the SU(2) breaking], a D mass of 1.91 GeV is predicted,[14] while the choice of $\eta' \equiv X(958)$ yields a D mass of 2.18 GeV. Therefore, the recently established value[15] of the D mass $\simeq 1.865$ GeV also seems to favor the assignment of $\eta' \equiv E$ in SU(4). The effect of SU(2) breaking—neglected in the results (masses and decay rates) of Ref. 11—will be significant, especially for the 16-plet whose structure is close to ideal. This is because of the presence of the selection rules associated with the ideal structure. For example, the value of the G-forbidden axial-vector matrix element of $\langle \rho^0 | \rho^+ \rangle$ obtained in this paper implies $g_{J/\psi \to \rho\pi} \lesssim g_{\rho\rho\pi}$, since the rate of $J/\psi \to \rho\pi$ is extremely small. This demonstrates that for the sum rules involving the 1^{--} 16-plet the G-forbidden axial-vector matrix elements cannot be neglected.[14]

If $\eta' \equiv E$, the simplest possible assignment[16] of $X(958)$ [if $X(958)$ is really a 0^{-+} meson] will be the lightest member of the radially excited 0^{-+} 16-plet of SU(4).

Note Added. We have recently received an unpublished report by G. J. Gounaris and S. B. Sarantakos. These authors also found that the SU(4) symmetry partner of η is forced to lie in the mass region of $E(1420)$ rather than of $\eta'(958)$.

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[5]$\lambda_r = \cos\theta \cos\phi$, $\sigma = \sin\theta \cos\omega - \cos\theta \sin\phi \sin\omega$, $\mu = \sin\theta \sin\omega + \cos\theta \sin\phi \cos\omega$, $\lambda' = -\sin\theta \cos\phi$, $\sigma' = \cos\theta \cos\omega + \sin\theta \sin\phi \sin\omega$, $\mu' = \cos\theta \cos\omega - \sin\theta \sin\phi \cos\omega$, $\lambda'' = -\sin\phi$, $\sigma'' = -\cos\phi \sin\omega$, and $\mu'' = \cos\phi \cos\omega$.

[6]Others are (suppressing r); $f_{K^-\pi^0} = f_{K^+\pi^0} = (\frac{1}{2})^{1/2}\lambda + (\frac{3}{2})^{1/2}\sigma$, $f_{\bar{K}^0\pi^0} = -f_{K^0\pi^0} = (\frac{1}{2})^{1/2}\lambda - (\frac{3}{2})^{1/2}\sigma$, $f_{K^-\eta} = f_{K^+\eta} = (\frac{1}{2})^{1/2}\lambda' + (\frac{3}{2})^{1/2}\sigma'$, $f_{\bar{K}^0\eta} = (\frac{1}{2})^{1/2}\lambda' - (\frac{3}{2})^{1/2}\sigma'$, and $f_{K^-\eta'} = f_{K^+\eta'} = (\frac{1}{2})^{1/2}\lambda'' + (\frac{3}{2})^{1/2}\sigma''$, $f_{\bar{K}^0\eta'} = (\frac{1}{2})^{1/2}\lambda'' - (\frac{3}{2})^{1/2}\sigma''$

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[14]In Ref. 11, the value of $D \simeq 2.11$ GeV was obtained. However, this value is obtained through the mass of D^* which is gotten from the sum rules involving the J/ψ. As mentioned in the end of Sec VI, these sum rules produce a large error because of the presence of the selection rules associated with the J/ψ, unless the effect of SU(2) breaking is included.

[15]G. Goldhaber *et al.*, Phys. Rev. Lett. 37, 255 (1976).

[16]For another assignment of E(1420), see for example, L. A. Copley and P. J. S. Watson, Phys. Lett. 61B, 477 (1976).